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NOTE ON μ SCATTERING

A. INTRODUCTION

A μ-meson scattering experiment at the PS has been proposed some time ago[1]. It requires a special beam. Since the construction of this beam has been delayed, the possibility of a preliminary experiment in a "compromise beam" has been envisaged by us[2]. It has now been suggested that a rough experiment could even been done in an existing beam. The use of heavy spark chambers for this purpose has been recommended.

In the following we try to analyse what type of experiments would increase our present knowledge about the μ meson, and what are the requirements for these experiments from the point of view of intensity and purity of the beam and precision of detection.

In order to compare experiments it is useful to introduce a figure of merit. One parameter which is frequently being used is the "μ-meson vertex cut-off Λ^{-1}" introduced in the discussions on breakdown of QED. It corresponds to a μ-form factor of which the important term is proportional to q^2. In a μ-scattering experiment at a momentum transfer q and with a standard error $\Delta\sigma/\sigma$ in the cross-section we can put, with 95% confidence, the following limit on Λ^{-1}.

$$\Lambda^{-1} \leq \frac{1}{q}\sqrt{\frac{\Delta\sigma}{\sigma}} \qquad (1)$$

The "r.m.s. radius of the charge distribution" is bigger than this quantity by a factor $\sqrt{6}$.

From the SC experiment ($q = 260$ MeV/c $= 1.3$ fermi^{-1}, $\Delta\sigma/\sigma \approx 10\%$) we expect a limit of ≈ 0.24 fermi. A similar limit is given by the g-2 experiment ($\Lambda^{-1} = 0.15\sqrt{2}$ fermi for 95% confidence).

A substantial improvement of this knowledge would require a decrease in this parameter by at least a factor 2 ($\Lambda^{-1} = 1.2$ fermi) This could be achieved in one of the following ways:

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I a precise experiment at $q = 0.5$ GeV/c $\Delta\sigma/\sigma = 10\%$

II a quantitative experiment at $q = 1.0$ GeV/c $\Delta\sigma/\sigma = 40\%$

It is, of course, possible that the μ-p interaction cannot be described by the simple picture which leads to Eq. (1). In the most general case the deviation from the electromagnetic cross-section can be a function of both the momentum transfer q^2 and the total c.m. energy W. But only if the q^2 independent term of a power series in q^2 increases strongly with W, Eq. (1) breaks down completely. In case the coefficients of higher powers in q, e.g., q^4 or q^6 become large, the square root in Eq. (1) would have to be replaced by a higher root, i.e., the precision would be of somewhat less importance and we should aim at a higher momentum transfer instead. This approach is, of course, limited by our knowledge of the proton form factors at high momentum transfers. Although, for this reason, a result of such an experiment would not have a unique interpretation at present, we will include this possibility in our list, as

III a qualitative experiment at $q = 2$ GeV/c $\Delta\sigma/\sigma = 100\%$
$\Lambda^{-1} = 0.10$ fermi

We will further distinguish experiments in these three classes by subscripts indicating the momentum of the incident μ meson in GeV/c.

B. INTENSITY CONSIDERATIONS

In this section we want to deal with two questions:

1) What is the minimum μ-meson flux required in order to carry out one of the experiments listed above ?
2) Which is the optimum momentum of the incident particles from the point of view of obtaining a high rate of scattering events at a given momentum transfer ?

The rate of μ mesons elastically scattered in hydrogen per incident μ meson and per hydrogen atom is given by

$$n(p,\vartheta) = \sigma(p,\vartheta)\ \Delta\Omega = \sigma(p,\vartheta)\ \sin\vartheta\ \Delta\vartheta\ \Delta\psi \qquad (2)$$

where

$$\sigma(p,\vartheta) = \frac{C}{p^2} \frac{\cos^2\vartheta/2}{\sin^4\vartheta/2} \frac{1}{1 + \frac{2p}{Mc}\sin^2\vartheta/2} F^2(q^2,\vartheta) \qquad (3)$$

is the differential cross-section for elastic scattering of fast ($1-\beta \ll 1$) Dirac particles against protons, with

