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TOPICS IN SUPERGRAVITY AND SUPERSYMMETRY *)

Bruno Zumino
CERN -- Geneva

A B S T R A C T

We discuss the supersymmetric Higgs effect which occurs when a supersymmetric matter system, in which supersymmetry is spontaneously broken, is coupled with supergravity. A second topic discussed is the formulation of supergravity as a geometry of superspace.

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Supersymmetric Higgs effect

Rigorous supersymmetry implies the existence of supermultiplets made up of fermions and bosons with equal masses. If supersymmetry is to be relevant for the physical world, it must be broken, either softly or spontaneously. Spontaneous breaking of global supersymmetry gives rise to the appearance of a Goldstone fermion. When global supersymmetry is promoted to a local invariance by coupling supersymmetric matter to supergravity, the Goldstone fermion disappears as a consequence of a phenomenon analogous to the Higgs effect of ordinary gauge theories. We describe now in some detail this supersymmetric Higgs effect and consider its possible application to the construction of realistic models. We follow some recent work by S. Deser and the author [1], to appear shortly*). Observe that the supersymmetric Higgs effect gives a possible solution to the problem of the apparent non-existence in Nature of the Goldstone fermion of spontaneously broken supersymmetry. As we know, this cannot be identified with the electron neutrino, because it would satisfy low energy theorems which contradict observed properties of the neutrino spectrum [3].

Spontaneous breaking of global supersymmetry generates a Majorana spin $1/2$ Goldstone fermion [4,5], which we shall call λ. Irrespective of the particular field theory in which it arises, its properties can be described, following Volkov and Akulov [4], by means of the non-linear realization of global supersymmetry

$$\delta\lambda = \frac{1}{a}\alpha + ia\,\bar{\alpha}\gamma^m\lambda\,\partial_m\lambda \tag{1}$$

where α is the infinitesimal supersymmetry parameter and a is a constant which measures the strength of the spontaneous breaking of supersymmetry. The non-linear Lagrangian for λ, invariant (up to a divergence) under (1), is given by

$$\begin{aligned} L_\lambda &= -\frac{1}{2a^2}\det\left(\delta_m{}^n + ia^2\bar{\lambda}\gamma^n\partial_m\lambda\right) \\ &= -\frac{1}{2a^2} - \frac{i}{2}\bar{\lambda}\gamma\cdot\partial\lambda + \cdots \end{aligned} \tag{2}$$

This description is perfectly analogous to that used for the pion in non-linear pion dynamics. However, while the chiral group SU(2) × SU(2) is also broken explicitly by a pion mass term, if we assume that supersymmetry is broken only spontaneously the above description is expected to be rigorous and to be actually valid for a suitably defined field λ in any renormalizable model in which a Goldstone fermion emerges.

*) Volkov and Soroka [2] were the first to point out the possible occurence of a supersymmetric Higgs effect. However, in spite of its formal similarity, their point of view is essentially different from ours.

Let us now try to promote (1) to a local transformation with parameter $\alpha(x)$ and to make (2) invariant under it by coupling λ to the supergravity fields $e_m{}^a$ and ψ_m. Without describing the complete Lagrangian, which is rather complicated, one can easily find the first terms in an expansion in the coupling constants a and κ (gravitational constant). Under the transformation laws

$$\begin{aligned} \delta\lambda &= \frac{1}{a}\,\alpha(x) + \cdots \\ \delta e_m{}^a &= -i\kappa\,\bar{\alpha}\gamma^a\psi_m \\ \delta\psi_m &= -\frac{2}{\kappa}\,\partial_m\alpha + \cdots \end{aligned} \qquad (3)$$

the Lagrangian

$$\begin{aligned} L_\lambda &= -\frac{1}{2a^2}\,e - \frac{i}{2}\,\bar{\lambda}\gamma\cdot\partial\lambda - \frac{i}{2a}\,\bar{\lambda}\gamma\cdot\psi + \cdots \\ e &= \det e_m{}^a \end{aligned} \qquad (4)$$

changes by a divergence. To (4) one must add the usual supergravity Lagrangian [6]

$$L = -\frac{1}{2\kappa}\,eR - \frac{i}{2}\,\varepsilon^{lmnr}\,\bar{\psi}_l\gamma_5\gamma_m D_n\psi_r \,. \qquad (5)$$

The sum $L + L_\lambda$ is invariant under (3). The transformation low for the field λ shows that it corresponds to a pure gauge degree of freedom and that it can be transformed to zero by means of a suitably chosen local supersymmetry transformation. In other words, the field λ can be absorbed into a redefinition of the fields $e_m{}^a$ and ψ_m The resulting theory is described by the Lagrangian (5) of supergravity plus a cosmological term (plus possible additional terms from the supersymmetric matter part which gave rise to spontaneous symmetry breaking).

