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THEORY OF INCLUSIVE PROCESSES †)

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The scope of the talk was determined by two factors:

- a) Owing to considerable overlapping with other Rapporteurs' talks an effort was made to avoid discussing subjects already covered.
- b) Since the subject is rather broadly defined, many contributions were presumably to be discussed in this Session. Taking into account that several review papers presented at Cornell are still up to date, it was decided to cover some topics hitherto undiscussed in previous symposia, as well as very new developments.

This choice implies no value judgement; it simply reflects one of the many viewpoints possible in this rapidly developing field.

The talk is organized as follows:

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1. DUALITY

1.1 Phenomenology of deep inelastic scattering as a function of  $\omega$

In strong interaction physics and in the energy region where we have photon experiments, the relevance of duality ideas seems established<sup>1,2)</sup>. Not only is the correlation of amplitudes for large and small  $\nu$  at work and the relation between s- and t-channel quantum numbers, but recently the presence of duality zeros in  $\pi\pi$  amplitudes has been verified as well<sup>3)</sup>, in the expected pattern. Phenomenologically, the duality properties of the imaginary part of the amplitude are hence satisfied though the real parts are more complicated<sup>1)</sup>.

In electroproduction the observation that averaging occurs was discussed for fixed  $q^2$ <sup>4)</sup> and for fixed  $\omega = 2m\nu/q^2$ <sup>5)</sup>. Since form factors are rapidly varying functions at fixed  $\omega$ , different resonances contribute with varying strengths to make averaging non-trivial.

Since scaling starts early, using duality, one can show that scaling holds even for real photons<sup>6)</sup> with the variable  $\omega_W$

$$\omega_W = \frac{2m\nu + m_W^2}{q^2 + a^2} \quad (1)$$

where  $a = 0.38 \pm 0.02 \text{ GeV}^2$  and  $m_W^2 = 1.4 \pm 0.3 \text{ GeV}^2$ . This is all known and was discussed at the Cornell and Oxford Conferences. New results at this Conference show that the averaging works with the same accuracy for the structure functions of neutron targets. The evidence is obtained through sum rules<sup>7)</sup> and averaging uses the same parameters for  $\omega_W$ . Using a theoretical model<sup>8)</sup>, as discussed below, the same conclusions obtain.

Hence, local duality seems to work well in electroproduction. The next step is to discuss quantum number correlations.

Since both  $\sigma_{\gamma p}$  and  $\sigma_{\gamma n}$  are scaling, their difference is also a function of  $\omega_W$ . This quantity is pure  $I = 1$  in the t-channel, and according to two-component duality it must be, s-channel-wise, only resonances. In this channel only  $I = 1/2$  baryon resonances can couple and the difference must be generated by states that can be excited by isoscalar photons since<sup>8)</sup>

$$\Delta = \langle \gamma_0 N | T | \gamma_1 N \rangle = \sum_n \langle \gamma_0 N | T | n \rangle \langle n | T | \gamma_1 N \rangle \quad (2)$$

where N stands for nucleon,  $\gamma_0$  for isoscalar, and  $\gamma_1$  isovector virtual photon. Then

$$\widetilde{\nu W}_2^{p-n} = \frac{\omega}{\omega_W} \nu W_2 = 2m(\nu - q^2/m^2) \sigma_{tot}^{p,n} \cdot \frac{1}{4\pi^2 \alpha} \frac{1}{\omega_W} \quad (3)$$

where the function  $\widetilde{\nu W}_2^{p-n}$  has a smooth  $q^2 \rightarrow 0$  limit<sup>5)</sup>. The multipoles are given by<sup>9)</sup>

$$\begin{aligned} \sigma_{tot}^p - \sigma_{tot}^n = & -\frac{16\pi q}{k} \sum_{n=0}^{\infty} \left\{ (n+1) \left[ \text{Re}(A_{n+}^{V*} + A_{n+}^S) + \text{Re}(A_{(n+1)-}^{V*} - A_{(n+1)-}^S) \right] + \right. \\ & \left. + \frac{n(n+1)(n+2)}{4} \left[ \text{Re}(B_{n+}^{V*} + B_{n+}^S) + \text{Re}(B_{(n+1)-}^{V*} - B_{(n+1)-}^S) \right] \right\} \quad (4) \end{aligned}$$

Hence, in regions below  $E_\gamma \sim 0.9$  GeV, it can be shown that a few exclusive channel contributions completely build up the difference. Using the Moorhouse-Oberlack phase-shifts<sup>12)</sup> one concludes without any model-dependence but using crucially  $\omega_W$  that:

- a) The averaging over the resonances reproduces the deep inelastic measurements completely in sign, magnitude, and  $\omega$  dependence. In particular, the presence of a difference for  $\omega \sim 12$  and the flattening of the difference as a function of  $1/\omega_W$  are fully reproduced<sup>8)</sup>.
- b) The difference is generated by resonant multipoles<sup>9)</sup> only, and all non-resonant multipoles, though big, cancel<sup>10,11)</sup>. Hence, no other singularities contribute; the standard duality prescription works well in the region  $2 < \omega < 10$ .

Further extension seems difficult because Eq. (2) becomes untractable due to the complexity of final states.

### Conclusions

Phenomenological duality works very well for virtual Compton scattering, both for neutrons and protons. Resonances survive the high  $q^2$  regime, but there is background as well that contributes only to  $I = 0$  t-channel amplitudes. The relative importance of  $I = 1$  and  $I = 0$  contributions as a function of  $q^2$  is unknown.

The observed persistence of  $R \sim 0.15$  in deep inelastic scattering is what is observed from resonance behaviour also<sup>13)</sup>. In the known  $\omega$  region it seems that Regge fits do not adequately describe the difference, though the  $\omega$  interval is quite small.

