Ref.TH.2736-CERN

TESTS OF QCD FOR THREE JET SEMI-LEPTONIC PROCESSES BASED ON GENERALIZED MOMENTS

Chang Chao-hsi *)

CERN - Geneva

ABSTRACT

A new kind of moment for testing three-jet semi-leptonic processes are discussed. A double moment defined in terms of a new fragmentation variable Z_{out} ($Z_{out} \equiv 2|P_{out}|/\sqrt{Q^2}$, P_{out} is the so-called out-momentum of an outgoing hadron) instead of a usual one[1], Z, and "triple" moments which are made of both Z and Z_{out} are introduced. These (newly) introduced moments have contributions from processes with three or more jets (because $Z_{out} = 0$ for two jet processes if one neglects non-perturbative effects) and can obviously be expressed by the moments of the structure functions and fragmentation functions. This is the advantage of introducing such moments. How to make a quantitative comparison with experiment is discussed, taking the neutrino experiment as an example.

*) On leave of absence from the Institute of High Energy Physics Peking, China

Ref.TH.2736-CERN
31 August 1979

1. INTRODUCTION

QCD is the only serious candidate theory of the strong interactions up to now, and it has many qualitative successes, especially the so-called perturbative QCD approach which even has some quantitative successes; it gives us some insight understanding of the scaling violation of the deep inelastic scattering and two - jet phenomena and so on[2]. (We will call these processes the lowest QCD processes later on.) Its predictions fit the data very well for these processes, which means the perturbative QCD for the lowest processes works well. So what about higher order processes, those due to a hard gluon bremsstrahlung or hard quark pair production? This becomes one of the next interesting problems. In this direction, there are several papers[3),4)], but here we emphasize the use of moments to test QCD to see whether it works well or not for the higher order processes. We find some generalized moments, which are calculable, relate to the moments of structure functions and fragmentation functions obviously and can be measured directly.

Moment analyses are preferable where possible, as opposed to direct structure function and fragmentation function analyses. The reasons are that the low values of x and z for the structure function and fragmentation functions, respectively, are experimentally inaccessible but do not contribute importantly to high moments $(N \geq 2)$ and the Q^2 dependence behaviour of the moments have an analytically simple asymptotic form which can provide a solid basis for quantitative comparisons with data. So we are interested in finding some generalized moments for testing high order QCD processes, say three jet processes. One more advantage of using moments is that higher order QCD processes are always related to the structure function and fragmentation function or their moments, but now the measurements for the moments are better than for functions, while the generalized moments depend on the moments only.

The component, perpendicular to the leptonic plane, of the momentum of an outgoing hadron, the so-called out-momentum, P_{out}, (Fig. 1) has contributions partly from the transverse momentum P_T of the incoming quark (or gluon) inside a target hadron and partly from the deviation of the fragment from the jet axis, namely, the so-called non-perturbative effects, mainly from the third jet effect if we consider a three-jet event. The former is Q^2 independent and we neglect it; the latter is what we are interested in. We can neglect the non-perturbative effects when either Q^2 is large enough and if the quantities which we are interested in are insensitive to the small Z_{out} region, or the effects can cancel experimentally. Sometimes it is possible to show this using the property of the Q^2 independence.

In this paper we first give the differential cross-section of semi-inclusive semi-leptonic processes, and based on them, give the definitions of generalized moments. To illustrate the basic idea and for simplicity, we take processes in which $\nu(\bar{\nu})$ produce three jets as an example. We consider a non-singlet incoming parton and a non-singlet fragment parton; we give formulae in detail and some curves of the generalized moments for getting some feeling for the subject. Finally, we discuss how to use them to test QCD experimentally.

2. DIFFERENTIAL CROSS-SECTION AND MOMENTS

As mentioned above, we are interested in higher order QCD processes, namely, three-jet processes, so we first give the differential cross-section of the semi-inclusive semi-leptonic processes:

$$\ell(k) + A(P) \rightarrow \ell'(k') + \pi^{\pm}(P') + \cdots \qquad (1)$$

The differential cross-section can be written as follows*),

$$\frac{d\sigma}{dx\,dy\,dz\,dP_T'\,d\Phi} = \sum_{ij}\int dx_p\,dz_p\,dp_T\,d\xi\,d\xi'\,\delta(x-\xi x_p)\,\delta(z-\xi' z_p)\,\delta(P_T'-\xi' p_T)$$
$$f_i(\xi,Q^2)\,\frac{d\sigma_{ij}}{dx_p\,dy\,dz_p\,dp_T\,d\phi}\,D_j^{\pi^{\pm}}(\xi',Q^2) \qquad (2)$$

where we choose the definitions (Fig. 1):

$$q \equiv k-k',\quad Q^2 \equiv -q^2,\quad x \equiv \frac{Q^2}{2(Pq)},\quad y \equiv \frac{(Pq)}{(Pk)},\quad z \equiv \frac{(PP')}{(Pq)},\quad P_T',\quad \Phi,\quad P_{out}' = P_T' \sin\Phi \qquad (3)$$

and $f_i(\xi,Q^2)$ is the momentum distribution function of the i parton (quark or gluon) in the target hadron A, and $D_j^{\pi^{\pm}}(\xi',Q^2)$ is the fragmentation function of the j parton to $\pi^{\pm}$. The sub-process differential cross-section:

$$\frac{d\sigma_{ij.a}^{\nu,\bar{\nu}}}{dx_p\,dy\,dz_p\,dp_T\,d\phi} = \frac{G_F^2 Q^2}{(2\pi)^3 y}\,K(2\pi)\,\delta(x_p-1)\,\delta(z_p-1)\,\delta(p_T)\left\{[1+(1-y)^2] \pm [1-(1-y)^2]\right\} \qquad (4)$$

for the two-jet process (Fig. 2), and

$$\frac{d\sigma_{ij.a}^{\nu,\bar{\nu}}}{dx_p\,dy\,dz_p\,dp_T\,d\phi} = \alpha_s(Q^2)\,\frac{G_F^2 Q^2}{(2\pi)^3 y}\,\delta\Big(p_T - \Big[\frac{Q^2 z_p(1-z_p)(1-x_p)}{x_p}\Big]^{1/2}\Big)\cdot$$
$$\cdot\, C_a\left\{[\underset{\sim}{A_a} \pm \underset{\sim}{A_a'}] + [\underset{\sim}{B_a} \pm \underset{\sim}{B_a}]\cos\phi + \underset{\sim}{C_a}\cos 2\phi\right. \qquad (5)$$

*) We give the formulae in detail here; we can see they are consistent with Ref. 4).

for the three-jet process (Fig. 3), where the suffix $a = 1,2,3$ stands for diagrams of Fig. 3a, 3b and 3c, respectively. In Eqs. (4) and (5) the factor K includes the coupling between the current and the interacting quark, for instance, $K = \cos\theta_c$ for $\langle d\rangle \to \langle u\rangle$, and the sign $\pm$ corresponds to neutrinos and anti-neutrinos. $\alpha_s(Q^2)$ is the running coupling constant:

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2f)\ln(Q^2/\Lambda^2)} . \tag{6}$$

The numerical factor C_a is due to colour

$$C_1 = C_2 = \frac{4}{3} , \quad C_3 = \frac{1}{2} . \tag{7}$$

According to the definitions, we have

$$p_a = \xi P , \quad P' = \zeta' p_b , \quad \phi = \Phi , \tag{8}$$

neglecting the non-perturbative effects.