This result is puzzling and disappointing. It is puzzling because the disappearance of the Goldstone particle (Higgs effect) gave rise to a cosmological term, instead of generating a mass term for the spin $3/2$ gauge field, as one would have expected. It is disappointing because the empirical smallness of the cosmological constant $-1/2a^2$ seems to destroy any hope that the spontaneous breaking of supersymmetry will be large enough to be responsible for the observed mass splitting between bosons and fermions. In the following we discuss these points.

The above puzzle is immediately resolved if one observes that, in presence of a cosmological term, one cannot quantize in a Minkowski background, but one must take instead as a background space a solution of the Einstein equations with cosmological term. The simplest and most natural is the corresponding de Sitter space. Now, in de Sitter space the concept of mass is rather delicate.

It has been recently observed [7,8] that one can add to the supergravity Lagrangian (5) the sum of a cosmological term and of a spin $^3\!/\!_2$ mass term

$$3\frac{\mu^2}{\kappa^2}e - \frac{i}{2}\mu\,\varepsilon^{\ell m n r}\,\bar{\psi}_\ell\,\gamma_5\,\Sigma_{mn}\,\psi_r\,, \qquad (6)$$
$$\Sigma_{mn} = \frac{1}{4}[\gamma_m,\gamma_n]\,,$$

without spoiling local supersymmetry. Indeed the sum of (5) and (6) is invariant under a modified supersymmetry transformation in which the usual transformation law

$$\delta\psi_m = -\frac{2}{\kappa}D_m\alpha \qquad (7)$$

is replaced by

$$\delta\psi_m = -\frac{2}{\kappa}D_m\alpha - \frac{\mu}{\kappa}\gamma_m\alpha \qquad (8)$$

(there is a corresponding change in $\delta\omega_{mab}$). The existence of this local supersymmetry shows that, in spite of the apparent mass term in (6), the spin $^3\!/\!_2$ field has the number of degrees of freedom appropriate to the massless case. It is also easy to see that the Lagrangian (5) plus (6) admits a global supersymmetry, obtained by taking α independent of x. It is the global supersymmetry of the corresponding de Sitter space of radius μ^{-1}, which has O(3,2) as the maximal Lie subalgebra*). When the cosmological term and the spin $^3\!/\!_2$ mass term are related in the particular way given in (6) there is local supersymmetry and, with a sensible definition of mass, the spin $^3\!/\!_2$ field is massless. When they are not related as in (6), there is no local supersymmetry and the spin $^3\!/\!_2$ field is massive (this is true in particular when there is only a cosmological term, which resolves the above mentioned paradox). Then the equations of motion imply the constraints

$$\gamma^m\psi_m = 0 \qquad (9)$$
$$\partial_m\psi^m + \dots = 0\,,$$

where the dots are terms of higher order in κ. These constraints are exactly of the kind that gives the right number of degrees of freedom for a massive spin $^3\!/\!_2$ field. The classical equations of motion are still consistent, even though there is no local supersymmetry, and no anomalous propagation hypersurfaces occur.

*) Here we disagree with Ref. 7, where the spin $^3\!/\!_2$ mass term is interpreted as giving rise to a breaking of global supersymmetry.

The existence of the invariant (6) can be used to resolve the second problem mentioned above. We observe that the sign of the cosmological term in (6) is fixed, and corresponds to a de Sitter space with O(3,2) invariance. On the other hand, that of the cosmological term in (4) is also fixed and is the opposite. Adding (4), (5) and (6), one can adjust the constants so that the cosmological terms cancel. Before, however, one must modify (4) so as to make it invariant under the new transformation law (8). This is not difficult, to the order considered here, and requires adding to (4) a term

$$-i\mu\bar{\lambda}\lambda + \cdots \qquad (10)$$

The meaning of this term can be understood as belonging to the invariant Lagrangian for a Goldstone spinor in a de Sitter space of radius μ^{-1}.

