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A COMMENT ON SPONTANEOUS SYMMETRY BREAKING IN THE REGGEON FIELD THEORY

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A B S T R A C T

We show, using functional techniques, that spontaneous symmetry breaking in the Reggeon field theory is, in general, not possible because of the requirement of an imaginary triple coupling.

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1. INTRODUCTION

It is by now well known that Gribov [1], by studying model field theories, was able to obtain a Reggeon calculus and a non-relativistic field theory useful for studying ℓ plane poles and cuts and their interaction, and which is such that the partial wave amplitudes constructed from the Green's functions of such a field theory automatically satisfy the so-called Reggeon unitarity equations [2].

In such a field theory a Reggeon is a quasiparticle in two-space and one-time dimensions; the space dimensions being conjugate to the two-dimensional momentum $\underset{\sim}{q}(\underset{\sim}{q}^2 = -t)$; and the time being essentially the rapidity, conjugate to the angular momentum of the Reggeon $E(E = 1 - j)$. The field theory is non-relativistic because the energy momentum relation usually chosen is:

$$E = \alpha_0' \underset{\sim}{q}^2 + \Delta_0 ,$$

where $\Delta_0 = 1 - \alpha_0$, α_0 being the "bare" intercept of the Reggeon.

The massless theory $(\Delta_0 = 0)$ has been investigated in the infra-red region $(E = q^2 \approx 0)$ by use of the renormalization group [3] with well-known results.

Recently people [4],[5] have considered Reggeon field theories with quartic couplings where the bare intercept α_0 is greater than one, i.e., $\Delta_0 < 0$. A classical examination of the non-relativistic theory indicated that the theory exhibited spontaneous symmetry breaking and that, when the original Reggeon field was shifted with respect to its developed non-zero vacuum expectation value, two things appeared to happen:

i) the intercept α_0 was pushed back to one as a consequence of the spontaneous breaking of the symmetry, and

ii) the theory developed a triple-Reggeon coupling.

The purpose of this note is to examine these points in more detail, going beyond the classical level by using functional techniques to develop Ward identities and to examine the Goldstone theorem [6] for this problem. Specifically, we discuss the compatibility of the reality of the vacuum expectation value of the field and spontaneous symmetry breaking with the existence of an imaginary triple Reggeon coupling, known to be necessary in Reggeon field theories with triple coupling in order to give the correct sign of the two-Reggeon cut.

We conclude that such a compatibility does not exist by showing that if one component of the original Reggeon field develops a non-vanishing vacuum expectation value (VEV), the component of the original field with the VEV is unstable asymptotically, or has zero wave function renormalization constant. This gives an equality constraint between the effective triple coupling and the VEV which will give an imaginary triple Pomeron coupling only if the VEV is imaginary, but this will be shown to contradict the original boundary condition used to pick out the symmetry breaking solution in the first place.

In Section 2 we write down the Green's functional for a Reggeon field theory with quartic couplings and $\Delta_0 < 0$. We indicate how one can pick out the solution which spontaneously breaks the symmetry by adding a term to the Green's functional which has less symmetry than the original Lagrangian. We also derive a Ward identity relating the vacuum expectation value of the real part of the field, ψ, to that of the product of two χ's (imaginary part of the field).

In Section 3 we derive the Goldstone theorem from one of the Ward identities and make the compatibility check mentioned earlier.

2. FUNCTIONAL TECHNIQUES

The Lagrangian for our problem is

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I ,$$

with

$$\mathcal{L}_0 = \frac{1}{2} i \phi^\dagger(\underline{x},t) \overleftrightarrow{\partial_t} \phi(\underline{x},t) - \alpha_0' \nabla\phi^\dagger(\underline{x},t)\cdot\nabla\phi(\underline{x},t) - \Delta_0 \phi^\dagger(\underline{x},t)\phi(\underline{x},t) \tag{1}$$

and

$$\mathcal{L}_I = -\lambda [\phi^\dagger(\underline{x},t)\phi(\underline{x},t)]^2 . \tag{2}$$

The generating functional is

$$W[J] = \frac{1}{N}\int [d\phi][d\phi^\dagger] \exp\Big\{ i\int dt\, d^2\underline{x}\, \big[\mathcal{L}[\phi(\underline{x},t)] + J^\dagger(\underline{x},t)\phi(\underline{x},t) + J(\underline{x},t)\phi^\dagger(\underline{x},t)\big]\Big\} , \tag{3}$$

with

$$N = \int [d\phi][d\phi^{+}] \exp \left\{ i \int dt \, d^2x \quad \mathcal{L}[\phi(x,t)] \right\} . \tag{4}$$

The Lagrangian, given in Eqs. (1) and (2), obviously has the phase symmetry

$$\mathcal{L}[e^{i\theta}\phi(x,t)] = \mathcal{L}[\phi(x,t)],$$

for $\theta =$ a constant. The reflection symmetry usually discussed in theories with quartic couplings would have $\theta = \pi$.