### 1.1.1 Phenomenological duality and vector dominance

Sakurai has discussed the possibility of relating high and small time-like  $q^2$  results using generalized sum rules<sup>14)</sup>. Assuming scaling one gets the formula:

$$R = \frac{\sigma_{e^+e^- \rightarrow \text{hadrons}}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}} = \frac{2\pi}{f_{\rho\pi}^2/4\pi} \simeq 2.5 \quad (5)$$

where  $f_{\rho\pi\pi}$  is the  $\rho$  coupling constant. Bjorken has discussed this in his talk.

One writes a spectral representation for the cross-section and, assuming saturation with an infinity of vector mesons with well-defined couplings (a very strong assumption), one obtains an approximation formula for deep inelastic scattering in terms of photoproduction cross-sections. In the large  $\omega$  region it is of the form

$$\sigma_T(q^2, W) = \left[ \frac{m_{th}^2}{q^2 + m_{th}^2} \right] \sigma_{\rho p}(W) \quad (6)$$

One then obtains an identification of this  $m_{th}$  (an effective threshold of the integral in the spectral representation) with the  $\omega_W$  parameter  $a^2$ <sup>14)</sup> in the diffractive region.

Another interesting application<sup>14)</sup> is a relation of  $\rho$  production in  $p + p \rightarrow \rho + \text{anything}$ , with heavy mass lepton production. Using duality, one relates the small lepton masses to the large virtual masses. A generalized Drell-Yan law is obtained, and the dependence of the scaling function  $f(S/M^2) \sim \ln(S/M^2)$  is predicted. Numerical predictions for ISR lepton production are also presented.

### 1.2 Theoretical duality

Theoretical models that satisfy large numbers of constraints are known for strong interactions, at least in the tree approximation. For currents further requirements are to be imposed<sup>15)</sup>:

- conservation laws,
- current algebra restrictions,
- compatibility at vector meson poles with purely hadronic amplitudes,
- a spectrum with no new states coming from photons,
- Bjorken scaling and good "resonance scaling" ( $\omega_W$ ),
- power law form factors,
- gauge conditions.

The problem is difficult since some of the requirements are best expressed in field theoretical language and there is no really consistent field theoretical formulation of dual models.

The most recent efforts are mathematically very beautiful<sup>16,17)</sup> but are still far from satisfactory, since Bjorken scaling demands exponential form factors.

In our view the problem requires fresh ideas. The dual models are related to infinitely dense Feynman diagrams with an exponentially growing number of states<sup>18)</sup>. This compositeness is not compatible with the "finite" compositeness seen by the photon<sup>19)</sup>.

Essentially, the elementarity of photons must appear in the theory in a more explicit fashion, while present constructions are purely "vector dominance-like".

Also gauge invariant electric Born models<sup>20,21)</sup> are very successful in the resonance region and they are not simply related to the known dual models that satisfy gauge invariance as an infinite conspiracy.

Perhaps the two-component duality ideas must be generalized, and duality between mass singularities and other structures should be envisaged; fixed poles are natural candidates since they also are peculiar to elementary external states.

### Conclusion

The situation of duality for photon initiated process both time-like and space-like seems very promising. In contrast to the case for hadron physics, a Born model for currents is still unavailable. The presence of large real parts for the  $\gamma$ -initiated reactions<sup>21)</sup> in the resonance region is imposed by gauge invariance and will have to be incorporated into dual theory.

## 2. DEEP INELASTIC SCATTERING IN A MODEL WITH CONSTITUENT AND CURRENT QUARKS

There is good evidence that  $SU(6)_W \otimes O(3)$  may finally be a very useful concept for classification, and now there is a revival of its use for transition operators<sup>22)</sup>. Combining these ideas with a parton model in which the constituents preserve their identity but they themselves are composed of point-like constituents has been proposed<sup>23)</sup>.

In this scheme the nucleons are classified in representations that are mixtures of  $(56,0)$   $(70,1^-)$ , ... in the infinite momentum frame, but each quark possesses structure.

The nucleon structure functions are of the form

$$\frac{1}{X} F_2^{aP} = \frac{4}{9} [p(x) + \bar{p}(x)] + \frac{1}{9} [n(x) + \bar{n}(x) + \lambda(x) + \bar{\lambda}(x)] \quad (7)$$

where  $p(x)$  is the average number of quarks of type  $p(x)$  and similarly for the other quarks  $\lambda, n$ . The function  $p(x)$  obeys the equation

$$p(x) = \int_x^1 \frac{dz}{z} \left[ p_0(z) \phi_{pp} \left( \frac{x}{z} \right) + n_0(z) \phi_{np} \left( \frac{x}{z} \right) \right] \quad (8)$$

where  $\phi_{AB}$  is the structure function that specifies the number of partons of type B within A, with fraction  $x/z$  of momentum,  $p_0, n_0$  are the distribution functions of the constituents within the proton. In the infinite momentum frame the authors make a choice of  $O(3)$ . This choice, very simple, defines the dynamical model. The physical state  $\Psi_{\text{Born state}} = \text{function} [q_{\perp i}, x_i, \text{SU}(6) \text{ quantum}]$  will be written

$$\Psi_{ABC} = \phi_{ABC}(V_1, V_2, V_3) e^{-k/2 \sum V_i \cdot V_i} \quad (9)$$

where  $\phi$  is an  $SU(6)$  eigenstate, the  $O(3)$  acts upon the vectors  $V_i$  and the radial part is parametrized by harmonic oscillator wave functions. The rotation group chosen in this cylindrical frame demands

$$V_i = Q_i - \frac{1}{2} (Q_j + Q_k) \quad i, j, k = 1, 2, 3 \quad (10)$$

where

$$\vec{Q}_k = [ \vec{q}_{k\perp}, m \ln x_k ] \quad (11)$$

( $m$  is a parameter) .

The proton is written as

$$P = a|A\rangle + b|B\rangle \quad (12)$$

where

$$A = |56\rangle, \quad B = |70\rangle \quad (13)$$

( $b = 0$  means no mixing) .