Now one can cancel the cosmological terms between (4) and (6)

$$\frac{1}{2a^2} = \frac{3\mu^2}{\kappa^2} \quad . \qquad (11)$$

If we assume that the spontaneous supersymmetry breaking is responsible for the observed mass splittings between mesons and baryons, the order of magnitude of the constant a must be given by a hadronic mass, say the proton mass,

$$\frac{1}{a} \sim m_p^2 \quad , \qquad (12)$$

We find

$$\mu \sim (\kappa m_p)\, m_p \quad , \qquad (13)$$

where $\kappa m_p \sim 10^{-19}$. The mass of the spin $3/2$ field is very small, but we have hadronic mass splittings of reasonable magnitude and zero cosmological constant*).

Geometry of superspace

We shall now describe how supergravity can be obtained from the geometry of superspace. If one takes the differential geometry of superspace to be (super) Reimannian [10,11], the connection with the space-time formulation of supergravity [6] is not very direct and requires a limiting process in superspace [12]. This is due to the fact that the field equations in Riemannian superspace do not admit as solution the flat superspace [4,13] of ordinary global supersymmetry. It was

*) The compensation between cosmological terms of opposite sign was considered by Freedman and Das [7,9] in a specific model. In that model difficulties seem to arise when one attempts to complete the combined Lagrangian to a locally supersymmetric invariant (private communication from D. Freedman).

for this very reason that a different differential geometry in superspace was introduced by the Wess and the author [14] and, independently, by Akulov, Volkov and Soroka [15]. The superspace of global supersymmetry is a special case of this kind of superspace. Furthermore, the equations of supergravity take a very simple form. We describe this below, following a recent paper by Wess and the author [16].

We begin by considering a general affine superspace. Its points are parametrized by coordinates $z^M \equiv (x^m, \theta^\mu)$ where the x^m are the commuting space-time coordinates while θ^μ are anticommuting variables. More precisely, x^m are even and θ^μ odd elements of a Grassmann algebra. Latin letters will denote vectorial (bosonic), Greek letters spinorial (fermionic) indices. The supervierbein matrix $E_M{}^A(z)$, where $A = (a,\alpha)$ and its inverse $E_A{}^M$, can be used to transform world tensors into tangent space tensors and vice versa. The submatrices $E_m{}^a$ and $E_\mu{}^\alpha$ consist of bosonic, $E_m{}^\alpha$ and $E_\mu{}^a$ of fermionic elements. The superconnection $\Phi_{M,A}{}^B$ and the supervierbien can be viewed as the coefficients of two one-forms

$$E^A = dz^M E_M{}^A \,, \quad \Phi_A{}^B = dz^M \Phi_{M,A}{}^B \,. \tag{14}$$

Here the differentials dz^m are taken to anticommute with each other and with the $d\theta^\mu$, while the $d\theta^\mu$ commute with each other. Similarly, E^a anticommute with each other and with E^α, while E^α commute with each other. Under a linear transformation in the tangent space

$$\delta v^A = v^B X_B{}^A \,, \quad \delta u_A = - X_A{}^B u_B \,, \tag{15}$$

the supervierbein $E_M{}^A$ transforms like the vector v^A, while the connection transforms as

$$\delta \Phi_A{}^B = \Phi_A{}^C X_C{}^B - X_A{}^C \Phi_C{}^B - d X_A{}^B \,, \tag{16}$$

and one can define the covariant differentials

$$\mathcal{D} v^A = dv^A + v^B \Phi_B{}^A \,, \quad \mathcal{D} u_A = du_A - \Phi_A{}^B u_B \,. \tag{17}$$

The notation of differential forms is compact and convenient and takes automatically into account all the sign changes due to the Grassmann nature of our variables. Many properties of Cartan forms generalize in a simple and natural way to our differential forms with Grassmann variables [14]. Because we write the differentials on the left, the differentiation operator $d = dz^M \partial/\partial z^M$ operates on a product starting from the right. For instance, if Ω_2 is a p-form, $d(\Omega_1\Omega_2) = \Omega_1 d\Omega_2 - (-1)^p d\Omega_1 \Omega_2$.