We add a term to the Lagrangian to pick out the broken symmetry solution $i\epsilon|\phi(\underset{\sim}{x},t) - c|^2$, assuming first that c is a non-vanishing real constant. We set $\epsilon = 0$ at the end of our calculations. The Green's functional becomes

$$W[J,\epsilon] = \frac{1}{N_\epsilon} \int [d\phi][d\phi^{+}] \exp \left\{ i \int dt \, d^2x \, [\mathcal{L}[\phi(x,t)] \right.$$

$$\left. + J^{+}(x,t)\phi(x,t) + J(x,t)\phi^{+}(x,t)] + i\epsilon \int |\phi(x,t) - c|^2 dt \, d^2x \right\}, \tag{5}$$

and

$$N_\epsilon = \int [d\phi][d\phi^{+}] \exp \left\{ i \int dt \, d^2x \quad \mathcal{L}[\phi(x,t)] \right.$$

$$\left. + i\epsilon \int |\phi(x,t) - c|^2 dt \, d^2x \right\} . \tag{6}$$

We write

$$\phi = \frac{1}{\sqrt{2}}(\psi + i\chi)$$

and

$$J = \frac{1}{\sqrt{2}}(J_1 + iJ_2) , \tag{7}$$

where $<\chi> = 0$ due to the $\chi \to -\chi$ symmetry as a result of the boundary condition implied by the $i\epsilon$ term.

Making the change of variables

$$\phi(\underset{\sim}{x},t) \to e^{i\theta}\,\phi(\underset{\sim}{x},t)$$

and using

$$\left.\frac{\partial W[J]}{\partial\theta}\right|_{\theta=0} = 0 \tag{8}$$

we obtain the basic identity

$$\begin{aligned} &i\int dt\, d^2\underset{\sim}{x}\quad \langle J_2(\underset{\sim}{x},t)\,\Psi(\underset{\sim}{x},t) - J_1(\underset{\sim}{x},t)\,\chi(\underset{\sim}{x},t)\rangle_{\varepsilon,J} \\ &= \sqrt{2}\,\varepsilon\, i\int dt\, d^2\underset{\sim}{x}\quad \langle\chi(\underset{\sim}{x},t)\rangle_{\varepsilon,J}\quad; \end{aligned} \tag{9}$$

$<F>_{\epsilon,j}$ represents the vacuum expectation value of a function of fields in the presence of the $i\epsilon$ term and the sources J_1, J_2.

Differentiating (9) with respect to J_2, we obtain the Ward identity

$$\langle\Psi(x)\rangle_\varepsilon = \sqrt{2}\,\varepsilon\, i\int[dy]\ \langle\chi(x)\,\chi(y)\rangle_\varepsilon\ , \tag{10}$$

and $x=(\underset{\sim}{x},t)$.

3. THE GOLDSTONE THEOREM AND $Z_\rho = 0$

We may do a Fourier decomposition of the two-point function in (10)

$$\langle\chi(x)\,\chi(y)\rangle_\varepsilon = \frac{1}{(2\pi)^3}\int dE\, d^2p\ e^{-iE(t_x-t_y)}\,e^{ip\cdot(\underset{\sim}{x}-\underset{\sim}{y})}\,G_\chi(E,p,\varepsilon)\ , \tag{11}$$

where we may write generally:

$$G_\chi(E,p,\varepsilon) = \frac{Z_\chi}{E-\alpha' p^2-\Delta_\chi+i\varepsilon} + (\text{continuum contribution})\ . \tag{12}$$

Defining

$$\langle\Psi(x)\rangle_\varepsilon \equiv v(\varepsilon)\ , \tag{13}$$

we obtain

$$v(\varepsilon) = i\sqrt{2}\ \varepsilon c\ G_\chi\ (0,0,\varepsilon)\ . \tag{14}$$

If $v = \lim_{\epsilon \to 0} v(\epsilon) \neq 0$, then

$$v = \sqrt{2}\ c\ Z_\chi\ , \qquad \text{for } \Delta_\chi = 0$$

$$v = 0\ , \qquad \text{for } \Delta_\chi \neq 0\ . \tag{15}$$

This is the Goldstone theorem, i.e., to have a non-vanishing vacuum expectation value of the field, ϕ, its imaginary part must have zero mass. This says nothing about the massiveness or masslessness of the real part of the field. In general, the real part of the field would be massive giving different intercepts for the real and imaginary parts of the field and changing the form of the propagator [4].

