On the decidability of the $\exists^*\forall^*$ prefix class in Set Theory

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Abstract. In this talk I will describe the set-theoretic version of the Classical Decision Problem for First Order Logic. I will then illustrate the result on the decidability of the satisfiability problem class of purely universal formulae ($\exists^*\forall^*$-sentences) on the unquantified language whose relational symbols are membership and equality. The class we studied is, in the classical (first order) case, the so-called Bernays-Schoenfinkel-Ramsey (BSR) class. The set-theoretic decision problem calls for the existence of an algorithm that, given a purely universal formula in membership and equality, establishes whether there exist sets that substituted for the free variables will satisfy the formula. The sets to be used are pure sets, namely sets whose only possible elements are themselves sets. Much of the difficulties in solving the decision problem for the BSR class in Set Theory came from the ability to express infinity in it, a property not shared by the classical BSR class. The result makes use of a set-theoretic version of the argument Ramsey used to characterize the spectrum of the BSR class in the classical case. This characterization was the result that motivated Ramsey celebrated combinatorial theorem.