Obligation versus Factual Conditionals under the Weak Completion Semantics

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Abstract

Conditionals play a prominent role in human reasoning and, hence, all cognitive theories try to evaluate conditionals like humans do. In this paper, we are particularly interested in the *Weak Completion Semantics*, a new cognitive theory based on logic programming, the weak completion of a program, the three-valued Łukasiewicz logic, and abduction. We show that the evaluation of conditionals within the *Weak Completion Semantics* as defined so far leads to counterintuitive results. We propose to distinguish between obligation and factual conditionals with necessary or sufficient conditions, and adapt the set of abducibles accordingly. This does not only remove the previously encountered counterintuitive results, but also leads to a new model for the Wason Selection Task.

1 Introduction

Conditionals are sentences of the form *if condition, then consequence*. They can be categorized in many different types and classes, but there are two main groups of conditionals: indicative and subjunctive (also known as counterfactual) conditionals. In indicative conditionals both, the condition and the consequence, can be either true, false or unknown. But if the condition is true, then the consequence is also asserted to be true. In counterfactuals, the consequence can again be either true, false or unknown, but the condition is known to be false. Besides that, in the counterfactual circumstance of the condition being true, the consequence is asserted to be true.

There are numerous examples of conditionals which have been extensively studied in the literature like *if she has an essay to write, then she will study late in the library* [1], *if there is a D on one side of the card, then there is a 3 on the other side* [24], or *if the prisoner is alive, then the captain did not signal* [20]. Cognitive theories try to evaluate conditionals like human do, yet they deviate from the answers provided by humans. Human syllogistic reasoning is just one of many examples [15].

In this paper we focus on the Weak Completion Semantics (WCS) [13]. WCS is a new cognitive theory which has been successfully applied to various human reasoning tasks like the suppression task [9], the selection task [10], and the belief-bias effect [4, 19]. WCS is rooted in the work by Keith Stenning and Michiel van Lambalgen [23] but corrects a technical bug by switching from three-valued Kripke-Kleene [11] to three-valued Łukasiewicz [18] logic [14].

WCS has also been applied to conditional reasoning [5, 7]. In this approach, the background knowledge is formalized in a logic program and, because the weak completion of such a program admits a least model under Łukasiewicz logic, this model is used in the evaluation of a conditional. In particular, if the condition of a given conditional is true under this model, then the truth value assigned to the conditional is the truth value assigned to the consequence of the conditional by this model. If the condition is false, then the background knowledge is revised with respect to the false condition.

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F	$\neg F$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	$\top$	U
$\bot$	$\bot$	U	$\top$

Table 1: The truth tables for the connectives under the three-valued Łukasiewicz logic, where $\top$, $\bot$, and U denote *true*, *false*, and *unknown*, respectively.

If the condition is unknown, then a technique called *minimal revision followed by abduction* (MRFA) is applied, where the program is revised as little as possible and abduction is applied afterwards in order to explain the condition of the conditional. MRFA was able to solve all conditionals in the *firing squad scenario* described in [20].

However, as we will show in Section 3 of this paper, MRFA may lead to counterintuitive results and does not generate the expected results in some cases. This happens because MRFA does not consider the semantics of the conditionals, which is a crucial aspect when it comes to their evaluation. In particular, in Section 4 we discuss obligation and factual conditionals with necessary or sufficient conditions. Based on the semantics of a given conditional, in Section 4.3 we vary the set of abducibles used in the abduction process and demonstrate, that by using this technique, the counterintuitive results obtained before can be avoided. In Section 5 we verify that the new technique does not alter the answers given by WCS for the suppression task [9]. Section 6 considers the selection task and shows that by varying only the set of abducibles the abstract case [24] and the social case [12] of Wason's selection task can be adequately modeled. This significantly improves the results obtained in [10], where the different representation of the two cases had to be modeled outside of the framework.

The main results of this paper are embedded into preliminaries presented in Section 2, the formal specification of MRFA in Section 2.6 as well as conclusions in Section 7.

2 Preliminaries

2.1 The Three-Valued Łukasiewicz Logic

In a three-valued logic, the truth values are not only *true* or *false*, symbolized by $\top$ and $\bot$, respectively. But there exists also a third value, which, in the sequel, we will call *unknown* and use the symbol U to denote it. More specifically, we will be using the three-valued Łukasiewicz (or Ł-) logic. As shown in Table 1, the expressions $U \leftarrow U$ and $U \leftrightarrow U$ are evaluated to *true* under the Ł-Logic. This is the main difference between this and the three-valued logics introduced by Kleene [16] and used by Fitting [11].

2.2 Programs

Clauses are expressions of the forms $A \leftarrow L_1 \wedge ... \wedge L_n$ (called *rules*), $A \leftarrow \top$ (called *facts*), and $A \leftarrow \bot$ (called *assumptions*), where $n \geq 1$, A is an atom, and each L_i, $1 \leq i \leq n$, is a literal. A is called the *head* and $L_1 \wedge ... \wedge L_n$ as well as $\top$ and $\bot$ are called *bodies* of the corresponding clauses. A *(propositional logic) program* $\mathcal{P}$ is a finite set of clauses. A is defined in $\mathcal{P}$ if and only if $\mathcal{P}$ contains a clause of the form $A \leftarrow Body$. A is *undefined in* $\mathcal{P}$ if and only if A is not defined in $\mathcal{P}$. The definition of A in $\mathcal{P}$ is defined as $def(A, \mathcal{P}) = \{A \leftarrow Body \mid A \leftarrow Body$ is a clause in $\mathcal{P}\}$.

A set of literals is *consistent* if it does not contain an atom and its negation. Let $\mathcal{S}$ be a finite and consistent set of literals in $rev(\mathcal{P}, \mathcal{S}) = (\mathcal{P} \setminus def(\mathcal{P}, \mathcal{S})) \cup \{A \leftarrow \top \mid A \in \mathcal{S}\} \cup \{A \leftarrow \top \mid \neg A \in \mathcal{S}\}$, where A denotes an atom. $rev(\mathcal{P}, \mathcal{S})$ is called the *revision of* $\mathcal{P}$ *with respect to* $\mathcal{S}$.

