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DETERMINATION OF THE G COMMUTATOR TERM FROM πN SCATTERING

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ABSTRACT

Using once subtracted fixed t dispersion relations, the value of the isospin even scattering amplitude at the Adler point (ν=0, ν_B=0) is evaluated by means of the most recent phase shift analysis and total cross-section data. The value obtained in this calculation is not very different from that in an earlier work by Höhler, Strauss and the author, but still considerably smaller than the result of Cheng and Dashen, although these authors used essentially the same data. It is pointed out that our method provides more consistency within the total energy range where πN data are available, in contrast to the work of Cheng and Dashen where this is achieved only in a rather restricted interval.

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Starting from rather different approaches, many authors [1)-4)] have attempted to determine the value of the σ commutator term, whose magnitude is crucial for specific models of chiral symmetry breaking. The determination of this term, however, implies the evaluation of an amplitude outside the physical region and, in addition to that, in the soft pion limit. Von Hippel and Kim [1)] applying off-mass-shell dispersion relations and πN scattering data, as well as Ericson and Rho [4)], who made use of the π nuclear scattering amplitude and a Fubini-Furlan mass extrapolation technique, have found small values for the σ term. This has to be contrasted with the four times larger result of Cheng and Dashen (CD) [2)], which has been derived from fixed t dispersion relations, using πN phase shifts and an expansion of the amplitude in powers of m_π^2 neglecting contributions of higher order. The various difficulties which the conventional concept of chiral symmetry breaking encounters in view of this large value are discussed in a recent review by Renner [5)]. However, the validity of the Cheng-Dashen value has been doubted in two respects. First of all, Höhler et al. [3)] have obtained a result which amounts only to a third of the CD value, although they used essentially the same data and their method was quite similar to that of CD [*)]. While this controversial situation will be attempted to be settled in the present note, another serious objection, dealing with the CD assumptions on the off-shell behaviour, has been pointed out recently by Schnitzer [6)]. Starting from a Ward identity representation for πN scattering amplitudes, and taking into account explicitly the dependence on the one-particle reducible part of the σ field, he was able to show that the CD calculation gives three times the σ term, rather than the σ term itself. His conclusion is based on t channel unitarity and assumptions on the background of $\pi\pi$ scattering. It would be interesting to know, whether these assumptions are compatible with the result of Strauss [7)] who found that the dispersion integral over the imaginary part of the $\pi\pi N\bar{N}$ s wave has a zero at $t = -25\, m_\pi^2$, indicating that $\mathrm{Im}\, f_0^+$ consists of at least two contributions of comparable magnitudes and opposite signs.

As the off-shell behaviour is far from being clarified, we shall confine ourselves in the present note to determine the amplitude for physical pions at $(\nu = 0,\ \nu_B = 0)$, in a way which provides consistency with all available πN data in the phase shift region.

Let us start with an expansion of the isospin even πN scattering amplitude

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*) Actually, the authors of Ref. 3) started from Ward identities, but there is no difference from CD as far as the mass extrapolation is concerned.

$$A^+(\nu,t) = a_1^+ + a_2^+ t + a_3^+ \nu^2 + a_4^+ \nu^2 t + \ldots\ldots \quad (1)$$

which coincides with the amplitude

$$T^+(\nu,\nu_B,q^2,q'^2) = A^+ + \nu B^+ \quad (2)$$

considered by CD at the kinematical point of interest ($\nu = 0$, $\nu_B = 0$) up to a constant g^2/M, g is the πN coupling constant, $\nu = (s-u)/4M$, $4M\nu_B + 2 = t$ if both pions are on-shell, M denotes the nucleon mass and s,t,u are the usual Mandelstam variables. Natural units $\hbar = c = m_\pi = 1$ are used unless stated otherwise.

