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A B S T R A C T

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*) On sabbatical leave from the University of California, Berkeley.

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HADRONIC WIDTHS IN CHARMONIUM

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Abstract: Theoretical estimates for the hadronic widths of the s-wave and p-wave states in charmonium (based on lowest order perturbation theory in QCD) are compared with indirect inferences from experiment. For the p-states, the expected ratios of 15:$\sim$ 1:4 for $J^{PC} = 0^{++}, 1^{++}, 2^{++}$, respectively, are in accord with the present, rather crude experimental evidence, and the theoretically less reliable absolute magnitudes are within a factor of three or four. For the pseudoscalar ($\ell = 0$) states the situation is less clear. Identification of the $\chi(3454)$ as η_c' causes difficulties for QCD.

Résumé : On compare aux indications expérimentales indirectes les estimations théoriques des largeurs hadroniques des états du charmonium de moment angulaire orbital $\ell = 0$ et $\ell = 1$. Pour $\ell = 1$, les rapports calculés sont de 15:$\sim$ 1:4 pour $J^{PC} = 0^{++}, 1^{++}, 2^{++}$ respectivement, ce qui est en accord avec les données expérimentales, d'ailleurs assez grossières. Les valeurs absolues de ces largeurs, pour lesquelles le calcul théorique est moins sûr, sont correctes à un facteur trois ou quatre près. La situation des états pseudoscalaires n'est pas aussi claire. L'identification de l'état $\chi(3454)$ avec η_c' cause quelques difficultés à la chromodynamique quantique.

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1. INTRODUCTION

The presently known states of charmonium below the charm threshold of approximately 3.8 GeV are shown in Fig. 1. The two $J = 1^{--}$ states, $\psi(3095)$ and $\psi'(3684)$, are well established in all their quantum numbers. The existence of the three states called $\chi(3414)$, $\chi(3508)$ and $\chi(3552)$ is beyond doubt; the assignments $J^{PC} = 0^{++}, 1^{++}, 2^{++}$ are consistent with all observations and are more or less strongly implied[1]. The remaining two states, X(2.83) and $\chi(3454)$, have been more firmly established recently[2,3], but so far have no quantum numbers assigned, apart from C = +1. The number of states and the rough ordering are just what is expected from a confined positronium-like spectrum, with n = 1 and n = 2 $^1S_0$ and $^3S_1$, plus n = 1 ^3P_J states (the $^1P_1$ state is reachable from the ψ' only via two-photon emission and is therefore expected to be seen only with difficulty). We adopt this interpretation throughout. Only in discussing the $\chi(3454)$ do we mention another possibility, still within the "positronium" framework.

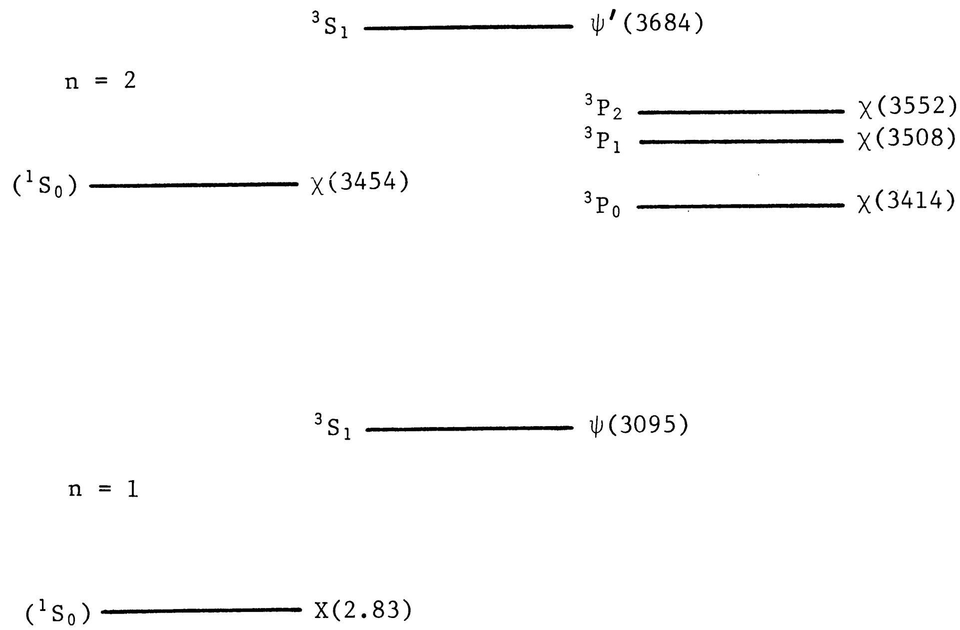

Fig. 1 Energy levels of charmonium

The basic properties of the spectrum, the size of the wave functions, and the magnitude of matrix elements not involving spin-flip can all be understood in terms of almost anyone's non-relativistic Schrödinger-equation description of a massive charmed quark-antiquark pair ($\bar{c}c$). Relativistic corrections (Fermi hyperfine interaction, tensor force, etc.) are less satisfactorily given, with the observed $^3S_1$-$^1S_0$ splitting being a factor of three to five larger than expected, but that is another subject.

