Ref.TH.3138-CERN

MASSIVE NEUTRINOS ?

A. De Rújula
CERN - Geneva

Various aspects of the subject of neutrino masses and their implications are outlined: Dirac and Majorana masses, suggestions from unified gauge theories, limits and hints from cosmology, galactic neutrinos and their detectability, experiments involving weak decays. The main voluntary lacunae are oscillations and double beta decay.

A talk given at the
"IX International Conference on High Energy Physics
and Nuclear Structure"
Versailles, July 1981

Ref.TH.3138-CERN
14 August 1981

1. Introduction

Should we know what neutrino masses are, and should we understand why they are what they are, this talk would presumably last only for a few minutes. This being very far from the case, one can talk forever. The subject of neutrino masses is vast and rapidly developing. It involves cosmogenesis, cosmology, astrophysics, ultra-violet and radio astronomy, nucleosynthesis, star theory, cosmic ray physics, geology, grand unification, the general mystery of the origin of masses, atomic and nuclear physics, chemistry and a long etc. My intention is to rush over the general arguments involved in a subset of these subjects, in an attempt to present a necessarily vague global picture.

2. Dirac and Majorana masses

Two kinds of electrically neutral bosons exist: those that, like the photon and the π_0, are their own antiparticle; and those that, like the neutral kaon, are not. An (approximately) conserved additive quantum number is associated only with non-self-conjugate bosons: strangeness in the case of the kaon. Self-conjugate bosons can be described by a real field, i.e., $\pi_0(x)$, and their mass is described by a term $m^2\pi_0^2$ in the Lagrangian. This term, involving a real field, is not invariant under the transformation $\pi_0 \rightarrow e^{i\alpha}\pi_0$ that would, via Madame Noether's theorem, correspond to a conserved "pion number". A particle-antiparticle pair of neutral bosons can be described by a complex field, i.e., $K_0(x)$, and their common mass is described by $m^2K_0^*K_0$ at the Lagrangian level. Here, Noether's theorem is applicable.

Could the above-mentioned scenario be repeated in the case of neutral fermions ? The answer is yes. Neutrinos could either be Dirac particles (analogous to kaons) or Majorana[1] particles (analogous to pions).

A Dirac field ν is different from the corresponding charge conjugate field ν_c and describes four objects: left- and right-handed neutrinos and antineutrinos. The mass term $m\bar{\nu}\nu$ for a Dirac particle is invariant under $\nu \rightarrow e^{i\alpha}\nu$ and a conserved lepton number can be defined. Neutrino-less double β-decay, a process in which two neutrons would exchange a neutrino to become two proton-electron pairs, is forbidden. This is in analogy to the process in which two neutrons would exchange a K^0 to become two (pK^-) pairs, a $\Delta S = 2$ process forbidden at the strong interaction level.

A Majorana neutrino field $\mathcal{N}$ is by definition identical to its own charge conjugate field: $\mathcal{N} = \mathcal{N}_c$. This Majorana condition reduces the number of objects described by the Dirac equation from four to two: the two helicity states of a single self-charge-conjugate fermion. This is most transparent and analogous to the boson case in the Majorana representations of the γ matrices, in which $(i\gamma_\mu)^* = i\gamma_\mu$. In these representations the Dirac operator $(i\gamma_\mu\partial/\partial x_\mu - m)$ is real and the Dirac equation admits real solutions, for which the Majorana condition is simply a reality condition $\mathcal{N} = \mathcal{N}^* \equiv \mathcal{N}_c$. The Majorana condition is incompatible with the transformations $\mathcal{N} \rightarrow e^{i\alpha}\mathcal{N}$ that would serve to define a conserved lepton number L. A Majorana mass term: $m\mathcal{N}^2$, is not invariant under such transformations and carries $\Delta L = 2$. The process wherein two neutrons exchange a π^0 to become two $(p\pi^-)$ pairs is allowed. Similarly, the neutrino-less double β decay process, wherein two neutrons exchange a Majorana neutrino to become two (pe^-) pairs is allowed. Since 1957, this statement must be slightly elaborated upon. Suppose that, as indicated by experiment and advocated by theory, the neutrino that couples to e^- is purely left-handed and the one that couples to e^+ is right-handed. (The prefix "anti", meaningless to Majoranas, has been quietly dropped.) The neutrino-less double β decay process is then possible only inasmuch as the neutrino may flip its handedness in flight. The handedness flip operator is the mass term in the Lagrangian. Thus, in a theory with maximal parity violation, the zero $\nu\ \beta\beta$ amplitude is proportional to the Majorana neutrino mass.

There is an old, instructive, elegant, convincing and false argument that neutrinos *must* be massless. Here it goes.

(i) The charged and neutral currents of neutrinos only involve right-handed antineutrinos and left-handed neutrinos.

(ii) A massive fermion has two helicity (or handedness) states. Should it be left-handed, it is always possible to overtake it with a reference system from which it would appear right-handed.

(iii) Since the right-handed neutrino has not been seen, it may not exist. Let is not exist.

(iv) The only way to reconcile (ii) and (iii) is to invalidate (ii) by letting $m = 0$, in which case the reference system that may overtake the neutrino simply does not exist. Thus neutrinos are massless.

The fallacy of the above argument2) is that massive Majorana neutrinos offer another outcome to the same line of reasoning. The loophole is the following: give up the neutrino-antineutrino distinction, in which case the observed right-handed "anti"-neutrino may be the right-handed neutrino demanded by premise (ii) of the aforementioned syllogism. This is more than a mere theorists' hat trick: Majorana masses are the neutrino masses suggested by most "grand" unified models.

