MEASURING THE RUNNING OF THE ELECTROMAGNETIC COUPLING ALPHA IN SMALL ANGLE BHABHA SCATTERING [a]

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We propose a method to determine the running of α_{QED} from the measurement of small-angle Bhabha scattering. The method is suited to high statistics experiments at e^+e^- colliders, which are equipped with luminometers in the appropriate angular region. We present a new simulation code predicting small-angle Bhabha scattering. A detailed description of this idea can be found in ref.[1]

1 Introduction

The electroweak Standard Model $SU(2) \otimes U(1)$ contains Quantum Electrodynamics (QED) as a constitutive part. The running of the electromagnetic coupling α is determined by the theory as

$$\alpha(q^2) = \frac{\alpha(0)}{1 - \Delta\alpha(q^2)}, \tag{1}$$

where $\alpha(0) = \alpha_0$ is the Sommerfeld fine structure constant, which has been measured to a precision of 3.7×10^{-9} [2]; $\Delta\alpha(q^2)$ positive arises from loop contributions to the photon propagator. The numerical prediction of electroweak observables involves the knowledge of $\alpha(q^2)$, usually for $q^2 \neq 0$. For instance, the knowledge of $\alpha(m_Z^2)$ is relevant to the evaluation of quantities measured by the LEP experiments. This is achieved by evolving α from $q^2 = 0$ up to the Z-mass scale $q^2 = m_Z^2$. The evolution expressed by the quantity $\Delta\alpha$ receives contributions from leptons, hadrons and the gauge bosons. The hadronic contribution to the vacuum polarization, which cannot be calculated from first principles, is estimated with the help of a dispersion integral and evaluated [3] by using total cross section measurements of $e^+e^- \rightarrow$ hadrons at low energies. Therefore, any evolved value $\alpha(q^2)$ particularly for $|q^2| > 4m_\pi^2$, is affected by uncertainties originating from hadronic contributions. The uncertainty on $\alpha(m_Z^2)^{-1}$ induced by these data is as small as ± 0.09 [3]; nevertheless it turned

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out [4] that this limits the accurate prediction of electroweak quantities within the Standard Model, particularly for the prediction of the Higgs mass.

While waiting for improved measurements from BEPC, VEPP-4M and DAFNE as input to the dispersion integral, intense efforts are made to improve on estimating the hadronic shift $\Delta\alpha_{had}$, as for instance [5–8], and to find alternative ways of measuring α itself. Attempts have been made to measure $\alpha(q^2)$ directly, using data at various energies, such as measuring the ratio of $e^+e^-\gamma/e^+e^-$ [9] or more directly the angular distribution of Bhabha scattering [10]. In this article the running of α is studied using small-angle Bhabha scattering. This process provides unique information on the QED coupling constant α at low *space-like* momentum transfer $t = -|q^2|$, where

$$t = -\frac{1}{2}\ s\ (1 - \cos\theta) \qquad (2)$$

is related to the total invariant energy $\sqrt{s}$ and to the scattering angle θ of the final-state electron. The small-angle region has the virtue of giving access to values of $\alpha(t)$ without being affected by weak contributions. The cross section can be theoretically calculated with a precision at the per mille level. It is dominated by the photonic t channel exchange and the non-QED contributions have been computed [11] and are of the order of 10^{-4}; in particular, contributions from boxes with two weak bosons are safely negligible.

In general, the Bhabha cross section is computed from the entire set of gauge-invariant amplitudes in both the s and t channels. Consequently, two invariant scales s and t govern the process. The different amplitudes are functions of both s and t and also the QED coupling α appears as $\alpha(s)$ resp. $\alpha(t)$ [12]. However, the restriction of Bhabha scattering to the kinematic regime of small angles results in a considerable simplification, since the s channel then gives only a negligible contribution, as is quantitatively demonstrated [1]. Thus, the measurement of the angular distribution allows us indeed to verify directly the running of the coupling $\alpha(t)$. For the actual calculations, $\theta \gg m_e/E_{beam}$ and $E_{beam} \gg m_e$ must be satisfied. Obviously, in order to manifest the running, the experimental precision must be adequate. This idea can be realized by high-statistics experiments at e^+e^- colliders equipped with finely segmented luminometers, in particular by the LEP experiments, given their large event samples, by SLC and future Linear Colliders. The relevant luminometers cover the t-range from a few GeV^2 to order 100 GeV^2. The analysis follows closely the procedure adopted in the luminosity measurement, which is described in detail [13], and elaborates on the additional aspect related to the measurement of a differential quantity. To this aim the luminosity detector must have a sufficiently large angular acceptance and adequate fine segmentation. The variable

t (eq. 2) is reconstructed on an event-by-event basis. The cross section for the process $e^+e^- \to e^+e^-$ can be conveniently decomposed into three factors :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{\mathrm{d}\sigma^0}{\mathrm{d}t} \left(\frac{\alpha(t)}{\alpha(0)} \right)^2 (1 + \Delta r(t)). \tag{3}$$

All three factors are predicted to a precision of 0.1% or better. The first factor on the right-hand side refers to the effective Bhabha Born cross section, including soft and virtual photons[11], which is precisely known, and accounts for the strongest dependence on t. The vacuum-polarization effect in the leading photon t channel exchange is incorporated in the running of α and gives rise to the squared factor in eq. 3. The third factor, $\Delta r(t)$, collects all the remaining real (in particular collinear) and virtual radiative effects not incorporated in the running of α. The experimental data after correction for detector effects have to be compared with eq. 3. The t dependence is rather steep, thus migration effects may need attention.

This goal is achieved by using a newly developed program based on the already existing semianalytical code `NLLBHA` [11,14]. A complete bibliography of this code called `SAMBHA` is given in ref.[1]. It is convenient to confront the fully corrected measured cross section with the Bhabha cross section, including radiative corrections in the factorized form given by eq. 3. The physical cross section is infrared safe [11]. This decomposition is neither unique nor dictated by a compelling physical reason; rather it allows the separation of the different sources of t dependence in a transparent way without introducing any additional theoretical uncertainty. The various factors as well as the relative accuracy of the QED corrections (see for example refs [15]-[16]) and the MonteCarlo tool to implement them are discussed in ref.[1].

2 Conclusions

A novel approach to access directly and to measure the running of α in the space-like region is proposed. It consists in analysing small-angle Bhabha scattering. Depending on the particular angular detector coverage and on the energy of the beams, it allows a sizeable range of the t variable to be covered. The feasibility of the method has been put in evidence by the use of a new tool, `SAMBHA` , to calculate the small-angle Bhabha differential cross section with a theoretical accuracy of better than 0.1%. The information obtained in the t channel can be compared with the existing results of the s channel measurements. This represents a complementary approach, which is direct, transparent and based only on QED interactions and furthermore free of some of the drawbacks inherent in the s channel methods. The method outlined can be readily applied to the experiments at LEP and SLC. It can also be exploited by future e^+e^- colliders as well as by existing lower energy machines. As an example the q^2 values $[GeV^2]$ as the $E_{Center\ of\ Mass}$ energy varies between 0 and 10^3 GeV within a range of acceptance for the emitted radiation between the angles from 1.8 to 7.2 degrees range between 0 and $4 \cdot 10^3$ GeV^2.

An extremely precise measurement of the loop contributions to the QED running coupling $\Delta\alpha(t)$ for small values of t may be envisaged with a dedicated luminometer even at low machine energies.

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