Thus we get

$$T^+(0,0,1,1) = a_1^+ - \frac{g^2}{M} + 2a_2^+ \quad (3)$$

which will be evaluated in two steps :

1. consider the fixed t dispersion relation for the amplitude

$$C^+(\nu,t) = A^+(\nu,t) + \frac{4M^2\nu}{4M^2 - t} B^+(\nu,t) \quad (4)$$

in the forward direction

$$\mathrm{Re}\, C^+(\nu,0) = A^+(0,0) - \frac{g^2}{M}\frac{\nu^2}{\nu^2-\nu_B^2} + \frac{2\nu^2}{\pi}\int_1^\infty \frac{d\nu'}{\nu'}\,\frac{k'\sigma^+(k')}{\nu'^2-\nu^2} \quad (5)$$

where we have subtracted at $\nu = 0$ and made use of the optical theorem

$$\mathrm{Im}\, C^+(\nu,0) = k\,\sigma^+(k) \quad (6)$$

$k = \sqrt{\nu^2 - 1}$ is the lab momentum of the incident pion and σ^+ the average of $\pi^\pm p$ total cross-sections.
By rearranging Eq. (5) and using Eq. (1), we arrive at :

$$a_1^+ - \frac{g^2}{M} = \mathrm{Re}\, C^+(\nu,0) + \frac{g^2}{4M^3}\frac{1}{\nu^2 - \frac{1}{4M^2}} - \frac{2\nu^2}{\pi}\int_1^\infty \frac{d\nu'}{\nu'}\,\frac{k'\sigma^+(k')}{\nu'^2-\nu^2} \quad (7)$$

We have calculated the principal value integral from the experimental σ^+ values given for example in the Table of Höhler and Strauss [8] and $\mathrm{Re}\, C^+(\nu,0)$ using the most recent phase shift analysis from CERN [9]. Although there have been incorporated three times more data than in the previous analysis, done by this group, the consistency of the resulting real parts and of Eq. (7) is not very good, as can be seen from Fig. 1. The origin of this inconsistency was already discussed in Ref. 3), therefore we only want to point out here that the real parts calculated from the phase shift analysis are certainly too big below and above the first resonance. This has also been found by Nielsen [10] in his attempt to improve the partial waves in the low energy region. The rapid variations of the right-hand side of Eq. (7) at higher energies reflect the fact that the resonance positions resulting from phase shifts are slightly different from those given by the total cross-section. Consequently, we discard completely the region below 215 MeV as well as the most extreme values in the neighbourhood of certain resonances, claiming that the remainder represents a suitable set of values to form an average. Our result is

$$a_1^+ - \frac{g^2}{M} = -1.53 \pm 0.2 \qquad (8)$$

which corresponds to an s wave scattering length value of

$$a_0^+ = -0.014 \pm 0.014 \qquad (9)$$

where we have inserted our result (8) in the relation following from Eq. (7)

$$a_1^+ - \frac{g^2}{M} - \frac{g^2}{M(4M^2-1)} + \frac{2}{\pi}\int_0^\infty \frac{\sigma^+ dk}{k^2+1} = 4\pi \frac{M+1}{M} a_0^+ \qquad (10)$$

It should be noted that the error of the scattering length is only due to the uncertainty of the number in Eq. (8), because the integral occurring in Eq. (10) is strongly convergent and its error is smaller than 2%.

2. The second coefficient of the expansion (1) can be most reliably determined by a study of the dispersion relation for the derivative of the amplitude $C^+(\nu,t)$ at $t=0$. Assuming the usual parametrization for the derivative of the imaginary part at higher energies

$$\frac{\partial}{\partial t} \mathrm{Im}\, C^+(\omega,t)\Big|_{t=0} = \frac{1}{2} b^+(\omega)\, k\, \sigma^+(k) \qquad (11)$$

where ω is given by $\omega = \nu - t/4M$ and $b^+(\omega)$ denotes the slope of the diffraction peak, we arrive at the following relation

$$\frac{\partial}{\partial t} \mathrm{Re}\, C^+(\omega,t)\Big|_{t=0} - \frac{\partial}{\partial t} C_N^+(\omega,t)\Big|_{t=0} - \frac{2\omega^2}{\pi}\int_1^{\bar\omega} \frac{d\omega'}{\omega'} \frac{\partial}{\partial t} \frac{\mathrm{Im}\, C^+(\omega',t)\big|_{t=0}}{\omega'^2 - \omega^2}$$