The theorem $d(d\Omega) = 0$ is valid and its inverse also applies with obvious restrictions on the topology of the domain of variation of the bosonic variables. In terms of covariant derivatives, defined by $\mathcal{D} = dz^M \mathcal{D}_M$, the formulae (17) become

$$\begin{aligned} \mathcal{D}_M v^A &= \frac{\partial}{\partial z^M} v^A + (-)^{bm} v^B \Phi_{M,B}{}^A \\ \mathcal{D}_M u_A &= \frac{\partial}{\partial z^M} u_A - \Phi_{M,A}{}^B u_B \quad , \end{aligned} \tag{18}$$

where the sign factor $(-)^{bm}$ is defined by the convention that $m = 0$ if M is vectorial and $m = 1$ if M is spinorial, and similarly for B. The torsion and the curvature are defined by

$$\begin{aligned} T^A &= dE^A + E^B \Phi_B{}^A = \tfrac{1}{2} dz^N dz^M T_{MN,}{}^A = \tfrac{1}{2} E^C E^B T_{BC,}{}^A \\ R_A{}^B &= d\Phi_A{}^B + \Phi_A{}^C \Phi_C{}^B = \tfrac{1}{2} dz^N dz^M R_{MN,A}{}^B = \tfrac{1}{2} E^C E^D R_{DC,A}{}^B \end{aligned} \tag{19}$$

As a consequence of their definition, they satisfy the Bianchi identities

$$\begin{aligned} dT^A + T^B \Phi_B{}^A - E^B R_B{}^A &= 0 \\ dR_A{}^B + R_A{}^C \Phi_C{}^B - \Phi_A{}^C R_C{}^B &= 0 \quad . \end{aligned} \tag{20}$$

Written out in terms of coefficients tensors the Bianchi identities take the form

$$\begin{aligned} E^C E^B E^A \left(\mathcal{D}_A T_{BC,}{}^D + T_{AB,}{}^{C'} T_{C'C,}{}^D - R_{AB,C}{}^D \right) &= 0 \\ E^C E^B E^A \left(\mathcal{D}_A R_{BC,D}{}^F + T_{AB,}{}^{C'} R_{C'C,D}{}^F \right) &= 0 \ , \end{aligned} \tag{21}$$

where

$$\mathcal{D}_A = E_A{}^M \mathcal{D}_M \quad .$$

In order to give more structure to our superspace we must specialize the group in the tangent space. The simplest would be to require the tangent space to be a super-Minkowski space. This would mean, assuming the existence of an invariant numerical tensor, $\eta_{AB} = (-1)^{ab} \eta_{BA}$ and, with the further restriction of vanishing torsion, would lead to a Riemannian superspace. Except for the use of vierbein and connection, the geometry would be equivalent to that described in [10,11] directly in terms of the metric tensor. For the reasons explained above, we choose a more restricted group in the tangent space and require the existence of a basis in which the matrices $X_A{}^B$ satisfy the relations

$$X_a{}^\beta = X_\alpha{}^b = 0 \, , \quad X_a{}^b = L_a{}^b(z)$$
$$X_\alpha{}^\beta = \frac{1}{2} L_a{}^b (\Sigma_b{}^a)_\alpha{}^\beta \, , \tag{22}$$

where $L_a{}^b$ is an infinitesimal Lorentz matrix, $L_{ab} = -L_{ba}$ and $\Sigma_b{}^a = 1/4[\gamma_b, \gamma^a]$. In words, $L_\alpha{}^\beta$ describes the same Lorentz transformation as $L_a{}^b$ when applied to spinors. Our group consists therefore of ordinary Lorentz transformations, but dependent on both x and θ. Since the connection and the curvature $R_A{}^B$ are matrices belonging to the algebra of the tangent space group they both satisfy the same restrictions as $X_A{}^B$. In particular $R_{CD,a\beta} = R_{CD,\alpha b} = 0$,

$$R_{CD,ab} = -R_{CD,ba} \, , \quad R_{CD,\alpha\beta} = \frac{1}{2} R_{CD,ab} (\Sigma^{ab})_{\alpha\beta} \, .$$