Our concern is the annihilation of these states into ordinary hadrons within the framework of QCD, a non-Abelian gauge theory of flavourful quarks interacting via colourful, massless, vector gluons. As is well known, such a theory possesses "asymptotic freedom", that is, the property that the effective coupling constant g(p) becomes smaller, the larger is the energy or momentum scale (p) being considered. By uncertainty principle arguments this leads to the hope that at short distances of separation of the quarks the coupling constant $\alpha_s = g^2/4\pi$ will be small enough to permit the use of perturbation theory. This hope is the basis of the terminology "charmonium", and the discussion of annihilation of the new particles into ordinary hadrons in analogy to the annihilation of positronium into photons.

Very much in the same spirit as the calculation of the expected value of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma_{QCD}(e^+e^- \to \mu^+\mu^-)$ from the materialization of the virtual photon into a quark-antiquark pair, without inquiry into the messy details of how the quarks manage to convert themselves into ordinary hadrons, the annihilation of a ($\bar{c}c$) bound state is estimated in QCD by calculating the rate of transformation into the minimum number of gluons permitted by selection rules. The assumption is that the gluons are sufficiently close to their mass shell (even though they never escape) that the subsequent conversion into ordinary hadrons occurs with unit probability. This is certainly an act of faith that can be challenged, but the plateaux seen in R as a function of energy indicate that this basic philosophy may not lead to gross error in estimating total rates of annihilation into hadrons[4].

2. S-STATE ANNIHILATION

For $^1S_0$ and $^3S_1$ positronium states, the lowest order annihilation is into two and three photons, respectively. The rates are[5]

$$\Gamma(^1S_0 \to \gamma\gamma) = \frac{4\alpha^2}{M^2}|R(0)|^2 \tag{1}$$

$$\Gamma(^3S_1 \to \gamma\gamma\gamma) = \frac{16}{9\pi}(\pi^2-9)\frac{\alpha^3}{M^2}|R(0)|^2 \tag{2}$$

where $\alpha = 1/137$, M is the mass of the decaying system and R(r) is the s-wave radial wave function of the bound state. For bound states of quark-antiquark these decay rates into *photons* must be multiplied by a colour factor of three and a factor of e_Q^2 for each power of α.

Another rate of interest is the annihilation of a massive charged fermion-antifermion bound state into e^+e^-. For quarks with colour, this rate is

$$\Gamma_e = \frac{4\alpha^2 e_Q^2}{M^2} |R(0)|^2 \tag{3}$$

The transcription of the above *positronium* formulae, (1) and (2), to $q\bar{q} \to$ $\to$ gluons involves certain factors arising from the non-Abelian nature of QCD, namely the traces of the product of n SU(3) matrices $(\lambda^a/2)$. For n = 2, the recipe is $\alpha^2 \to 2\alpha_s^2/3$; for n = 3 it is $\alpha^3 \to 5\alpha_s^3/18$. Here $\alpha_s = g^2/4\pi$ is the QCD "fine structure" constant. For a bound $^3S_1$ $c\bar{c}$ state such as $\psi(3095)$, the direct annihilation into hadrons is thus assumed to be given by

$$\Gamma_{direct}(\psi \to h) \simeq \Gamma(\psi \to ggg) = \frac{40}{81\pi}(\pi^2 - 9)\frac{\alpha_s^3}{M^2}|R(0)|^2 \tag{4}$$

The coupling of ψ to e^+e^- is given by Eq. (3). The ratio of Eqs. (4) and (3), experimentally determined to be 10 ± 1.5 [1], is

$$\frac{\Gamma_{direct}(\psi \to h)}{\Gamma_e(\psi)} = \frac{5(\pi^2-9)}{18\pi}\frac{\alpha_s^3}{\alpha^2}\left[\frac{2/3}{e_Q}\right]^2$$

with the conventional assumption, $e_Q = 2/3$, the experimental ratio yields the QCD coupling constant,

$$\alpha_s = 0.187 \pm 0.10$$

The error in α_s is just from the experimental error in the widths -- the systematic (theoretical) error is unknown! We can be gratified that $\alpha_s \simeq 0.2$ for systems of $M \simeq 3^+$ GeV. So far, nothing is proved about QCD, but at least is has not been shown to be patently ridiculous.

In passing we note that we can now estimate various rates for the pseudoscalar partner of the ψ, and also, from the observed ratio of Γ_e's for ψ' and ψ, for the $\psi'(3684)$ and its pseudoscalar partner:

$$\Gamma(P \to \gamma\gamma) = \frac{4}{3}\left|\frac{R_P(0)}{M_P}\right|^2 \cdot \left|\frac{M_V}{R_V(0)}\right|^2 \cdot \Gamma_e(V) \tag{6}$$

$$\Gamma(P \to h) = \frac{9}{8}\left(\frac{\alpha_s}{\alpha}\right)^2 \cdot \Gamma(P \to \gamma\gamma) \simeq 738\,\Gamma(P \to \gamma\gamma) \tag{7}$$

$$\Gamma_{direct}(V' \to h) = \frac{\Gamma_e(V')}{\Gamma_e(V)} \cdot \Gamma_{direct}(V \to h) \tag{8}$$

In Eqs. (6) and (7), P and V stand for the pseudoscalar and vector partners (η_c,ψ or η_c',ψ'). From Eq. (8) and the observed $\Gamma_e(\psi') = 2.1$ keV, we conclude that $\Gamma_{direct}(\psi' \to h) \simeq 21$ keV, implying that only $21/228 \simeq 9\%$ of ψ' decays go directly into ordinary hadrons. This is quite consistent with the book-keeping of the various ψ' decays[1].