The pattern of lepton number violation implied by theories with massive Majorana neutrinos and one-handed weak currents is not dissimilar to the known pattern of parity violation in the weak interactions. Suppose one has two electrons at rest, spinning up. Accelerate one of them *down* to some high energy $E \gg m_e$. When it hits the floor it is approximately left-handed (up to corrections of order m_e/E) and it will, via the charged weak currents, make neutrinos with the full weak interaction strength. Accelerate the other electron *up* to hit the roof. Since it is now an approximately right-handed state, it will not make neutrinos, but for corrections of order m_e/E in amplitude. Repeat the argument with spin up massive Majorana neutrinos. The ones hitting the roof will make positrons, the ones hitting the floor electrons. Electrons on the roof or positrons on the floor only appear to order $m_\nu/\sqrt{s}$ in amplitude. Clearly, the new and interesting lepton number violating effects are to be sought in *low* energy experiments.

As the neutrino mass tends to zero, the distinction between the two Majorana states (right- and left-handed) and the two states described by Weyl's equations (left-handed neutrinos and right-handed antineutrinos) becomes purely semantic2). Lepton number violation effects vanish. Formally, this corresponds to the fact that the axial current generated by $\mathcal{N} \rightarrow e^{i\gamma_5\alpha}\mathcal{N}$ transformations becomes a conserved current in the Majorana $m \rightarrow 0$ limit and can be used to define a conserved lepton number.

3. Neutrino masses in grand unified theories

Limits on neutrino masses are generally quoted under the assumption that a single mass eigenstate couples via the weak current to each charged lepton. Prior to the seminal experiment of the ITEP group[3], (that contends that 14 eV $< m(\nu_e) <$ $<$ 46 eV, with 99% confidence) only upper limits existed. They were:

$m(\nu_\tau) < 250$ MeV[4]

$m(\nu_\mu) < 570$ keV[5]

$m(\nu_e) < 55$ eV[6]

The electron neutrino, for instance, must be at least four orders of magnitude lighter than the lightest massive particle: the electron. Once upon a time, the wise advocated that only three numbers could naturally arise from theory (0, 1 and π). If a number, like $m(\nu_e)/m(e)$, had to be much smaller, the reaction would be to set it to zero "for aesthetical reasons". Tastes change, and for good reasons. With the rise and success of gauge theories and the discovery of new particles we have learned two things. First, exact results (like the masslessness of the photon) follow from mystifying but explicit principles (like exact gauge invariance). Second, small numbers, like $m_e/m_\tau \sim 3\ 10^{-4}$ or the predicted ratio of Λ to proton lifetimes ($\sim 10^{-48}$) are certainly there, waiting to be fully understood. Without a compelling explicit reason, only a fool would set a number to zero because it is bound to be small. As of today, no such general reason is known for neutrinos to be massless. QED: *neutrinos are massive !*

We believe the weak interactions to be "weak" (relative to electromagnetism) only as a consequence of the historical accident that they have been observed at low energies (relative to the intermediate vector boson masses, M). At E >> M weak and electromagnetic interactions in unified theories are one and the same and have the same coupling strength of order $e = (4\pi\alpha)^{1/2} \simeq 0.3$. At E << M the weak interactions manifest themselves as effective non-renormalizable current-current coupling such as

$$\mathcal{L}_{eff}(\mu \text{ decay}) = \sqrt{2}G_F(\bar{\nu}_\mu\gamma_\alpha\mu_L)(\bar{e}\gamma^\alpha\nu_L)$$

where the Fermi coupling constant $G_F \sim e^2/M^2$ is "small" ($\sim 10^{-5} m_p^{-2}$).

The origin of small non-vanishing neutrino masses advocated by strong-weak-electromagnetic "grand" unified theories is quite analogous to the origin of the "small" Fermi coupling constant. Let the Georgi mass, the grand unified mass scale, be denoted $M_G \sim 10^{15}$ GeV[7]. At E >> M_G the masses of all intermediate vector bosons, including the ones responsible for proton decay, are negligible, and all coupling constants are one and the same. At E << M_G some effects survive as effective non-renormalizable couplings. An example is

$$\mathcal{L}_{eff}(p \text{ decay}) = G_G(\bar{d}_c\gamma_\mu e_L^-)(\bar{u}_c\gamma_\mu u_L) + \ldots$$

with $G_G \sim e^2/M_G^2$. Another example are Majorana neutrino masses. A Majorana mass $m\nu_L\nu_L$ of the observed left-handed neutrinos is not an SU(2) singlet and is not allowed as a direct term in the Lagrangian. But it can arise as an effective low energy non-renormalizable coupling [from a singlet term of the form $(\phi^0\nu_L - \phi^+ e_L)^2 \to \langle\phi_0\rangle^2\nu_L\nu_L$, where ϕ are the usual elementary scalars and the brackets denote vacuum expected values]. This mass generation[8] occurs in most grand unified models, that predict neutrino masses of the general order of magnitude

$$m_\nu = y\,\frac{1}{G_F M_G} = (0.1\ \text{eV})y$$

Different models suggest very different values for the quantity y, ranging from 10^{-2} to 10^{-5} ($m_\nu \sim 10$ eV to 10^{-6} eV). In some models the values of y for different neutrinos are proportional to up quark masses m_i (implying large neutrino mass ratios) or to m_i^2 (enormous mass ratios). Only one model deserves special attention[9]: "minimal" SU(5). In it, baryon minus lepton number is an accidental exact global symmetry, that forbids a $\Delta L = \Delta(B-L) = 2$ Majorana mass. The model goofs in another of its naïve statements[10] about masses, $m_d m_\mu = m_s m_e$. Whether this debilitates its sad prediction for neutrinos, I do not know.

To summarize, grand unified theories suggest non-vanishing Majorana neutrino masses as a low energy manifestation (similar to proton decay) of ultra-short distance dynamics. They indicate a range of masses that is neither absurd, nor completely hopeless experimentally. In all but one of the models the deep reason why neutrinos are not massless is that there is no deep reason why they should be massless.