The equations of supergravity can be stated as simple restrictions on the torsion tensor in superspace. We take

$$T_{\alpha\beta}{}^c = 2i(\gamma^c)_{\alpha\beta} \, , \quad T_{\alpha\beta}{}^\gamma = 0 \tag{23}$$

$$T_{a\beta}{}^c = T_{a\beta}{}^\gamma = 0 \, , \quad T_{ab}{}^c = 0 \tag{24}$$

while we leave the components $T_{ab}{}^\gamma$ which correspond to the gauge invariant Rarita Schwinger field, undetermined. A number of relations can be immediately obtained by combining (23) and (24) with the Bianchi identities (21). Among them are

$$R_{\alpha\beta,c}{}^d = 0 \tag{25}$$

$$R_{\alpha b,\beta}{}^\delta + R_{\beta b,\alpha}{}^\delta - 2i(\gamma^c)_{\alpha\beta} T_{cb,}{}^\delta = 0 \tag{26}$$

$$R_{\alpha b,c}{}^d - R_{\alpha c,b}{}^d - 2i(\gamma^d)_{\alpha\beta} T_{bc,}{}^\beta = 0 \tag{27}$$

$$\mathcal{D}_\alpha R_{\beta b,c}{}^d + \mathcal{D}_\beta R_{\alpha b,c}{}^d + 2i(\gamma^{c'})_{\alpha\beta} R_{c'b,c}{}^d = 0 \tag{28}$$

Furthermore, with a little algebra, one can see that (26) actually implies

$$(\gamma^c)_{\alpha\beta} T_{cb,}{}^{\beta} = 0 \,, \tag{29}$$

and therefore also, from (27),

$$R_{\alpha b,c}{}^{b} = 0 \,. \tag{30}$$

This in turn, through (28) tells us that

$$R_{ab,c}{}^{b} = 0 \,. \tag{31}$$

Finally, combining (26) and (27) and remembering the relation

$$R_{\alpha b,\beta}{}^{\delta} = \frac{1}{2} R_{\alpha b,c}{}^{d} (\Sigma_d{}^c)_{\beta}{}^{\delta} \,,$$

one finds that

$$R_{\alpha b,cd} = 2i(\gamma_b)_{\alpha\beta} T_{cd,}{}^{\beta} \,. \tag{32}$$

The simplest way to extract field equations from our superfield equations is to observe that it is possible to choose a gauge such that, as $\theta \to 0$, the superconnection $\Phi_{m,ab}$ becomes the usual connection in four-space, the supervierbein becomes*)

$$\begin{aligned} E_m{}^a &= e_m{}^a(x) & \qquad E_m{}^{\alpha} &= \frac{1}{2}\psi_m{}^{\alpha}(x) \\ E_{\mu}{}^a &= 0 & \qquad E_{\mu}{}^{\alpha} &= \delta_{\mu}{}^{\alpha} \,, \end{aligned} \tag{33}$$

where $e_m{}^a$ is the usual vierbein in four-space and $\psi_m{}^{\alpha}$ the Rarita-Schwinger field. It follows that, in the same limit,

$$\begin{aligned} &T_{\mu\nu}{}^{\alpha} = T_{\mu n}{}^{\alpha} = R_{\mu\nu,a}{}^{b} = 0 \\ &R_{m\nu,a}{}^{b} = e_m{}^c \delta_{\nu}{}^{\beta} R_{c\beta,a}{}^{b} \,, \end{aligned} \tag{34}$$

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*) The higher powers in θ are also expressible in terms of the physical fields and their derivatives, provided one fixes the gauge appropriately. The coordinate transformations in superspace which preserve the choice of gauge can be expressed in terms of a coordinate transformation in ordinary space time and an x-dependent supersymmetry transformation (local supersymmetry transformation).

while $T_{mn,}{}^{a}$ and $R_{mn,a}{}^{b}$ become the four-space torsion and curvature tensor, which we shall denote by $\mathcal{T}_{mn,}{}^{a}$ and $\mathcal{R}_{mn,a}{}^{b}$. Using (33) and the relation $T_{MN,}{}^{C} = E_M{}^A E_N{}^B T_{BA}{}^C$ we see that (23) and (24) imply the connection between the torsion and the spin density

$$\mathcal{T}_{mn,}{}^{c} - \frac{i}{2} \psi_m{}^{\alpha} (\gamma^c)_{\alpha\beta} \psi_n{}^{\beta} = 0 . \tag{35}$$