For the parton distribution one uses duality and Regge ideas and finally three parameters are left. These roughly fix the energy dependence, the  $p_T$  fall-off of the oscillator wave functions, and the sea contributions near  $x \sim 1$  to be small. The results are seen in Figs. 1 and 2 for the structure functions and their difference. One can also compute the famous number

$$g_A/g_V = \frac{5}{3} a^2 \simeq 1.52 \quad (14)$$

where  $a^2$  is the probability for the nucleon being in  $56^{25)}$ . The compositeness of the constituents drives down the ratio to  $1.25^{26)}$ , and in this respect the scheme differs from the old mixing schemes that gave the full change in terms of mixing. Qualitatively the approach seems promising. Of course, finding the exact  $O(3)$ , and understanding the dynamics of quark proton scattering will not be easy<sup>27)</sup>, but if further phenomenological connections are found it will be useful. The method fails to explain  $\mu$ -pair production, but it is argued that this particular process is mixed with strong interactions and one should not hope for agreement. The standard parton model sum rules are fulfilled but the parameters of the sea are not the same as in the Kuti-Weisskopf model<sup>24)</sup> since in this model the sea quark concepts are not the same as in Ref. 24. The connection of the parameters used here and the ones obtained in the three-point function calculations<sup>22)</sup> should be understood, and this may clarify the structure of the theory.

### 3. PARTON MODEL

#### 3.1 Theoretical questions

The parton model<sup>28)</sup> has been widely accepted as the natural explanation of the persistence of large cross-sections in the deep inelastic region.

It became apparent that in field theory models<sup>29)</sup> the problem of keeping constituents with fractional quantum numbers from being produced in final states is very difficult. One wants to construct a model in which somehow these quark coordinates are totally absent in asymptotic states. The problem is serious and beyond simple-minded perturbation theory considerations. This can be seen in two different ways.

Consider an annihilation graph in which a time-like photon of mass  $Q^2$  splits into two partons. A short-range mechanism allows these partons to thermalize and one hopes to obtain an overlap of these cascades in space-time to annihilate fractional quantum numbers. The cascade, however, requires time to develop, proportional to  $Q^2$ , and since the partons are moving apart the overlap mechanism fails<sup>30)</sup>. Hence, each jet will inevitably show fractional additive quantum numbers.

With long-range forces one could presumably avoid the difficulty<sup>31)</sup> since the time argument is not present any more. Nevertheless, one must produce a non-perturbative solution to a model that expresses all physical states in terms of bound states, to avoid the unwanted quarks.

Two-dimensional models do indeed show a mechanism that may have some relevance<sup>32)</sup>. Consider Schwinger's two-dimensional electrodynamics<sup>33)</sup>. One starts with a theory in terms of fermions that obey



$$(\partial_{\pm} - e A_{\pm}) \Psi_{\pm} = 0 \quad ; \quad \Psi_{\pm} = e^{i\phi_{\pm}} \chi_{\pm} \quad (15)$$

where

$$x_{\pm} = x_{1\pm} x_0, \quad \partial_{\pm} \chi_{\pm} = 0, \quad \partial_{\pm} \phi_{\pm} = e A_{\pm} \quad (16)$$

These equations are so simple because of the restricted dimensionality of space-time.

Compute a typical two-point function in configuration space

$$\begin{aligned} \langle 0 | \bar{\psi}(x) \psi(0) \rangle &= \frac{1}{x^2} \left[ e^{a \Delta_F(0,x) - a \Delta_F(m,x^2)} \right] \\ &= e^{-a \Delta_F(m,x^2)} \end{aligned} \quad (17)$$

one finds out that the Wightman functions are exactly given by a free Bose field that has a mass  $e^2/\pi$ . Hence, the fermions have disappeared. At short distances the current being  $j_{\mu} = \epsilon_{\mu\nu} \partial_{\nu} \phi$ , the singularity of the matrix element is like a fermion. Such an interpretation is possible<sup>32)</sup>, but of course this is peculiar to the structure of the current in the model, and the dimensionality of space-time. Physically, the fermi charge has been shielded to infinity, helped by the peculiar nature of electromagnetism in two dimensions; the potential of which is not diminished by distance. So, the model gives a physical example of total shielding, but it is not clear that the mechanism is general. Several questions come to mind:

- a) How is the spread avoided in three dimensions?
- b) Which long-range forces are effective in nature?
- c) How is (b) to be modified if "photon" masses are finite?

A non-trivial model in four dimensions is needed!

Other approaches are field theories with massive quarks<sup>34)</sup> and thoughts using gauge theories<sup>35)</sup>, but it seems too early to decide on their effectiveness. Clearly, if partons are the answer, and if some of them carry quark quantum numbers a serious problem is still with us.

### 3.2 Parton model calculations

Let us forget the difficulties and try to calculate with the model. We discuss some predictions typical for the model and whenever possible discuss the experimental situation. However, this discussion is not intended to be a complete list of predictions.

### 3.2.1 Charge asymmetries

Consider the reaction " $\gamma$ " + N  $\rightarrow$  h $^\pm$  + anything. It is well known<sup>36)</sup> that the value of  $\omega = 2m\nu/q^2 = 1/x$  is related to x; where x is the fraction of the target momentum carried by the parton hit by the current.

Consider first  $\omega$  small. One is then testing the large x distribution that is presumably dominated by valence quarks, the sea being confined to  $x < 0.3$  or less. If one further looks to  $x_\pi \sim 1$ , one is then looking to the pion ejected from the current and there the neutron quark cannot contribute to  $\pi^+$  formation or the proton quark to  $\pi^-$  formation, since these may require multiple emission steps. One can immediately write down formulae to express these inclusive production cross-sections in terms of charges of partons, their x distribution and the emission probability D of a given  $\pi$ . They read<sup>37)</sup>