Identifying X(2.83) as η_c, Eqs. (6) and (7) give $\Gamma(X \to \gamma\gamma) \simeq 6$-$8$ keV, $\Gamma_t(X) \simeq 4.7$-5.7 MeV, depending on whether one takes $|R_p(0)/M_p| = |R_V(0)/M_V|$ or only $|R_p(0)| = |R_V(0)|$. The value $1/738 = 1.36 \times 10^{-3}$ for the branching ratio of $\eta_c \to \gamma\gamma$ is a firmer QCD expectation. For the η_c', the widths are reduced by a factor of ~ 0.44, modulo ratios of masses squared, from $\Gamma_e(\psi')/\Gamma_e(\psi)$. The present experimental situation on these widths for η_c and η_c' are discussed below.

3. P-STATE ANNIHILATION IN QCD

An opportunity for a *test* of the ideas of QCD, rather than merely a determination of parameters, is provided by the hadronic widths of the p-states[6,7] [χ(3414), χ(3508), and χ(3552) of Fig. 1]. For the $^3P_0(0^{++})$ and $^3P_2(2^{++})$ states, the lowest order annihilation is into two gluons, as indicated in Fig. 2a, while for the $^3P_1(1^{++})$ and the $^1P_1(1^{+-})$, the three-gluon or single-gluon plus light $q\bar{q}$ pair processes of Fig. 2b presumably dominate. Since the even J rates are proportional to α_s^2, while the J = 1 rates are proportional to α_s^3, and $\alpha_s \simeq 0.2$, we expect χ(3508) to show up more strongly in the cascade transitions, $\psi' \to \gamma_1\chi$, $\chi \to \gamma_2\psi$, and less strongly in the transitions, $\psi' \to \gamma_1\chi$, $\chi \to$ hadrons, compared with χ(3414) and χ(3552). This is indeed what is observed[1], and forms a qualitative success for the ideas of QCD.

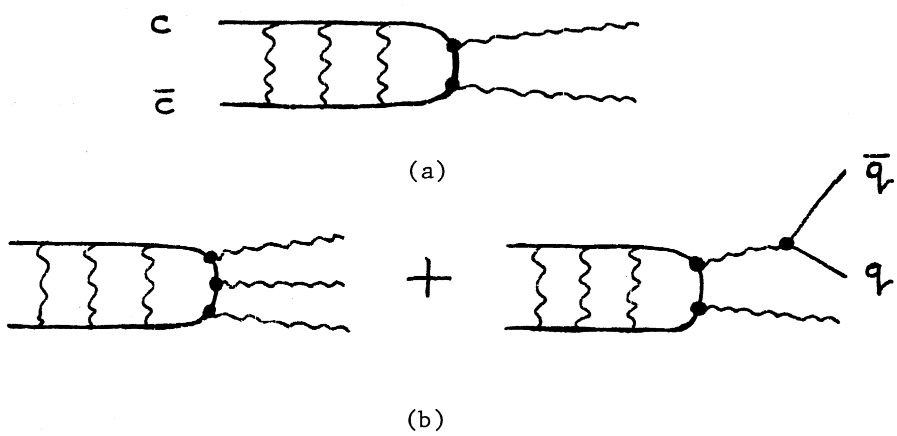

Fig. 2 a) Two-gluon annihilation for $^3P_0$ and $^3P_2$, b) Three-gluon or $q\bar{q}$-gluon annihilation of $^3P_1$ and $^1P_1$

The test can be made more quantitative[6,7]. First of all, the relative values of the hadronic widths of the p-states can be computed. Then, with less reliability, the absolute magnitudes can be determined from the properties of the bound-state wave functions. We sketch the basic idea for the two-gluon annihilation of the 0^{++} and 2^{++} states. For an exothermic reaction proceeding via the s-wave, it is well known that near threshold in the entrance channel the cross-section behaves as

$$\sigma_0 = \frac{1}{v} \times (\text{constant})$$

where v is the relative velocity (flux factor) and the constant is the transition probability at threshold, for normalization of one particle per unit volume in the incident beam. For higher partial waves, barring some peculiarity, one has

$$\sigma_\ell = \frac{1}{v} \cdot p^{2\ell} \times (\text{constant})$$

where p is the c.m.s. momentum in the entrance channel. For the same process proceeding from a weakly bound state of the incident pair, it can be shown[5] that the transition probability is given in first approximation by

$$\Gamma = \mathcal{S} \cdot \lim_{v\to 0}\left[\frac{v\sigma_\ell}{p^{2\ell}}\right] C_\ell \left| \frac{\partial^\ell R_\ell(r)}{\partial r^\ell}\bigg|_{r=0}\right|^2 \tag{9}$$

where $\mathcal{S}$ is a statistical factor that may occur because the spin and orbital angular population of the incident beam may differ from that of the bound state in question, C_ℓ is a numerical factor, and $R_\ell(r)$ is the unperturbed radial wave function of the bound state of angular momentum ℓ. For s-waves, $\ell = 0$, $C_0 = 1/4\pi$, and $\Gamma = \lim_{v\to 0}(v\sigma_0)|\psi(0)|^2$, the familiar result that led to Eqs. (1) to (3).