The $\underset{\sim}{A}_a$, $\underset{\sim}{A}'_a$, $\underset{\sim}{B}_a$, $\underset{\sim}{B}'_a$ and $\underset{\sim}{C}_a$ in Eq. (5) are functions of x_p, y and z_p, where

$$x_p \equiv \frac{Q^2}{2(p_a q)} , \qquad z_p \equiv \frac{(p_a p_b)}{(p_a q)} , \tag{9}$$

$$\underset{\sim}{A}_1 = 8(1-y)x_p z_p + [1+(1-y)^2][(1-x_p)(1-z_p) + \frac{1+x_p^2 z_p^2}{(1-x_p)(1-z_p)}] , \tag{10.1}$$

$$\underset{\sim}{A}_2 = 8(1-y)x_p(1-z_p) + [1+(1-y)^2][(1-x_p)z_p + \frac{1+x_p^2(1-z_p)^2}{(1-x_p)z_p}] , \tag{10.2}$$

$$\underset{\sim}{A}_3 = 16(1-y)x_p(1-x_p) + [1+(1-y)^2][x_p^2+(1-x_p)^2]\frac{z_p^2+(1-z_p)^2}{z_p(1-z_p)} , \tag{10.3}$$

$$\underset{\sim}{A}'_1 = y(2-y)[2(x_p+z_p) + \frac{x_p^2+z_p^2}{(1-x_p)(1-z_p)}] . \tag{10.4}$$

$$\underset{\sim}{A}'_2 = y(2-y)[2(1+x_p-z_p) + \frac{x_p^2+(1-z_p)^2}{(1-x_p)z_p}] , \tag{10.5}$$

$$A'_3 = y(2-y)[x_p^2+(1-x_p)^2]\frac{2z_p-1}{z_p(1-z_p)} , \tag{10.6}$$

$$\underset{\sim}{B}_1 = -4(2-y)(1-y)^{1/2}[\frac{x_p z_p}{(1-x_p)(1-z_p)}]^{1/2}[x_p z_p + (1-x_p)(1-z_p)] , \tag{10.7}$$

$$\underset{\sim}{B_2} = 4(2-y)(1-y)^{1/2}\left[\frac{x_p(1-z_p)}{(1-x_p)z_p}\right]^{1/2}\left[x_p(1-z_p)+(1-x_p)z_p\right], \tag{10.8}$$

$$\underset{\sim}{B_3} = -4(2-y)(1-y)^{1/2}\left[\frac{x_p(1-x_p)}{z_p(1-z_p)}\right]^{1/2}(1-2x_p)(1-2z_p), \tag{10.9}$$

$$\underset{\sim}{B_1'} = -4y(1-y)^{1/2}\left[\frac{x_p z_p}{(1-x_p)(1-z_p)}\right]^{1/2}(x_p+z_p-1), \tag{10.10}$$

$$\underset{\sim}{B_2'} = 4y(1-y)^{1/2}\left[\frac{x_p(1-z_p)}{(1-x_p)z_p}\right]^{1/2}(x_p-z_p), \tag{10.11}$$

$$\underset{\sim}{B_3'} = 4y(1-y)^{1/2}\left[\frac{x_p(1-x_p)}{z_p(1-z_p)}\right]^{1/2}(1-2x_p), \tag{10.12}$$

$$\underset{\sim}{C_1} = 4(1-y)x_p z_p, \tag{10.13}$$

$$\underset{\sim}{C_2} = 4(1-y)x_p(1-z_p), \tag{10.14}$$

$$\underset{\sim}{C_3} = 8(1-y)x_p(1-x_p). \tag{10.15}$$

From Eqs. (4) and (8), due to the factor $\delta(p_T)$, the two jet processes do not contribute anything for a $p_T' \neq 0$ event, if we neglect the non-perturbative effects, so we do so. In Eqs. (10.1) - (10.15), $\underset{\sim}{A_a'}$ and $\underset{\sim}{B_a'}$ come from the V,A interference. If we do the following substitution:

$$\frac{G_F^2 Q^2}{(2\pi)^3} \longrightarrow \frac{\alpha^2}{2\pi Q^2} \tag{11}$$

and put all $\underset{\sim}{A_a'}$ and $\underset{\sim}{B_a'}$ into zero, the Eqs. (2), (4), (5) and (10.1) - (10.15) turn to the corresponding formulae for an electron or a muon semi-inclusive process.

Because we are interested in the behaviour of the Q^2 dependence, we change the differential cross-section into a Q^2 fixed one:

$$\frac{d\sigma}{dx\,dz\,dQ^2\,dp_T'\,d\Phi} = \frac{1}{2xE_\nu M}\left.\frac{d\sigma}{dx\,dz\,dy\,dp_T'\,d\Phi}\right|_{y=\frac{Q^2}{2xE_\nu M}} \tag{12}$$

where $E_\nu = k_0$, the coming energy of $\nu(\bar{\nu})$. First, we consider the cases of non-singlet targets and fragments, namely:

$$\frac{d\sigma^{I}(ns)}{dx\,dz\,dQ^2dP_T'd\Phi} = \frac{d\sigma}{dx\,dz\,dQ^2dP_T'd\Phi}\Big((\nu p\to\pi^+ +\cdots)-(\nu p\to\pi^- +\cdots)+(\bar\nu p\to\pi^+ +\cdots)-(\bar\nu p\to\pi^- +\cdots)\Big) \quad (13)$$

or

$$\frac{d\sigma^{II}(ns)}{dx\,dz\,dQ^2dP_T'd\Phi} = \frac{d\sigma}{dx\,dz\,dQ^2dP_T'd\Phi}\Big((\nu p\to\pi^+ +\cdots)-(\nu p\to\pi^- +\cdots)-(\nu n\to\pi^+ +\cdots)+(\nu n\to\pi^- +\cdots)\Big) \quad (14)$$

where p = proton, and n = neutron. And we have

$$\frac{d\sigma^{I,II}(ns)}{dx\,dz\,dQ^2dP_T'd\Phi} = \frac{\alpha_s(Q^2)G_F^2}{(2\pi)^3}\int dx_p\,dz_p\,dp_T\,d\xi\,d\xi'\,\delta(x-\xi x_p)\,\delta(z-\xi' z_p)\,\delta(P_T'-\xi' p_T)$$
$$\cdot F_V(\xi,Q^2)\,\delta\Big(p_T-\Big[\frac{Q^2 z_p(1-z_p)(1-x_p)}{x_p}\Big]^{1/2}\Big)\,U^{I,II}\,D^{(\pi^+-\pi^-)}_{\langle u\rangle}(\xi',Q^2) \quad (15)$$

where, if we take $\theta_c = 0$ and charm is neglected:

$$U^{I} = -\frac{4}{9}\left(\underset{\sim}{A}_1+\underset{\sim}{B}_1\cos\Phi+\underset{\sim}{C}_1\cos 2\Phi\right)+\frac{4}{3}\left(\underset{\sim}{A}_1'+\underset{\sim}{B}_1'\cos\Phi\right) \quad (16)$$

for

$$\frac{d\sigma^{I}(ns)}{dx\,dz\,dQ^2dP_T'd\Phi},$$

and

$$U^{II} = -\frac{4}{9}\left[\underset{\sim}{A}_1+\underset{\sim}{B}_1\cos\Phi+\underset{\sim}{C}_1\cos 2\Phi\right]-\frac{4}{9}\left(\underset{\sim}{A}_1'+\underset{\sim}{B}_1'\cos\Phi\right) \quad (17)$$