$$\begin{aligned}
 \gamma + p &\longrightarrow \pi^+ \sim \left(\frac{2}{3}\right)^2 u(x) D_u^{\pi^+} + \left(\frac{1}{3}\right)^2 d(x) D_d^{\pi^+} \\
 \gamma + p &\longrightarrow \pi^- \sim \left(\frac{2}{3}\right)^2 u(x) D_u^{\pi^-} + \left(\frac{1}{3}\right)^2 d(x) D_d^{\pi^-} \\
 \gamma + n &\longrightarrow \pi^+ \sim \left(\frac{2}{3}\right)^2 d(x) D_d^{\pi^+} + \left(\frac{1}{3}\right)^2 u(x) D_u^{\pi^+} \\
 \gamma + n &\longrightarrow \pi^- \sim \left(\frac{2}{3}\right)^2 d(x) D_d^{\pi^-} + \left(\frac{1}{3}\right)^2 u(x) D_u^{\pi^-}
 \end{aligned} \tag{18}$$

where

$$\int u(x) dx = 2 \quad \int d(x) dx = 1 \tag{19}$$

Hence, from our previous considerations we obtain

$$R_{\pi^+/\pi^-}^p \xrightarrow[\substack{\omega \rightarrow 1 \\ x_\pi \rightarrow 1}]{\omega \rightarrow \infty} \infty \quad R_{\pi^+/\pi^-}^n \xrightarrow[\substack{\omega \rightarrow 1 \\ x_\pi \rightarrow 1}]{\omega \rightarrow \infty} 0 \tag{20}$$

where the notation is self-explanatory. When  $\omega \rightarrow \infty$  one is testing the wee partons and everyone predicts

$$R_{\pi^+/\pi^-}^p = R_{\pi^+/\pi^-}^n \longrightarrow 1 \tag{21}$$

Using more sophisticated means it can be shown<sup>38)</sup> that for the integrated  $x_\pi$  distribution

$$R_{\pi^+/\pi^-} - 1 \xrightarrow{\omega \rightarrow \infty} \frac{5}{3} \left[ 1 - \frac{\sigma_{ep}(\omega)}{\sigma_{en}(\omega)} \right] \quad (22)$$

a similar formula holding for other additive quantum numbers<sup>38)</sup>.

The intermediate  $\omega$  region is even more model-dependent. However, using the fact that  $F^{ep}/F^{en} \geq 1$ , one can predict that

$$\begin{aligned} \text{a) } R_{\pi^+/\pi^-}^p &> R_{\pi^+/\pi^-}^n && \text{always, and} \\ \text{b) } R_{\pi^+/\pi^-}^n &> 1 \end{aligned} \quad (23)$$

after some  $\omega$  and approaches 1 from above. Because of the presence of the sea one expects the effects to be sharpest at  $x_\pi \sim 1$ . Experimentally the situation is encouraging. There is a net excess of  $\pi^+$  in the current fragmentation region that seems to be most marked at small  $\omega$ .

The DESY experiments<sup>39)</sup> are perhaps too low energy and a conventional resonance model with pions overflowing into the forward hemisphere and  $\pi$  exchange may explain the data; the  $x_\pi$  dependence is not favourable for partons. SLAC experiments<sup>40)</sup> are perhaps better as a test. The  $x_\pi$  dependence is not the most favourable and large  $\omega$  still shows asymmetry. However, the large  $x_\pi$  neutron data fall below 1, as expected. This may be an interesting test once errors are narrowed down. Of course, the total charge asymmetry is a tiny effect. Most positive charge remains in the target fragmentation region. There are more detailed models for this effect<sup>41)</sup>.

### 3.2.2 $\phi$ -dependence in inclusive electroproduction

In the same experiment one can measure the angle of the hadron with respect to the plane defined by the photon and the leptons. In the target fragmentation region it is hard to expect anything but loss of memory of photon polarization. In the current fragmentation region the helicity cannot be flipped since the photon hits a parton of at most spin- $1/2$  and finite transverse mass, so one expects no  $\phi$  dependence either<sup>42)</sup>.

One predicts<sup>43)</sup>

$$W_3 \sim \frac{1}{q^2}, \quad W_4 \sim \left[ \frac{1}{q^2} \right]^{1/2} \quad (24)$$

where these are the structure functions appearing as coefficients of  $\cos \phi$  and  $\cos 2\phi$ , respectively. Though early experiments seemed to substantiate this prediction<sup>44)</sup>, results submitted at this Conference seem less promising<sup>45,46)</sup>.

### 3.2.3 Baryon number distributions

Nothing new has been reported; it remains a problem.

### 3.2.4 Multiplicities

In multiperipheral models<sup>47)</sup> and in soft parton field theories<sup>48)</sup> in which the partons couple to the photon, one expects a multiplicity law  $\log \omega$  and a constituent isolated at the edge of phase space. Bjorken<sup>49)</sup> has argued that consistency demands that in the spirit of the theoretical discussion the parton in the photon fragmentation region also cascades to fill the rapidity gap. One expects a law

$$\bar{n} \sim a \log \omega + b \log q^2 \quad (25)$$

where presumably this is the distribution of hadrons and not constituents.  $a$  is expected to be related to the hadronic plateau<sup>50)</sup> and  $b$  in some naive models<sup>51)</sup> to the annihilation plateau.

Recent experiments show<sup>46)</sup>

$$\bar{n} \sim \log s \quad (26)$$

implying  $a = b$ . However, since we are in a region of  $q^2$  and  $W$  where hadronic physics is far from having developed plateaux, I would consider the result of little relevance<sup>52)</sup>. A naive argument based on rapidity length 2 for fragmentation regions and  $\omega_W$  for connecting  $\omega$  to  $s$  illustrates the sad possibility that for exposing the whole rapidity axis one may require  $E_Y \sim 10.000$  GeV. Several problems concerning fractional quantum number migration are discussed by Bjorken<sup>53)</sup>.

There are several models using Mueller analysis<sup>54)</sup> and more detailed models for quark structure. Unless something specific is fed about the photon vertex, nothing new can come out. Nevertheless, as a guide for problems of scaling and approach to scaling these may be useful<sup>54)</sup>.

Finally, though it is not a parton model result it is worth pointing out that the light-cone does not predict constant multiplicities in annihilation<sup>55)</sup>.

The general connection between singularities in the matrix element and the multiplicity law has been related to the dimension of the source. As expected, soft theories have constant multiplicities, a result already established directly<sup>55)</sup>.