$$- \frac{\omega}{2\pi M}\int_1^{\infty} \frac{d\omega'}{\omega'^2} \frac{(2\omega'+\omega)\, k' \sigma^+}{(\omega'+\omega)^2} = a_2^+ + \frac{\langle b^+\rangle}{2}\,\xi \qquad (12)$$

with

$$\xi = \frac{2\omega^2}{\pi}\int_{\bar\omega}^{\infty} \frac{d\omega'}{\omega'} \frac{k'\sigma^+(k')}{\omega'^2 - \omega^2}$$

In deriving Eq. (12), we have taken into account a possible energy dependence of b^+ by an average value $\langle b^+\rangle$ *). The left-hand side of Eq. (12) can be calculated in the interval $1 < \omega \leq \bar\omega \approx 15$ from phase shifts [9] and total cross-sections [8], whereas ξ is given by total cross-section data alone. The result of our calculation is presented in Figs. 2 and 2a, where we plot the left-hand side of Eq. (12), versus ξ. According to Eq. (12), we have made a straight line fit, but following the arguments given above, we did not use the points below $\xi = 0.35$ (corresponding to $T_\pi \approx 220$ MeV). As it can be seen in more detail from Fig. 2a, they deviate systematically from the straight line behaviour of the remaining points. In the interval $0.35 \leq \xi \leq 35$ the best χ^2 was obtained for

$$a_2^+ = 1.11 \pm .02 \;;\; \langle b^+\rangle = .12 \simeq 6\ (\mathrm{GeV/c})^{-2} \qquad (13)$$

where the error of a_2^+ indicates the variation of this quantity when the number of points for fitting is changed. Therefore we conclude that, despite some local deviations from the straight line, this determination of a_2^+ leads to a reliable and very accurate value. Furthermore, it yields a value for $\langle b^+\rangle$ consistent with high energy data which obviously would not be the case if only points in the low energy region are considered.

Inserting now our results (8) and (13) in Eq. (3), we obtain

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*) In a more detailed investigation [11] we have recently shown that a logarithmic increase of b^+ which is consistent with the data from Coulomb interference experiments, would not change significantly the energy dependence of ξ in the interval $1 \leq \omega \leq \bar\omega$.

$$T^{+}(0,0,1,1) = .69 \pm .24 \tag{14}$$

in complete disagreement with the CD result

$$T^{+}_{CD}(0,0,1,1) = 1.7 \tag{15}$$

In order to show the origin of the above discrepancy and to justify our assumption in Eq. (1) that terms of higher order in t are negligible, we have studied the fixed t relation for $C^{+}(\nu,t)$ at $\nu_B = 0$ $(t=2)$.

Using Eq. (11) in the high energy part as it was also done by CD in their calculation, we can write

$$\mathrm{Re}\, C^{+}(\nu,2) + \frac{g^2}{M}\frac{1}{2M^2-1} - \frac{2\nu^2}{\pi}\int_{1+\frac{1}{2M}}^{\bar{\nu}} \frac{d\nu'}{\nu'}\,\frac{\mathrm{Im}\, C^{+}(\nu',2)}{\nu'^2-\nu^2} = \tag{16}$$

$$= T^{+}(0,0,1,1) + \langle e^{b^{+}} \rangle \xi$$

where, in this case, ξ is defined as

$$\xi = \frac{2\nu^2}{\pi}\int_{\bar{\nu}}^{\infty} \frac{d\nu'}{\nu'}\,\frac{k' G^{+}(k')}{\nu'^2-\nu^2} \tag{17}$$

and the above-mentioned energy dependence of b^{+} has been taken into account as in Eq. (12).

