Spinoza's Ontology

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Abstract. We examine the possibility of applying knowledge representation and automated reasoning in the context of philosophical ontology. For this purpose, we use the axioms and propositions in the first book of Spinoza's *Ethics* as knowledge base and a tableau-based satisfiability tester as reasoner. We are able to reconstruct most of Spinoza's system with formal logic, but this requires additional axioms which are assumed implicitly by Spinoza. This study illustrates how tools developed in computer science can be of practical use for philosophy.

1 Introduction

In this paper, we derive an ontology (knowledge base) from the ontology (metaphysics) of a philosopher. The problem of such an endeavour obviously lies in the fact that not only the word "ontology", but also most of the terminology related to reasoning is used differently in philosophy and knowledge representation. However, there are cases in which philosophers used the same formal strictness as in mathematics (or computer science): e.g., Baruch de Spinoza claims to have proven the propositions in his work *Ethics* [2] "in the geometric manner", i.e. with the same strictly logical approach as Euclid used in the *Elements*, his fundamental work on geometry and mathematics in general.

As expected, the translation of Spinoza's system into formal logic was not straightforward: in addition to the problems resulting from ambiguous formulation, it turned out that most of the theorems could not be proved with the literal translation of the axioms alone. In these cases, the examination of the proofs often revealed additional assumptions which Spinoza implicitly made and apparently considered as trivial. Since our intention was not to demonstrate the shortcomings of his work, but to reconstruct his ontology as adequately as feasible, we added these implicit axioms wherever possible to our knowledge base. Thus, the question examined in this paper is "How can the axioms be interpreted in such a way that the proofs hold" rather than "Which of the proofs hold with the intuitive interpretation of the axioms". We also do not attempt to reflect the historical discussion about the Ethics or the terms used (e.g. "substance") or discuss the legitimacy of the axioms. As we cannot examine all 36 propositions of the first book in detail, we restrict ourselves to the first 15 ones since they cover the essential statements of the first book [6] and climax in the principal statement that everything exists in and is conceivable through god.

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2 Translation of axioms and definitions

The structure of the Ethics is very clear: after sequences of definitions and axioms, a sequence of propositions together with proofs follows. Each of these is assigned a number, and thus we refer to a definition, axiom or proposition by the letters D, A, or P, respectively, together with its number.

Since our aim is to *effectively* decide the validity of Spinoza's proofs, we cannot use full first order predicate logic (FO), since it is undecidable. Instead, we use a decidable fragment of FO, namely the *Guarded Fragment* ($\mathcal{GF}$) [1, 4] which restricts the appearance of quantifiers to formulas of the kind

$$\forall \boldsymbol{x}(G(\boldsymbol{x},\boldsymbol{y}) \rightarrow \varphi(\boldsymbol{x},\boldsymbol{y})) \quad \text{or} \quad \exists \boldsymbol{x}(G(\boldsymbol{x},\boldsymbol{y}) \wedge \varphi(\boldsymbol{x},\boldsymbol{y})),$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ are tuples of variables; $G(\boldsymbol{x},\boldsymbol{y})$, called the *guard* of the formula, is an atom with variables $\boldsymbol{x}$ and $\boldsymbol{y}$; and $\varphi(\boldsymbol{x},\boldsymbol{y})$, called the *body* of the formula, is a guarded formula whose free variables are (a subset of) $\boldsymbol{x}$ and $\boldsymbol{y}$. Satisfiability of $\mathcal{GF}$ formulas is decidable in 2-ExpTime [3]. $\mathcal{GF}$ is quite expressive: for example, it allows for relations of arity greater than two, and it can express general axioms of the kind $\forall x\, \varphi(x)$ for a formula φ with one free variable x in the following way: $\forall x((x = x) \rightarrow \varphi(x))$.