for

$$\frac{d\sigma^{II}(ns)}{dx\,dz\,dQ^2dP_T'd\Phi}.$$

Now we introduce the so-called generalized moments. One of them, a double moment, is

$$E^{(i)}(n,m,Q^2) \equiv -\left(\frac{G_F^2}{2\pi}\right)^{-1}\int_0^1 dx\int_0^1 dz\int dP_T'\int d\Phi\; x^{n-1}\,z_{out}^{m-1}\,\frac{d\sigma^{(i)}}{dx\,dz\,dQ^2dP_T'd\Phi} \quad (18)$$

where i = I,II,...,

$$Z_{out} \equiv \frac{2|P_{out}|}{\sqrt{Q^2}} \quad . \tag{19}$$

We choose Z_{out}, not $Z_T = 2P_T'/\sqrt{Q^2}$, to make experimental measurement easier. Now we have

$$E^{I,II}(n,m,Q^2) = \frac{\alpha_s(Q^2)}{(2\pi)^2}\int_0^1 dx_p \int_0^1 dz_p\, x_p^{n-\frac{m+1}{2}}\, 2^{m-1}\left[z_p(1-z_p)(1-x_p)\right]^{\frac{m-1}{2}} \cdot W^{I,II}\, \hat{D}^{(\pi^+-\pi^-)}_{\langle u\rangle}(m,Q^2) \tag{20}$$

and

$$W^{I} = \frac{4}{9}\left[\alpha_{m-1}(Y_1+Y_2)+\beta_{m-1}Y_4\right] - \frac{4}{3}\alpha_{m-1}Y_3 \tag{21}$$

$$W^{II} = \frac{4}{9}\left[\alpha_{m-1}(Y_1+Y_2+Y_3)+\beta_{m-1}Y_4\right] \tag{22}$$

where

$$\alpha_{m-1} = \int_0^{2\pi} d\Phi\, |\sin\Phi|^{m-1} \quad , \tag{23}$$

$$\beta_{m-1} = \int_0^{2\pi} d\Phi\, |\sin\Phi|^{m-1}\cos 2\Phi \quad , \tag{24}$$

$$Y_1 = \left[\hat{F}_V(n,Q^2) - \frac{Q^2}{2x_pE_\nu M}\hat{F}_V(n-1,Q^2)\right]\left\{8x_pz_p + 2\left[(1-x_p)(1-z_p) + \frac{1+x_p^2z_p^2}{(1-x_p)(1-z_p)}\right]\right\} , \tag{25}$$

$$Y_2 = \hat{F}_V(n-2,Q^2)\left(\frac{Q^2}{2x_pE_\nu M}\right)^2\left\{(1-x_p)(1-z_p) + \frac{1+x_p^2z_p^2}{(1-x_p)(1-z_p)}\right\} , \tag{26}$$

$$Y_3 = \left[2\hat{F}_V(n-1,Q^2) + \frac{Q^2}{2x_pE_\nu M}\hat{F}_V(n-2,Q^2)\right]\frac{Q^2}{2x_pE_\nu M}\left\{2(x_p+z_p) + \frac{x_p^2+z_p^2}{(1-x_p)(1-z_p)}\right\} , \tag{27}$$

$$Y_4 = \left[\hat{F}_V(n,Q^2) - \frac{Q^2}{2x_pE_\nu M}\hat{F}_V(n-1,Q^2)\right]\cdot\left\{4x_pz_p\right\} , \tag{28}$$

and the moments

$$\hat{F}_V(n,Q^2) = \int_0^1 dx\, x^{n-1}\hat{F}_V(x,Q^2) , \tag{29}$$

$$\hat{D}^{(\pi^+-\pi^-)}_{\langle u\rangle}(m,Q^2) = \int_0^1 dz\, z^{m-1} D^{(\pi^+-\pi^-)}_{\langle u\rangle}(z,Q^2) . \tag{30}$$

From (20) - (30) we can see $E^{I,II}(n,m,Q^2)$ are expressed in terms of moments of the structure function and of the fragmentation function directly. Similarly, we can introduce "triple" moments (if $m > 1$):

$$H^{I,II}(n,m,k,Q^2) \equiv -\left(\frac{G_F}{2\pi}\right)^{-1}\int_0^1 dx \int_0^1 dz \int dP_T' \int d\Phi \, x^{n-1} z_{out}^{m-1} z^k \frac{d\sigma^{I,II}(ns)}{dx\,dz\,dQ^2\,dP_T'\,d\Phi}$$

$$= \frac{\alpha_s(Q^2)}{(2\pi)^2}\int_0^1 dx_p \int_0^1 dz_p \, 2^{m-1} x_p^{n-\frac{m+1}{2}} z_p^k [z_p(1-z_p)(1-x_p)]^{\frac{m-1}{2}} W^{I,II} \hat{D}_{<u>}^{(\pi^+-\pi^-)}(m+k,Q^2). \quad (31)$$

One point which we should like to note is that, from the definitions, only when $n \geq 2 + m/2$, are the generalized moments calculable.

3. DISCUSSION

The reason that we introduce the generalized moments based on the differential cross-section is that it can be compared to experimental data more easily. For E_ν and Q^2 given, the mean value of $x^{n-1}z_{out}^{m-1}$, $<x^{n-1}z_{out}^{m-1}>^{I,II}_{ns}$, is

$$\int_{x_{min}}^1 dx \int_0^1 dz \int dP_T' \int d\Phi \, x^{n-1} z_{out}^{m-1} \frac{d\sigma^{I,II}(ns)}{dx\,dz\,dQ^2\,dP_T'\,d\Phi}$$

where $x_{min} = Q^2/2ME_\nu$. It can be measured directly. Considering the differential cross-section property in the $x \sim 0$ region, $<x^{n-1}z_{out}^{m-1}>$ is insensitive to X_{min} when $X_{min} << 1$ and $n > 2 + m/2(m>1)$. So we have

$$E^{I,II}(n,m,Q^2) \sim \left(\frac{G_F}{2\pi}\right)^{-1} < x^{n-1} z_{out}^{m-1} >^{I,II}_{ns} , \quad (32)$$

if E_ν is very large, say $E_\nu \sim 150$ GeV, and $1 \text{ GeV}^2 \leq Q^2 \leq 20 \text{ GeV}^2$ when $n > 2 + m/2$. Similarly, we have

$$H^{I,II}(n,m,k,Q^2) \sim \left(\frac{G_F^2}{2\pi}\right)^{-1} < x^{n-1} z_{out}^{m-1} z^k >^{I,II}_{ns} . \quad (33)$$

To study the behaviour of the generalized moments (20) and (31), we choose as an example $E_\nu = 150$ GeV and use the latest results about moments, $\hat{F}_V(n,Q^2)$ and $\hat{D}_{<u>}^{\pi^+-\pi^-}(m,Q^2)$:

$$\hat{F}_V(n,Q^2) = \frac{3}{2}\cdot\frac{V_n}{(\ln Q^2/\Lambda^2)^{d_n}} \quad (34)$$

$$\hat{D}_{<u>}^{(\pi^+-\pi^-)}(m,Q^2) = \frac{C_m}{(\ln Q^2/\Lambda^2)^{d_m}} \quad (35)$$

the anomalous dimensions

$$d_n = \frac{4}{33-2f}\left[1 - \frac{2}{n(n+1)} + 4\sum_2^n \frac{1}{j}\right] \quad (36)$$

and Λ = .75 GeV, the constants V_n and C_m are measured experimentally (see Table)[5]. We calculate some curves of $E^{I,II}(n,m,Q^2)$ and $H^{I,II}(n,m,k,Q^2)$ (Figs. 4 - 9). From these curves we can see that $E^{I}(n,m,Q^2)$ and $H^{I}(n,m,k,Q^2)$ decrease very quickly when Q^2 increases in the region $Q^2 \sim 20$ GeV because the sign of the last term in (21) is minus, so that to use $E^{II}(n,m,Q^2)$ and $H^{II}(n,m,k,Q^2)$ to test QCD is better than $E^{I}(n,m,Q^2)$ and $H^{I}(n,m,k,Q^2)$ because when $E(n,m,Q^2)$ and $H(n,m,k,Q^2)$ are very small, the non-perturbative effects become more important.