The evaluation of the left-hand side of Eq. (16) is quite analogous to that of Eq. (12). Taking $C^{+}(\nu,2)$ from phase shifts, we have plotted the left-hand side of Eq. (16) versus ξ which depends only on total cross-sections. A least square fit to the points in Fig. 3 in the interval $0.35 \leq \xi \leq 35$ resulted in

$$T^{+}(0,0,1,1) = .72 \qquad \langle e^{b^{+}} \rangle = 1.13 \tag{18}$$

which agrees with our previous result. By varying the number of fitted points, we are led to the following errors :

$$T^{+}(0,0,1,1) = .72 \; {}^{+.2}_{-.1} \tag{19}$$

Figure 3a shows in more detail our result in the low energy region. It demonstrates immediately that the CD result in fact may be obtained if the extrapolation to $\xi = 0$ is done with a straight line fit, taking into account only the points from the restricted interval $\{\xi_1(\nu_1), \xi(\nu_2)\}$ *). This interval corresponds exactly to the energy region which was selected by CD to give the best information on the real parts for the broad area subtraction method. However, as we already have pointed out above, it is just this energy interval where the real parts from phase shifts must be considered as doubtful.

Moreover, the straight line extrapolation of the points from the interval $\{\xi_1, \xi_2\}$ to the CD value yields a negative value for $\langle e^{b^+} \rangle$ which is of course unphysical. This happens despite of the fact that CD inserted in their dispersion relation the same high energy assumption [Eq. (11)] which requires $\langle e^{b^+} \rangle$ at least to be greater than 1. This is obviously an internal inconsistency of the broad area subtraction method which cannot occur in our analysis, because the physical requirement

$$\langle e^{b^+} \rangle > 1 \qquad (20)$$

can be used as a supplementary condition on the extrapolation to $\xi = 0$.

Therefore we conclude that our result for $T^+(0,0,1,1)$ should be preferred to that of CD because it provides consistency of fixed t dispersion relations with the input in the whole energy region where phase shifts are available. The price one has to pay if the CD result is accepted can be read off Figs. 3 and 3a.

Finally, we want to make some remarks on a recent paper by Altarelli, Cabibbo and Maiani [12], where they suggested to calculate the G term from a combination of s and p wave scattering lengths. We do not think that this method leads to a reliable value [for details, see Ref. 13]. Furthermore, their determination of the coefficients of the expansion for

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*) The fact that the analysis of CD was based on an evaluation of T^+ as defined in Eq. (2), instead of C^+ does not affect the conclusion at all, because the results for the left-hand side of Eq. (16) would differ by less than 1%. Furthermore, it may be argued that CD have obtained their value with different phase shifts, but this obviously does not account for the discrepancy as we have carried out our analysis for four different solutions [CERN-PIP, Ref. 9), presented here, CERN-exp., CERN-theoret., Glasgow A] without finding our results substantially changed.

$$T^+(\nu, t, q^2, q'^2) = A + Bt + C(q^2+q'^2) + D\nu^2 \qquad (21)$$

is not quite consistent from our point of view. Instead of the Samaranayake-Woolcock [14] value for $T^+(0,0,1,1) = -1.46 \pm 0.02$(!) they should rather use $T^+(0,0,1,1) = -1.0 \pm 0.2$ which corresponds to the CD analysis (see Fig. 1). This would make their value for B (which is accurately known) more acceptable but changing considerably their conclusion on the magnitude of C whose smallness was considered as a support for the CD assumption on the off-shell behaviour.

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FIGURE CAPTIONS

Figure 1 : The right-hand side of Eq. (7) as calculated from CERN phase shifts and total cross-sections. The broken line indicates our average value.

Figure 2 : The left-hand side of Eq. (12) calculated from phase shifts and total cross-sections plotted versus ξ (definition given in the text). The solid line corresponds to the parameters of Eq. (13). At low energies only few points are displayed.

Figure 2a : The same as in Fig. 2, but with a different scale for ξ to demonstrate the behaviour in the low energy region.

Figure 3 : The left-hand side of Eq. (16) calculated from phase shifts. The solid line shows our linear fit. In the interval $0 \leq \xi \leq 1.5$ only a few points are displayed.

Figure 3a : The same as in Fig. 3 on a larger scale for ξ . ξ_1 and ξ_2 are the boundary values for the "broad area" of CD which corresponds to a rather small interval in Fig. 3. The broken line indicates the extrapolation of the points from the "broad area" to the CD value.