For $\mathcal{GF}$, there exists the tableau-based satisfiability tester Saga [5]. A satisfiability tester can be used to check the validity of an implication $\varphi \Rightarrow \psi$ in the following way: if the formula $\varphi \wedge \neg\psi$ is satisfiable, then the implication is not valid. Through this procedure, no formal proof is obtained for a valid implication, but in the opposite case a counter-model is found, which can be used to examine the reason for the failure of the implication. Moreover, since this algorithm is a decision procedure, we can be sure that if no model is found by the algorithm, then there exists none and therefore the implication is valid. One of the main practical advantages of tableau algorithms is their performance, which is much better in practice than their worst-case complexity suggests.

Although Spinoza's style bears a high similarity with mathematical language, his statements require interpretation before they can be translated into formal logic. We will illustrate this with two examples. Firstly, there is no clear distinction between universal and existential quantification: by D3, "a substance is conceivable through itself". If this is interpreted as existential quantification, it reads: "Substance is *at least* conceivable through itself (but possibly also through something different)." With universal quantification, it means: "Substance is *only* conceivable through itself (but possibly not conceivable at all)." In this case and in several similar ones, we think that only both statements together reflect Spinoza's intention, since this eliminates the possibilities in the parentheses. Secondly, in the Latin text there are two words which translate into English as "or": "vel" (e.g. A1, P4) stands for a logical alternative, and therefore has to be translated into formal logic as "$\vee$". In contrast, "sive" (e.g. D1, D4, A5) means rather "or also", i.e. it is followed by an alternative definition. Thus logically, this "or" has to be translated into "$\wedge$".

In order to express the axioms and definitions, we use nine unary, eight binary and two ternary relations, which are explained in Table 1. In the following, we present our translation of the definitions and axioms. Starting with (the English translation of) Spinoza's original formulation, we present our understanding first in ordinary language, then in formal logic.

Table 1. Relations

Unary Relations	Meaning	Bi-/ternary Relations	Meaning
$\mathsf{S}(x)$	x is a substance	$\mathsf{sn}(x, y)$	x has the same nature as y
$\mathsf{A}(x)$	x is an attribute	$\mathsf{ao}(x, y)$	x is an attribute of y
$\mathsf{M}(x)$	x is a mode	$\mathsf{mo}(x, y)$	x is a mode of y
$\mathsf{E}(x)$	x is existing	$\mathsf{ei}(x, y)$	x exists in y
$\mathsf{C}(x)$	x is conceivable	$\mathsf{ct}(x, y)$	x is conceivable through y
$\mathsf{G}(x)$	x is (a) god	$\mathsf{sc}(x, y)$	x has something in common with y
$\mathsf{F}(x)$	x is finite in its kind	$\mathsf{lb}(x, y)$	x can be limited by y
$\mathsf{CA}(x)$	x is a cause	$\mathsf{co}(x, y)$	x is cause of y
$\mathsf{EF}(x)$	x is an effect	$\mathsf{ca}(x, y, z)$	x and y have the common attribute z
		$\mathsf{di}(x, y, z)$	x can be divided into y and z

D1 *By that which is "self-caused" I mean that of which the essence involves existence, or that of which the nature is only conceivable as existent.* This is rather an axiom than a definition, since "self-caused" is a complex term: something has itself and only itself as cause if and only if its existence follows from its conceivability. Note that this implies that everything which exists and everything which is not conceivable is self-caused.

$$\forall x((x = x) \rightarrow ((\mathsf{co}(x, x) \wedge (\forall y(\mathsf{co}(y, x) \rightarrow (x = y)))) \leftrightarrow (\mathsf{C}(x) \rightarrow (\mathsf{E}(x)))))$$

D2 *A thing is called "finite in*[1] *its kind" when it can be limited by another thing of the same nature [...]* A thing is finite in its kind if and only if there exists a different thing with the same nature by which it can be limited.

$$\forall x((x = x) \rightarrow (\mathsf{F}(x) \leftrightarrow \exists y(\mathsf{lb}(x, y) \wedge \mathsf{sn}(x, y) \wedge (x \neq y))))$$

D3 *By "substance" I mean that which is in itself, and is conceived through itself [...]* A substance exists in itself (and only in itself), and it is conceivable through itself (and only through itself).