The procedure is thus to compute $\sigma_\ell(p)$, isolate the $p^{2\ell}$ dependence at threshold, and hence find $\mathcal{S}(v\sigma_\ell/p^{2\ell})$. The relative values for different J (but the same ℓ) will then give the ratios of widths for the exothermic process. For ${}^3P_J \to gg$, we have $\ell = 1$ and $J = 0^{++}, 2^{++}$. For the collision process, $c\bar{c} \to g_1 g_2$, we examine the helicity amplitude $\langle \lambda_1\lambda_2|\mathcal{M}|\tfrac{1}{2},\tfrac{1}{2}\rangle$. Calculation shows that at threshold, the two independent amplitudes are

$$\frac{1}{p}\langle|\mathcal{M}|\rangle = \begin{cases} -C & \text{for } \lambda_1 = \lambda_2 \\ +C\sin^2\theta & \text{for } \lambda_1 = -\lambda_2 \end{cases} \tag{10}$$

where θ is the angle of the momentum of one gluon relative to the beam direction. Since helicity amplitudes have the angular variation $d^J_{\lambda\mu}(\theta)$, $\lambda = \lambda_1 - \lambda_2$, $\mu = \lambda_c - \lambda_{\bar{c}}$, we expect $d^J_{00}(\theta)$ for $\lambda_1 = \lambda_2$ and $d^J_{20}(\theta)$ for $\lambda_1 = -\lambda_2$. Inspection of a table of $d^J_{MM'}$ shows that the matrix element (10) has $J = 0$ for $\lambda_1 = \lambda_2$ and $J = 2$ for $\lambda_1 = -\lambda_2$.

We can now compute the ratio of widths for the 2^{++} and 0^{++} states to annihilate into two gluons. There are two factors: (i) the integration over angles in the final state, (ii) the different probabilities of the incident beam of $c\bar{c}$ to have J = 0 and J = 2:

$$\text{Final state ratio} = \frac{\int d\Omega \sin^4\theta}{\int d\Omega} = \frac{32\pi/15}{4\pi} = \frac{8}{15}$$

The initial state of the $c\bar{c}$ plane wave with $\lambda_c = \lambda_{\bar{c}}$ and $\ell = 1$, S = 1 has a spin-angular wave function,

$$Y_{10}\chi_{10} = -\frac{1}{\sqrt{3}}(J=0) + \sqrt{\frac{2}{3}}(J=2)$$

The ratio of J = 2 to J = 0 in the incident beam is thus 2:1. For pure J = 2 or pure J = 0 bound states, we must therefore replace the "collision" matrix elements (10) by $\sqrt{3}C$ and $-\sqrt{3/2}C\sin^2\theta$ for J = 0 and J = 2, respectively. This introduces a relative factor of 1/2 in the ratio of widths. Thus we obtain finally,

$$\frac{\Gamma(2^{++}\to gg)}{\Gamma(0^{++}\to gg)} = \frac{8}{15}\cdot\frac{1}{2} = \frac{4}{15} \tag{11}$$

This result is fairly specific to QCD, with two massless vector gluons in the final state. If, for example, the gluons were scalar, the matrix element (10) would be replaced by the single amplitude, $C_s(\cos^2\theta - 2)$. This is a mixture of J = 0 and J = 2 Legendre polynomials in the ratio 5:2 in amplitude, leading to a final state ratio of

$$\left(\frac{2}{5}\right)^2 \frac{\int d\Omega\, P_2^2}{\int d\Omega\, P_0^2} = \left(\frac{2}{5}\right)^2 \frac{4\pi/5}{4\pi} = \frac{4}{125}$$

Scalar gluons would thus lead to 2/125 for the ratio in Eq. (11).

Keeping track of all the factors leads to the absolute rate for the 0^{++} p-state[6],

$$\Gamma(0^{++}\to gg) = \frac{96\alpha_s^2}{M^4}|R'(0)|^2 \tag{12}$$

where $R'(0)$ is the derivative of the p-state radial wave function, evaluated at the origin.