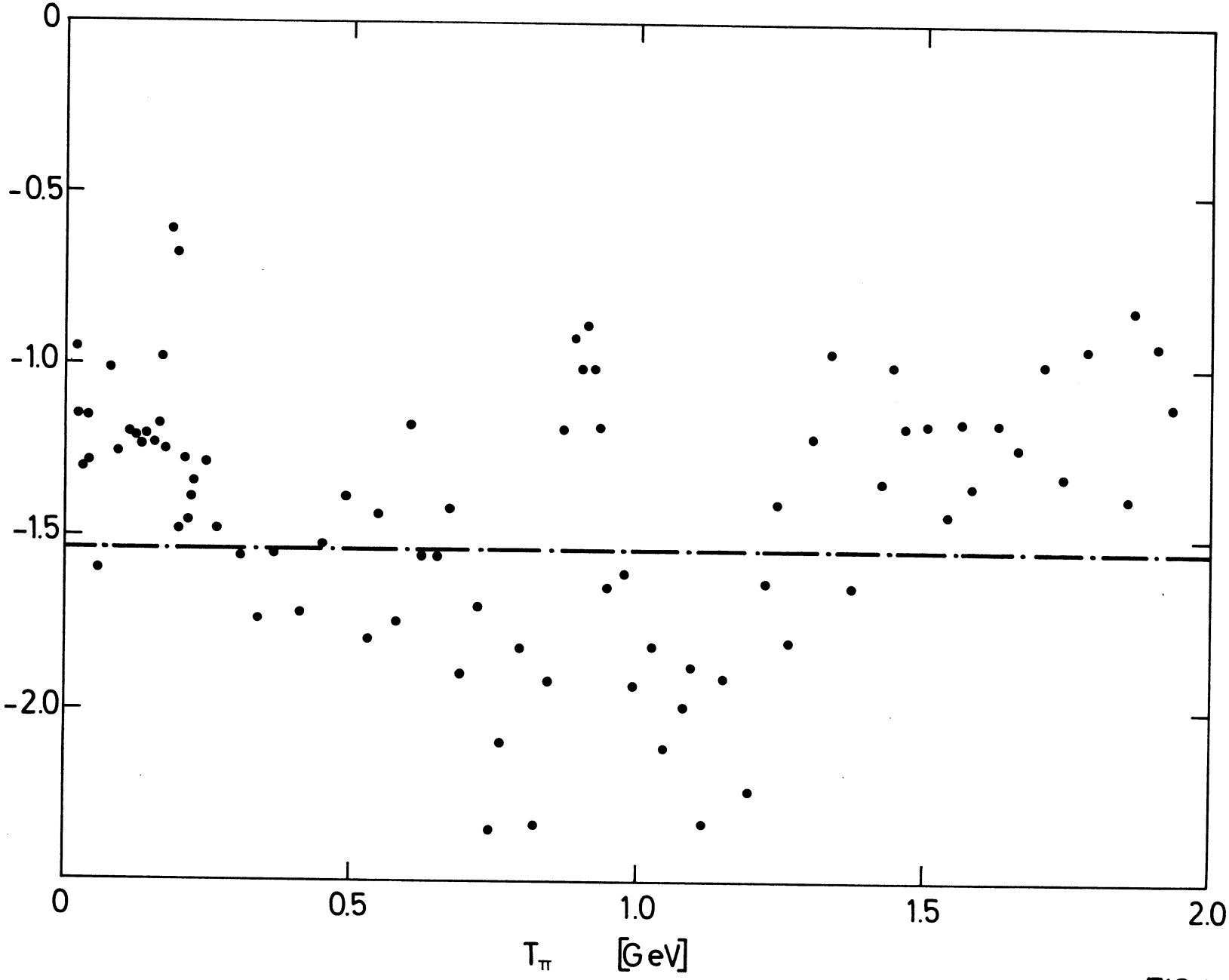

FIG. 1

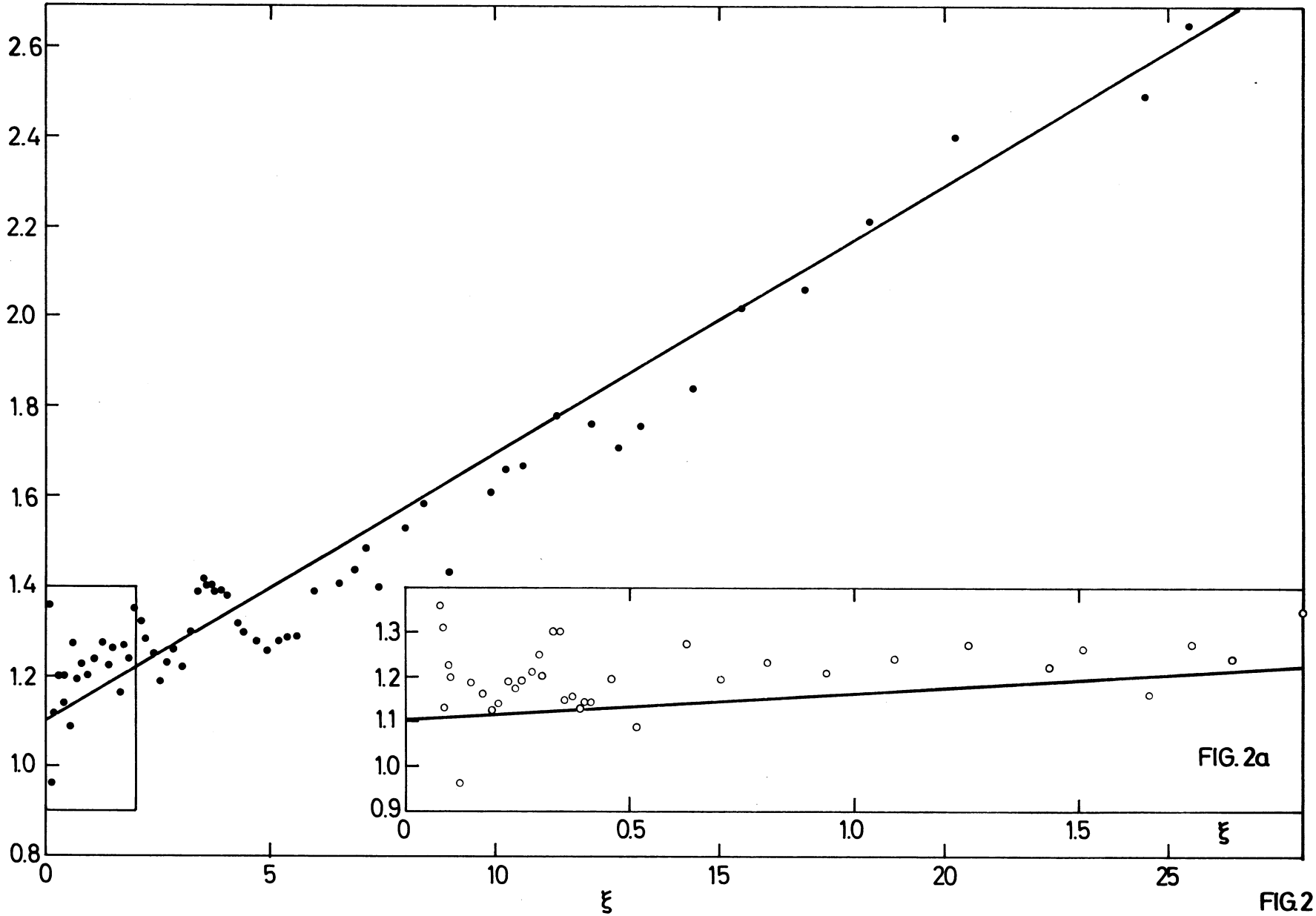