2.3 The (Weak) Completion of Programs

The definitions bellow are based on [3]. Let $\mathcal{P}$ be a program and consider the following transformation:

1. All clauses with the same head $A \leftarrow Body_1$, $A \leftarrow Body_2$, ... are replaced by the single formula $A \leftarrow Body_1 \vee Body_2 \vee ...$
2. An assumption $A \leftarrow \bot$ is added for each atom A which is not the head of any clause in $\mathcal{P}$
3. All occurrences of $\leftarrow$ are replaced by $\leftrightarrow$

The resulting set of formulas is called the *completion* of $\mathcal{P}$ or $c\mathcal{P}$ and, if the second step is omitted, then the resulting set is called the *weak completion* of $\mathcal{P}$ or $wc\mathcal{P}$.

2.4 Interpretations and Models

The declarative semantics of a logic program is given by a model-theoretic semantics of formulas in the underlying language. We represent interpretations by pairs $\langle I^\top, I^\bot\rangle$, where the set $I^\top$ consists of all atoms which are mapped to $\top$, the set $I^\bot$ consists of all atoms which are mapped to $\bot$, and $I^\top \cap I^\bot = \emptyset$. All atoms which occur neither in $I^\top$ nor $I^\bot$ are mapped to U. The logical value of formulas can be derived from Table 1 as usual.

Let I be an interpretation of a language L and let F be a formula of L. I is a model of F iff F is *true* with respect to I (i.e., $I(F) = \top$). Let $\mathcal{P}$ be a program of a language L and let I be an interpretation of L. We say I is a model of $\mathcal{P}$ iff I is a model of each clause in $\mathcal{P}$. Two formulas F and G are said to be semantically equivalent if and only if both have the same truth value under all interpretations. The least model of $wc\mathcal{P}$ can be obtained as the least fixed point of the semantic $\Phi_\mathcal{P}$ operator [23]: Let $I = \langle I^\top, I^\bot\rangle$ be an interpretation. $\Phi_\mathcal{P}(I) = \langle J^\top, J^\bot\rangle$, where

$$\begin{aligned} J^\top &= \{A \mid \text{there exists } A \leftarrow Body \in \mathcal{P} \text{ with } I(Body) = \top\},\\ J^\bot &= \{A \mid \text{there exists } A \leftarrow Body \in \mathcal{P} \text{ and for all } A \leftarrow Body \in \mathcal{P} \text{ we find } I(Body) = \bot\}.\end{aligned}$$

As has been shown in [14], the least fixed point of $\Phi_\mathcal{P}$ always exists.

Weak Completion Semantics (WCS) is the approach to consider weakly completed logic programs and to reason with respect to the least models of the weak completion of these programs. We write $\mathcal{P} \models_{wcs} F$ iff formula F holds in $\mathsf{lfp}\,\Phi_\mathcal{P}$, which is identical to the least model of $wc\mathcal{P}$. In the sequel, $\mathcal{M}_\mathcal{P}$ denotes the least model of $wc\mathcal{P}$.

2.5 Abductive Framework

An *abductive framework* consists of a logic program $\mathcal{P}$, a set of *abducibles* $\mathcal{A} \subseteq \mathcal{A}_\mathcal{P}$, where

$$\mathcal{A}_\mathcal{P} = \{A \leftarrow \top \mid A \text{ is undefined in } \mathcal{P}\} \cup \{A \leftarrow \bot \mid A \text{ is undefined in } \mathcal{P}\},$$

and the entailment relation $\models_{wcs}$. An abductive framework is denoted by $\langle \mathcal{P}, \mathcal{A} \models_{wcs}\rangle$. One should observe that each $\mathcal{P}$ and, in particular, each finite set of positive and negative facts has an least model of the weak completion. For the latter, this can be obtained by mapping all heads occurring in this set to *true*. Thus, explanations as well as the union of a program and an explanation are always satisfiable.

An *observation* $\mathcal{O}$ is a set of literals; $\mathcal{O}$ is *explainable* in the framework $\langle \mathcal{P}, \mathcal{A}, \mathcal{IC}, \models_{wcs}\rangle$ iff there exists an $\mathcal{E} \subseteq \mathcal{A}$ called *explanation* such that $\mathcal{M}_{\mathcal{P}\cup\mathcal{E}} \models_{wcs} L$ for all $L \in \mathcal{O}$ and $\mathcal{P} \cup \mathcal{E}$ satisfies $\mathcal{IC}$. We require explanations to be *minimal*, i.e. they cannot be subsumed by any other explanation.

There are two possible ways of abductive reasoning: credulous and skeptical reasoning. Let $\langle \mathcal{P}, \mathcal{A}, \mathcal{IC}, \models_{wcs}\rangle$ be an abductive framework, $\mathcal{O}$ an observation and $\mathcal{F}$ a formula:

- $\mathcal{F}$ follows credulously from $\mathcal{P}$ and $\mathcal{O}$ iff there exists an explanation $\mathcal{E}$ for $\mathcal{O}$ such that $\mathcal{P} \cup \mathcal{E} \models_{wcs} \mathcal{F}$.
- $\mathcal{F}$ follows skeptically from $\mathcal{P}$ and $\mathcal{O}$ iff for all explanations $\mathcal{E}$ for $\mathcal{O}$ we find $\mathcal{P} \cup \mathcal{E} \models_{wcs} \mathcal{F}$.