There is no free parameter in $E(n,m,Q^2)$ and $H(n,m,k,Q^2)$ if we input the moments $\hat{F}_v(n,Q^2)$ and $D_{<u>}^{(\pi^+-\pi^-)}(m,Q^2)$, so when fitting data, it is meaningful to study not only the Q^2 dependence, but also the normalization.

Only the non-singlet case is discussed because only the non-singlet moments $\hat{F}_v(n,Q^2)$ and $D_{<u>}^{(\pi^+-\pi^-)}(m,Q^2)$ are measured well. For the singlet case, it is easy to obtain the corresponding generalized moments; this is a trivial generalization but it can test the contribution from Fig. 3b and 3c as well as from Fig. 3a.

As pointed out above, one needs to know the energy of the incoming lepton when fitting experiments, so for a $\nu(\tilde{\nu})$ bubble chamber experiment we must pick out the events with a fixed, high energy of the $\nu(\tilde{\nu})$ from others for a wide-band beam experiment or, alternatively, one can use a narrow-band beam. It is difficult to collect many events, although it is possible in principle. It seems that, using the generalized moments to test QCD for three jet processes, the most accessible experiments are muon deep inelastic scattering experiments, semi-inclusive or ideally exclusive ones, namely, ones in which every outgoing hadron momentum is measured. In the case of a muon beam the above formula should be changed a little bit. For example, we only have case II for non-singlet case and

TABLE

N	2	3	4	5	6	7
V_N	0.519 ±0.029	0.167 ±0.008	0.073 ±0.004	0.040 ±0.002	0.024 ±0.002	0.015 ±0.001
C_N	0.318 ±0.014	0.209 ±0.011	0.158 ±0.010	0.132 ±0.009	0.120 ±0.00[illegible]	0.112 ±0.008

$$U^{II}_{(\mu)} = \frac{1}{9}\left[\underset{\sim}{A}_1 + \underset{\sim}{B}_1 \cos\Phi + \underset{\sim}{C}_1 \cos 2\Phi\right] \tag{37}$$

instead of (17). Also, we now have

$$E^{II}_{(\mu)}(n, m, Q^2) \equiv (Q^2)^2 \left(\frac{2\pi}{\alpha^2}\right) \int_0^1 dx \int_0^1 dz \int dP'_T \int d\Phi\, x^{n-1} z_{out}^{m-1} \frac{d\sigma^{II}_{(n.s)}}{dx\, dz\, dQ^2\, dP'_T\, d\Phi} \tag{38}$$

instead of (18) and

$$E^{II}_{(\mu)}(n, m, Q^2) \sim \langle [Q^2]^2 x^{n-1} z_{out}^{m-1} \rangle^{II}_{(\mu)} \cdot \left(\frac{2\pi}{\alpha^2}\right) \tag{39}$$

instead of (32). Similarly, for the "triple" moments, we have

$$H^{II}_{(\mu)}(n, m, k, Q^2) \sim \langle [Q^2]^2 x^{n-1} z_{out}^{m-1} z^k \rangle^{II}_{(\mu)} \cdot \left(\frac{2\pi}{\alpha^2}\right). \tag{40}$$

ACKNOWLEDGEMENTS

We are grateful to J. Ellis for stimulating and valuable discussions and suggestions, to W.G. Scott for private communications and discussions and to C. Sachrajda for reading the manuscript and making suggestions. We would like to thank E. Simopoulou for information on the situation of the neutrino experiments.

REFERENCES

1) J. Ellis, M.K. Gaillard and W.J. Zakrzewski, Phys. Lett. 81B (1979) 224.

2) Yu.L. Dokshitzer et al., Proc. XIII Winter School of the Leningrad B.P. Konstantinov Institute of Nuclear Physics (Leningrad, 1978);
C.H. Llewellyn Smith, Oxford preprint 47/78, Lectures at Schlamding (1978);
G. Altarelli and G. Martinelli, Nucl. Phys. B126 (1977) 298;
G. Sterman and S. Weinberg, Phys. Rev. Lett. 39 (1977) 1436.

3) H. Georgi and H.D. Politzer, Phys. Rev. Lett. 40 (1978) 3;
E.G. Floratos, Nuovo Cimento 43A (1978) 241;
G. Altarelli and G. Martinelli, Phys. Lett. 76B (1978) 89;
P. Binétruy and G. Girardi, CERN preprint TH. 2611 (1979).

4) A. Méndez, Nucl. Phys. B145 (1978) 199.

5) W.G. Scott, private communication.

Figure captions

Fig. 1 : Configuration of the semi-inclusive process $\ell N \to \ell h X$ and the definitions of momenta in lab. system.

Fig. 2 : The lowest perturbative QCD diagram.

Fig. 3 : Perturbative QCD diagrams:
a) a harder gluon bremsstrahlung and the hadron fragmentation from the quark;
b) a harder gluon bremsstrahlung and the hadron fragmentation the gluon;
c) a hard pair production from a gluon inside N and the hadron fragmentation from the quarks.

Fig. 4 : Some curves of the double moment $E^{I}(n,m,Q^2)$.

Fig. 5 : Some curves of the double moment $E^{I}(n,m,Q^2)$.

Fig. 6 : Some curves of the triple moment $H^{I}(n,m,k,Q^2)$.

Fig. 7 : Some curves of the douhle moment $E^{II}(n,m,Q^2)$.

Fig. 8 : Some curves of the double moment $E^{II}(n,m,Q^2)$.

Fig. 9 : Some curves of the triple moment $H^{II}(n,m,k,Q^2)$.