On the other hand (29) gives the Rarita-Schwinger equation in the form

$$(\gamma^c)_{\alpha\beta} \mathcal{T}_{cd,}{}^{\beta} = 0 , \quad \mathcal{T}_{cd,}{}^{\beta} = e_c{}^m e_d{}^n \left(D_m \psi_n{}^{\beta} - D_n \psi_m{}^{\beta} \right) , \tag{36}$$

where D_m is the covariant derivative used in the second of Ref. 5, and (31) gives the Einstein equation in the form

$$\mathcal{R}_{ab,c}{}^{b} + \frac{i}{2} e^{bm} \psi_m{}^{\alpha} (\gamma_a)_{\alpha\beta} \mathcal{T}_{bc,}{}^{\beta} = 0 . \tag{37}$$

The equations (36), and (37) are equivalent, but not identical, to those given in Ref. 5. To establish the equivalence of the two forms of the Einstein equation observe that the Rarita-Schwinger equation implies that

$$(\gamma_5)_{\alpha\beta} \mathcal{T}_{ab,}{}^{\beta} = \frac{1}{2} \varepsilon_{ab}{}^{cd} \mathcal{T}_{cd,\alpha} . \tag{38}$$

Instead of taking all of (23) and (24) as basic equations, one can take some of them and some of the equations we have derived from them through the Bianchi identities. A particularly interesting choice includes (23) and (25) among the basic equations.

Finally, let us observe that the above formalism permits the construction of superspace scalars. On the other hand, one knows how to transform a scalar into a density. One multiplies it by the det $E_M{}^A$, where the determinant of a matrix with commuting and anticommuting elements is defined in Ref. 11. By this procedure one can construct invariant actions in superspace.

REFERENCES

[1] S. Deser and B. Zumino, in preparation.
B. Zumino, Lectures at the 1976 Scottish Universities Summer School, August 1976.

[2] D.V. Volkov and V.A. Soroka, JETP Letters 18 (1973) 312.

[3] W. Bardeen, unpublished;
B. de Wit and D.Z. Freedman, Phys. Rev. Letters 35 (1975) 827.

[4] D.V. Volkov and V.P. Akulov, Phys. Letters 46B (1973) 109.

[5] J. Iliopoulos and B. Zumino, Nuclear Phys. B76 (1974) 310.
A. Salam and J. Strathdee, Phys. Letters 49B (1974) 465.
P. Fayet and J. Iliopoulos, Phys. Letters 51B (1974) 461.
L. O'Raifertaigh, Nuclear Phys. B96 (1975) 331.

[6] D.Z. Freedman, P. van Nieuwenhiuzen and S. Ferrara, Phys. Rev. D 13 (1976) 3214.
S. Deser and B. Zumino, Phys. Letters 62B (1976) 335.

[7] D.Z. Freedman and A. Das, Stony Brook preprint ITP-SB-76-64.

[8] S. MacDowell and F. Mansouri, private communication.

[9] D.Z. Freedman, Stony Brook preprint ITP-SP-76-50.

[10] R. Arnowitt and P. Nath, Phys. Letters 56B (1975) 117.

[11] R. Arnowitt, P. Nath and B. Zumino, Phys. Letters 56B (1975) 81.

[12] P. Nath and R. Arnowitt, Phys. Letters 65B (1976) 73.

[13] A. Salam and J. Strathdee, Nuclear Phys. 76B (1974) 477.
S. Ferrara, J. Wess and B. Zumino, Phys. Letters 51B (1974) 239.

[14] See, B. Zumino *in* Proc. of the Conf. on Gauge Theories and Modern Field Theory, Northeastern University, September 1975, eds. R. Arnowitt and P. Nath (MIT Press).

[15] V.P. Akulov, D.V. Volkov and V.A. Soroka, JETP Letters, 22 (1975) 396.

[16] J. Wess and B. Zumino, University of Karlsruhe preprint, to be published in Phys. Letters.