$$\forall x((x = x) \rightarrow (\mathsf{S}(x) \leftrightarrow \mathsf{ei}(x, x) \wedge \mathsf{ct}(x, x) \wedge \forall y(\mathsf{ei}(x, y) \rightarrow (x = y)) \wedge \forall y(\mathsf{ct}(x, y) \rightarrow (x = y))))$$

D4 *By "attribute" I mean that which the intellect perceives as constituting the essence of substance.* An attribute is conceivable, and it is an attribute of a substance (and only of substances), which are also conceivable. Moreover, every substance has an attribute.

$$\forall x(\mathsf{S}(x) \rightarrow \exists y(\mathsf{ao}(y, x)))$$

$$\forall x((x = x) \rightarrow (\mathsf{A}(x) \leftrightarrow \mathsf{C}(x) \wedge \exists y(\mathsf{ao}(x, y)) \wedge \forall y(\mathsf{ao}(x, y) \rightarrow (\mathsf{S}(y) \wedge \mathsf{C}(y)))))$$

D5 *By "mode" I mean the modifications ("affectiones") of substance, or that which exists in, and is conceived through, something other than itself.* A mode exists (only) in and is (only) conceivable through a substance.

$$\forall x((x = x) \rightarrow (\mathsf{M}(x) \leftrightarrow \mathsf{C}(x) \wedge \exists y(\mathsf{mo}(x, y)) \wedge \forall y(\mathsf{mo}(x, y) \rightarrow (\mathsf{ei}(x, y) \wedge \mathsf{ct}(x, y) \wedge \mathsf{S}(y)))))$$

D6 *By "God" I mean a being absolutely infinite—that is, a substance consisting in infinite attributes, of which each expresses eternal and infinite essentiality.* If a god is absolutely infinite, he must also be infinite in his kind. We omit the predicate "eternal", since it is not used in any other proposition considered in this paper. Regarding the meaning of "infinite attributes", see our explanation for I1 below.

$$\forall x((x = x) \rightarrow (\mathsf{G}(x) \leftrightarrow (\mathsf{S}(x) \wedge \neg\mathsf{F}(x))))$$

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[1] We think that the literal translation "in" reflects Spinoza's intention better than "after".

A1 *Everything which exists, exists either in itself or in something else.* Everything existing exists in something, which leaves open the possibility that it is something else or the same. $\forall x(\mathsf{E}(x) \rightarrow \exists y(\mathsf{ei}(x,y)))$

A2 *That which cannot be conceived through anything else must be conceived through itself.* Everything conceivable is conceivable through something (possibly itself) which, as is suggested by D3 and D5, must be a substance.

$$\forall x(\mathsf{C}(x) \rightarrow \exists y(\mathsf{ct}(x,y) \wedge \mathsf{S}(y)))$$

A3 *From a given definite cause an effect necessarily follows; and, on the other hand, if no definite cause be granted, it is impossible that an effect can follow.* Every cause has an effect, and vice versa.

$\forall x(\mathsf{CA}(x) \rightarrow \exists y(\mathsf{co}(x,y) \wedge \mathsf{EF}(y)))$ $\forall x(\mathsf{EF}(x) \rightarrow \exists y(\mathsf{co}(y,x) \wedge \mathsf{CA}(y)))$

A4 *The knowledge of an effect depends on and involves the knowledge of a cause.* Since "knowledge" does not appear in any other axiom or definition and is used synonymously with "conception" in the corollary of P6, we treat the two terms as synonymous. Thus, something is a conceivable effect if and only if it has a cause and is conceivable through its causes.

$$\forall x((x=x) \rightarrow ((\mathsf{EF}(x) \wedge \mathsf{C}(x)) \leftrightarrow (\exists y(\mathsf{co}(y,x)) \wedge \forall y(\mathsf{co}(y,x) \rightarrow (\mathsf{ct}(x,y))))))$$

A5 *Things which have nothing in common cannot be understood, the one by means of the other; the conception of one does not involve the conception of the other.* Expressed positively, this means: if x is conceivable through y, then x and y have something in common. $\forall x,y(\mathsf{ct}(x,y) \rightarrow \mathsf{sc}(x,y))$

We omit D7, D8 and A6 since they are not used for the propositions considered in this paper, and A7 because it is merely a repetition of D1.