2.6 Evaluation System for Conditionals

We consider conditionals of the form *if* $\mathcal{C}$ *then* $\mathcal{D}$, where $\mathcal{C}$ and $\mathcal{D}$ are finite and consistent sets of literals viewed as conjunctions of literals. Conditionals are evaluated with respect to some background information specified as a program. Let $\mathcal{P}$ be a program, $\mathcal{M}_\mathcal{P}$ be the least model of $wc\mathcal{P}$ and *if* $\mathcal{C}$ *then* $\mathcal{D}$ be a conditional. [5, 8] introduced an abstract reduction system for conditionals (ARSC) where the states are either the truth values or tuples containing a program and two consistent and finite sets of literals.[1] The initial state for a given program $\mathcal{P}$ and a conditional *if* $\mathcal{C}$ *then* $\mathcal{D}$ is $\langle \mathcal{P}, \mathcal{C}, \mathcal{D}\rangle$. Final states are *true*, *false* and *unknown*. The set of rules of ARSC is $\{\longrightarrow_a, \longrightarrow_r, \longrightarrow_s, \longrightarrow_c\}$:

- $\langle \mathcal{P}, \mathcal{C}, \mathcal{D}\rangle \longrightarrow_s \mathcal{M}_\mathcal{P}(\mathcal{D})$ iff $\mathcal{M}_\mathcal{P}(\mathcal{C}) = true$.
- $\langle \mathcal{P}, \mathcal{C}, \mathcal{D}\rangle \longrightarrow_c \langle rev(\mathcal{P}, \mathcal{S}), \mathcal{C} \setminus \mathcal{S}, \mathcal{D}\rangle$ iff $\mathcal{M}_\mathcal{P}(\mathcal{C}) = false$, where $\mathcal{S} = \{L \in \mathcal{C} \mid \mathcal{M}_\mathcal{P}(L) = \bot\}$.
- $\langle \mathcal{P}, \mathcal{C}, \mathcal{D}\rangle \longrightarrow_a \langle \mathcal{P} \cup \mathcal{E}, \mathcal{C}, \mathcal{D}\rangle$ iff $\mathcal{M}_\mathcal{P}(\mathcal{C}) = unknown$, $\mathcal{O} \subseteq \mathcal{C}$, $\mathcal{O} \neq \emptyset$, for each $L \in \mathcal{O}$ we find $\mathcal{M}_\mathcal{P}(L) = unknown$, and $\mathcal{E}$ explains $\mathcal{O}$ in the abductive framework $\langle \mathcal{P}, \mathcal{A}_\mathcal{P}, \models_{wcs}\rangle$.
- $\langle \mathcal{P}, \mathcal{C}, \mathcal{D}\rangle \longrightarrow_r \langle rev(\mathcal{P}, \mathcal{S}), \mathcal{C} \setminus \mathcal{S}, \mathcal{D}\rangle$ iff $\mathcal{M}_\mathcal{P}(\mathcal{C}) = unknown$, $\mathcal{S} \subseteq \mathcal{C}$, $\mathcal{S} \neq \emptyset$, for all $L \in \mathcal{S}$ we find $\mathcal{M}_\mathcal{P}(L) = unknown$.

[5, 8] proposed the following strategy for the evaluation of conditionals: *if* $\mathcal{C}$ *then* $\mathcal{D}$ is evaluated as follows:

1. If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \top$ then *if* $\mathcal{C}$ *then* $\mathcal{D}$ is assigned to $\mathcal{M}_\mathcal{P}(\mathcal{D})$. $(\longrightarrow_s)$
2. If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \bot$, then *if* $\mathcal{C}$ *then* $\mathcal{D}$ is evaluated with respect to $\mathcal{M}_{rev(\mathcal{P},\mathcal{S})}$, where $\mathcal{S} = \{L \in \mathcal{C} \mid \mathcal{M}_\mathcal{P}(L) = \bot\}$. $(\longrightarrow_c)$

[1] The original version also considered integrity constraints. For simplicity and as we do not use them here, we leave them out.

3. If $\mathcal{M}_\mathcal{P}(C) = \text{U}$, then *if C then $\mathcal{D}$* is evaluated with respect to $\mathcal{M}_{\mathcal{P}'}$, where

   (a) $\mathcal{P}' = rev(\mathcal{P}, \mathcal{S}) \cup \mathcal{E}$, $(\longrightarrow_r)$

   (b) $\mathcal{S}$ is a smallest subset of C and $\mathcal{E} \subseteq \mathcal{A}_{rev(\mathcal{P},\mathcal{S})}$ is a minimal explanation for $C \setminus \mathcal{S}$ such that $\mathcal{M}_{\mathcal{P}'}(C) = \top$. $(\longrightarrow_a)$

If the condition C of a conditional is *true*, then the conditional is an indicative one and is evaluated as implication in Ł-logic. If C is *false*, then the conditional is a counterfactual one and revision is applied in order to revise the truth value of the literals in C, which are mapped to *false*. If C is *unknown*, then we propose to split C into two disjoint subsets $\mathcal{S}$ and $C \setminus \mathcal{S}$, where the former is treated by revision and the latter by abduction. In case C contains some literals, which are *true* and some, which are *unknown* under $\mathcal{M}_\mathcal{P}$, then the former will be part of $C \setminus \mathcal{S}$ because the empty explanation explains them. As we assume $\mathcal{S}$ to be minimal, this approach is called *minimal revision followed by abduction* (MRFA).

3 Reasoning about Conditionals

For the following examples in this section, the conditionals will be evaluated by means of MRFA and we will assume that the following conditionals are known:

If it rains, then the streets are wet. (1)

If it rains, then she takes her umbrella (2)

The logic program representing this background knowledge is encoded as:

$$\mathcal{P} \;=\; \{\; wet\_streets \leftarrow rain \wedge \neg ab_1, \quad ab_1 \leftarrow \bot, \quad umbrella \leftarrow rain \wedge \neg ab_2, \quad ab_2 \leftarrow \bot \quad \}.$$

The set of abducibles is $\mathcal{A}_\mathcal{P} = \{rain \leftarrow \top, rain \leftarrow \bot\}$ and the least model of wc$\mathcal{P}$ is $\mathcal{M}_\mathcal{P} = \langle \emptyset, \{ab_1, ab_2\}\rangle$.

3.1 Denying the Consequent

Assume that *the streets are not wet*. Do humans conclude that *it did not rain*? Or differently said, given the background knowledge $\mathcal{P}$, how should we evaluate the following conditional?

