

A Mechanism Design Approach for Allocation of Commodities [★]

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Abstract. We deploy a mechanism design approach for allocating a divisible commodity (electricity in our example) among consumers. We consider each consumer with an associated personal valuation function of the energy resource during a certain time interval. We aim to select the optimal consumption profile for every user avoiding consumption peaks when the total required energy could exceed the energy production. The mechanism will be able to drive users in shifting energy consumptions in different hours of the day. We start by presenting a very basic Vickrey-Clarke-Groves mechanism, we discuss its weakness and propose several more complex variants. This is an extended abstract, for additional details we provide a technical report [1].

1 VCG Mechanisms

Mechanism Design [3–6] is based on the concept of social choice that is simply an aggregation of the preferences of the different participants toward a single collective decision. Mechanism Design attempts to implement desired social choices in a strategic setting, assuming that players act rationally in a game theoretic sense.

Definition 1 (Player’s Valuation Function) *Let us consider a set of players $N = \{1, \dots, n\}$ and a set of alternatives or outcomes A . Every player i has a preference over alternatives that is described by a valuation function:*

$$v_i : A \rightarrow \mathbb{R}$$

where $v_i(a)$ denotes the valuation that player i assigns to outcome a . Furthermore, $v_i \in V_i$ where $V_i \subseteq \mathbb{R}^{|A|}$ is a set of possible valuation functions for player i .

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Definition 2 (Social Choice Function) *The social choice function selects an alternative (or outcome) from the set of alternatives A according to the vector of users' valuation functions:*

$$f : V_1 \times \cdots \times V_n \rightarrow A$$

So, an outcome a , from the set A of alternatives, depends on each possible profile $v = (v_1, v_2, \dots, v_n)$:

$$a = f(v)$$

This outcome is called social choice for that profile.

When considering a mechanism with money, that is a mechanism where there are money transfers between the mechanism and players, a payment function computes money transfers for every players.

Definition 3 (Direct Revelation Mechanism) *A direct⁴ revelation mechanism \mathcal{M} is composed of:*

$$\mathcal{M} = \langle f, p_1, \dots, p_n \rangle$$

where f is the social choice function with A as the possible outcomes and p_1, \dots, p_n are the payment vectors where $p_i : V_1 \times \cdots \times \dots \times V_n \rightarrow \mathbb{R}$ is the amount that user i pays to the mechanism.

Definition 4 (Utility Function) *Considering a mechanism $\mathcal{M} = \langle f, p_1, \dots, p_n \rangle$, the valuation set $v = (v_1, \dots, v_n)$ and the alternative chosen $a = f(v_1, \dots, v_n)$, the utility for every user i is:*

$$u_i(a) = v_i(a) - p_i(v_i(a), v_{-i}(a)) \quad (1)$$

where v_{-i} is the $(n-1)$ -dimensional vector in which the i 'th coordinate is removed.

The most famous direct revelation mechanism is the Vickrey-Clarke-Groves (VCG) Mechanism [5].

Definition 5 (VCG Mechanism) *A VCG mechanism determines $f(v)$:*

$$f(v) \in \operatorname{argmax}_{b \in A} \sum_{j=1}^n v_j(b) \quad (2)$$

and $p_i(v)$ such that:

$$p_i(v) = h_i(v_{-i}) - \sum_{j \neq i}^n v_j(f(v)) \quad (3)$$

for some function h_1, \dots, h_n where $h_i : V_{-i} \rightarrow \mathbb{R}$.

⁴ There exists also the indirect revelation mechanism with money, that differs for the fact that players have private information (player's preferences) and select strategies according to this information set. In this work, we do not consider indirect revelation mechanism because we assume that the users' valuations functions are known.

Now, there are several versions of the model according to the choice of $h_i(v_{-i})$. Important to note that the function h_i can be any arbitrary function but it must not depend on the v_i . One of the most important versions is the VCG mechanism with the Clarke pivot rule, introduced by Clarke [2].

Definition 6 (VCG Mechanism with Clarke Pivot Rule) *A VCG mechanism with Clarke pivot payments determines $h_i(v_i)$:*

$$h_i(v_{-i}) = \max_a \sum_{j \neq i}^n v_j(a) \quad (4)$$

so $p_i(v)$ becomes:

$$p_i(v) = \max_b \sum_{j \neq i}^n v_j(b) - \sum_{j \neq i}^n v_j(f(v)) \quad (5)$$

where b is the selected alternative if the i -th player is not present in the system⁵.

By choosing this kind of $h_i(v_{-i})$ Clarke wants to let the buyer paying only the influence that he has in the system. In fact, an user influences the system when the outcome changes depending on the absence or presence of player i . If the outcome changes significantly when player i is removed, it means that player i strongly affects the system, so he has to pay for his influence (from this concept derives the name "Clarke pivot rule").