C 5.20 mb $(\mathrm{MeV/c})^2$

p the momentum of the incident particles

ϑ the scattering angle

M the proton rest mass

$$q = \frac{2p \sin\vartheta/2}{\sqrt{1 + \frac{2p}{Mc}\sin^2\vartheta/2}} \quad \text{the four-momentum transfer} \qquad (4)$$

$$F^2(q^2,\vartheta) = \left[F_1^2(q^2) + \frac{q^2}{4M^2c^2}F_2^2(q^2)\right] + \frac{q^2}{2M^2c^2}\left[F_1(q^2) + F_2(q^2)\right]^2 \tan^2\vartheta/2 \qquad (5)$$

$$\equiv A(q^2) + B(q^2)\tan^2\vartheta/2$$

$F_1(q^2)$ and $F_2(q^2)$ are the two conventional form factors which describe the electromagnetic properties of the proton. These can be derived from electron scattering experiments. We have based our estimates on results of Wilson et al.[3]. Up to $q \approx 1.2$ GeV/c these are compatible with the expressions for the form factors suggested by Fubini et al.[4], namely

$$F_1 = 1 - \frac{q^2}{2}\left(\frac{\alpha_v}{q^2 + q_v^2} + \frac{\alpha_s}{q^2 + q_s^2}\right)$$

$$F_2 = 1.793 - q^2\left(\frac{\beta_v'}{q^2 + q_v^2} + \frac{\beta_s'}{q^2 + q_s^2}\right) \qquad (6)$$

with

$$\alpha_v = 1.1 \quad \beta_v' = 2.11 \quad q_v^2 = 8.3 \times 10^{26}\ \mathrm{cm}^2$$

$$\alpha_s = 0.58 \quad \beta_s' = 0.09 \quad q_s^2 = 4.4 \times 10^{26}\ \mathrm{cm}^2 \qquad (7)$$

We have used these formulae also for extrapolating beyond the range where they are checked experimentally. This means, roughly, assuming form factors which tend to a constant value for very high q^2 and give cross-sections of the order of 0.1 times the point cross-sections.

$\Delta\psi$ in (2) is the range of azimuthal angle accepted; it will be determined by practical considerations.

$\Delta\vartheta$ is the range of scattering angles accepted. It should be chosen in such a way that we accept always a fixed band of relative momentum transfer, i.e.

$$\frac{\Delta q}{q} = \text{const.} = \alpha, \text{ say.} \tag{8}$$

(8) and (4) yield

$$\Delta\vartheta = 2\alpha \tan\vartheta/2 \left(1+\frac{2p}{Mc}\sin^2\vartheta/2\right). \tag{9}$$

Substituting (3 and (9) into (2) we find

$$n(p,\vartheta) = \frac{4\alpha C}{p^2}\cot^2\vartheta/2\ F^2(q^2,\vartheta)\Delta\psi, \tag{10}$$

and with (5)

$$n(p,\vartheta) = \frac{4\alpha C}{p^2}\left[A(q^2)\ \cot^2\vartheta/2 + B(q^2)\right]\Delta\psi. \tag{11}$$

In order to study the dependance of this quantity on p, for a given q, we eliminate ϑ, using the relation (4), which yields

$$\cot^2\vartheta/2 = \frac{4p^2}{q^2}\left[1-\frac{q^2}{4p^2}-\frac{q^2}{2pMc}\right] \tag{12}$$

Then we have

$$n(p,q) = 4\alpha C\ \Delta\psi\left\{\frac{4A(q^2)}{q^2}\left(1-\frac{q^2}{4p^2}-\frac{q^2}{2pMc}\right)+\frac{B(q^2)}{p^2}\right\}. \tag{13}$$

The right hand side of (12), put equal to zero ($\vartheta = \pi$) gives the kinematic threshold in p for a given q.

$$p_{min} = \frac{q}{4Mc}\left(q + \sqrt{q^2 + 4M^2c^2}\right) \qquad (14)$$

Figure 1 gives the kinematic threshold p_{min} as a function of q. In Fig. 2 we have plotted $n(p,q)/\alpha\,\Delta\psi$ [Eq. (13)] as a function of p, for q = 0.5, 1 and 2 GeV/c. It must be stressed that the curve for q = 2 GeV/c is already an extrapolation *). The approximate shape of the curve for q = 3 GeV/c is also indicated although the extrapolation in the form factor is too bold here to justify a detailed calculation.