Example 1 *If the streets are not wet, then it did not rain.*

According to MRFA, we obtain

$$\langle \mathcal{P}, \neg wet\_streets, \neg rain\rangle \longrightarrow_a \langle \mathcal{P} \cup \{rain \leftarrow \bot\}, \neg wet\_streets, \neg rain\rangle \longrightarrow_s true,$$

where first, as $\mathcal{M}_\mathcal{P}(\neg wet\_streets) = \text{U}$, rule 3b $(\longrightarrow_a)$ from the MRFA evaluation of the previous section applies, where $\mathcal{E} = \{rain \leftarrow \bot\}$ is the only explanation for $\neg wet\_streets$. Given that $\mathcal{M}_{\mathcal{P}\cup\mathcal{E}} = \langle \emptyset, \{rain, wet\_streets, umbrella, ab_1, ab_2\}\rangle$, $\neg wet\_streets$ is evaluated to $\top$, thus rule 1 $(\longrightarrow_s)$ applies, i.e. the conditional is evaluated with respect to the truth value of $\neg rain$ under $\mathcal{M}_{\mathcal{P}\cup\mathcal{E}}$, which is $\top$. Summing up, the conditional *if $\neg wet\_streets$ then $\neg rain$* is evaluated to *true*.

Let's assume that *she did not take her umbrella*. Do humans conclude that *it did not rain*? Or differently said, given the background knowledge $\mathcal{P}$, how should we evaluate the following conditional?

Example 2 *If she did not take her umbrella, then it did not rain.*

Analogously to the previous example, given that *if $\neg umbrella$ then $\neg rain$*, according to MRFA, we obtain

$$\langle \mathcal{P}, \neg umbrella, \neg rain\rangle \longrightarrow_a \langle \mathcal{P} \cup \{rain \leftarrow \bot\}, \neg umbrella, \neg rain\rangle \longrightarrow_s true.$$

This conditional has the same structure with respect to $\mathcal{P}$ as the previous one, thus the evaluation strategy is identical.

3.2 Affirming the Consequent

Similarly to the previous examples we can ask, given that *the streets are wet*, do humans conclude that *it rained*?

Example 3 *If the streets are wet, then it rained.*

Given that *if $wet\_streets$ then $rain$*, according to MRFA, we obtain

$$\langle \mathcal{P}, wet\_streets, rain\rangle \longrightarrow_a \langle \mathcal{P} \cup \{rain \leftarrow \top\}, wet\_streets, rain\rangle \longrightarrow_s true,$$

Now the explanation for $wet\_streets$ is $rain \leftarrow \top$ and *if $wet\_streets$ then $rain$* is evaluated to *true*.

Let's consider the last case, where *she took her umbrella* is given. Do humans conclude that *it rained*?

Example 4 *If she took her umbrella, then it rained.*

Analogously to the previous example, given that *if $umbrella$ then $rain$*, according to MRFA, we obtain

$$\langle \mathcal{P}, umbrella, rain\rangle \longrightarrow_a \langle \mathcal{P} \cup \{rain \leftarrow \top\}, umbrella, rain\rangle \longrightarrow_s true,$$

and thus the conditional *if $umbrella$ then $rain$* is evaluated to *true*.

4 A Semantic Approach to the Evaluation of Conditionals

Although MRFA evaluates all the conditionals shown in Section 3 to *true*, it is not clear that given the information *the streets are not wet*, humans would conclude *it did not rain* in the same way that it would be concluded if the information that *she did not take her umbrella* would be given. Similarly, this is the case for the examples in Section 3.2: Given the information *she took her umbrella*, it is again not clear that one would conclude *it rained* in the same way that it would be concluded if the information that *the streets are wet* would be given.

We will now look at some semantic definitions which will allow us to explain the difference between these conditionals and show that they should not be handled in the same way as it is done by the current MRFA approach.

4.1 Obligation versus Factual Conditionals

The difference between *obligation* and *factual conditionals* is that in the *obligation conditional* the consequence is obligatory if the condition of the conditional is true, while in the *factual conditionals* this is not the case. More precisely, given an *obligation conditionals* it can never be the case that the condition is true and the consequence is not true.

As explained in [2], a conditional where the consequent is denied is more likely to be evaluated to *true* if it is an *obligation conditional*. This happens because for this type of conditionals, people keep in mind a forbidden possibility where condition and not consequence happens together and, in this case, if the consequence is known to be *false*, then it cannot be the case that condition is *true*, otherwise the forbidden possibility is violated. Thus, *not condition* is concluded. But, since in a *factual conditional* this forbidden possibility does not exist, conditionals with the consequence denied should be evaluated to *unknown*.

Consider again the conditions assumed as the background knowledge introduced in Section 3. Let's first consider conditional (1): How plausible is it, that *it rained* (condition is *true*) and *the streets are not wet* (consequent is *false*)? Someone could think of a roofed street, yet this is rather an exception to the usual case. On the other hand, let's consider conditional (2): How plausible is it, that *it rained* (condition is *true*) and *she did not take her umbrella* (consequent is *false*)? In contrast to conditional (1), it seems to us, that it is more plausible that, even if it's raining, she forgets her umbrella or just doesn't have one available, than that the streets were roofed.

Therefore, we classify the conditionals (1) as an *obligation conditional* and (2) as a *factual conditional*.

4.2 Necessary versus Sufficient Conditions

If a conditional has a *necessary condition*, then the consequence cannot be *true* unless its condition is *true*. But if a conditional has only a *sufficient condition*, then the circumstance where this condition is *true* gives us the adequate grounds to conclude that the consequence is *true* as well, but there is no necessity involved.

This means that, when the consequent of a conditional is affirmed, those conditions which are necessary can be derived, but not necessarily the sufficient ones.