As mentioned in Section 1, we added implicit axioms, i.e. axioms which follow immediately from the semantics of the words used (e.g. if x is the cause of y, then x is a cause and y is an effect), or assumptions which are only revealed in the proofs (e.g. in the proof of P4, it is argued that substances and modes are the only existing things). In the following list, we enumerate and justify these additional axioms.

I1 In D6, god is described as a substance having "infinite attributes". In the proof of P14, it becomes clear that not the attributes themselves are infinite, but their number, or more precisely that god has all attributes: "god is a being [...], of whom no attribute [...] can be denied (by D6)". Thus, every attribute is an attribute of god. Moreover, since every substance has an attribute, it has a common attribute with god.[2]

$\forall x(\mathsf{A}(x) \rightarrow \exists y(\mathsf{ao}(x,y) \wedge \mathsf{G}(y)))$ $\forall x(\mathsf{S}(x) \rightarrow \exists y,z(\mathsf{ca}(x,y,z) \wedge \mathsf{G}(y)))$

I2 In the proof of P4, Spinoza states that "outside[3] of the intellect, there is nothing except substance and its modifications." Hence, everything existing is either substance or mode. $\forall x(\mathsf{E}(x) \rightarrow (\mathsf{S}(x) \vee \mathsf{M}(x)))$

I3 In the proof of P2, Spinoza (implicitly) assumes that if two things are not conceivable through each other, then they have nothing in common, i.e. that the implication of A5 also holds in the opposite direction. Therefore, we also add

[2] We have to state this explicitly since we cannot fully express P5 in $\mathcal{GF}$, see our explanation for P5 below.

[3] We think that translating "extra" literally as "outside" instead of "in addition to" makes more sense in this context, since then "extra intellectum" simply means "in nature", whereas otherwise its meaning becomes unclear (see [7], p. 14).

the axiom that if two things have something in common, then one is conceivable through the other. $\forall x, y(\mathsf{sc}(x,y) \rightarrow (\mathsf{ct}(x,y) \vee \mathsf{ct}(y,x)))$

I4 In P5, Spinoza mentions "substances having the same nature or ('sive') attribute", which indicates that the term "same nature" in D2 has to be interpreted as "same attribute". For this purpose, we introduce the relation $\mathsf{ca}(x,y,z)$, with the semantics that both x and y have attribute z, and they have something in common (namely z). $\forall x, y(\mathsf{sn}(x,y) \rightarrow \exists z(\mathsf{ca}(x,y,z)))$

$$\forall x, y, z(\mathsf{ca}(x,y,z) \rightarrow (\mathsf{ao}(z,x) \wedge \mathsf{ao}(z,y) \wedge \mathsf{co}(x,y)))$$

I5 The relation sc is symmetric and reflexive.

$\forall x, y(\mathsf{sc}(x,y) \rightarrow \mathsf{sc}(y,x))$ $\qquad \forall x((x = x) \rightarrow \mathsf{sc}(x,x))$

I6 If x is conceivable through (or exists in) y, then both x and y are conceivable (or existing, respectively). If x is the cause of y, then x is a cause and y is an effect. Analogous axioms hold for the relations ao and mo.

$\forall(x,y)(\mathsf{ct}(x,y) \rightarrow (\mathsf{C}(x) \wedge \mathsf{C}(y)))$ $\qquad \forall(x,y)(\mathsf{ao}(x,y) \rightarrow (\mathsf{A}(x) \wedge \mathsf{S}(y)))$

$\forall(x,y)(\mathsf{ei}(x,y) \rightarrow (\mathsf{E}(x) \wedge \mathsf{E}(y)))$ $\qquad \forall(x,y)(\mathsf{mo}(x,y) \rightarrow (\mathsf{M}(x) \wedge \mathsf{S}(y)))$

$\forall(x,y)(\mathsf{co}(x,y) \rightarrow (\mathsf{CA}(x) \wedge \mathsf{EF}(y)))$

I7 The sets of substances, attributes and modes are pairwise disjoint.