4. Cosmological limits and hints on neutrino masses

If the Universe obeys the laws of the standard big bang cosmology, its average energy density ρ_0 can be related to other observable parameters: Newton's gravitational constant G, and the contemporary values of the Hubble constant H_0 and the deceleration parameter q_0:

$$\rho_0 = \frac{3H_0{}^2 q_0}{4\pi G} \tag{4.1}$$

The measured values of q_0 and H_0 are not precise or uncontroversial enough to determine ρ to better than 100% error. Yet, *upper limits* on ρ_0 following from the above expression (or from lower limits on the age of the Universe) can presumably be trusted. It is not possible to fill the Universe with arbitrarily many arbitrarily massive neutrinos[11]. To say something about their mass, we must believe something about their numbers[12]. The 2.7°K background microwave radiation is one of the two triumphant predictions of the standard cosmological theories. The observed number density of photons is of the order $n_\gamma \sim 400/\text{cm}^3$. The observed number density of baryons, averaged to a uniform distribution, is much much smaller: $n_B/n_\gamma \sim 10^{-9\pm1}$. Much like photons, neutrinos should once have been in thermal equilibrium with matter and radiation, at the time when the Universe was so hot and dense that weak interactions could copiously produce and consume neutrinos. In thermal equilibrium the number density n_ν of light neutrinos of each type was the same, to within a statistical factor of order one, as the number of photons. As the Universe cools the neutrinos decouple and as it expands their number density decreases. With some delay, photons undergo a similar history. Nothing in this evolution substantially alters the number ratio n_ν/n_γ, but for a factor of O(1) boosting photons at the time of e^+e^- annihilation. Thus, the present value of n_ν/n_γ is predicted to be of order unity, and $n_\nu/n_B \sim 10^{9\pm1}$. It follows that neutrinos will contribute as much as baryons to the universal number density provided their mass is $m_\nu \sim 10^{-9\pm1}\, m_p \sim 0.1$ to 10 eV. (The neutrino temperature is predicted to be 1.9°K and neutrinos in the above-mentioned mass range will be approximately non-relativistic.) The condition that the energy density not exceed the allowed upper limits for ρ_0, measured as in Eq. (4.1), reads[11]

$$\sum_i m(\nu_i) < \frac{\rho}{n_\nu} \lesssim 50 \text{ eV} \tag{4.2}$$

where i runs over neutrino eigenmass types. This only applies to neutrinos light enough not to affect the primordial thermal equilibrium ($m(\nu) \lesssim 3$ MeV) and stable enough not to have decayed during the age of the Universe[13]. One can doubt, perhaps even within an order of magnitude, the numerics of the above argument. But within the standard cosmology, and given the successful prediction of the microwave radiation, the line of argument is not in doubt. More controversial and titillating are the cosmological arguments that suggest certain neutrino mass ranges, rather than upper limits, to which I now turn your attention.

There is a hierarchy of cosmological missing mass problems, related to objects of different sizes, from single galaxies to the Universe itself. The problems present themselves always in a similar fashion. The mass (or energy density) as computed from gravitational mechanics turns out to be larger than the one computed from the luminosity of the objects (that allows an estimate of baryonic mass in its most common form: stars).

The largest problematic object is the Universe itself, though the numerics are presumably the least convincing. The comparison of the universal density ρ_0, as estimated in Eq. (4.1), with the average universal baryon density ρ_B, as computed from starlight, results in $\rho_0 > \rho_B$ by about one order of magnitude. We will momentarily come back to the possibility of there being large amounts of "dark" ordinary matter.

The smallest problematic objects are single galaxies 14) whose missing mass problem stands on very good footing. The mass m(R) in a sphere of radius R centred on a given galaxy can be deduced, via Newtonian mechanics, from the observed velocity v(R) of objects gravitating in circular orbits at the distance R ($Gm(R)/R^2 = v^2(R)/R$). The most complete data come from the observation of Doppler shifts in the 21 cm hyperfine line of atomic hydrogen. The results are astonishing. For all but three peculiar galaxies, of the more than two score galaxies studied this way, v(R) stays put at 200-300 km/sec, out to distances several times the visible radius of the galaxies, at which the signal becomes too faint to detect. Should the mass be concentrated in the visible part of the galaxy, the large distance behaviour would have been $v(R) \sim R^{-1/2}$, rather than the observed $v(R) \simeq$ constant. This means that galaxies have an invisible halo several times as big as their visible radius. More dramatic, the haloes are at least five or ten times more massive than the visible region of the galaxy. *There is much more to a galaxy than meets the eye.*

There are also similar missing mass problems, hovering around a factor of ten in mechanical to visible mass, for objects of intermediate sizes: clusters, small groups and binary galaxies. Why not solve them all, from galactic haloes upwards in size, just by unimaginatively assuming that there are large amounts of dark conventional matter in the form of dust, golfballs, bricks, defunct stars, small black holes, etc. ? The second great success of the standard cosmology, the calculation of primordial nucleosynthetic abundances, limits the escape path along this direction. Should one increase the nuclear density ρ_B, the calculated densities of ^{4}He and ^{7}Li increase, as the ones of ^{2}H and ^{3}He decrease. For large enough ρ_B, theory and observation disagree. It is argued 15) that ρ_B cannot be more than 12% of ρ_c, the value of ρ_0 in Eq. (4.1) that would make the Universe critical ($q_0 = 1/2$, with which the observed values $q_0 \simeq 0.94 \pm 0.4$ are roughly compatible). It is thus not easy to blame the missing mass problems on unobservable baryons. What could the stuff of this mass be ? You guessed it !