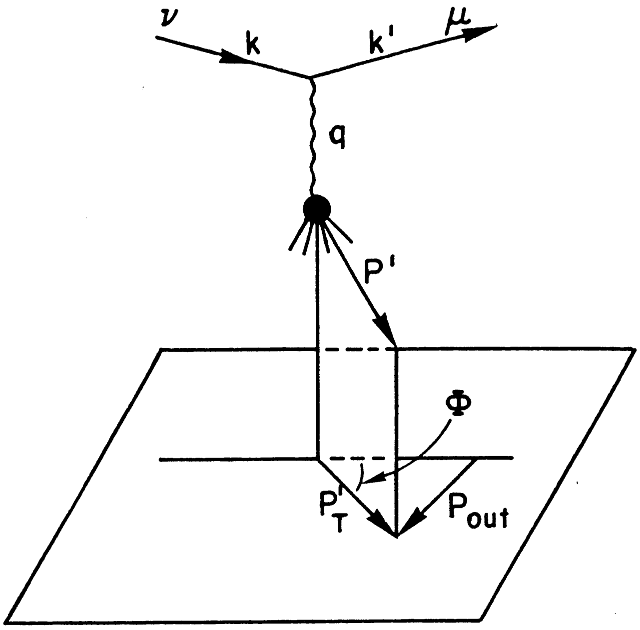


FIG. 1

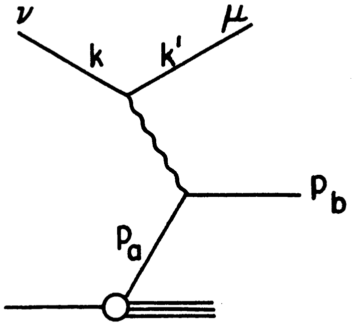


FIG. 2

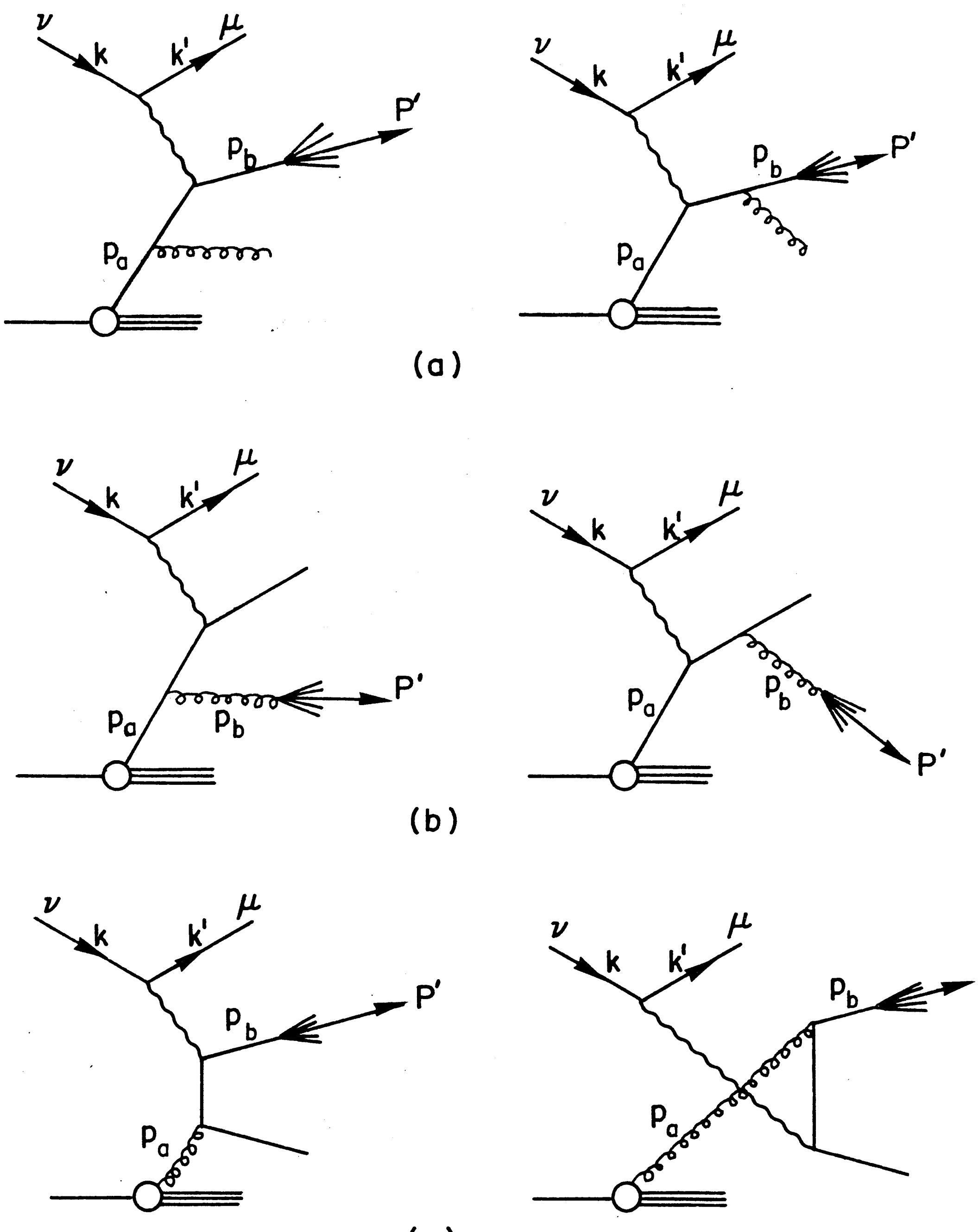
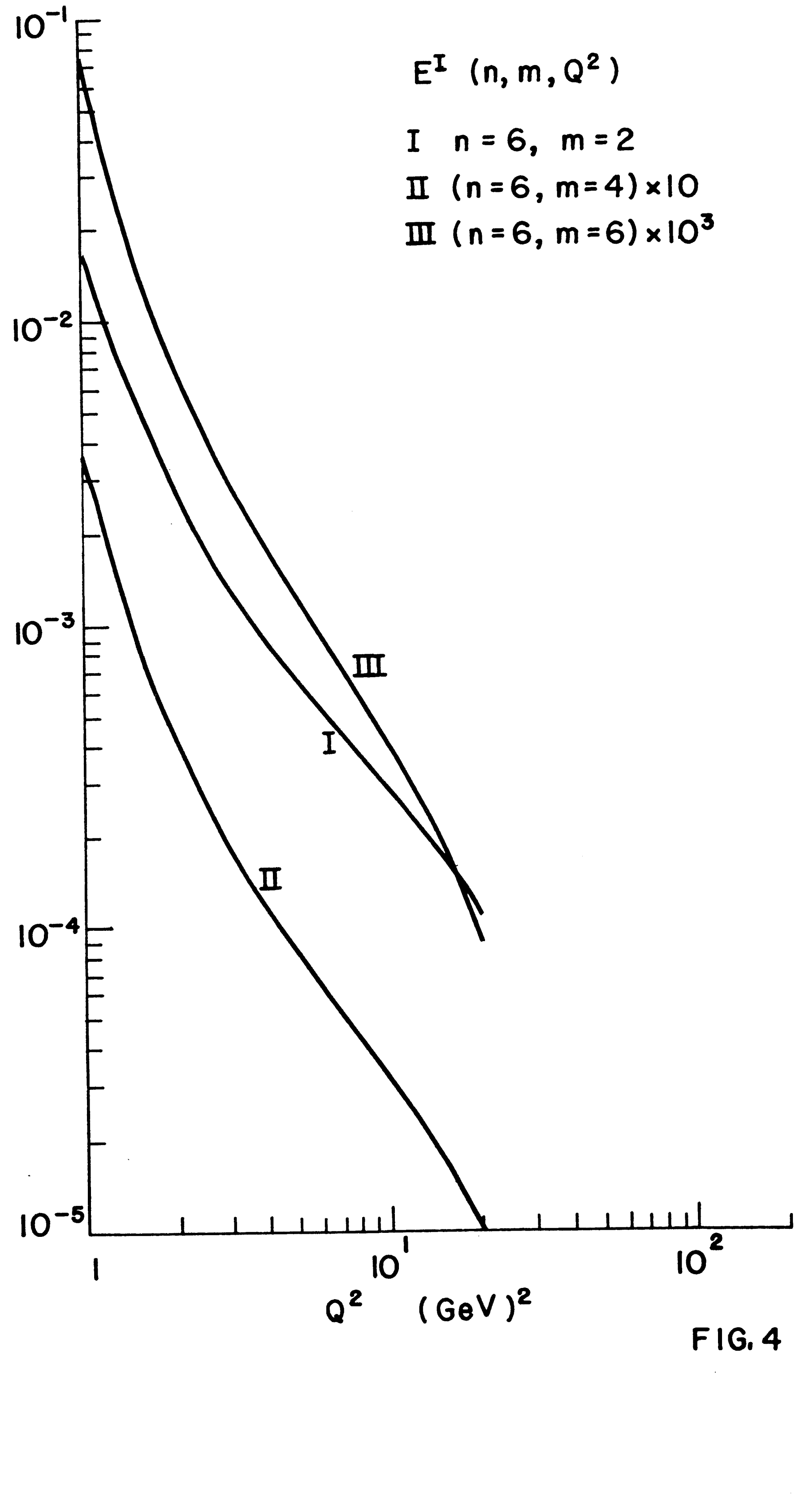
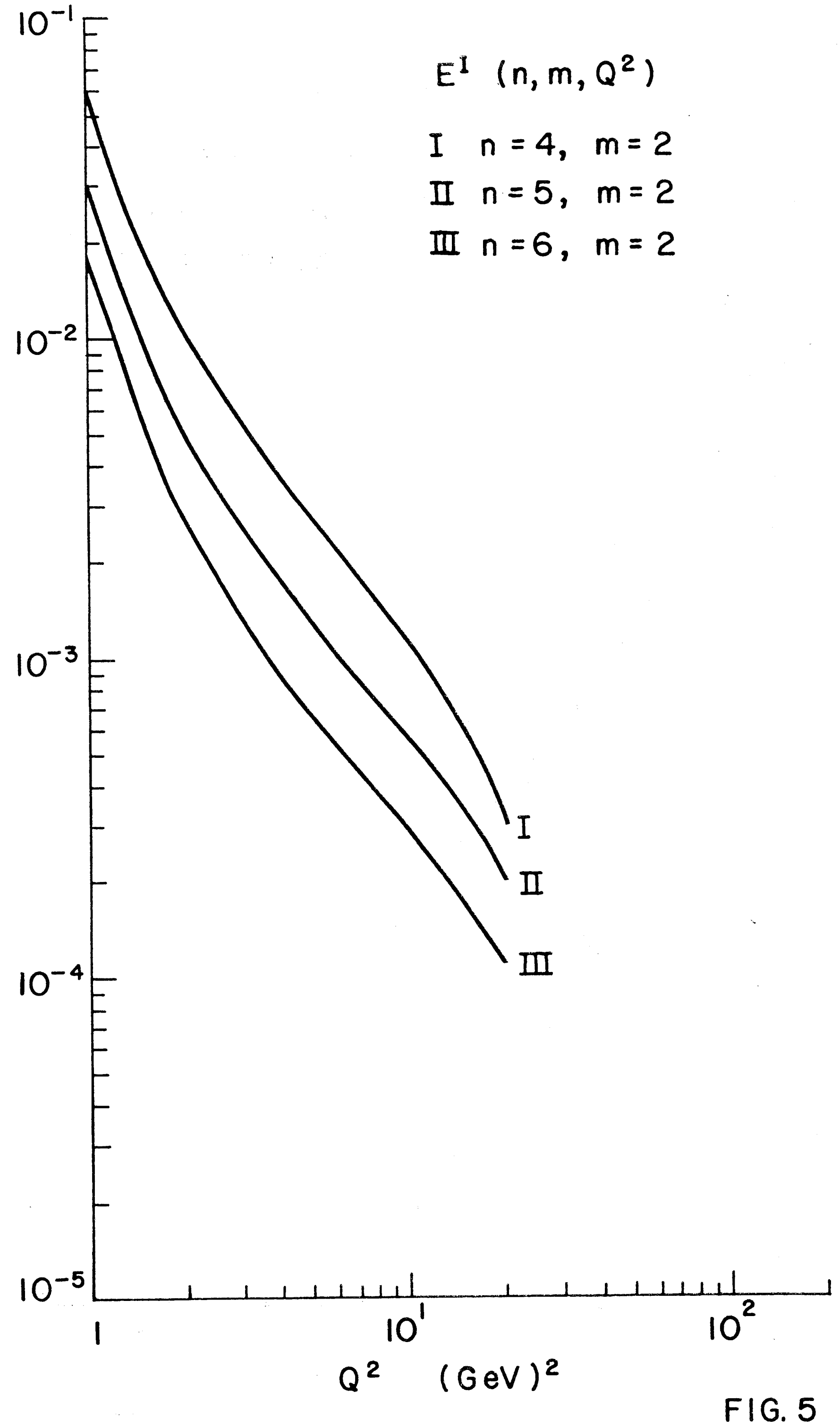