$\forall x(\mathsf{S}(x) \rightarrow \neg\mathsf{A}(x))$ $\qquad \forall x(\mathsf{S}(x) \rightarrow \neg\mathsf{M}(x))$ $\qquad \forall x(\mathsf{A}(x) \rightarrow \neg\mathsf{M}(x))$

I8 We express the relation "x is divisible into y and z" as used in P12 and P13 as "x is cause of two different things y and z, but not equal to either."

$$\forall x, y, z(\mathsf{di}(x,y,z) \rightarrow (\mathsf{co}(x,y) \wedge \mathsf{co}(x,z) \wedge (x \neq y) \wedge (x \neq z) \wedge (y \neq z)))$$

3 Translation and proof of propositions

In this section, we consider the propositions in detail. As in Section 2, we first give the original formulation by Spinoza, followed by our interpretation and the translation into $\mathcal{GF}$. In order to improve readability, we use the notation $\alpha \rightarrow \beta$ for $\neg\alpha \vee \beta$ (material implication), and $\varphi \Rightarrow \psi$ for the (logical) implication claimed in the proposition, though the translation of both into $\mathcal{GF}$ is identical. Consequently, the satisfiability of $\varphi \wedge \neg\psi$ was tested by Saga.

P1 *Substance is by nature prior to its modifications.* The term "prior" is not formally defined in the axioms and definitions. If we interpret it logically and not temporally (which is suggested by the proof, since time is not mentioned in D3 and D5), we obtain the following: if there exist a mode, then there also exists a substance. This proposition really follows from D5, and thus Saga cannot find a counter-model. $\mathsf{M}(x) \Rightarrow \exists y(\mathsf{S}(y))$

P2 *Two substances, whose attributes are different, have nothing in common.* If z is an attribute of the substance x, but not of the substance y, then x and y have nothing in common. Here, Spinoza only proves that the conception of x does not involve the conception of y. Without I3, Saga finds a counter-model with two substances which have something in common, but are not conceivable through each other. If we add I3, this proposition follows from the fact that in order to have something in common, one of the substances has to be conceivable through the other one, but every substance is only conceivable through itself.

$$(\mathsf{S}(x) \wedge \mathsf{S}(y) \wedge \mathsf{A}(z) \wedge \mathsf{ao}(z,x) \wedge \neg\mathsf{ao}(z,y)) \Rightarrow \neg\mathsf{sc}(x,y)$$

P3 *Things which have nothing in common cannot be one the cause of the other.* Spinoza's proof relies on A4, but this axiom is only applicable if conceivability of the effect is assumed. However, if the effect is not conceivable, it is (by

D1) self-caused, and thus (by I5) has something in common with its cause, so the proposition also holds in this case. $\neg\mathsf{sc}(x, y) \Rightarrow \neg\mathsf{co}(x, y)$

P4 *Two or more distinct things are distinguished one from the other, either by the difference of the attributes of the substances, or by the difference of their modifications.* Two things x, y having a common attribute z and a common mode w are identical. This proposition holds; the explanation is similar to the one for P5 below. $(\mathsf{ca}(x, y, z) \wedge \mathsf{mo}(w, x) \wedge \mathsf{mo}(w, y)) \Rightarrow (x = y)$

P5 *There cannot exist in the universe two or more substances having the same nature or attribute.* The intuitive translation of this proposition into FO is: $\forall x, y, z((\mathsf{S}(x) \wedge \mathsf{S}(y) \wedge \mathsf{A}(z) \wedge \mathsf{ao}(z, x) \wedge \mathsf{ao}(z, y)) \Rightarrow (x = y))$, which is not guarded, since x and y appear together in the body, but not in the guard. In order to express this in $\mathcal{GF}$, we introduce the relation ca and the axiom I4. Then, we can express the proposition as follows: two substances with a common attribute are identical. This results from P2 and therefore also depends on I3: without this additional axiom, SAGA finds a counter-model with two finite substances having a common attribute. $(\mathsf{S}(x) \wedge \mathsf{S}(y) \wedge \mathsf{ca}(x, y, z)) \Rightarrow (x = y)$