Consider again conditional (1) from Section 3: How plausible is it that *the streets are wet* (consequent is *true*) even though *it did not rain* (condition is *false*)? We may be able to imagine cases like a flooding or a tsunami has occurred, but we would expect that such an extraordinary reason would have been mentioned in the context. Similarly, while considering conditional (2): How plausible is it that *she took her umbrella* (consequent is *true*) even though *it did not rain* (condition is *false*)? It seems plausible that, for instance, the weather forecast was wrong and she took her umbrella even though it did not rain. In another hand, we cannot easily imagine a situation where the streets are wet and it did not rain.

Therefore, we classify the conditionals (1) and (2) as conditionals with *necessary* and *sufficient conditions*, respectivally.

4.3 A Semantic Approach to MRFA

In order to allow MRFA to distinguish between the different types of conditionals discussed in Section 4.2 and 4.1, the following two modifications will be introduced to the approach:

1. The conclusion of a given conditional will be evaluated by means of skeptical reasoning.
2. The set of abducibles is extended as follows:

$$\begin{array}{rcl} \mathcal{A}_\mathcal{P} & = & \{A \leftarrow \top \mid A \text{ is undefined in } \mathcal{P}\} \cup \{A \leftarrow \bot \mid A \text{ is undefined in } \mathcal{P}\} \\ & \cup & \{A \leftarrow \top \mid \text{A is the head of a conditional with a sufficient condition in } \mathcal{P}\} \\ & \cup & \{ab_i \leftarrow \top \mid \ ab_i \text{ is in the body of a factual conditional in } \mathcal{P}\}. \end{array}$$

C	$\mathcal{D}$	MRFA steps	Revised MRFA steps
$\neg wet\_streets$	$\neg rain$	$\mathcal{S} = \emptyset$ and $\mathcal{E} = \{rain \leftarrow \bot\}$ *true*	$\mathcal{S} = \emptyset$ and $\mathcal{E} = \{rain \leftarrow \bot\}$ *true*
$\neg umbrella$	$\neg rain$	$\mathcal{S} = \emptyset$ and $\mathcal{E} = \{rain \leftarrow \bot\}$ *true*	$\mathcal{S} = \emptyset$ and $\mathcal{E}_1 = \{rain \leftarrow \bot\}$ or $\mathcal{E}_2 = \{ab_2 \leftarrow \top\}$ *unknown*
$wet\_streets$	$rain$	$\mathcal{S} = \emptyset$ and $\mathcal{E} = \{rain \leftarrow \top\}$ *true*	$\mathcal{S} = \emptyset$ and $\mathcal{E} = \{rain \leftarrow \top\}$ *true*
$umbrella$	$rain$	$\mathcal{S} = \emptyset$ and $\mathcal{E} = \{rain \leftarrow \top\}$ *true*	$\mathcal{S} = \emptyset$ and $\mathcal{E}_1 = \{rain \leftarrow \top\}$ or $\mathcal{E}_2 = \{umbrella \leftarrow \top\}$ *unknown*

Table 2: Results of examples 1 to 4 using MRFA and revised MRFA.

4.4 Reasoning about Conditionals Revisited

We reconsider the examples discussed in Section 3.1 and 3.2 with the newly introduced semantic approach, where we take the following assumption: *if it rains, then the streets are wet* is an *obligation conditional* with a *necessary condition* and *if it rains, then she takes her umbrella* is a *factual conditional* with a *sufficient condition*. Thus, the set of abducibles is defined as follows:

$$\mathcal{A}_{\mathcal{P}} = \{rain \leftarrow \top,\ rain \leftarrow \bot\} \cup \{umbrella \leftarrow \top\} \cup \{ab_2 \leftarrow \top\}.$$

Example 1 *If the streets are not wet, then it did not rain.*

The evaluation of this conditional remains the same, since the explanation used for the condition $\neg wet\_streets$ is unique and, thus, the consequence $\neg rain$ follows skeptically from the program $\mathcal{P}$ and the observation $\neg wet\_streets$. Because of this, the conditional is still evaluated to *true* by following the same steps showed in Section 3.1.

Example 2 *If she did not take her umbrella, then it did not rain.*

We have two possible explanations $\mathcal{E}_1 = \{rain \leftarrow \bot\}$ and $\mathcal{E}_2 = \{ab_2 \leftarrow \top\}$ for $\mathcal{O} = \{\neg umbrella\}$, where $\mathcal{P} \cup \mathcal{E}_1 \models_{wcs} \neg rain$, but $\mathcal{P} \cup \mathcal{E}_2 \not\models_{wcs} \neg rain$. According to semantic MRFA, we obtain the following two evaluation strategies:

$$\langle \mathcal{P}, \neg umbrella, \neg rain\rangle \longrightarrow_a \langle \mathcal{P} \cup \mathcal{E}_1, \neg umbrella, \neg rain\rangle \longrightarrow_s true,$$
$$\langle \mathcal{P}, \neg umbrella, \neg rain\rangle \longrightarrow_a \langle \mathcal{P} \cup \mathcal{E}_2, \neg umbrella, \neg rain\rangle \longrightarrow_s unknown.$$

The consequence does not follow from all explanations, and therefore, the conditional is evaluated to *unknown*. This complies with the point of view expressed in Section 4.1.

Example 3 *If the streets are wet, then it rained.*

This conditional is still evaluated to *true* by following the same steps as before for the reasons described for Example 1.

Example 4 *If she took her umbrella, then it rained.*

Similar to Example 1, we have two explanations $\mathcal{E}_1 = \{rain \leftarrow \top\}$ and $\mathcal{E}_2 = \{umbrella \leftarrow \top\}$, where $\mathcal{P} \cup \mathcal{E}_1 \models_{wcs} rain$, but $\mathcal{P} \cup \mathcal{E}_2 \not\models_{wcs} rain$. According to semantic MRFA, we obtain the following two evaluation strategies:

$$\langle \mathcal{P}, umbrella, rain\rangle \longrightarrow_a \langle \mathcal{P} \cup \mathcal{E}_1, umbrella, rain\rangle \longrightarrow_s true,$$
$$\langle \mathcal{P}, umbrella, rain\rangle \longrightarrow_a \langle \mathcal{P} \cup \mathcal{E}_2, umbrella, rain\rangle \longrightarrow_s unknown.$$

Because we are reasoning skeptically, the conditional is now evaluated to *unknown*. This complies with the point of view expressed in Section 4.2.