A universally uniformly distributed neutrino sea, with the predicted number density and an average mass of few eV, would suffice to solve the Universe's missing density problem, if there is one. The neutrino solution to the missing mass problems in smaller objects is more interesting, debatable and debated. In principle, massive neutrinos could form self-consistent bound systems, analogous to neutron stars, bound in by gravitation and held out by the exclusion principle. Galactic haloes could just be neutrino balls, with the visible galaxy just a small collapsed core. Practically everybody reacts to this statement with the following criticism: how could weakly interacting neutrinos have lost enough of their energy to be trapped in a gravitational well ? The catch is that not even ordinary matter seems to be sufficiently viscous to do it. The prevailing theory of galaxy formation assumes density fluctuations in the primordial soup. The higher density regions expand less fast than the average Universe and result in galaxies. This argument invokes no other interaction than gravitation and applies equally well to neutrinos and ordinary matter. The subject of the origin of galaxies is controversial, some think that massive neutrinos naturally solve the problem 16), others think they do not 17), and yet others believe that galaxies are entirely impossible. Those who believe in galactic neutrino haloes constrain neutrino masses to the range of 20 eV $\lesssim m_\nu \lesssim$ 200 eV. The smaller m_ν is, the bigger the bound neutrino balls are. For balls of cluster size, that would solve the corresponding missing mass problem, $m_\nu \sim O(5 \text{ eV})$. For binaries or small groups $m_\nu \sim O(10 \text{ to } 20 \text{ eV})$ 18). Curiously enough, all of these numbers fall again in the usual electron Volt range.

5. Properties and (un?)detectability of the fossil neutrino sea

The predicted temperature of the neutrinos left over from the big bang is $T_\nu \sim 1.9^0$K, corresponding to an average velocity of $v = (2kT/m)^{1/2} \sim 3\ 10^{-3}c$ for a 30 eV neutrino mass. This velocity is of the same order of magnitude as that of the solar system in our galaxy. Should neutrinos be uniformly distributed in the Universe, their number density for each neutrino type (= flavour = mass) is predicted to be $n_\nu \sim 150/\text{cm}^3$. Should neutrinos be the stuff of galactic haloes, their number density (at our position in our galaxy) is expected to be at least three to to five orders of magnitude higher.

The observed microwave radiation reflects properties, (i.e., the isotropy) of the Universe at the time when photons last scattered ($t_\gamma \sim 10^6$ years) and the Universe was ∿10 000 times younger. Observation of the fossil neutrino background radiation would tell us about the Universe when neutrinos decoupled ($T \sim$ ∿ 1 MeV, $t_\nu \sim 1$ second !) There are at least half a million and perhaps as many as 50 American billion neutrinos in an empty bottle of wine. Yet their cross-sections are so small that their direct detection - perhaps the greatest low energy physics challenge - will not be easy to meet. A possible line of thought is the following. Because neutrinos are expected to move at velocities comparable to that of the solar system, there must be a considerable diurnal asymmetry and a small seasonal asymmetry in the neutrino "wind". Wind can be concentrated, by its total deflection at grazing incidence[19] or its small refraction on a prism[20], in the case of neutrinos. An enormous array of prisms or grazing mirrors could focus a fraction of the directional neutrino wind towards a supersensitive torsion balance, set in motion by the momentum transferred by a similar deflection. My most optimistic dreams in this direction fail by a few orders of magnitude. This is excellent, when compared with the prospects of the many detectors set to detect gravitational waves, that presumably fail by twice as many orders of magnitude.

There may be hope in *indirect* signatures for massive relic cosmological neutrinos. If neutrinos have masses, no doubt there will be neutrinos of different masses. Much as a Λ sometimes decays into $n\gamma$, (or a $\Sigma^+ \to p\gamma$) a heavy neutrino ν_H should decay into a lighter neutrino ν_L and a photon. The lifetimes for this channel are very model-dependent and estimated (for $m(\nu_H) >> m(\nu_L)$) to be in the range[21,22]

$$\tau(\nu_H \to \nu_L\gamma) \sim 10^{26\pm3} \left(\frac{30\ \text{eV}}{m(\nu_H)}\right)^5 \text{years} \qquad (5.1)$$

with the shorter lifetimes being more contrived. For $m(\nu_H) \sim 10 \to 100$ eV, $\sigma(\nu_H) \sim$ ∿ $10^{-2} \to 10^{-3}c$ and the ultra-violet photon should be monochromatic to better than 1%. This monochromatic signature is lost for photons originating so far that cosmological redshifts play a role. But if the halo of our galaxy or Andromeda is made of ν_H, one may expect an UV glow of the sky, whose monochromatic character may make it directly detectable[22]. It has been argued that the broad enhancement at ∿7 eV of the UV light observed from satellites may be due to the decay of $m(\nu_H) = 14$ eV neutrinos[23]. But the lack of the resolution necessary to observe a monochromatic signature and the surprising low ν lifetime required by this interpretation may make it premature. Very recently Sciama and Mellot[24] have used the galactic halo neutrino radiative decay hypothesis to explain a curious observational fact. The very ionized states C_{IV} and Si_{IV} are observed several kiloparsecs away from our galactic plane. Some electromagnetic radiation must ionize them. But that radiation is not energetic enough to produce N_V. This brackets its energy so that $96\ \text{eV} < m(\nu_H) < 144$ eV, should the radiation be due to $\nu_H \to$ $\to \gamma\nu_L$ decay, with $m(\nu_H) >> m(\nu_L)$. The pleasant aspect of this beautifully wild argumentation is that the computed lifetime[24], $\tau(\nu_H) \sim 10^{27}$ to 10^{28} years, is within the range of Eq. (5.1). This need not have happened.

6. Down to earth measurements of ν masses

Four types of experiments have been devised so far to directly or indirectly measure neutrino masses in the Earth or its neighbourhood.

(i) Neutrino oscillation experiments in principle offer the possibility of investigating the smallest mass range: fractions on an eV for reactor or very dedicated accelerator experiments, millivolts for "deep mine" experiments sensitive to atmospheric neutrinos produced by cosmic rays, microvolts for solar neutrinos. Oscillation experiments have been recently discussed and reviewed at length25), with the result that no convincing positive conclusions have arisen. Because of this, and the time limit in this talk, I will not discuss this subject.