The curves of Fig. 2 show that the dependance of n on p is very weak, once we are well above the kinematic threshold. From the intensity point of view the best conditions for an experiment aiming at a certain q are thus given by the strongest beam available at any momentum well above the kinematic threshold. This will in general be a beam of low momentum, due to the higher abundance of lower momentum π mesons and their bigger decay probability per unit path length. On the other hand, due to the influence of the kinematic threshold, an experiment aiming at a very wide range of q should be done at a high momentum.

More quantitative information can be obtained by further specifying the experimental conditions. The number of counts N is given by

$$N = \Phi\, t\, A\, d\, \rho\, \frac{n(p,\vartheta)}{\alpha\ \Delta\psi}\, \alpha\ \Delta\psi \equiv \Phi\, t\, A\, d\, \rho\, n'(p,\vartheta)\alpha\ \Delta\psi$$

where

- Φ is the μ flux per burst
- t is the length of the experiment (in bursts). We have assumed 10 shifts of data taking (60 hours) for one point as a maximum tolerable time, since any decent experiment will require several runs of data taking plus a lot of test- and check runs. This yields t = 72 000 bursts.
- A Avogadro's number $6.0 \cdot 10^{23}$

*) This extrapolation depends critically also on the behaviour of F_2 at large q^2.

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d thickness of target 100 cm

ρ density of hydrogen 0.07 g cm^{-3}

α $= \Delta q/q = 0.2$. Due to the strong q dependence of n it would not be meaningful to multiply a differential cross-section by a wider q-band. An integration would show, that the contribution of scatters with a q more than 20% above a certain q_{min} is not big (except where the form factor flattens off completely)

$\Delta\psi$ $= \pi$ for a reasonable detector geometry.

Under these assumptions we will work out the required flux Φ for various types of experiments, as listed in the introduction. We find

$$\Phi = 5.2 \cdot 10^{-30} \frac{N}{n'} /\text{burst} \qquad (16)$$

This yields the figures for various experiments as given in Table I. The fluxes are also summarized in Table II.

In how far such fluxes of π mesons can be made available without a special effort has to be studied in more detail. The high-energy μ-mesons beam used by Hyams et al.[5] had a flux of 2000/burst. At first sight it seems highly unlikely that an intensity of more than 10 times this figure can be "found" anywhere in the present beams. From this point of view only experiment I could perhaps be done without a special beam. We will see, however, that it requires a very high precision in momentum and angles.

C. PURIFICATION

An accurate estimate of the permissible π contamination would require a detailed analysis of π-scattering results, leading to curves similar to those of Fig. 2. For the time being we will limit ourselves to a rough estimate, starting from the following assumptions:

1) In a given momentum interval in the π beams there will be about 2% μ mesons.
2) Amongst the scattered events we want to tolerate no more than 10% π scatters.

3) The differential (elastic and part of the inelastic) π-scattering cross-sections σ_π are of the order of 1 mb/sterad.

The purification factor P required will then be given by

$$P = 500 \times \frac{\sigma_\pi}{\sigma_\mu} = \frac{5 \times 10^{-25}}{\sigma_\mu}$$

σ_μ varies between 10^{-30} and 10^{-33} cm^2 for the various cases considered. This leads to Table III, where P is listed as a function of σ_μ. If we wish to obtain this purification by the use of absorber only, and if we assume, following Hyams[5)], that the pion attenuation length in light material is 130 g cm^{-2}, we obtain the absorber thicknesses listed. For a 10 GeV/c incident π beam we can furthermore list the resulting final momentum $p_{r_{10}}$ and the r.m.s. multiple scattering angle α_{10}. The same has been done for 6 GeV/c incident momentum (p_{r_6}, α_6). This shows that:

1) in many cases a determination of the direction of the incident particle will be necessary, in order to obtain the precision required (see next section), even if one starts with a "parallel beam";

2) the losses in intensity in the absorber and in any momentum analyser behind it will be severe. They will be more severe at low momenta. On the other hand the tolerable beam spread for a given momentum resolution will be lower at high energies for the same magnet. Also at lower momenta other means of discrimination (Čerenkov counters) can still be used, thus eliminating the need for part of the absorber.