5 Byrne's Suppression Task

Byrne's suppression task [1] is a famous psychological study from the literature, consisting of two parts. We reconsider the computational logic approach of the second part of this task from [9] and show that the results still hold within our new approach. We do not need to reconsider the first part as modeling the first part does not require abduction, and therefore our new approach does not affect the results in [9]. Consider the following three conditionals:

1. *If she has an essay to write, then she will study late in the library.*
2. *If she has a textbook to read, then she will study late in the library.*
3. *If the library stays open, then she will study late in the library.*

Abstract	Case				Social	Case		
D	F	3	7		*beer*	*coke*	22*yrs*	16*yrs*
89%	16%	62%	25%		95%	0.025%	0.025%	80%

Table 3: The results of the abstract and social cases of the selection task.

In [1], three groups of participants had been given optionally the information that *she will study late in the library* or that *she will not study late in the library*. Additionally, the first group had been given conditional 1, the second group had been given conditional 1 and 2, and the third group had been given conditional 1 and 3. Given the information that *she will study late in the library*, the majority of the participants of the first and the third group concluded that *she has an essay to write*, whereas only 16% of the participants of the second group derived the same conclusion. Yet, given the information that *she will not study late in the library*, the majority of the participants of the first and the second group concluded that *she does not have an essay to write*, whereas only 44% of the participants of the third group derived the same conclusion. The representation as logic programs from [23] for the first, second and third group is respectively $\mathcal{P}_e = \{\ell \leftarrow e \wedge ab_1,\ ab_1 \leftarrow \bot\}$, $\mathcal{P}_{e+Alt} = \{\ell \leftarrow e \wedge ab_1,\ \ell \leftarrow t \wedge ab_2,\ ab_1 \leftarrow \bot,\ ab_2 \leftarrow \bot\}$ and $\mathcal{P}_{e+Add} = \{\ell \leftarrow e \wedge ab_1,\ \ell \leftarrow o \wedge ab_3,\ ab_1 \leftarrow \neg o,\ ab_3 \leftarrow \neg e\}$, together with either the observation $O_\ell = \{\ell \leftarrow \top\}$ or $O_{\neg\ell} = \{\ell \leftarrow \bot\}$.

The sets of abducibles originally defined in [9] are the facts and assumptions for all undefined atom of the respective programs. Following the conditional classification in the Section 4, conditonals 1 and 2 are classified as *obligation conditionals* whereas conditional 3 is classified as a *factual conditional*. Individually, each of the three conditionals have a *necessary condition*. However, in the second group when conditionals 1 and 2 happen together, their conditions are no longer *necessary*, but *sufficient*. Therefore, for $\mathcal{P}_e$, the set of abducibles remains as in [9] and the results coincide with the majority of participants' answers, as shown in [9].

Yet, for $\mathcal{P}_{e+Alt}$, the set of abducibles, originally defined as $\mathcal{A}_{\mathcal{P}_{e+Alt}} = \{e \leftarrow \top, e \leftarrow \bot, t \leftarrow \top, t \leftarrow \bot\}$ is now extended by $\{\ell \leftarrow \top\}$, because in the second group the conditionals have *sufficient conditions*. Consider $\mathcal{P}_{e+Alt}$ together with the observation $O_\ell = \{\ell \leftarrow \top\}$: There are three possible explanations for O, $\mathcal{E}_1 = \{e \leftarrow \top\}$, $\mathcal{E}_2 = \{t \leftarrow \top\}$ and $\mathcal{E}_3 = \{\ell \leftarrow \top\}$. As $\mathcal{P}_{e+Alt} \cup \mathcal{E}_2 \not\models_{wcs} e$, [2] *she has essay to write* does not follow skeptically from $\mathcal{P}_{e+Alt}$ and O_ℓ. Consider $\mathcal{P}_{e+Alt}$ together with the observation $O_{\neg\ell} = \{\ell \leftarrow \bot\}$: The only possible explanation for O is $\mathcal{E} = \{e \leftarrow \bot, t \leftarrow \bot\}$ and $\mathcal{P}_{e+Alt} \cup \mathcal{E} \models_{wcs} \neg e$. Thus, *She does not have an essay to write* follows skeptically from $\mathcal{P}_{e+Alt}$ and $O_{\neg\ell}$.

Moreover, for $\mathcal{P}_{e+Add}$, the set of abducibles, originally defined as $\mathcal{A}_{\mathcal{P}_{e+Add}} = \{e \leftarrow \top,\ e \leftarrow \bot,\ o \leftarrow \top,\ o \leftarrow \bot\}$ is now extended by $\{ab_3 \leftarrow \top\}$, because conditional 3 is a *factual conditional*. Consider $\mathcal{P}_{e+Add}$ together with the observation $O_\ell = \{\ell \leftarrow \top\}$: The only possible explanation for O is $\mathcal{E} = \{e \leftarrow \top,\ o \leftarrow \top\}$ and $\mathcal{P}_{e+Add} \cup \mathcal{E} \models_{wcs} e$. Thus, *She has an essay to write* follows skeptically from $\mathcal{P}_{e+Add}$ and O_ℓ. Consider $\mathcal{P}_{e+Add}$ together with the observation $O_{\neg\ell} = \{\ell \leftarrow \bot\}$: O has three possible explanations, $\mathcal{E}_1 = \{e \leftarrow \bot\}$, $\mathcal{E}_2 = \{o \leftarrow \bot\}$ and $\mathcal{E}_3 = \{ab_3 \leftarrow \top\}$. As $\mathcal{P}_{e+Add} \cup \mathcal{E}_2 \not\models_{wcs} \neg e$,[2] *She does not have an essay to write* does not follow skeptically from $\mathcal{P}_{e+Add}$ and $O_{\neg\ell}$.