(ii) The long disdained subject of double β decay has recently began to resurface. Because Ettore Fiorini will review it at this conference, I skip it.

(iii) P. Baisden et al., at Princeton, have began an experiment to infer neutrino masses from the relative rates of the very low Q value electron capture from different orbitals in ^{163}Ho. Because I do not know the details of this proposal26), I do not feel confident to discuss it at this point.

(iv) A variety of analyses of spectral shapes in decays involving neutrinos offer the oldest and most studied direct ways of measuring neutrino masses. I discuss them in the following three sections.

7. The middlepoint of β decay

The hadronic charged weak currents couple the up quark to a unitary linear combination $\sim d \cos \theta_C + s \sin \theta_C$; or the charmed quark to the orthogonal combination (for the sake of discussion, I have neglected heavier quarks). Here the d and s or the u and c quarks are "flavours", defined as mass eigenstates. Similarly, if neutrino "flavours" can be defined, e.g., if there are neutrinos with different masses, one expects the electron to couple to a linear combination $\nu_e = a_{e_1}\nu_1 + a_{e_2}\nu_2 + \ldots$ with a_{ei} a unit complex vector; the muon to couple to an orthogonal combination, etc. Oscillation experiments are sensitive to rather small mass differences, relatively large mixings ($|a_{e_1}a^*_{e_2}|^2 \ldots$ not too small compared with unity). Shrock28) has recently emphasized that β decay experiments, in particular meson two-body decays, are very sensitive to a different domain of parameter space: relatively large mass differences, tiny mixing. Indeed, consider π_{μ_2} ($\pi \rightarrow \mu\nu_\mu$) or K_{e_2} ($K \rightarrow e\nu_e$) decays. If ν_μ or ν_e are superpositions of two or more mass eigenstates, the outgoing charged lepton momentum spectrum will not consist of a single peak. Analysis of experiments *not* done with this in mind reveals that

$$\pi_{\mu_2}\ \text{(SIN)} \qquad 0.6\ \text{MeV} < m_\nu < 6\ \text{MeV} \qquad \frac{|a_{\mu_1}a_{\mu_2}|^2}{|a_{\mu_2}|^2} < 5\%$$

$$K_{e_2}\ \text{(CERN-HEIDELBERG)} \quad 82\ \text{MeV} < m_\nu < 163\ \text{MeV} \qquad \frac{|a_{e_1}a_{e_2}|^2}{|a_{e_1}|^2} < 10^{-5}$$

where the second result is particularly impressive28). The neutrino mass ranges quoted are for a second neutrino that could be distinguished within the experimental resolution from the one whose mass is compatible with zero. Dedicated experiments along these lines are being proposed29).

8. The endpoint β decay

Half a century ago Perrin30) noted and Fermi31) analyzed how a neutrino mass would affect the high energy endpoint of the electron spectrum in a three-body β decay ($Z \rightarrow (Z+1) + e^- + \bar{\nu}_e$). The effect is "trivial" in the sense that it is governed by kinematics. The result is generally given in terms of a Kurie plot: the square root of the number of events corrected for Coulomb effects (F_C) and other trivial kinematical factors31):

$$K(E_e) \equiv \sqrt{\frac{d\omega/dp_e}{F_c(E_e)p_e^2}} \propto (E_e^{max} - E_e)^{1/2}([E_e^{max} - E_e]^2 - m_\nu^2)^{1/4} \qquad (8.1)$$

in a self-explanatory notation where, for simplicity, I have assumed the electron (anti) neutrino to be a pure eigenstate of mass m_ν. Thus plotted, the data of a perfect resolution experiment would end in a straight line for $m_\nu = 0$ (the full line of Fig. 1a) or with a vertical tangent for $m_\nu \neq 0$ (the full line of Fig. 1b). The best experiments analyze ^{3}H β decay, the nuclide that with a reasonably short lifetime has the smallest E_e^{max}, and thus the largest relative number of events in the potentially interesting region of width m_ν at the endpoint. It is in tritium experiments that the best limits or measurements have been made so far

$$^3H \rightarrow {}^3He^+ + e^- + \bar{\nu}_e \begin{cases} m_\nu < 60 \text{ eV} & \text{(90\% c.l.) Bergkvist}^{6)} \\ 14 \text{ eV} < m_\nu < 46 \text{ eV} & \text{(99\% c.l.) Lyubimov et al.}^{3)} \\ m_\nu < 65 \text{ eV} & \text{(95\% c.l.) Simpson}^{32)} \end{cases}$$

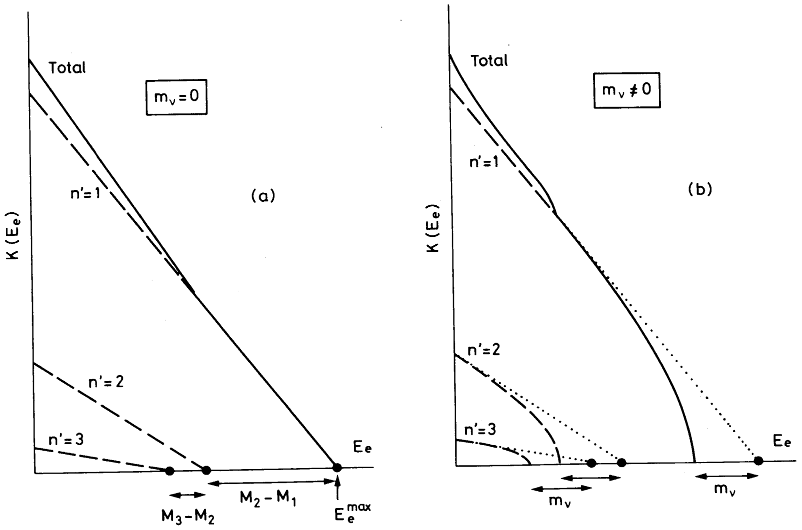

Fig. 1

Two of the limitations of these experiments are[6,33]:

(i) Electrons have the disgusting habit of scattering, radiating or being affected by surface potentials on their way out of a source, thereby gaining or losing small amounts of energy. This renders very delicate the calibration, the estimate of the resolution function and the determination of the experimental line shape.