For the J = 1 states ($^3P_1$ and $^1P_1$), annihilation proceeds as in Fig. 2b. Here things are somewhat trickier than for the even J states. For both ggg and $g(q\bar{q})$ final states, there occurs a singularity at zero binding, i.e. the matrix element for the collision, divided by p, is not analytic for $p \to 0$. Our simple argument must be augmented by more elaborate considerations[7]. An approximate result for the annihilation rate is

$$\Gamma \simeq N \frac{\alpha_s^3}{M^4} |R'(0)|^2 \ln\left(\frac{4m_c^2}{4m_c^2 - M^2}\right) \tag{13}$$

where

$$N = \begin{cases} 128/3\pi = 13.6 & \text{for } ^3P_1 \\ 320/9\pi = 11.3 & \text{for } ^1P_1 \end{cases}$$

[$g(q\bar{q})$ dominates the $^3P_1$ rate, while ggg dominates $^1P_1$]. The presence of the logarithm is a signal that Eq. (13) is more approximate than Eq. (12), even granting all the QCD assumptions. Terms of order one relative to the logarithm have been neglected. The logarithm can be estimated in a number of different ways. One is to say that the binding energy is of the order of $\alpha_s^2 m_c$. Then

$$\ln\left(\frac{4m_c^2}{4m_c^2 - M^2}\right) \simeq 2\ln\left(\frac{1}{\alpha_s}\right) \simeq 3.3$$

Another is to say that the binding energy is the difference between the charm threshold of $\sim$ 3.75 GeV and the χ_1 mass of 3.51 GeV. This gives 2.1. The ratio of $J = 1^{++}$ annihilation into hadrons to $J = 2^{++}$ annihilation into hadrons is thus, from Eqs. (12) and (13),

$$\frac{\Gamma(1^{++} \to h)}{\Gamma(2^{++} \to h)} \simeq \frac{13.6\,\alpha_s \ln(\ )}{\frac{4}{15} \cdot 96} \simeq 0.10 \ln(\ ) \simeq 0.3$$

The QCD prediction for the relative values of the hadronic widths of the triplet p-states of charmonium is therefore

$$\Gamma_h(0^{++}) : \Gamma_h(1^{++}) : \Gamma_h(2^{++}) \simeq 15 : \sim 1 : 4 \tag{14}$$

Much less reliable are the actual values of the hadronic widths from Eqs. (12) and (14). With $\alpha_s = 0.187$ and the favourite wave functions of Ref. 6, one finds

$$\Gamma_h(0^{++}) \simeq 2.2 \text{ MeV} \tag{15}$$

Other plausible estimates of the derivative of the p-state wave function at $r = 0$ give values from 1.4 to 1.8 MeV. Since $|R'(0)|^2$ depends on the inverse length scale parameter ξ as ξ^5, and is also sensitive to the detailed shape of the potential, the value (15) should be taken only as indicative of the general order of magnitude.

4. COMPARISON WITH EXPERIMENTS FOR THE P-STATE

We are now ready for a confrontation with experiment. There are data on the branching ratios B_1 for $\psi' \to \gamma_1\chi_J$ for the ^3P_J final states[3,8] and also on the products B_1B_2 for the cascade $\psi' \to \gamma_1\chi_J \to \gamma_1\gamma_2\psi$[3], as shown in Fig. 3. From these data, the branching ratio $B_2(J)$ for $\chi_J \to \gamma_2\psi$ can be deduced for each of the three ^3P states.

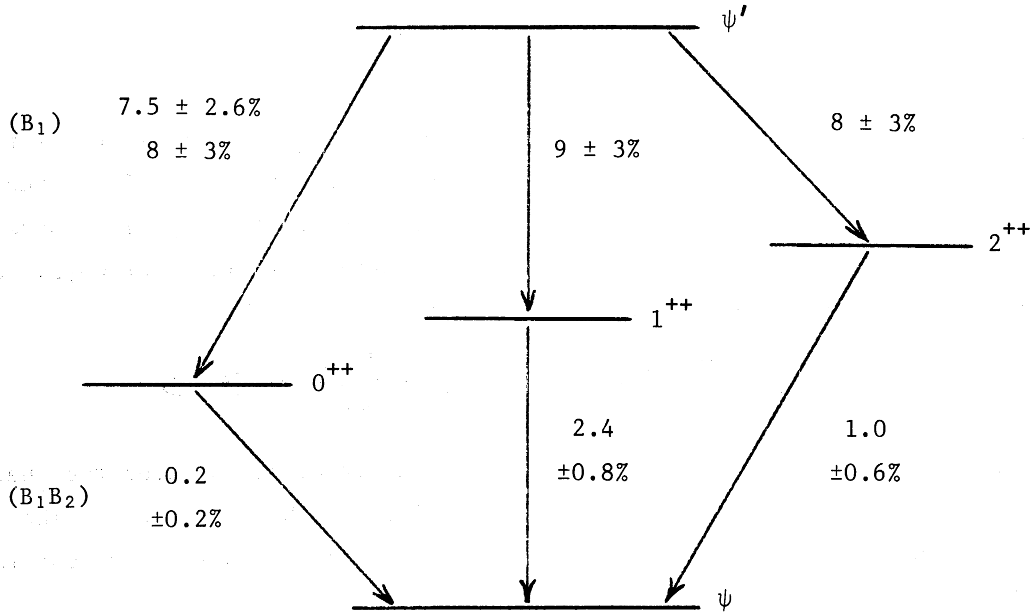

Fig. 3 (Top) Branching ratios B_1 in per cent for the radiative transitions, $\psi' \to \gamma_1\chi_J$. (Bottom) Product of branching ratios B_1B_2 in per cent for the cascade transitions, $\psi \to \gamma_1\chi_J \to \gamma_1\gamma_2\psi$