In both programs above, the derived result for the observations $O_\ell = \{\ell \leftarrow \top\}$ and $O_{\neg\ell} = \{\ell \leftarrow \bot\}$ considered complies with the majority of the participants' answers.

6 The Wason Selection Task

Wason's [24] is yet another famous psychological task, where participants had to select a given conditional statement on some instances. Participants were given the conditional

if there is a D on one side of the card, then there is 3 *on the other side*

and had to consider four cards, showing the letters D and F as well as the numbers 3 and 7. Furthermore, they were told that each card had a letter on one side and a number on the other side. Next, the participants where asked which cards must be turned to prove that the conditional holds.

From a classical logic point of view, the conditional can be represented as the implication $D \rightarrow 3$. From a classical logical point of view we must turn the cards showing D (modus ponens) and 7 (modus tollens). As repeated experiments have shown consistently (see Table 3), from a classical logical point of view, the majority of the participants correctly chose card D. However, they failed to choose card 7 and incorrectly chose card 3. In other words, the overall correctness of the answers for the abstract selection task if modeled by an implication in classical two-valued logic is pretty bad.

[12] developed an isomorphic representation of the problem in a social context, and surprisingly almost all of the participants solved this task classical logically correctly. Analogously, participants again were given a conditional

if a person is drinking beer, then the person must be over 19 years of age

[2] Or $\mathcal{E}_3$ alternatively.

and again had to consider four cards, showing drinking beer, drinking coke, 22 years old and 16 years old. Also, they were told that on one side there is the person's age and on the other side of the card what the person is drinking, and had to answer the question, which drinks and persons must be checked to prove that the conditional holds. Table 3 shows the results represented in [12] and one can see that, in this case, the participants seemed to solve the task classical logically correctly.

One explanation for the differences between both cases can be found in [17], namely that people saw the conditional in the abstract case as a belief. For instance, the participants perceived the task to examine whether the conditional is either *true* or *false*. On the other hand, in the social case, the participants perceived the conditional as a social constraint, a conditional that ought to be *true*. People intuitively aim at preventing the violation of such a constraint, which is normally done by observing whether the state of the world complies with the rule.

In [10] a computational logic approach under the Weak Completion Semantics has been proposed to model the two cases of the task. However, the approach presented there, does not distinguish between both conditionals, but instead, models the different interpretations outside of the logical framework. The different results for these two tasks with the same structure confirms that the semantics of conditionals is relevant for their evaluation. Taking into account Kowalski's [17] explanations and the semantics presented in Section 4, we understand the conditional in the abstract case as a *factual conditional* with a *necessary condition*, while the social case is an *obligation conditional* with a *sufficient condition*. In the following, we will model the two cases within one logical framework by distinguishing the two cases with respect to the classification of the conditionals

6.1 Modeling the Abstract and the Social Case

To generalize the problem, let's consider cards with type X on one side and type Y on the other side. Given the conditional *if X, then Y* we would like to decide which cards must be turned in order to assure that the conditional holds. The program representing the background knowledge is given by $\mathcal{P} = \{Y \leftarrow X \wedge \neg ab, ab \leftarrow \bot\}$ and the set $\mathcal{A}_{\mathcal{P}}$ of abducibles depends on the type of the given conditional.

Given a card where the observed side has type X, this card will be turned if

1. X and Y follow skeptically from $\mathcal{P}$ and $\mathcal{O}$, or
2. For all explanations $\mathcal{E}$ for observation $\mathcal{O} = \{X\}$ given $\mathcal{P}$, either $\mathcal{E} = \{Y \leftarrow \top\}$ or $\mathcal{E} = \{Y \leftarrow \bot\}$.

The case where the observed side has type Y can be solved likewise. We encode and evaluate both abstract and social cases using this approach. Table 4 and Table 5 contain the evaluation of the observations for each case and shows that in both cases the results of our approach coincide with the experimental results.

6.2 The Abstract Case

In the abstract case we have cards with type *number* on one side and *letter* on the other side. The background knowledge is represented by the program

$$\mathcal{P}_{abstract} = \{3 \leftarrow D \wedge \neg ab,\ ab \leftarrow \bot\}.$$

As $3 \leftarrow D \wedge \neg ab$ is classified as a factual conditional, the set of abducibles is

$$\mathcal{A}_{\mathcal{P}_{abstract}} = \{D \leftarrow \top,\ D \leftarrow \bot,\ ab \leftarrow \top\}.$$

Table 4 shows that in the cases where D and *3* are observed, the cards are turned. In both cases, $\mathcal{E} = \{D \leftarrow \top\}$ is the only explanation and, in the two cases, $\mathcal{P}_{abstract} \cup \mathcal{E} \models_{wcs} D$ and $\mathcal{P}_{abstract} \cup \mathcal{E} \models_{wcs} 3$, satisfying condition 1. On the other hand, the cards are not turned in the cases where F and *7* are observed. For the case where F is observed, $\neg D$ must to be explained and the only possible explanation is $\mathcal{E} = \{D \leftarrow \bot\}$. Because $\mathcal{P}_{abstract} \cup \mathcal{E} \not\models_{wcs} D$ and $\mathcal{P}_{abstract} \cup \mathcal{E} \not\models_{wcs} 3$, condition 1 is not satisfied, and, further as *7* is not the head of a rule in $\mathcal{E}$, condition 2 is also not satisfied. Finally, in the case where *7* is observed, besides the explanation $\mathcal{E}$ above, there is another possible explanation $\mathcal{E}' = \{ab \leftarrow \top\}$. But, because we already know that $\mathcal{E}$ does not satisfy any of the conditions, it cannot be the case that one of them would be satisfied by an additional explanation. In the first case, because we reason skeptically, and in the second one, because the condition must hold for all explanations.