In view of these difficulties it looks tempting to place some of the absorber into the scattered beam where the multiple scattering is of no consequence. There exist two limitations to this scheme:

1) Low-energy charged secondaries which may accompany a nuclear cascade started off by a π meson. By requiring a coincidence between two counters separated by some absorber these can be eliminated.

2) Decay in flight of π mesons after the scattering event. The probability for this is roughly equal to the decay probability of the π

between the scatterer and the final detector. For a path length of a few m, this is of the order of 1%. So a factor 100 is the highest purification rate we can obtain here corresponding to 130 ln 100 = = 600 g cm^{-2} of absorber, which can be placed into the scattered beam, while the absorber in the primary beam can be reduced accordingly. The values α_6' listed in Table III correspond to this case. This possibility is of special importance in the case of a low-energy beam, e.g., I_1.

We can conclude that any beam lay-out should be made in such a way as to minimize the losses due to multiple scattering. (Alternation of absorber and focusing elements etc.).

D. PRECISION REQUIREMENTS

In order to obtain information about the μ meson one has to be able to compare the experimental results with predictions based on electron scattering data. Such a comparison can, at present, not be done for $q > 1.2$ GeV/c, since no electron scattering data are available.

But also for the q region where the proton-electron scattering cross-sections are known, a comparison can only be carried out if sufficiently precise data are available about the scattering events. In order to obtain quantitative information on this point, we have computed a number of derivatives of the cross-section and of n [Eq. (2)], with respect to pairs of observable quantities, p, ϑ and the recoil angle φ. We will extend the calculations to other pairs of variables. It seems important to measure at least three variables in order to obtain a check on unelastic events. The derivatives are listed in Table II for the various experimental conditions. Case III ($q = 2$ GeV/c) is again an extrapolation. We have also listed the uncertainty $\Delta p/p$, $\Delta\psi$ and $\Delta\varphi$, which would by itself lead to an error equal to the precision aimed at in the experiment. It follows, that the uncertainties have to be kept well below the limits given in the table. It is seen that in experiment I an extreme precision both in the determination of the momentum and the angles is required. The latter would only be possible using thin spark chambers arranged for nearly perpendicular incidence of the tracks, or cloud chambers. The precision requirements for the other cases are more normal. But two points are apparent from this:

1) In all cases a momentum analysis has to be made in front of the target, if any direct information on p is wanted.

2) An angular precision of a few degrees is wanted for all cases except III. This can not be achieved, unless thin plates (a few g cm^{-2} total mass) are used in the spark chambers, which detect the recoil proton, otherwise the result will be unreliable because of possible nuclear small angle scatterings.

E. CONCLUSIONS

The only experiment which seems feasible from the intensity point of view without a special beam is the low-q experiment I (q = = 0.5 GeV/c). It could possible be done in a beam like the Hyams beam, derived from d_9. I_1 could possibly also be done in a_1. The precision requirements are extreme. Heavy spark chambers would be useless.

In order to reach a higher q and have less stringent precision requirements, it would be desirable to make a better intensity μ beam above 2 GeV/c. Such a beam could probably be derived from a medium momentum (5 to 9) GeV/c π beam of about 10^6 π/burst. This is, incidentally the kind of beam for which the Brookhaven experiment is planned. (It relies on a direct measurement of q by range of the recoil, and of φ). Light spark chamber would be preferable to heavy ones.

An attempt to reach high momentum transfers in an existing low intensity beam does not look promising, unless one hopes for a cross-section about 10 times higher than that expected for heavy electrons.