### 3.2.5 Scaling in inclusive production

Having a field theory parton model one can study the structure functions  $\bar{W}_{1,2}$  as a function of four independent variables<sup>56)</sup>. In simple models one finds that one obtains scaling in two independent ratios for fixed values of the other

invariants<sup>56,57)</sup>. This scaling cannot be generated by light-cone considerations and whether it obtains or not is a very interesting problem. Data is as yet insufficient to test it and "non-asymptotic" modifications<sup>58)</sup> are not very significant. One must wait for further data and a larger span of  $W$  and  $q^2$ .

### 3.2.6 Exclusive channels

Work along these lines exists but is very model-dependent<sup>59)</sup>.

### 3.2.7 Inclusive $\mu$ -pair production from real photons

Calculations by parton methods<sup>60,61)</sup> disagree violently with measured production (see Bloom's report).

## Conclusions

Besides the problems of principle, I think that it is fair to say that there is little support in the present energy and  $q^2$  regime for parton model predictions. Nevertheless, some results may be more general and indeed of fundamental validity. Good examples are Drell-Yan threshold relations<sup>62)</sup> and  $\mu$ -pair scaling laws<sup>63)</sup>. Anyhow, it is a useful way to think about experiments.

## 4. SCALING AND SCALING BREAKING

### 4.1 Models for scaling breakdown including field theory calculations of anomalous dimensions

It has become possible to test scaling in a detailed fashion<sup>64)</sup> and all information is compatible with no breakdown, though logarithmic terms with small coefficients are allowed. The rapid onset of scaling is a cause of embarrassment to the asymptotic arguments which anyhow are never a general proof of scaling.

Renormalization group arguments were first used to predict scaling breakdown<sup>65)</sup>. Since then field theory models<sup>66)</sup>, parton structure<sup>67)</sup>, sound state models<sup>68)</sup>, and large  $p_T$  hadronic physics<sup>69)</sup> have given support to these ideas.

In super-renormalizable models scaling can be arranged<sup>70)</sup> but in renormalizable theories one is led to expect anomalous dimensions for some tensors. Only quantities that must remain canonical do remain so. As a consequence some observables like the annihilation remain to scale while the integrals<sup>71)</sup>

$$\int_1^{\infty} \frac{\nu W_2}{\omega h} d\omega \Big|_I \propto q^2 - d_A(n, I)/2 \quad (27)$$

become functions of  $q^2$ .

In some models  $\lambda\phi^4$ ,  $g(\bar{\psi}\psi)^2$  the anomalous dimensions of tensor operators like  $\phi^2$ ,  $\phi\nabla_1 \dots \nabla_n \phi$  can be computed using the elegant  $\epsilon$  expansion method<sup>72)</sup>. In particular  $\lambda\phi^4$  in less than four dimensions gives anomalies that are very small<sup>71)</sup>. This would indicate support for small breaking. However, other models (like gluon models) give large breaking<sup>73)</sup>. Since in any case these theories become trivial at four dimensions, one should take the results with a grain of salt. This of course relates to the problem of solving non-trivial theories, since it is well known that scaling must break down in perturbation theory solutions.

#### 4.2 A Thirring model as a conformal bootstrap

Another interesting result about scaling, again in two dimensions, concerns a Thirring model with currents that have internal  $SU(n)$  symmetry<sup>74)</sup>. As is well known<sup>75)</sup>, the Thirring model is always scale-invariant for any coupling strength but with anomalous dimensions.

The model with non-abelian currents is still solvable, and if one demands a scale-invariant solution it turns out that the answer is that only one finite value of the coupling allows for a solution:  $g_v = 4\pi/n + 1$ . This is a sort of conformal bootstrap. Unfortunately, the peculiar factorization seen in Section 3 plays a crucial role here too and it is difficult to see how general the result is.

### 5. LAST MOVEMENT: PRESTISSIMO

Here we discuss a few items briefly. How is scaling built in terms of exclusive channels? Certainly exclusive channels do not scale, at least not in the duality sense. Also, diffraction channels are not scaling and in particular one must consider the possibility that  $\nu W_2$  may go to 0. The average  $p_T$  seems to be increasing and this problem is related to the height of the photon fragmentation plateau. Finally, there is the problem of connections between inclusive annihilation and total electroproduction. No continuation is in general possible<sup>76)</sup>. However, in perturbation theory if the amplitude behaves as  $A(\omega - 1)^P$  on one side when  $\omega \rightarrow 1$ , it behaves like  $A(1 - \omega)^P$  as a limiting relation<sup>77)</sup> on the other. The Gribov-Lipatov<sup>78)</sup> relations relating  $F(\omega)$  to  $F(1/\omega)$  are probably formal due to the previous argument. They hold as a formal identity in the leading logarithmic approximation in large classes of renormalizable field theories<sup>78,79)</sup>. New bounds have been obtained in deep inelastic processes<sup>80)</sup>.

Conclusions

The field of photon dynamics is a most fascinating one and it may hold some of the clues of the hadron structure.

We have very little understanding through models of the behaviour of the amplitude as a function of  $q^2$ . Scaling seems to hold quite well, and some hadronic concepts like duality seem to be at work.

The question of point-like structure is open, and further confirmation of spectacular findings, like increasing annihilation cross-section, may well give us the hint to enable us to unravel the mysteries of photons, leptons, and hadrons.

It is a pleasure to acknowledge discussions with G. Altarelli, N. Cabibbo, J. Ilioupoulos, A. Schwimmer and Y. Zarmi.