FIG. 3



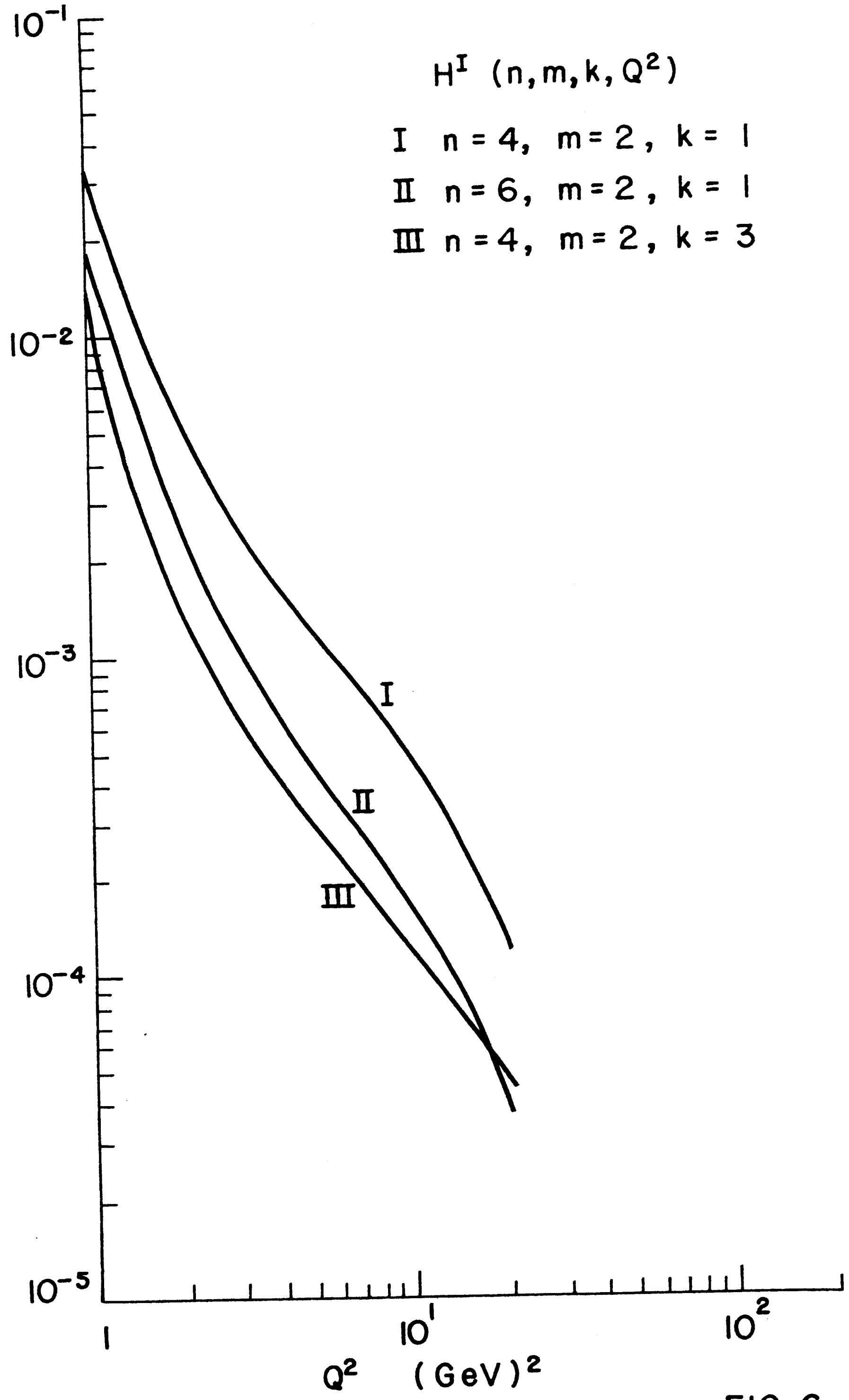
$E^I(n, m, Q^2)$
I n = 6, m = 2
II (n = 6, m = 4) × 10
III (n = 6, m = 6) × 10³
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10^1
10^2
Q^2 (GeV)²
I
II
III