Now the vacuum expectation value, v, should be insensitive to the magnitude of c as long as $c \neq 0$, i.e.,

$$\lim_{\varepsilon \to 0} \frac{\partial v}{\partial c} = 0$$

or

$$\lim_{\varepsilon \to 0} \frac{\partial}{\partial c} \langle \psi(x) \rangle_\varepsilon = 0\ . \tag{16}$$

One can show that

$$\frac{\partial}{\partial c} \langle \psi(x) \rangle_\varepsilon = \sqrt{2}\ \varepsilon c \int [dy]\ \langle \rho(x)\ \rho(y) \rangle_\varepsilon\ . \tag{17}$$

We write

$$G_\rho(p,\varepsilon) = \frac{Z_\rho}{p^2 - \Delta_\rho + i\varepsilon} + (\text{continuum contribution})\ , \tag{18}$$

where $p^2 = E - \alpha_0' \underset{\sim}{p}^2$. If $\Delta_\rho \neq 0$, then

$$\lim_{\varepsilon \to 0} \frac{\partial}{\partial c} \langle \psi(x) \rangle_\varepsilon = \frac{\partial v}{\partial c} = 0\ . \tag{19}$$

The Goldstone theorem does not require the mass of ρ to be zero, however, if $\Delta_\rho = 0$ (which would be interesting for a true triple Pomeron coupling, since the coupling ρXX is naturally generated by the spontaneous symmetry breakdown), then

$$\lim_{\epsilon \to 0} \frac{d}{d\epsilon} \langle \psi(x) \rangle_\epsilon \neq 0 , \tag{20}$$

unless $Z_\rho = 0$, [cf. Eq. (18)].

If we consider a Reggeon field theory with interaction Lagrangian $\mathcal{L}_I = -\lambda(\phi^+\phi)^2$, then after the shift of fields : $\psi = \mathrm{Re}\,\phi = \rho + v$ and $\chi = \mathrm{Im}\,\phi = X$, the Lagrangian has the form

$$\mathcal{L}(x) = \rho(x)(\Box + M^2)\rho(x) + \mathcal{L}_I(\rho) + g_0\,\rho\,K(x) + \mathcal{L}_1(\chi(x)) , \tag{21}$$

where

$$\Box = i\frac{\partial}{\partial t} - \alpha_0'\nabla^2 , \qquad K(x) = [\chi(x)]^2 \tag{22}$$

$$g_0 = -\lambda v , \qquad M^2 = \tfrac{3}{2}\lambda v^2 + \Delta_0 . \tag{23}$$

It is known that [7] (at least in the chain approximation) $Z_\rho = 0$ may be realized in a way which keeps the theory local by the conditions :

$$\begin{aligned} M^2 &\to \infty \\ g_0^2 &\to \infty \end{aligned} \tag{24}$$

such that $\frac{1}{2}(g_0^2/M^2) \to -\lambda_0 < \infty$, since Z_ρ can be shown to be

$$Z_\rho = \frac{1}{1 - g_0^2 \dfrac{d\Pi_\rho^{(0)}(\Delta_\rho)}{d\Delta_\rho}} , \tag{25}$$

where $\pi_\rho^{(0)}$ is the self-energy for ρ and $d\pi_\rho^{(0)}/d\Delta_\rho$ is finite for finite (hopefully zero) Δ_ρ ; (Δ_ρ is the renormalized ρ mass).

Using the Lagrangian in Eq. (33), the action may be written

$$\frac{1}{N'} \exp\left\{ i \int [dx]\, \mathcal{L}_I \left(\frac{1}{i} \frac{\delta}{\delta K(x)} \right) + \mathcal{L}_1(\chi(x)) - \frac{1}{2} g_0^2 \int [dx][dy]\, K(x)\, \Delta(x-y)\, K(y) \right), \tag{26}$$

with N defined in Eq. (6). The term

$$- \frac{g_0^2}{2} \int [dy]\, K(x)\, \Delta(x-y)\, K(y)$$

becomes in the limit given in Eq. (24)

$$-\lambda_0\, K(x)\, K(x) = -\lambda_0\, \chi^4(x)\ ; \tag{27}$$

we get a local theory with quartic couplings. For consistency λ_0 must equal the quartic coupling constant, $\lambda/4$, which is the coupling of four χ fields in the original Lagrangian. Equations (23) and (24) yield