REFERENCES AND FOOTNOTES

- †) This is really a written version of the talk. The only expansion concerns the references. They should be used as a guide. They do not claim to be complete and are certainly not intended to assign priorities.
- 1) See, for example, the Tel Aviv Duality Conference, edited by E. Gotsman.
  - 2) For theoretical ideas, see G. Veneziano, Physics Reports, to be published; for photons see Ref. 5.
  - 3) P. Estabrooks et al., CERN preprint TH 1661.
  - 4) E. Bloom and F. Gilman, Phys. Rev. Letters 25, 1140 (1970); and F. Gilman's talk in Ref. 1.
  - 5) V. Rittenberg and H.R. Rubinstein, Phys. Letters 35 B, 50 (1971). For a detailed discussion see: H.R. Rubinstein, Springer Tracts of Modern Physics, 62, 72 (1972).
  - 6) W. Brasse et al., Nuclear Physics B39, 421 (1972).
  - 7) W. Köbberling et al., Paper 299 submitted to this Conference.
  - 8) G. von Gehlen, H.R. Rubinstein and H. Wessel, Phys. Letters 42 B, 365 (1972).
  - 9) H. Kowalski, H. Römer, H.R. Rubinstein, Nuclear Physics B, 59, 589 (1973).
  - 10) There is a small discrepancy in the region where  $2\pi$  inelastic channels open. This can be related to the failure of the isovector-isoscalar Drell-Hearn-Gerasimov sum rule (Ref. 11).
  - 11) I. Karliner, SLAC preprint 1179 (TH), to appear in Physical Review.
  - 12) P.G. Moorhouse and H. Oberlack, as reported by G. von Gehlen; see also R. Moorhouse, H. Oberlack and A.H. Rosenfeld, to be published.
  - 13) See Bloom's report, this Conference.
  - 14) J.J. Sakurai, 1973 Varenna Lectures, to be published, and references therein.
  - 15) For a review of other models, see M. Ademollo, Springer Tracts of Modern Physics, Vol. 59.
  - 16) A. Neveu and J. Scherck, Nuclear Physics B41, 365 (1972).
  - 17) A. Cremmer and J. Scherck, Nuclear Physics B58, 557 (1973).
  - 18) H. Nielsen, 1969 Copenhagen manuscript, unpublished.
  - 19) D. Amati, R. Jengo, H.R. Rubinstein, G. Veneziano and M. Virasoro, Phys. Letters 27 B, 267 (1968).
  - 20) D. Lücke and P. Söding, Springer Tracts on Modern Physics 59, 39 (1971).
  - 21) P.V. Collins, H. Kowalshi, H. Römer and H.R. Rubinstein, Phys. Letters B44, 183 (1973).



- 22) For recent work, see: F. Gilman and M. Kugler, Phys. Rev. Letters 30, 518 (1973).  
F. Gilman, M. Kugler and S. Meshkov, Phys. Letters 45 B, 481 (1973).  
W. Hey and J. Weyers, CERN preprint TH-1614 (1973).  
For classification see: D. Faiman, Talk at the Aix-en-Provence Conference (1973).
- 23) C. Altarelli, N. Cabibbo, L. Maiani and G. Petronzio, CERN preprint TH-1727 (1973).
- 24) J. Kuti and V. Weisskopf, Phys. Rev. D4, 3418 (1973).
- 25) H. Harari, Phys. Rev. Letters 16, 964 (1966); 17, 56 (1966).  
G. Altarelli, R. Gatto, L. Maiani and G. Preparata, Phys. Rev. Letters 16, 918 (1966).  
R. Gatto, L. Maiani and G. Preparata, Phys. Rev. Letters 16, 377 (1966).  
I. Gerstein and B. Lee, Phys. Rev. Letters 16, 1060 (1966).
- 26) The authors have computed this value. G. Altarelli, private communication.
- 27) The possible differences introducing generalized vectors

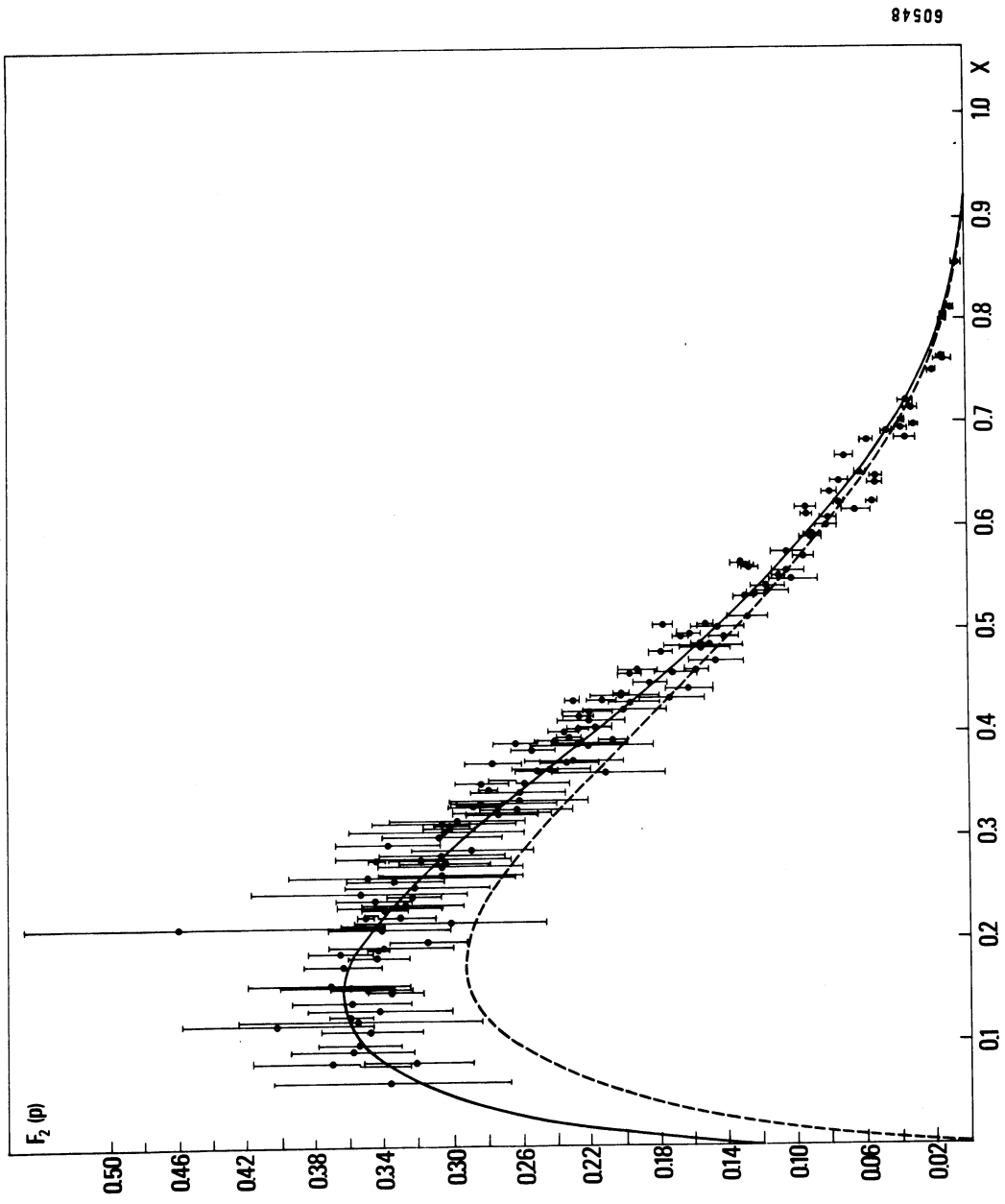
$$Q_i = [q_{k\perp}, m k x_k + |h(q_{k\perp})|]$$