(ii) When the nucleus of ^{3}H decays into the nucleus of ^{3}He, its charge increases by a factor of two. The expectant electron that was there in ^{3}H does not feel so much at home in the radically changed Coulomb field. For free tritium atoms, one can compute[33] that only in 70% of the decays does the electron remain in the original Bohr orbital, n = 1. In 25% of the decays it jumps to n = 2, an atomic state 41 eV heavier, and so on. The electron spectrum in the decay to the heavier n = 2 atomic state ends 41 eV below the spectrum in the decay to the ground state. Thus there is a structure of the spectrum in the region of interest of a few tens of eV from the endpoint, as shown in Fig. 1. Since the structure is computable and computed, why worry ? The problem is that experiments have not been done with tritium or tritons, but with tritium coated on aluminium6) or contained in valine ($C_5H_{11}NO_2$)3). The surface atomic and molecular properties and excitation spectrum of these substances have not been theoretically computed, perhaps for no good reason. This introduces a considerable uncertainty in the numerical analysis of experimental results. As seen in Fig. 1, atomic effects and neutrino mass effects introduce opposite curvatures in the spectrum. If a neutrino mass is still needed to fit the data under the extreme assumption that the decay is dominated by a single final atomic state, the situation is satisfactory. This is the case at the low end of the neutrino mass range given by the ITEP group3).

The above two criticisms do not apply to "calorimetric" experiments, like Simpson's32). Here tritium is implanted into a Si(Li) detector that measures *all* of the energy released, but that of the neutrino. To put it peculiarly, the experiment in its ground state decays into another ground state (with one less neutron, one more proton and electron), plus a neutrino, plus the measured calorimetric energy: a clean three-body decay. The main limitation of this type of experiment is resolution, and the control of the way it is affected by the source implantation.

9. Radiative orbital electron capture34)

A neutron deficient nucleus (A,Z) may capture an atomic electron to become the previous nucleus in the atomic table $(A,Z-1)^H$, with the emission of an electron neutrino. The atom $(A,Z-1)^H$ is left with a hole $H \equiv \{nL_j\}$ in the orbital from which the electron is captured. An electron from a higher orbital will subsequently fall into H with the emission of a monochromatic X ray or Auger electron. The energy of this secondary emission carries the signature for the orbital from which the primary electron is captured. The rate of these processes contains no practical information on the neutrino mass, unless the released energies are so small as to be comparable with m_ν26).

In a small fraction of orbital captures a bremsstrahlung photon is emitted. This "IBEC" process (internal bremsstrahlung in electron capture) is a three-body decay:

$$Z \to (Z-1)^H + \gamma(k) + \nu_e \qquad (9.1)$$

The photon spectrum at the endpoint is sensitive to the neutrino mass in the same way as the electron spectrum in β decay:

$$\sqrt{\frac{d\omega}{kdk}} \propto (k^0_{max} - k)^{1/2}((k^0_{max} - k)^2 - m^2_\nu)^{1/4} \qquad (9.2)$$

Thus, if the rates are sufficient and the resolution in photon energy measurements is good enough, the process may compete with β decay in measuring the neutrino mass. Moreover, the problems labelled (i) and (ii) in the previous section lose their bite:

(i) Photons are much less likely than electrons to change their energy by a small fraction as they interact with the environment.

(ii) In the electron capture process only the charge distribution, and not the charge, changes in the nuclear neighbourhood. The troublesome outer electrons that may jump by a few eV and modify the spectral endpoint have small wave functions close to the nucleus and are not significantly affected by the radiative or non-radiative capture.

The trouble conservation theorem implies that two new problems arise:

(iii) Electrons may be captured from different atomic orbitals. Let Q denote the mass difference between the ground state atoms Z and Z-1, and let $E[nL_j]$ denote the (positive) ionization energy of the electron in the orbital from which capture occurs. The $m_\nu = 0$ maximum photon energy in the IBEC process (9.1) is

$k_{max} = Q - E[nL_j]$, a function of the emptied orbital. Thus the photon spectrum endpoint is again a messy superposition of different channels, as pictured in Fig. 2. Were one to attempt a neutrino mass measurement at the endpoint of the $nL_j = 1S_{1/2}$ spectrum, there would be an enormous "background" due to higher orbital capture. The solution of this problem is (in principle) trivial: do the experiment in coincidence with the subsequent X rays or Auger electrons, in which case capture from the different orbitals can be extricated. Many such coincidence experiments have been done[35].

(iv) A second, more serious problem, is one of rate. Let $\omega^{NB}[1S]$ stand for the dominant rate of non-radiative capture from the 1S orbital ("K capture"). Let $d\omega^B/dk$ be the corresponding IBEC spectrum. The goodie-goodie fraction G of potentially interesting IBEC endpoint events is

$$G = \int_{k^0_{max}-m_\nu}^{k^0_{max}} dk \; \frac{d\omega^B(1S)}{\omega^{NB}(1S)dk} \sim \alpha \, \frac{m_\nu^3}{\pi m_e^2 3k^0_{max}}$$