$$v^2 = -\frac{\Delta_0}{\lambda} \tag{28}$$

$$g_0^2 = -\lambda\, \Delta_0\ . \tag{29}$$

To satisfy (29) for $g_0^2 < 0$ (i.e., g_0 imaginary) would require $\Delta_0 < 0$ and $\lambda < 0$, or $\Delta_0 > 0$ and $\lambda > 0$. Since Eq. (28) says

$$v = \left(-\frac{\Delta_0}{\lambda} \right)^{1/2}, \tag{28'}$$

what we wish to emphasize is the compatibility of the reality or non-reality of g_0 with our boundary condition for picking a spontaneous broken solution. The two criteria for $g_0^2 < 0$ would imply that $v = iV$, i.e., is imaginary. This contradicts our boundary condition embodied in the $i\epsilon$ term.

If we go back to the $i\epsilon$ term, we have :

$$|\phi - c|^2.$$

We originally assumed c to be real which means that v is real (since the phase of c is the phase of v) *),

$$|\phi - c|^2 = |\psi - c|^2 + \chi^2 = \psi^2 + \chi^2 + c^2 - 2c\psi \, , \tag{30}$$

which means the real part of the field breaks the symmetry and it was on this basis that we derived all our results (including $Z_\rho = 0$). So if v is imaginary, implying c is imaginary, we would have

$$|\phi - c|^2 = \psi^2 + |\chi - c|^2 = \psi^2 + \chi^2 + c^2 - 2c\chi \, , \tag{31}$$

which means that the imaginary part of the field breaks the symmetry. This is a contradiction of our original choice of boundary condition and hence the symmetry with respect to $\chi \to -\chi$ which we know must exist in the theory as implied by the derivation of the Ward identities.

4. DISCUSSION

We have examined a Reggeon field theory of a complex scalar field with real quartic coupling. We find that the spontaneous breaking of phase symmetry in this problem leads, in addition to the Goldstone theorem, to the fact that the component of the field with non-vanishing vacuum expectation value, if massless, must have vanishing wave function renormalization constant. This gives a certain criterion which relates the effective triple coupling, the bare mass in the problem, and the quartic coupling constant.

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*) Replace c by $e^{i\alpha}c$; restore the ϵ term to original form by the change $\phi \to e^{i\alpha}\phi$. Since the Lagrangian is phase invariant, the new Green's functional differs from the old by $J(x) \to J'(x) = e^{i\alpha}J(x)$. Then $v' = e^{i\alpha}v$.

We find that the reality of the vacuum in the theory with spontaneous symmetry breaking implies the satisfaction of this criterion with real triple coupling. To the extent that a Reggeon field theory must have imaginary triple coupling (to give the correct sign of the two-Reggeon cut relative to the pole), our analysis indicates that a Reggeon field theory (such as we have described - including one with higher couplings) cannot undergo spontaneous symmetry breaking [8].

We expect these conclusions to stand for any Reggeon field theory with interactions which are even in the number of fields with real couplings, since the basic criterion stems from the necessity of having an imaginary triple Pomeron coupling after symmetry breakdown.

Another point we wish to make is the apparent utility of using functional techniques to examine certain problems in the Reggeon field theory, just as one uses them in standard relativistic field theories.

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REFERENCES

1) V.N. Gribov - Soviet Phys. JETP 26, 414 (1968).

2) A.R. White - Nuclear Phys. B50, 93 (1972).

3) H.D.I. Abarbanel and J.B. Bronzan - Phys.Letters 48B, 345 (1974) ; Phys. Rev. D9, 2397 (1974) ; Phys.Rev. D9, 3302 (1974).

4) H.D.I. Abarbanel - Phys.Letters 49B, 61 (1974).

5) S.S. Pinsky and V. Rabl - Ohio State University Preprint COO-1545-140 (1974).

6) J. Goldstone - Nuovo Cimento 19, 154 (1961).

7) B. Jouvet - Nuovo Cimento 3, 1133 (1956) ; 5, 1 (1957) ;
J.C. Houard and B. Jouvet - Nuovo Cimento 18, 466 (1960) ;
M.T. Vaughn, R. Aaron and R.D. Amado - Phys.Rev. 124, 1258 (1961) ;
R. Zimmerman - Phys.Rev. 141, 1554 (1966) ;
H. Hayashi, M. Hirayama, T. Muta, N. Seto and T. Shirafuja - Fortschr. Phys. 15, 625 (1967).

8) J. Ellis and R. Savit (to be published) arrive at a similar conclusion using different techniques and assumptions.