FIG. 2a

FIG. 2

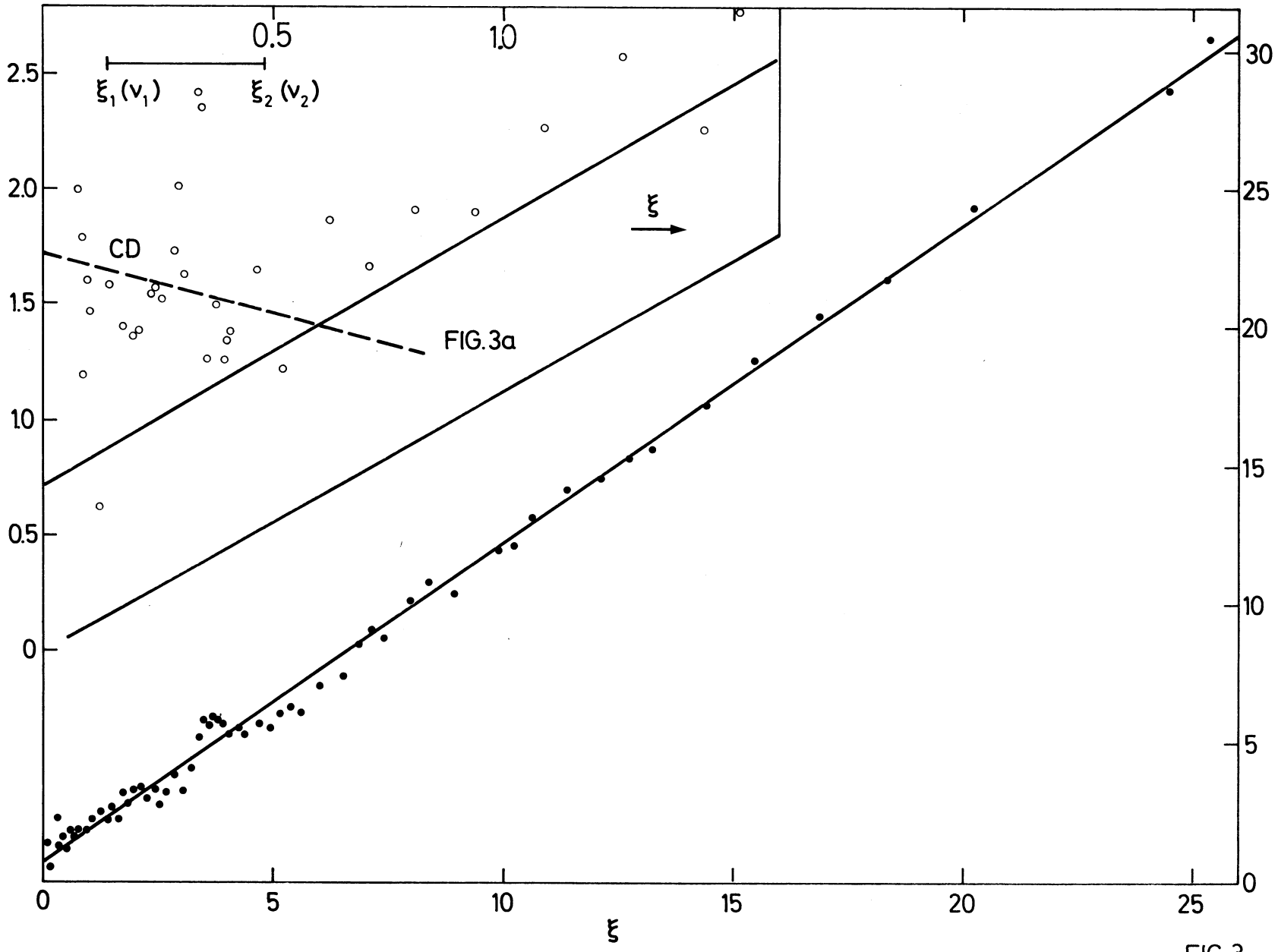
0.5
1.0
$\xi_1(\nu_1)$
$\xi_2(\nu_2)$
CD
FIG. 3a
ξ
2.5
2.0
1.5
1.0
0.5
0
30
25
20
15
10
5
0
0
5
10
15
20
25
ξ

FIG. 3