FIG. 4



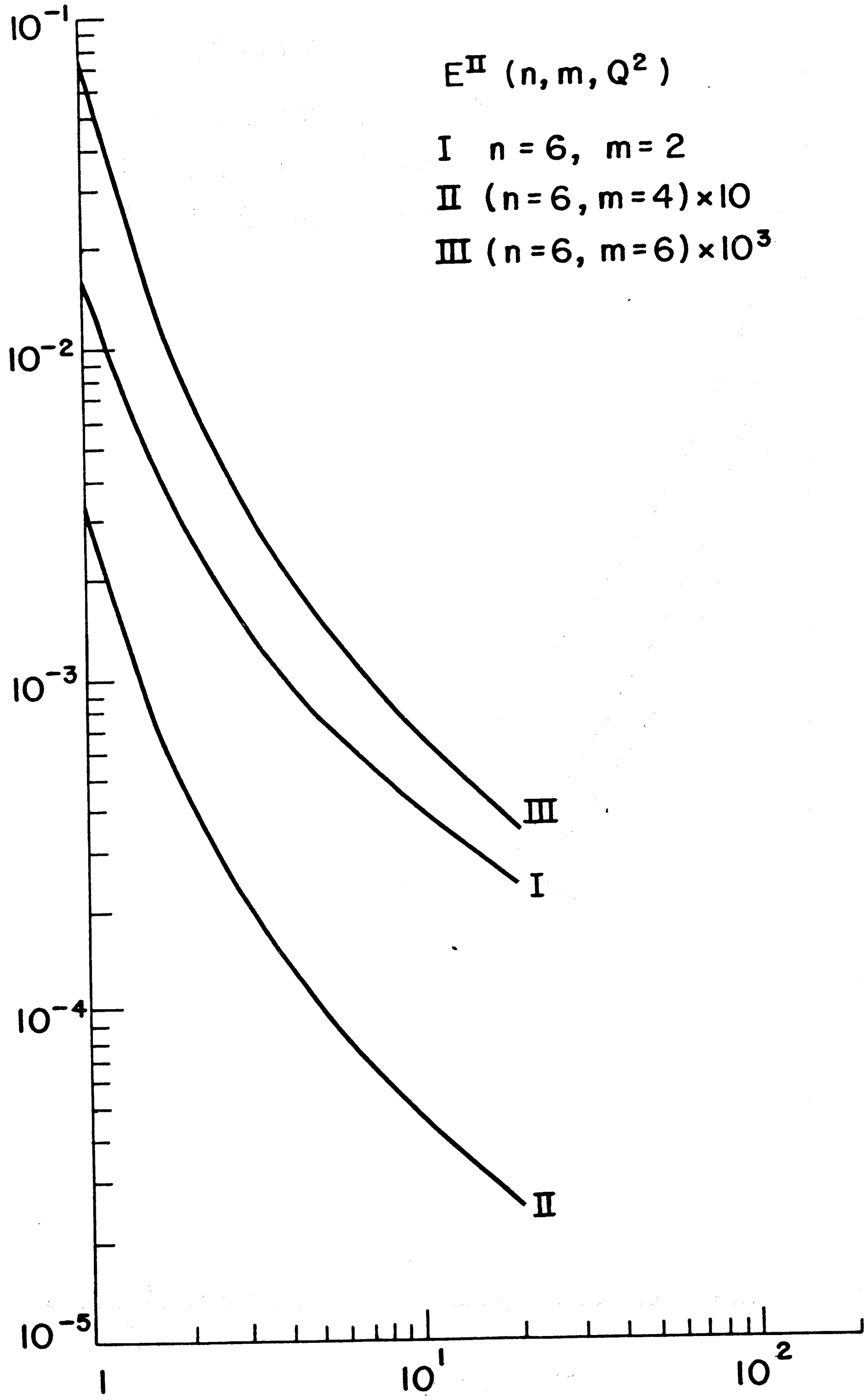
$E^{I}(n, m, Q^2)$
I n = 4, m = 2
II n = 5, m = 2
III n = 6, m = 2
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10^{1}
10^{2}
Q^2 (GeV)^2
I
II
III

FIG. 5



$H^{I}(n,m,k,Q^2)$
I n = 4, m = 2, k = 1
II n = 6, m = 2, k = 1
III n = 4, m = 2, k = 3
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
I
II
III
1
10^{1}
10^{2}
Q^2 (GeV)^2

FIG. 6



10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$E^{II}(n, m, Q^2)$
I n = 6, m = 2
II (n = 6, m = 4) × 10
III (n = 6, m = 6) × 10^3
III
I
II
1
10^1
10^2
Q^2 (GeV)^2