Assuming that the only significant competing processes are $\chi_J \to \gamma_2\psi$ and $\chi_J \to$ hadrons, the width into hadrons can be expressed as

$$\Gamma_h(J) = \left(\frac{1}{B_2(J)} - 1\right)\Gamma_{\gamma_2}(J)$$

If *the radiative width were known,* we would have an experimental determination of $\Gamma_h(J)$. Unfortunately, we have no direct information on $\Gamma_{\gamma_2}(J)$. The next best thing is to use some plausible estimate of the relative values of $\Gamma_{\gamma_2}(J)$ for different J and thereby estimate the relative hadronic widths for comparison with Eq. (14)[9]. Specifically, we have

$$\frac{\Gamma_h(J')}{\Gamma_h(J)} = \frac{\Gamma_{\gamma_2}(J')}{\Gamma_{\gamma_2}(J)}\left[\frac{B_1(J')}{B_1B_2(J')} - 1\right] \Bigg/ \left[\frac{B_1(J)}{B_1B_2(J)} - 1\right] \tag{16}$$

The transitions $\psi' \to \gamma_1\chi_J$ and $\chi_J \to \gamma_2\psi$ are plausibly predominantly electric dipole transitions. For a group of initial states (χ_J) in the same Russell-Saunders multiplet, the dipole matrix elements can be taken as approximately equal. Then the relative rates for a radiative transition to a common final state are just proportional to k^3. With this k^3 assumption and the data shown in Fig. 3, the numbers are

$$\frac{\Gamma_h(0^{++})}{\Gamma_h(1^{++})} = \left(\frac{0.304}{0.389}\right)^3\left[\frac{7.7}{0.2} - 1\right] \Bigg/ \left[\frac{9}{2.4} - 1\right] \simeq 6.5$$

$$\frac{\Gamma_h(2^{++})}{\Gamma_h(1^{++})} = \left(\frac{0.428}{0.389}\right)^3\left[\frac{8}{1} - 1\right] \Bigg/ \left[\frac{9}{2.4} - 1\right] \simeq 3.4$$

We thus find

$$\Gamma_h(0^{++}) : \Gamma_h(1^{++}) : \Gamma_h(2^{++}) \simeq 6.5 : 1 : 3.4$$

for comparison with Eq. (14). The agreement is quite acceptable, considering the poor statistics on the experimental values of B_1B_2. Note, in particular, that for $J = 0^{++}$, $B_1B_2 = 0.2 \pm 0.2\%$ (based on one event). The ratio 6.3:1 is therefore really something like $(6.3 {}^{+\infty}_{-3.2})$:1! Clearly, better data are needed before a strict test can be envisioned. The scalar gluon ratio of 125/2 for $0^{++}:2^{++}$ cannot even be excluded at the moment.

To test the absolute magnitudes of the hadronic widths it is necessary to have $\Gamma_{\gamma_2}(J)$. One estimate can be made by using some bound-state model. If such a model gives reasonable values for the absolute radiative rates $\psi' \to \gamma_1\chi_J$, it is plausible to trust its predictions for the second transition. Even cruder is to take a harmonic oscillator estimate, reduced by a factor of two to account for the distribution of the oscillation strength over several levels rather than just one[10]. This gives $\Gamma_{\gamma_2}(J) \simeq 100$ keV, 230 keV and 300 keV for J = 0, 1, 2, respectively. Then one finds $\Gamma_h(J) \approx 4$ MeV, 0.6 MeV, 2 MeV, in the same ordering. The corresponding QCD values, from Eqs. (14) and (15), are 2.2 MeV, 0.15 MeV, 0.6 MeV.

Another method of estimating the magnitude of $\Gamma_{\gamma_2}(J)$ is by the use of dipole sum rules[11]. With some assumptions about the underlying dynamics of the bound states, these give upper and lower bounds on $\Gamma_{\gamma_2}(J)$. The ranges are found to be 160-240 keV, 230-400 keV, and 280-480 keV, for J = 0, 1, 2, respectively, yielding hadronic widths $\Gamma_h(J) \simeq$ 6-9 MeV, 0.6-1.1 MeV, 2.0-3.4 MeV. These are somewhat larger than the cruder estimate, but not much[12].

For the ^3P_J states the situation at present is that the relative hadronic widths seem in accord with the expectations of QCD, although the data are subject to rather large uncertainties at present. The absolute magnitudes, inferred somewhat indirectly from experiment, are a factor of perhaps four larger than the rather uncertain QCD estimates.