P6 *One substance cannot be produced by another substance.* The proof only shows that one substance cannot be the cause of another one, which suggests that the terms "cause of" and "produced by" are used synonymously, and thus we can express this as follows: if two substances are different, then one cannot be the cause of the other; this follows from A4. $(\mathsf{S}(x) \wedge \mathsf{S}(y) \wedge (x \neq y)) \Rightarrow \neg\mathsf{co}(x, y)$

P7 *Existence belongs to the nature of substances.* Every substance is existing. This follows directly from D3 and I6. $\mathsf{S}(x) \Rightarrow \mathsf{E}(x)$

P8 *Every substance is necessarily infinite.* This indeed follows from D2 and P5, which state that a thing can only be limited by another thing of the same nature, but substances with the same nature are identical. $\mathsf{S}(x) \Rightarrow \neg\mathsf{F}(x)$

P9 *The more reality or being a thing has, the greater the number of its attributes.* We do not see a possibility of expressing this in $\mathcal{GF}$ or even in FO. Since this proposition is not used in any other proof in the first book of the ethics, it probably serves merely as a justification for a substance to have several attributes. Hence, we do not consider it any further.

P10 *Each particular attribute of the one substance must be conceived through itself.* Spinoza's proof shows only that the attribute of a substance must be conceived through this substance. Still, no counter-model can be found: by I5, everything has something in common with itself, and by I3, this implies that it is conceivable through itself. $\mathsf{A}(x) \Rightarrow \mathsf{ct}(x, x)$

P11 *God [...] necessarily exists.* There can be no model in which there is no individual with the predicate G. To avoid an empty structure[4], we add an individual x with the predicate $C(x)$, so that our proposition reads "If something is conceivable, then there exists a god", which also reflects the structure of the ontological argument used by Spinoza.[5] $C(x) \Rightarrow \exists y(\mathsf{G}(y))$

P12 *No attribute of substance can be conceived from which it would follow that substance can be divided.* The main problem of this and the next proposition is the interpretation of "division". The reference to P6 indicates that if a

[4] In an empty structure, even a formula like $\forall x(C(x) \wedge \neg C(x))$ is satisfiable.

[5] Note that the naive translation $\mathsf{G}(x) \Rightarrow \mathsf{E}(x)$ follows directly from the definition of substance, and it does not show that god *necessarily* exists.

substance x is divided into y and z, then it must be the cause of y and z, but not equal to either. Then, neither y nor z can be a substance (by P6), nor can both be something different and thus x cease to exist (P7).

$$\mathsf{A}(w) \wedge \mathsf{C}(w) \wedge \mathsf{S}(x) \wedge \mathsf{ao}(w,x) \wedge \mathsf{di}(x,y,z) \Rightarrow \neg\mathsf{S}(y) \wedge \neg\mathsf{S}(z) \wedge \mathsf{E}(x)$$

P13 *Substance absolutely infinite is indivisible.* The absolutely infinite substance is god. Beyond this, P13 is merely a repetition of P12 without the attribute w.

$$\mathsf{G}(x) \wedge \mathsf{di}(x,y,z) \Rightarrow \neg\mathsf{S}(y) \wedge \neg\mathsf{S}(z) \wedge \mathsf{E}(x)$$

P14 *Besides God no substance can be granted or conceived.* The argument relies on the assumptions that god has all attributes (D6, I1), that two substances x and y which have something (e.g. an attribute) in common are conceivable through each other (I3), and that every substance is only conceivable through itself (D3), such that x and y must be identical. Again, without I3 there exists a counter-model with a finite substance that has a common attribute with god, but is not conceivable through him.

$$\mathsf{S}(x) \wedge (\mathsf{E}(x) \vee \mathsf{C}(x)) \Rightarrow \mathsf{G}(x)$$

P15 *Whatsoever is, is in God, and without God nothing can be, or be conceived.* This follows quite easily from P14, since everything existing (i.e. substances and modes) exists in and is conceivable through a substance, and god is the only substance.