FIG. 7

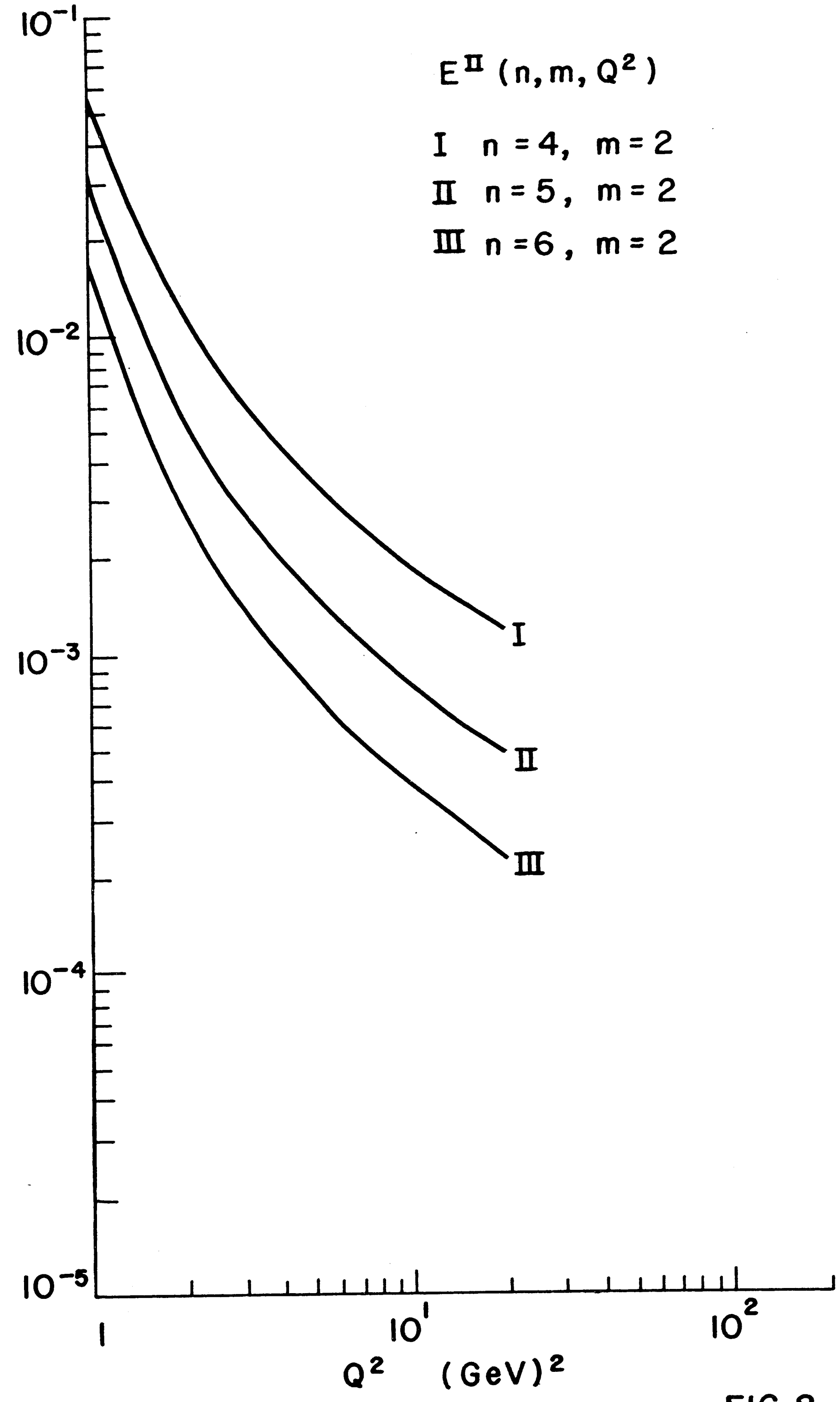

$E^{II}(n, m, Q^2)$
I n = 4, m = 2
II n = 5, m = 2
III n = 6, m = 2
I
II
III
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10^{1}
10^{2}
Q^2 (GeV)^2

FIG. 8

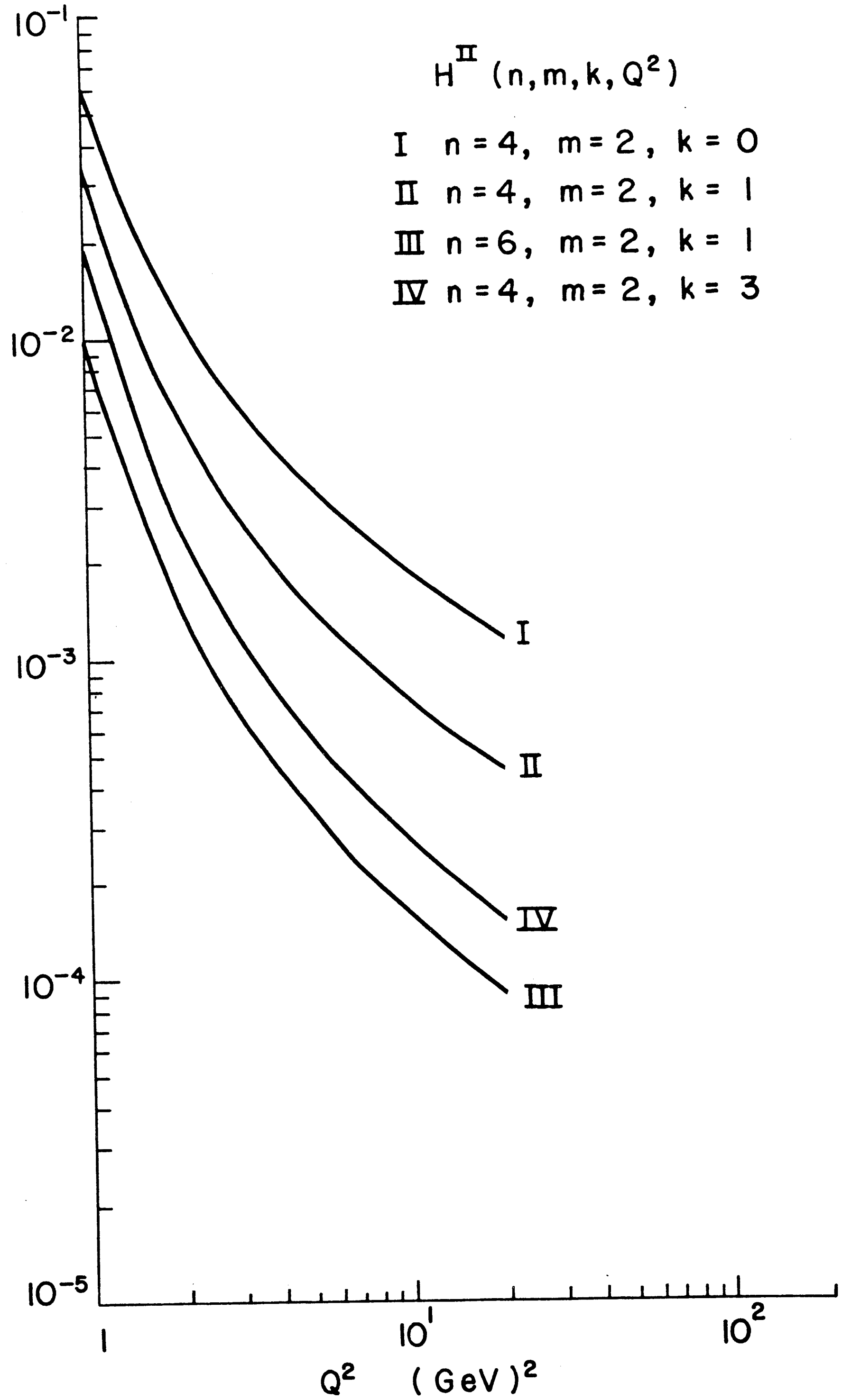

$H^{II}(n,m,k,Q^2)$
I n = 4, m = 2, k = 0
II n = 4, m = 2, k = 1
III n = 6, m = 2, k = 1
IV n = 4, m = 2, k = 3
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
I
II
IV
III
1
10^1
10^2
Q^2 $(GeV)^2$

FIG. 9