For typical K capturing nuclei, and $m_\nu = 30$ eV, $G \sim 4\ 10^{-16}$. But do not despair. IBEC spectra in a typical K capturing isotope (^{55}Fe) are shown in Fig. 3. The 2P spectrum has an enormous peak, towering several orders of magnitude outside the figure, at a photon energy corresponding to an X ray energy $k^{RES}[2P] = E[1S] - E[2P]$. This is because at that energy, bremsstrahlung in 2P capture and 1S capture followed by a 2P → 1S X ray (*atomic resonant*) transition, are indistinguishable[36]. Both processes have the same initial state and the same final state (the atom with a 2P hole, a neutrino and a photon of given energy). All of the 2P spectrum below the peak is enormously enhanced. Now a second premise. The endpoint of the 2P spectrum is at $k^{max}[2P] = Q - E[2P]$. Because Q knows about nuclear masses, this is independent of the resonant energy $E[1S] - E[2P]$. Should nature provide isotopes such that $Q - E[2P] \lesssim E[1S] - E[2P]$, the endpoint of the spectrum, where the neutrino mass hunt takes place, would be enormously enhanced by the neighbouring resonance (for this it is not necessary that the condition be met within a few X ray widths). The relevant isotopes are obviously those for which electron capture is energetically allowed, but not from the 1S orbital. Half a dozen exist, and an example is ^{193}Pt. The most promising nuclide is ^{163}Ho, for which both n = 1 and 2 capture are forbidden and the Q value is a record low. The predicted fractional endpoint counting rates for these and a few other nuclides are sufficient to have triggered experimentalists to give it a try[37].

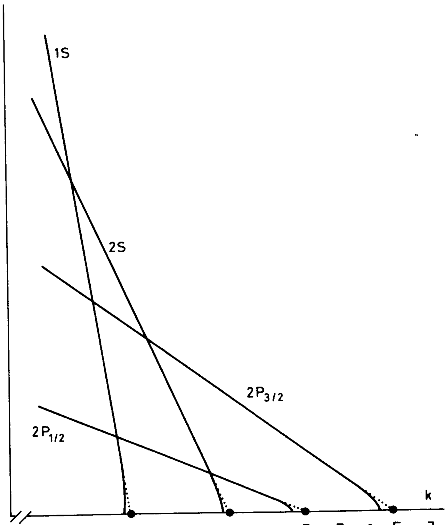

Fig. 2

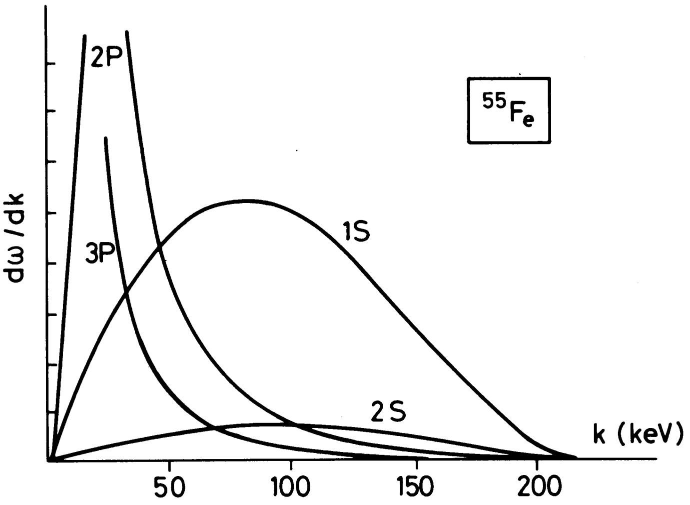

Fig. 3

10. Conclusions ?

Neither theory nor experiment have converged to a convincing statement on neutrino masses (the ITEP experiment is hopefully right, but so far it is unique). Nonetheless, you have no doubt noticed an absolutely extraordinary coincidence: all positive statements concern masses in the eV range. If this is more than a coincidence, the considerable experimental efforts dedicated to this theme will not be in vain. As of today, and to the shame of theorists, it must be admitted that neutrino weight-watching is mainly an experimental science.

The detection of neutrino masses in the eV range would have vast implications from the cosmological domain of the very large ($d \sim 10^{28}$ cm) to the domain of unified theories that, though "grand", deals with the very small ($d \sim 10^{-28}$ cm).

References

1) E. Majorana, Nuovo Cimento 14 (1937) 171;
G. Racah, Nuovo Cimento 14 (1937) 322.

2) J.A. McLennan, Phys. Rev. 106 (1957) 821;
K.M. Case, Phys. Rev. 107 (1957) 307;
For a review of these arguments in modern parlance, see
E. Witten, talk presented at the First Workshop on Grand Unification, New England Centre, Univ. of New Hampshire, April 1980, Harvard Univ. preprint HUTP-80/A051.

3) V.A. Lyubimov et al., Phys. Lett. 94B (1980) 266.

4) J. Kirkby, in Proc. of the International Symposium on Lepton and Photon Interactions at High Energies, FNAL, Batavia, Illinois 1979.

5) M. Daum et al., Phys. Lett. 74B (1978) 126.

6) K.E. Bergkvist, Nucl. Phys. B39 (1972) 317.

7) H. Georgi and S.L. Glashow, Phys. Rev. Letters 32 (1974) 438;
H. Georgi, H. Quinn and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451;
A. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B135 (1978) 66;
D. Ross, Nucl. Phys. B140 (1978) 1.

8) M. Gell-Mann, P. Ramond and R. Slansky, in "Supergravity", Proc. of the Supergravity Workshop at Stony Brook, ed. by P. Van Nieuwenhuizen and D.Z. Freedman (North Holland, Amsterdam, 1979), p. 315;
H. Georgi and D.V. Nanopoulos, Nucl. Phys. B155 (1979) 52;
S.L. Glashow, Cargèse Lectures (1979), to be published;
R. Barbieri, J. Ellis and M.K. Gaillard, Phys. Lett. 90B (1980) 249;
R. Barbieri et al., Phys. Lett. 90B (1980) 91;
E. Farhi, Phys. Lett. 97B (1980) 229;
S. Pakvasa and K. Tennakone, Phys. Rev. Lett. 28 (1972) 1415;
A. De Rújula, H. Georgi and S.L. Glashow, Annals of Physics 109 (1977) 258;
E. Witten, Phys. Lett. 91B (1980) 81;
R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912;
General arguments have been given by
S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566.