6.3 The Social Case

In the social case we have cards with type *drink* on one side and *age* on the other side. The background knowledge is represented by

$$\mathcal{P}_{social} = \{over19 \leftarrow beer \wedge \neg ab,\ ab \leftarrow \bot\}.$$

As $over19 \leftarrow beer \wedge \neg ab$ is classified as an obligation conditional the set of abducibles is defined as

$$\mathcal{A}_{\mathcal{P}_{social}} = \{beer \leftarrow \top,\ beer \leftarrow \bot,\ over19 \leftarrow \top\},$$

$\mathcal{O}$	$\mathcal{E}$	$\mathcal{M}_{\mathcal{P}_{abstract} \cup \mathcal{E}}$	Experimental results
D	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, \{ab\} \rangle \rightsquigarrow$ turn	89%
F ($\neg D$)	$\{D \leftarrow \bot\}$	$\langle \emptyset, \{D, 3, ab\} \rangle$	16%
3	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, \{ab\} \rangle \rightsquigarrow$ turn	62%
7 ($\neg 3$)	$\{D \leftarrow \bot\}$	$\langle \emptyset, \{D, 3, ab\} \rangle$	25%
	$\{ab \leftarrow \top\}$	$\langle \{ab\}, \{3\} \rangle$	

Table 4: The approach for the abstract case of the selection task.

$\mathcal{O}$	$\mathcal{E}$	$\mathcal{M}_{\mathcal{P}_{social} \cup \mathcal{E}}$	Experimental results
beer	$\{beer \leftarrow \top\}$	$\langle \{beer, over19\}, \{ab\} \rangle \rightsquigarrow$ turn	95%
coke ($\neg beer$)	$\{beer \leftarrow \bot\}$	$\langle \emptyset, \{beer, over19, ab\} \rangle$	0.025%
22yrs ($over19$)	$\{beer \leftarrow \top\}$	$\langle \{beer, over19\}, \{ab\} \rangle$	0.025%
	$\{over19 \leftarrow \top\}$	$\langle \{over19\}, \{ab\} \rangle$	
16yrs ($\neg over19$)	$\{beer \leftarrow \bot\}$	$\langle \emptyset, \{beer, over19, ab\} \rangle \rightsquigarrow$ turn	80%

Table 5: The approach for the social case of the selection task.

because *beer* is undefined in the program $\mathcal{P}_{social}$ and *over19* is the head of a sufficient conditional.

Table 5 shows the results with respect to the social case, where the cards are turned when *beer* and *16yrs* are observed, while for the observation *coke* and *22yrs*, the cards are not turned. When *beer* is observed, $\mathcal{E} = \{beer \leftarrow \top\}$ is the only possible explanation and, because $\mathcal{P}_{social} \cup \mathcal{E} \models_{wcs} beer \wedge over19$, condition 1 is satisfied. If *coke* is observed, $\neg beer$ must be explained and $\mathcal{E} = \{beer \leftarrow \bot\}$ is the only possible explanation. Because neither $\mathcal{P}_{social} \cup \mathcal{E} \models_{wcs} beer \wedge over19$ holds nor *over19* is the head of a rule in $\mathcal{E}$, none of the conditions are satisfied. In the case where *22yrs* is observed, *over19* must be explained and there are two possible explanations: $\mathcal{E}_1 = \{beer \leftarrow \top\}$ and $\mathcal{E}_2 = \{over19 \leftarrow \top\}$. Because $\mathcal{P}_{social} \cup \mathcal{E}_1 \models_{wcs} beer \wedge over19$, but $\mathcal{P}_{social} \cup \mathcal{E}_2 \not\models_{wcs} beer \wedge over19$, *beer* and *over19* don't follow skeptically from $\mathcal{P}_{social}$ and *22yrs*. Thus, condition 1 is not satisfied. Besides this, because *beer* is the head of a rule in $\mathcal{E}_1$, but not in $\mathcal{E}_2$, condition 2 is also not satisfied. Finally, if *16yrs* is observed, then $\neg over19$ must be explained, which is possible only by explanation $\mathcal{E} = \{beer \leftarrow \bot\}$. Since *beer* is the head of a rule in $\mathcal{E}$, condition 2 is satisfied.

To conclude, the modeling proposed in this section can be seen as an improvement of the approach in [10]. The main difference between this two approaches is that the one here proposed deals with the so-called first step of modeling human reasoning, which means reasoning with respect to an adequate representation, while the other one doesn't. The approach proposed here has a well defined modeling process and both cases use the same structure, still leading to the expected different results. Besides this, the representation of the task in this approach fits the semantic notions introduced earlier in this paper, allowing us to not only generate the expected results but also to explain them.

7 Conclusion

We started out by analyzing the MRFA approach with respect to how conditionals are evaluated based on the Weak Completion Semantics and identified some of its limitations. Our hypothesis is that the semantics of these conditionals play a crucial role when it comes to their evaluation in the context of human reasoning. We have shown examples that support our hypothesis and saw that MRFA could not evaluate these examples as expected. Therefore, we propose a semantic approach to the evaluation of conditionals, in a way that we can specify different types of conditionals, viz. obligation or factual conditionals with necessary or sufficient conditions, based on their meaning. From a technical point of view, this new approach consists of extending the set of abducibles and varying this set with respect to the semantics of the conditionals. The suppression task can still be adequately modeled. Furthermore, a way of modeling the abstract and the social case of the selection task without changing the background knowledge can be done and improves the approach in [10].

Yet much remains to be done. Do humans reason with multi-valued logics and, if they do, which multi-valued logic are they using? Can an answer *I don't know* be qualified as a truth value assignment or is it a meta-remark? At least, it appears that three-valued logics are needed for modeling human reasoning [21]. Do humans apply abduction and/or revision if the condition of a conditional is unknown and, if they apply both, do they prefer one over the other? Do they prefer skeptical over credulous abduction? Do they prefer minimal revision? Do they prefer minimal explanations? Our answers to these questions are only very partial ones and we need much more experimental data to answer them in more detail.

All the conditionals considered in this paper have only one condition. What will happen if conditionals have more than one condition, and one of the conditions is necessary whereas another one is sufficient? How important is the order in which multiple conditions of a conditional are considered? From human spatial reasoning [6, 22] we know that order plays an important role in the construction of models and we expect that this holds for the evaluation of conditionals with multiple conditions as well.

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