$$(\mathsf{E}(x) \Rightarrow \exists y(\mathsf{ei}(x,y) \wedge \mathsf{G}(y))) \wedge (\mathsf{C}(x) \Rightarrow \exists y(\mathsf{ct}(x,y) \wedge \mathsf{G}(y)))$$

Summary. Out of the 15 propositions under consideration, 12 can be proved using Spinoza's axioms and definitions and the implicit axioms mentioned above. P11 can be proved with some appropriate pre-condition. P5 cannot be expressed fully in $\mathcal{GF}$, but it can be reformulated in a way that renders it provable. Only one proposition remains: the meaning of P9 is very vague and cannot be expressed in $\mathcal{GF}$. The implicit axiom I3 ("if two things have something in common, then one of them is conceivable through the other") is crucial for the high number of provable propositions. Although it is not explicitly formulated by Spinoza and its legitimacy is questionable from a philosophical point of view, it is clearly assumed by Spinoza in the proof of P2, and without it, counter-models for the propositions 2, 4, 5, 8, 10, 14 and 15 can be found. This dependency is also reflected in Spinoza's proofs: P14 relies on P5, which in turn relies on P2.

In Figure 1, the subsumption hierarchy of the unary relations is shown: an arrow $A \rightarrow B$ indicates that the relation A is *subsumed* by B, i.e. that every instance of A is an instance of B. To improve readability, we show only direct subsumption; note that the subsumption relation is transitive and thus $A \rightarrow B \rightarrow C$ implies $A \rightarrow C$. Here, the symbol $\perp$ stands for an inconsistent relation, which is subsumed by all other ones, and $\top$ is the universal relation, which subsumes all other ones. The trivial arrows from $\perp$ and to $\top$ are shown as dashed. If two relations mutually subsume each other, they are displayed as one.

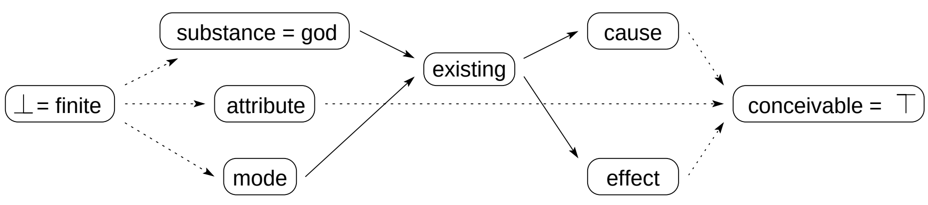

Fig. 1. Subsumption hierarchy

This hierarchy reveals additional propositions which follow from the set of axioms but are not mentioned or proved by Spinoza. Firstly, everything is conceivable, since everything has something in common with itself (I5), and is therefore conceivable through itself (I3). This result again demonstrates the problematic implications of I3. Secondly, 'finite' is inconsistent. It has been shown that substances are infinite (P8), but why cannot something else, e.g. a mode, be finite? The reason for this is D2, which defines 'finite' as something which can be limited by some other thing of the same nature. By I4, 'nature' means 'attribute', but only substances have attributes. Thus, anything other than a substance cannot be finite either. Thirdly, everything existing is cause as well as effect. This results from D1, since everything existing is self-caused. All other subsumptions are mentioned more or less explicitly by Spinoza. This is an indicator that our system is not over-constrained, i.e. through our translation and the additional axioms we did not introduce any conclusions which do not follow from Spinoza's system.

4 Conclusion

We developed an ontology based on the arguments made in the first part of Spinoza's *Ethics*. While testing the validity of the proofs provided, it turned out that most of them require additional axioms, which are only revealed inside the proofs. In particular, the main theorem depends crucially on an implicit assumption which is not justified anywhere. The counter-models provided by SAGA proved to be very helpful in the detection of these missing prerequisites. With the additional axioms, we are able to prove most of the propositions. The restrictions imposed by the language $\mathcal{GF}$ cause problems only in one case, and a workaround is possible. In addition to the propositions claimed by Spinoza, we are able to derive additional theorems following from the axioms. In summary, we think that this illustrates the use of automated reasoning tools for philosophy.

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