5. THE PSEUDOSCALARS

We now turn to the alleged pseudoscalar states, X(2.83) and $\chi(3454)$. First, the X(2.83). The experimental observation of this state is via three photons in the cascade, $\psi \to \gamma_1 X$, $X \to \gamma_2\gamma_3$. The observed branching ratio product is[2] $(1.2 \pm 0.5) \times 10^{-4}$. On the other hand, the radiative transition $\psi \to \gamma_1 X$ has not been seen at SPEAR in the inclusive photon spectrum from ψ decay[3,8]. The upper limit on the branching ratio is 3%. Thus we have the experimental inequality,

$$\frac{\Gamma(X \to \gamma\gamma)}{\Gamma_t(X)} > \frac{1.2 \times 10^{-4}}{3 \times 10^{-2}} = 4 \times 10^{-3}$$

to compare with $1/738 = 1.36 \times 10^{-3}$ from Eq. (6). Since the error on the 4 is ±1.7, there is no immediate cause for alarm. If the transition $\psi \to \gamma X$ is not seen at the 1% level, however, there will be an order of magnitude discrepancy between a fairly firm QCD prediction and experiment[13]. Something will then have to give!

The situation with respect to the identification of the $\chi(3454)$ as the η_c' is even less satisfactory. No hadronic decays have been observed so far. It is only seen in the cascade, $\psi' \to \gamma_1\chi$, $\chi \to \gamma_2\psi$ with a product of branching ratios[3], $B_1B_2 = (0.8 \pm 0.4) \times 10^{-2}$. An upper limit of 5% can be set for B_1 from the inclusive photon spectrum from ψ' decay[8]. This means that $B_2 > 0.16 \pm 0.08$, corresponding to a large radiative width or a small hadronic width. There are various ways of getting a handle on the magnitude of the hadronic width. The most naïve approach would be to say that both radiative transitions are allowed magnetic dipole transitions with the same matrix element. Then the radiative widths are in the ratio, $\Gamma_{\gamma_2}/\Gamma_{\gamma_1} = 3(0.340/0.223)^3 = 10.7$. From the upper limit, $\Gamma_{\gamma_1} < 0.05 \times 228 = 11.4$ keV, we infer $\Gamma_t < 10.7 \times 11.4$ keV$/0.16 \simeq 0.76$ MeV. This is a factor of three smaller than the QCD expectation of about 2.2 MeV [see below Eq. (8)].

Another and more intelligent way of proceeding is to observe that if the $\chi(3454)$ is the partner of the $\psi'(3684)$ we expect their spatial wave functions to be similar and so the first transition, $\psi' \to \gamma_1\chi$, to be an allowed M1 transition, with a radial overlap integral of roughly unity. The estimated rate is 18 keV, only a factor of two larger than the upper bound of 11 keV. But the second transition is another story. Since the ψ' and η' are radial excitations, their spatial wave functions are largely orthogonal to the ground state wave function of the $\psi(3095)$. The transition, $\chi \to \gamma_2\psi$, is thus an unfavoured M1 with a very small overlap integral. The unobserved transition, $\psi' \to \gamma X(2.83)$, is a similarly unfavoured transition. In fact, from the upper limit[3,8] of 1.1% for $\psi' \to \gamma X(2.83)$, corresponding to a width of less than 2.5 keV, one can expect the width for $\chi \to$ $\to \gamma_2\psi$ to be no more than $3 \times 2.5 = 7.5$ keV [14]. Then we conclude $\Gamma_t < 7.5/0.16 \simeq$ $\simeq 50$ keV.

Other ways of estimating or setting a bound on Γ_{γ_2} lead to numbers nearer the 50 keV figure for Γ_t than the over-simple 760 keV. Thus there appears to be a fairly serious discrepancy (by an order of magnitude or more) between QCD predictions and "experiment" *if* the $\chi(3454)$ is identified with the η_c'.

An alternative identification of $\chi(3454)$ as the $^1D_2$-state ($J^{PC} = 2^{-+}$) has been suggested[15]. This at first seems unpalatable, but in view of the apparently large "hyperfine" splitting between the X(2.83) and $\psi(3095)$ cannot be excluded. The triplet d-states are expected just above the ψ'; the singlet d-state *could* be depressed to 3454 MeV. The radiative transitions through the $^1D_2$-state are both unfavoured M1's, involving spin-flip and an overlap of s- and d-state radial wave functions, if the ψ and ψ' are assumed to be purely $^3S_1$-states. The expected transition rate for $\psi' \to \gamma_1\chi$ is so small as to be inconsistent with the observed branching ratio of $B_1B_2 = (0.8 \pm 0.4) \times 10^{-2}$. This $^1D_2$ interpretation only makes sense if there is an admixture of $^3D_1$ in the ψ and ψ'. This is quite plausible on general grounds, QCD having in it a tensor force contribution to the binding potential. With the simple quark model estimate[5] for allowed M1 transitions, one finds $\Gamma_{\gamma_1} \simeq 18\ \varepsilon_D'$ keV, $\Gamma_{\gamma_2} \simeq 187\ \varepsilon_D$ keV, where ε_D and ε_D' are the intensity fractions of d-state present in the ψ and ψ'. The upper limit of $\Gamma_{\gamma_1} < 11$ keV sets the bound, $\varepsilon_D' < 0.6$. The lower limit, $B_2 > 0.16$, merely implies $\Gamma_t < 1.2\ \varepsilon_D$ MeV. The percentage of d-state in the ψ is probably quite small ($\varepsilon_D \sim 0.01$?), but so is the expected width of the $^1D_2$ state[16]. There is thus no inconsistency apparent yet in the identification of $\chi(3454)$ as the $^1D_2$-state. This assignment means, of course, that the η_c' is still to be discovered! One problem has been replaced by another.