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TABLE I

Fluxes required for various experiments

Experiment		I_1	I_2	II_2	II_6	III_6	
Momentum transfer	q	0.5	0.5	1	1	2	GeV/c
Number of counts	N	120	120	40	40	10	
Incident momentum	p	1	2	2	6	6	GeV/c
Cross-section from Fig. 2	n′	110	120	7.2	8.8	1.4	nbarn
Required flux	Φ	5.7	5.2	29	24	36	10^3/burst

N is chosen 120 for a 10% statistics after some corrections
40 for a quantitative experiment
10 for an exploration

1 nbarn = 10^{-33} cm^2

Φ is calculated from Eq. (16)

TABLE II

Derivatives of cross-sections with respect to observed quantities and precision requirements in their observation

Experiment		I_1	I_2	II_2	II_6	III_6	
Momentum transfer	q	0.5	0.5	1	1	2	GeV/c
Incident momentum	p	1	2	2	6	6	GeV/c
Scattering angle	ϑ	31	15	34	10	25	°
Recoil angle	φ	60	68	46	57	31	°
Required flux (Table I)	Φ	5.7	5.2	29	24	36	10^3/burst
$\left(\frac{\partial \ln \sigma}{\partial \ln p}\right)_\vartheta \cong \left(\frac{\partial \ln n}{\partial \ln p}\right)_\vartheta$		-4	-4	-4	-4	-3	
$\left(\frac{\partial \ln \sigma}{\partial \vartheta}\right)_p$		-10	-17	-9	-32	-11	rad^{-1}
$\left(\frac{\partial \ln n}{\partial \vartheta}\right)_p$		-7	-9	-6	-21	-7	rad^{-1}
$\left(\frac{\partial \ln \sigma}{\partial \varphi}\right)_\vartheta \cong \left(\frac{\partial \ln n}{\partial \varphi}\right)_\vartheta$		+14	+16	+12	+5	+6	rad^{-1}
$\left(\frac{\partial \ln \sigma}{\partial \vartheta}\right)_\varphi$		+2	+4	+1	-8	-4	rad^{-1}
$\left(\frac{\partial \ln n}{\partial \vartheta}\right)_\varphi$		-5	+12	+4	+3	+1	rad^{-1}
Precision aimed at	$\frac{\Delta\sigma}{\sigma}$	10	10	40	40	100	%
$\frac{\Delta p}{p}$		2.5	2.5	10	10	30	%
$\Delta\vartheta$ (from n)		14	8	70	19	140	mrad
$\Delta\varphi$		7	6	33	80	170	mrad

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TABLE III

Purification requirements

Expt.	σ_μ cm^2	P	s g cm^{-2}	p_{r10} GeV/c	α_{10} mrad	p_{r6} GeV/c	α_6 mrad	α'_6 mrad
I	10^{-30}	5×10^5	1700	6.6	17	2.6	35	23
II_6	10^{-31}	5×10^6	1930	6.1	19	2.1	41	27
II_2	10^{-12}	5×10^7	2160	5.8	20	1.7	48	31
III_6	10^{-33}	5×10^8	2390	5.4	22	1.2	60	37

σ_μ typical cross-section

P purification required

s thickness of light absorber providing this purification

p_{r10} residual momentum of 10 GeV/c incident muons behind this absorber

α_{10} r.m.s. multiple scattering angle behind the absorber

p_{r6}, α_6 } the same for 6 GeV/c incident momentum

α'_6 r.m.s. multiple scattering angle behind the absorber in the primary beam, if 600 g cm^{-2} of the total absorber are placed into the scattered beam.

REFERENCES

1) Muon-nuclear scattering at the PS, Proposal by Citron Group, 6.2.61.

2) Muon-nuclear scattering experiment. A. Citron et al., 15.11.61.

3) R.M. Littauer, H.F. Schopper, R.R. Wilson. Phys.Rev.Letters 7, 141, 144 (1961).

4) S. Bergia, A. Stanghellini, S. Fubini, C. Villi. Phys.Rev. Letters 6, 367 (1961).

5) G. Knop. NP Internal Report 61-19.

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FIGURE CAPTIONS

Fig. 1 The kinematic threshold, i.e., the lowest incident momentum p_{min}, at which a four-momentum transfer q can be reached, as a function of q.

Fig. 2 The cross-section n for elastic scattering of Dirac particles of momentum p against hydrogen into a solid angle $\Delta\Omega$, divided by $\alpha\Delta\psi$. $\Delta\Omega = \sin\vartheta\,\Delta\vartheta\,\Delta\psi$ and $\Delta\vartheta$ is determined by the requirement, that the scattering should take place with a momentum transfer q within the (small) tolerance $\Delta q/q = \alpha$ [Eq. (9)].