9) See the first paper of Ref. 7).

10) M.S. Chanowitz, J. Ellis and M.K. Gaillard, Nucl. Phys. B128 (1977) 506;
A. Buras et al., Nucl. Phys. B135 (1979)

11) S.S. Gerstein and Ya.B. Zeldovich, Zh. Eksp. Teor. Fiz. Pisma Red. 4 (1966) 174;
R. Cowsik and J. McClelland, Phys. Rev. Lett. 29 (1972) 669;
A.S. Szalay and G. Marx, Astron. Astrophys. 49 (1976) 437;
B.W. Lee and S. Weinberg, Phys. Rev. Lett. 42 (1977) 407;
J.R. Bond, G. Efstathiou and J. Silk, Berkeley Univ. preprint (1980).

12) For a recent review, see
A.D. Dolgov and Ya.B. Zeldovich, Revs. Mod. Phys. 53 (1981) 1.

13) Models where neutrinos of a few keV would not have survived as long as the Universe can be constructed. See, for instance
M. Roncadelli and G. Senjanović, Max-Planck-Inst. Report MPI-PAE/PTh16/81 (1981).

14) For a thorough review, see
S.M. Faber and J.S. Gallagher in Annual Revs. of Astron. and Astrophys., ed. by G. Burbridge and J.G. Philips, (1979).

15) See D.N. Schramm and G. Steigman, Bartol Research Foundation preprint BA-80-17 and references therein.

16) A. Mellot, Univ. of Texas preprint (1981);
F. Klinkhamer and C. Norman, F. Klinkhamer Leiden preprints 1980-81;
R. Rufini, Lett. Nuovo Cim. 29 (1980) 167.

17) S. Gunn and J.E. Tremaine, Phys. Rev. Lett. 42 (1979) 407;
See also Ref. 15).

18) For reviews, see
R. Cowsik, E. Kolb, D. Schramm, M. Turner and J. Fry, all in the Proceedings of Neutrino Mass Miniconference, Wisconsin 1980, ed. by V. Barger and D. Olive;
A. Doroshkevich et al., Contribution to the Neutrino-80 Conference, Erice 1980 and Ref. 15).

19) R.R. Lewis, Phys. Rev. D21 (1980) 663.

20) R. Opher, Astron. Astrophys. 37 (1974) 135.

21) S. Pakvasa and K. Tennakone, Phys. Rev. Lett. 28 (1972) 1415;
T. Goldman and J.G. Stephenson, Phys. Rev. D16 (1977) 2256;
B. Cowsik, Phys. Rev. Lett. 39 (1977) 784.

22) A. De Rújula and S.L. Glashow, Phys. Rev. Lett. 45 (1980) 1460.

23) F.W. Stecker, Phys. Rev. Lett. 45 (1980) 1460.

24) D.W. Sciama and A.L. Mellot and D.W. Sciama, University of Texas preprints Austin (1981).

25) S.M. Bilenky and B. Pontecorvo, Phys. Rep. 41 (1978) 225;
S. Eliezer and D.A. Ross, Phys. Rev. D10 (1974) 3088;
S.M. Bilenky and B. Pontecorvo, Phys. Lett. 61B (1976) 248;
B. Pontecorvo, JETP Lett. 13 (1971) 199;
S.M. Bilenky and B. Pontecorvo, Lett. Nuov. Cimento 17 (1976) 569;
H. Fritzsch and P. Minkowski, Phys. Lett. 62B (1976) 72;
R. Davis Jr., J.C. Evans and B.T. Cleveland, Proc. of the International Neutrino Conference, Purdue University, U.S.A., 1978; for a review, see
J.N. Bahcall, Rev. of Mod. Phys. 50 (1978) 881;
F. Reines, H.W. Sobel and E. Pasierb, Phys. Rev. Lett. 45 (1980) 1307;
G. Boehm et al., Caltech preprint CALT-63-350 (1980);
A. De Rújula et al., Nucl. Phys. B164 (1980) 54;
L. Maiani, CERN preprint TH.2846 (1980);
V. Barger et al., Phys. Rev. Lett. 93B (1980) 194;
D. Silvermann and A. Soni, Univ. of California preprint UCLA/80/TEP/25 (1980);
V. Barger, in Proc. of the XX International Conference on High Energy Physics, Madison 1980, DOE-ER/00881-169 (1981) and references therein.

26) For a very brief description, see the "News" Section of the July 1981 issue of Physics Today, by Gloria Lubkin.

27) Z. Maki, N. Nakawaga and S. Sakata, Progr. Theor. Phys. 28 (1962) 870.

28) R.E. Schrock, Phys. Lett. 96B (1980) 159; see also
Y. Asano et al., KEK proposal E89, UTMSL-6 (1980).

29) T. Yamazaki et al., KEK proposal E89, UTMSL-6 (1980).

30) F. Perrin, Comptes Rendus 197 (1933) 1625.

31) E. Fermi, Nuovo Cimento 11 (1934) 1; Zeitschrift für Physik 488 (1934) 161.

32) J.J. Simpson, Phys. Rev. D23 (1981) 649.

33) K.E. Bergkvist, Phys. Scripta 4 (1971) 23.

34) A. De Rújula, CERN preprint TH.3045 (April 1st, 1981).

35) For a review, see
W. Bambynek et al., Revs. Mod. Phys. 49 (1977) 77.

36) R.J. Glauber and P.C. Martin, Phys. Rev. 104 (1956) 158.

37) J.V. Anderson et al., Proposal of the ISOLDE Committee, CERN, (Jan. 1981).