2 Problem Formulation and System Model

2.1 Problem Description

In our work, we model an energy allocation problem through a VCG mechanism approach. The main aim is to avoid blackouts when the users' requested energy exceeds the available energy and the energy network must be switched off due to overload. Fig. 1 describes a possible case for one-day time period with trends for the energy functions. The dashed lightblue line represents the distributor's available energy, the continuous darkblue line the energy requested from all consumers. The lines on the bottom represent the single consumption of every user (five users in this case). So in Fig. 1b, we can consider only the consumers "players" and the energy available function as a parameter for the mechanism. The produced energy is a constraint to take into account while maximizing the social welfare. Fig. 1a describes an example of a situation in which energy availability function (dashed lightblue line) and aggregate users' consumption (continuous darkblue line) are compared. Where the set of players contains the distributor

⁵ Here, " a " is the optimal assignment including player " i ", while " b " is the optimal assignment when we exclude player " i "

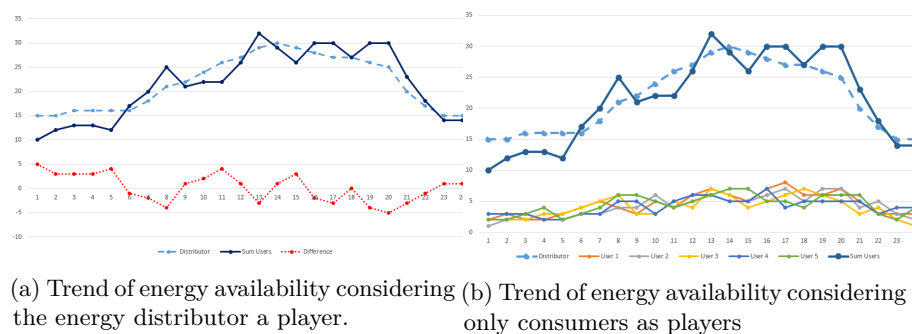


Fig. 1: Comparison between energy functions: users' desired trends, the aggregated desired trend of all of agent and the produced energy function.

and the difference between this two functions (production - consumption, the dotted red line in the graph) must be greater or equal than zero. In fact, another way to tackle this problem is that we can consider that the difference between the production and the aggregate consumption, must be greater or equal than zero for this purpose we can consider the distributor as a player. The main idea is to deploy a mechanism in order to drive users in shifting energy consumptions, according to the produced energy and the consumption preference of the community.

2.2 Model Description

Our objective is to determine a mechanism that will select the optimal energy according to the desired consumption of every player. We were initially inspired by the "Public Project" example provided in [5]. To better understand the idea, we could sum up our problem by drawing connections with the classical knapsack optimization problem. As first solution, we decide to put all into the knapsack only if the volume of the knapsack is sufficient, otherwise we do not put in anything. In the energy case, the distributor will provide energy only if the consumption is not greater than the available energy. For this reason, we consider a scenario with an energy distributor and n energy users ($n + 1$ players). The distributor has an amount of available energy and each consumer has a desired consumption, that is represented by a negative value due to the design of the social choice function of the VCG mechanism showed in Eq. 2. This mechanism has the positive aspect that it avoids blackout situations.

As a further solution, we assume that we have a set of object to put into the knapsack minimizing the empty space. We model the scenario removing the distributor from the set of players and considering the available energy a threshold resulting that the energy is provided only to a subset of users with aim to avoiding the waste of produced energy. Moreover, each consumer has a positive desired consumption avoiding a negative net utility introduced in Eq. 1. This

second model with respect to the first one allows to provide energy even if the aggregated consumption is greater than the production by selecting a subset of users. Regarding the next solution, we assume that we have a liquid instead of a set of objects, so we are able to fill the entire knapsack. So, we introduce a third solution that assigns to users all the available energy till reaching the total amount of produced energy. It means that for each user the mechanism calculates a portion of available energy not greater than his desired consumption. Here as well, the mechanism selects the consumption for a subset of users, however, unlike the second case, there is no wasted energy. The next improved mechanism is similar to the third solution and in addition we introduce the time variable. So, every energy function become a power function over time and the mechanism chooses the energy to be provided according to every user's preferences, allowing the shifting of the consumption in the time interval considering that each user must receive at least his aggregated requested energy.

Our final approach is based on the previous one in which we assign a part of available power thanks to a proportional allocation scheme that provides a positive amount of power to every user.

A final remark is that this model is essentially a game so players must be motivated to play by getting a positive utility. But, in our case study, energy allocation, a user usually has to consume and, consequently, play the game for this reason he can accept also an utility equal to zero.

In this work, we propose several configurations of the mechanism, starting from the simplest to a more complicated configuration which has the property of assigning the available energy to users according to their desired energy minimizing the energy wasting while maximizing the aggregate utility of all users.

A development is to find a different payment scheme that takes into account the actual consumption and energy consumption peaks. The final aim is to stimulate users to behave in a good energy way offering a discount on the electricity bill that will lead to get a positive net utility.

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