* * *

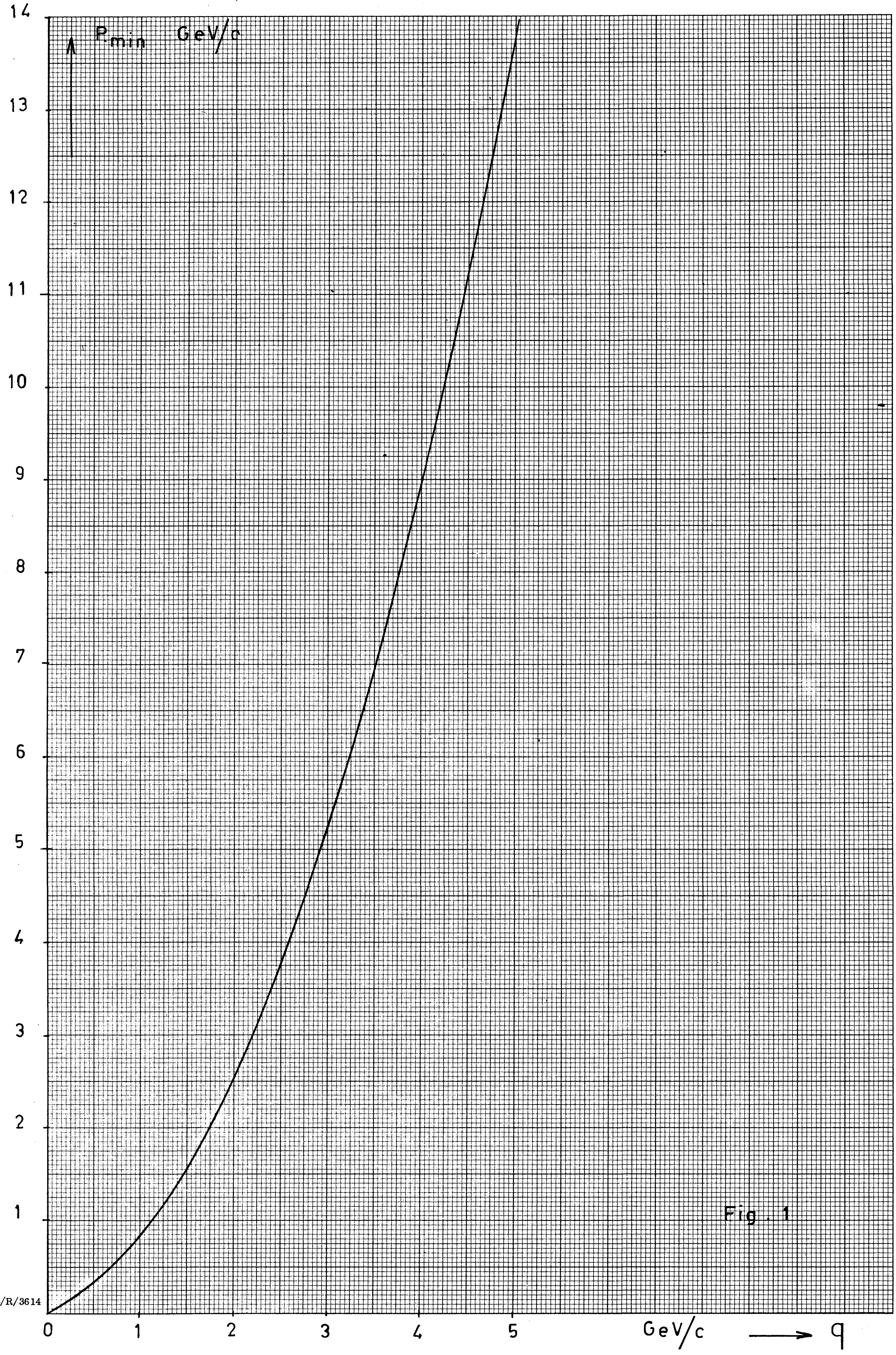

Fig. 1

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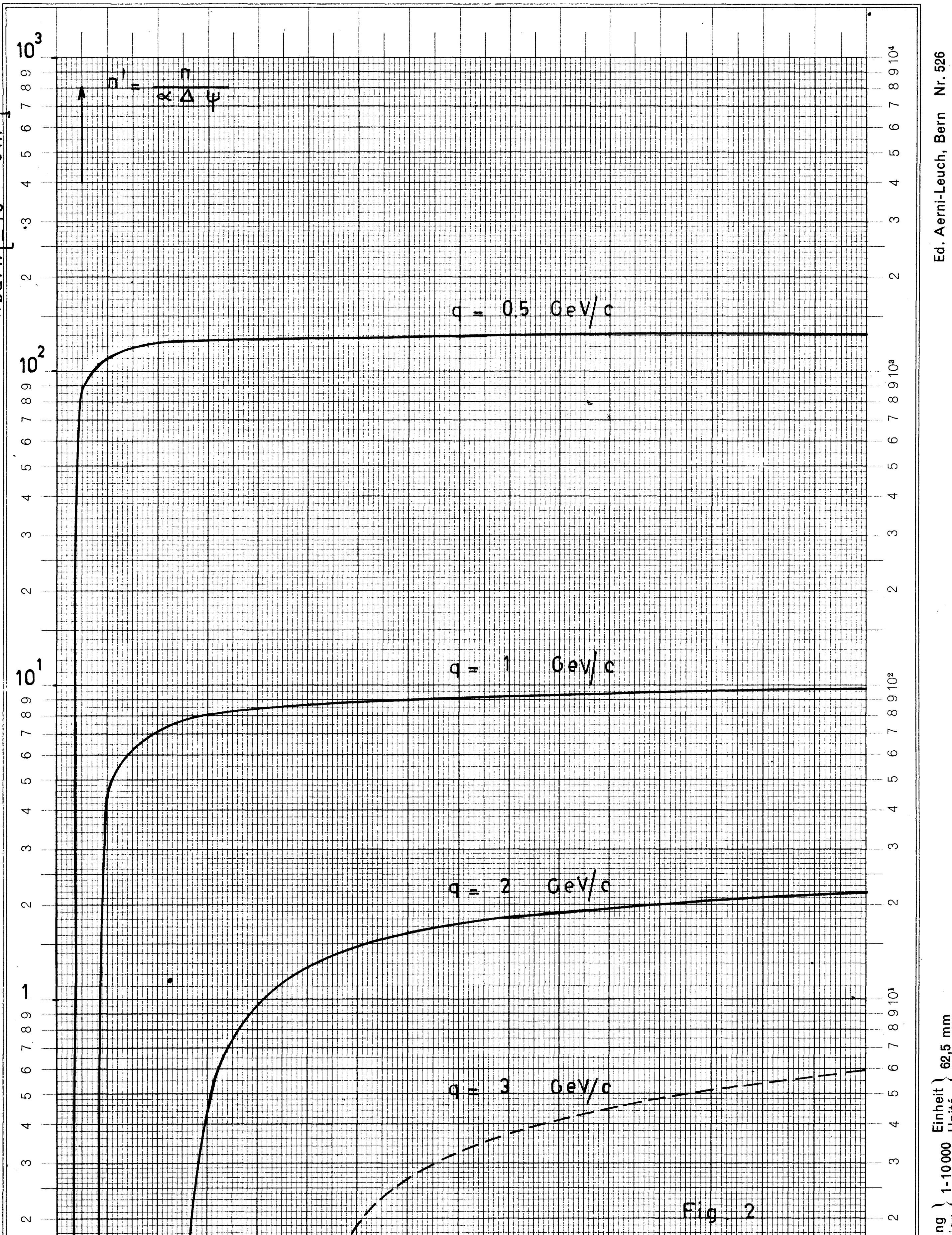
$n' = \frac{n}{\alpha \Delta \psi}$
nbarn [= 10^{-33} cm^2]
q = 0.5 GeV/c
q = 1 GeV/c
q = 2 GeV/c
q = 3 GeV/c
Fig. 2
P GeV/c
Ed. Aerni-Leuch, Bern Nr. 526
Logar. Teilung Division 1-10000 Einheit Unité 62,5 mm

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