where the function  $h$  is chosen different from 0 is being studied.

- 28) J.D. Bjorken and E.A. Paschos, Phys. Rev. 185, 1975 (1969).
- 29) S. Drell, D. Levy and T. Yan, Phys. Rev. Letters 22, 744 (1969), and later papers. This result is very general and applies to any kind of multi-peripheral diagram with elementary coupling of the photon to the constituents.
- 30) J. Kogut, D.K. Sinclair and L. Susskind, Institute for Advanced Studies, COO 2260-6 (1973).
- 31) This observation is due, like many others, to J.D. Bjorken.
- 32) A. Casher, J. Kogut and L. Susskind, Tel Aviv University preprint TAUP 373-73 (1973).
- 33) J. Schwinger, Phys. Rev. 128, 2425 (1962).
- 34) This point of view is argued by G. Preparata. See Bjorken's report and R. Gatto and G. Preparata, work submitted to this Conference.
- 35) Complicated shielding mechanisms seem possible in non-abelian gauge theories.
- 36) For a discussion of this point and for a comprehensive discussion of the parton model, see:  
R.P. Feynman, Photon-hadron interactions, W.A. Benjamin (1973).
- 37) These formulae are taken from M. Gronau, F. Ravndal and Y. Zarmi, Nuclear Physics B51, 611 (1973).
- 38) A. Schwimmer, Comments and addenda, Physical Review in press. Cal Tech 68-376 (1972).
- 39) I. Danam et al., DESY 72/71 (1972).

- 40) J. Dakin et al., SLAC preprint 1074 (1972), 1236 (1973).
- 41) M. Chaichian, S. Kitakado, S. Pallua and Y. Zarmi, Nuclear Physics B58, 140 (1973), and related papers.
- 42) F. Ravndal, Phys. Letters 43 B, 301 (1973).
- 43) The kinematics of this process requires four structure functions.
- 44) See J. Dakin SLAC preprint 1236 (1973).
- 45) See W. Brasse's report on the DESY streamer chamber experiments.
- 46) K. Berkelman et al., Contribution 115 to this Conference.
- 47) The log  $\omega$  answer is well known. See for example:  
G. Altarelli and L. Maiani, Nuclear Physics B51, 509 (1973).
- 48) P.V. Landshoff, J.C. Polkinhorne and R.D. Short, Nuclear Physics B19, 432 (1970).  
For a general view of soft field theories see the review by the first two authors, Physics Report 5 C, 1 (1972).
- 49) J. Bjorken, Invited Talk at the Irvine-California Conference on Particle Physics 1971, and other related papers.
- 50) R.N. Cahn, J.K. Cleymans and E.W. Colglazier, SLAC preprint (1972).
- 51) This result seems to be too naive since the cascading process must be complicated and must alter the coefficient  $a$ .
- 52) The hadronic plateau is seen somewhere at  $E_{lab} \sim 200$  GeV.
- 53) See for example, G. Ferrar and J. Rosner, CALTECH preprint, and R.N. Cahn, J.K. Cleymans and E.W. Colglazier, submitted to this Conference, for models of quantum number migration.
- 54) See, M. Chaichian, S. Kitakado, C.S. Lam and Y. Zarmi, University of Helsinki preprint (1973), and references therein for a Muller analysis. However, since the dynamics of the photon vertex is unknown, very little new is contributed by this approach.
- 55) J. Ellis and Y. Frishman, Phys. Rev. Letters 31, 135 (1973).  
R. Brandt and W. Ng, Stony Brook preprints 73-16, 73-34.  
C. Altarelli and L. Maiani, Phys. Letters 41 B, 480 (1972).
- 56) S.D. Drell and T.M. Yan, Phys. Rev. Letters 24, 855 (1970).
- 57) P. Landshoff and J.C. Polkinhorne, Nuclear Physics B28, 240 (1971).
- 58) E.W. Colglazier and F. Ravndal, Phys. Rev. D7, 1537 (1973).
- 59) P.V. Landshoff and J.C. Polkinhorne, Nuclear Physics B32, 541 (1971).  
J.H. Weis, Nuclear Physics B40, 562 (1970); Phys. Letters 36 B, 579 (1971).
- 60) J.D. Bjorken and E. Paschos, Phys. Rev. D1, 1450 (1970).
- 61) J. Brodsky and P. Roy, Phys. Rev. D3, 2914 (1971).
- 62) S.D. Drell and T. Yan, Phys. Rev. Letters 24, 855 (1970).