Added remark (in answer to a question of M. Jacob)

In estimating the hadronic widths in QCD one computes only the lowest order annihilation into gluons, treated as real massless vector particles with only two transverse states of polarization. Might not these estimates be modified drastically by the inclusion of the longitudinal polarization state of a gluon by making it off mass shell by coupling to a light $q\bar{q}$ pair? Admittedly, there is another power of α_s present, but perhaps the longitudinal contribution is so large as to cancel out the factor of $\alpha_s \simeq 0.2$.

The answer is that for an allowed process like $\eta_c \to gg$ the addition of the Dalitz pair contributions only contributes a small correction (ignoring the question of double counting). Specifically, the Dalitz-Kroll-Wada formula for the relative addition is

$$\rho = \frac{2\alpha_s}{3\pi}\left[\ln(M^2/m^2) - \frac{7}{2}\right]$$

where M is the mass of the decaying state and m is the light quark mass. With $M = 3.5$ GeV, $m = 0.34$ GeV and $\alpha_s = 0.187$, we have $\rho = 0.05$. Omitting the $-7/2$ term raises this to $\rho \simeq 0.20$, still a small correction.

REFERENCES AND FOOTNOTES

1) For the experimental facts and inferences from them, see G.J. Feldman, ψ spectroscopy, Proc. 1976 Summer Institute on Particle Physics, Stanford Linear Accelerator Center, SLAC-PUB-1851 (1976) and SLAC Report No. 198 (1976), or B.H. Wiik and G. Wolf, Electron-positron interactions, Les Houches Summer School, 1976, DESY 77/01 (1977).

2) W. Braunschweig et al., DESY 77/02 (1977).

3) J.S. Whitaker et al., Phys. Rev. Letters 37, 1596 (1976). The three or four events seen in this experiment as $\psi' \to \gamma_1\chi(3454)$, $\chi(3454) \to \gamma_2\psi$ were, until recently, the only evidence for the $\chi(3454)$. At this conference, V. Blobel has reported three events from PLUTO.

4) Critics point out that while the few-gluon annihilation of a massive state may involve a small coupling constant for each gluon, the many-gluon annihilation presumably has a large coupling constant for each, because in such a configuration each gluon is "soft". Why, they ask, can one then believe the lowest-order calculation? I have no solid answer for this, but simple-minded estimates for many-gluon rates indicate that if the effective coupling constant for each of N gluons does not grow with N too rapidly, for example $\alpha_N/\alpha_2 \sim N/2$, the sum of annihilations into an arbitrary number of gluons differs by only a modest numerical factor from the rate into the smallest number of gluons, at least for the charmonium value of $\alpha_2 \simeq 0.2$.

5) For an elementary discussion of the calculation of such rates, see J.D. Jackson, Lectures on the new particles, Proc. 1976 Summer Institute on Particle Physics, Stanford Linear Accelerator Center, Report No. 198 (1976).

6) R. Barbieri, R. Gatto and R. Kögerler, Phys. Letters 60B, 183 (1976).

7) R. Barbieri, R. Gatto and E. Remiddi, Phys. Letters 61B, 465 (1976).

8) D.H. Badtke et al., Contribution to the 18th Internat. Conf. on High-Energy Physics, Tbilisi, July 1976.

9) M.S. Chanowitz and F.J. Gilman, Phys. Letters 63B, 178 (1976).

10) This is discussed in Ref. 5. Such estimates agree well enough with model calculations, for example E. Eichten et al., Phys. Rev. Letters 34, 369 (1975); 36, 500 (1976); K.M. Lane (private communication).

11) J.D. Jackson, Phys. Rev. Letters 37, 1107 (1976).

12) The spread in values comes solely from the upper and lower bounds from the sum rules. The errors in the branching ratios have not been included.

13) There is already a discrepancy of a factor of perhaps 10-16 between the expected M1 rate for $\psi \to \gamma X$ and the 3% upper limit of 2.1 keV.

14) There is, in fact, a rapid energy dependence to the width of an unfavoured M1 transition. Naïvely, it is as k [7]. This would reduce the estimate for $\chi \to \gamma_2\psi$ drastically.

15) H. Harari, Phys. Letters 64B, 469 (1976).

16) We can appeal to Eq. (9) and estimate very crudely that $\Gamma_h(^1D_2) \simeq (2\xi/M)^2 \times \Gamma_h(^3P_2)$, where $\xi \simeq 0.5$ GeV is the scale parameter of the oscillator wave functions that fit the energy level spacing and electric dipole moments. This gives $\Gamma_h(^1D_2) \sim 50$ keV, an admittedly very rough estimate.