- 63) S.D. Drell and T.M. Yan, Phys. Rev. Letters 25, 316 (1970).
- 64) See Bloom's report.
- 65) K. Wilson, Phys. Rev. Letters 27, 690 (1971).
- 66) S. Drell and T.D. Lee, Phys. Rev. D6, 1738 (1972).
- 67) S. Drell and M. Chanowitz, Phys. Rev. Letters 30, 806 (1973).
- 68) This has been emphasized by T.D. Lee, 1973 Lectures at Weizmann Institute.
- 69) A. Casher, S. Nussinov and L. Susskind, Tel Aviv University preprint.
- 70) G. Altarelli and H.R. Rubinstein, Phys. Rev. 187, 2111 (1969),  
for  $\phi^3$ . For spin- $\frac{1}{2}$  particles see:  
S. Shei and D.M. Tow, Phys. Rev. D4, 2056 (1971).
- 71) K. Wilson, CLNS preprint 198 (1972).
- 72) C.G. Bollini and J. Giambiaggi, Phys. Letters 40 B, 566 (1972).  
t'Hooft and T. Veltman, Nuclear Phys. B44, 189 (1972).  
K. Wilson, see Ref. 71.  
J.F. Ashmore, Nuovo Cimento Letters 4, 289 (1972).
- 73) S.S. Shei and T.M. Yan, result quoted in CLNS 238 (1973).
- 74) R. Dashen and Y. Frishman, Institute for Advanced Studies preprint (1973).
- 75) K. Wilson, Phys. Rev. D2, 1473 (1970).  
J. Lowenstein, Com. Math. Phys. 16, 265 (1970).  
G.F. dell'Antonio, Y. Frishman and D. Zwanziger, Phys. Rev. D6, 988 (1972).
- 76) P. Landshoff and J. Polkinhorne, Phys. Rev. D6, 3708 (1972).
- 77) R. Gatto, G. Menotti and I. Vendramin, Nuovo Cimento Letters 5, 54 (1972).
- 78) N. Christ, B. Hasslacher and A. Mueller, Phys. Rev. D6, 3543 (1972).  
N. Gribov and N. Lipatov, Phys. Letters 37 B, 78 (1971).
- 79) S. Ferrara, R. Gatto and G. Parisi, Phys. Letters 44 B, 381 (1973).
- 80) O. Nachtmann, Institute for Advanced Studies preprints.



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Fig. 1

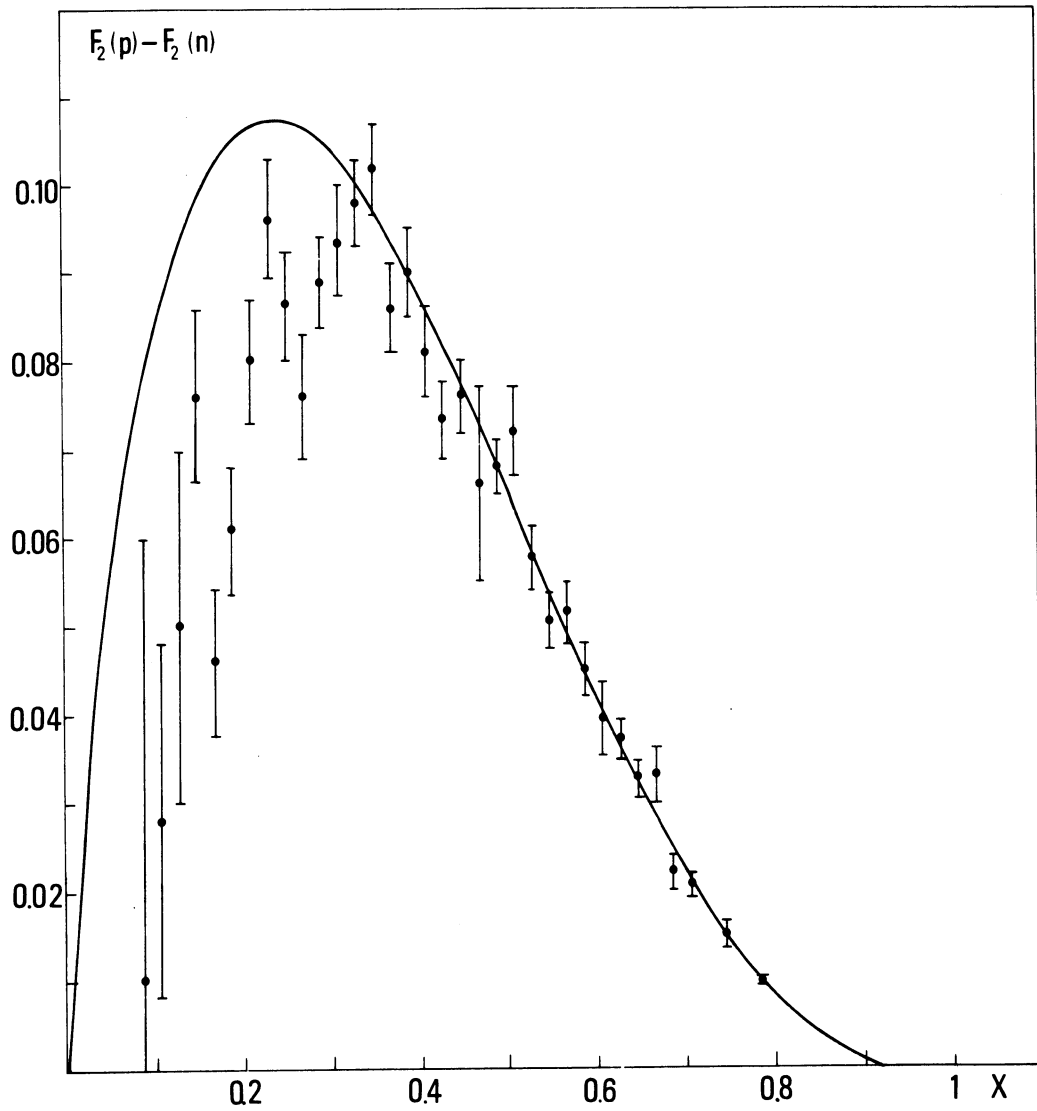


Fig. 2

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