



RADIATIVE DECAYS OF MESONS

M. Gourdin \*)  
CERN - Geneva

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\*) On leave from the Laboratoire de Physique  
Théorique, Orsay, France.

. - GENERAL CONSIDERATIONS

1) The subject of this introductory talk is the radiative decay of pseudoscalar mesons  $P$  and vector mesons  $V$

$$P \Rightarrow \gamma + \gamma$$

$$V \Rightarrow P + \gamma$$

$$P \Rightarrow V + \gamma$$

and a convenient theoretical framework for such a study in the broken  $SU(3)$  symmetry.

2) For electromagnetic interactions the notion of  $U$  spin invariance <sup>1),2)</sup> has been introduced in analogy with the  $I$  spin invariance of strong interactions. One of the best success of the  $U$  spin invariance is the Coleman-Glashow relation <sup>1)</sup> for the baryon octuplet  $J^P = \frac{1}{2}^+$  which agrees very well with experiment.

In the  $SU(3)$  theory of electromagnetic interactions the assumptions made for the photon are twofold :

- a) the photon is a  $U$  spin scalar ;
- b) the photon belongs to an adjoint representation of  $SU(3)$ .

We must notice that a large number of results are independent of the assumption b).

3) In the two cases of mesons considered here we have a nonet of particles <sup>3)</sup>, e.g., a direct sum of octuplet and singlet  $SU(3)$  representations. Because of the  $SU(3)$  breaking a configuration mixing can occur between the two isoscalar members of the nonets <sup>4)</sup>.

A nice way to have an idea about such a mixing is to use the Gell-Mann Okubo (GM-O) mass formula <sup>5)</sup> which is very successful for baryon multiplets like  $J^P = \frac{1}{2}^+$  and  $\frac{3}{2}^+$ . Unfortunately for meson nonets the mass formula involves an a priori unknown mixing parameter and loses its predictive power for comparing the masses.

4) For baryons the GM-0 mass formula is linear in the particle masses. For mesons the theoretical situation is somewhat confusing. Arguments based on Lagrangian models or on propagator methods suggest to use squared masses or inverse squared masses depending on the type of SU(3) breaking we introduce <sup>6),7)</sup>. In our opinion this question is still open and we shall consider here both possibilities of a mass mixing formalism linear (L) or quadratic (Q) in the pseudoscalar and vector meson masses.

5) More elaborated schemes of particle mixing have been proposed <sup>6),8),9)</sup> using two parameters instead of one. We shall disregard these possibilities here for simplicity because the mass mixing formalism is in agreement with the Orsay data on vector meson production in electron-positron annihilation <sup>10),11)</sup>.

The comparison between theory and experiment involves the mixing angle  $\theta_V$  for vector mesons and as an empirical fact the two values of  $\theta_V$  computed with the GM-0 mass formula using linear or quadratic masses are very close to each other

$$\theta_V^L = 37^\circ 7$$

$$\theta_V^Q = 40^\circ 2$$

so that a choice between these two directions is not possible in a process where only vector mesons are involved <sup>12)</sup>. Fortunately the situation is different for pseudoscalar mesons where the two values of the mixing angle  $\theta_P$  are enough separated

$$\theta_P^L = 23^\circ 4$$

$$\theta_P^Q = 10^\circ 1$$

to imply different predictions.

6) It is out of the limits of this report to discuss the general problem of SU(3) breaking for pseudoscalar and vector mesons. Let us just point out some difficulties.

- a) on what coupling constants must be applied the SU(3) or the U spin relations ? <sup>13)</sup> ;
- b) is it sufficient to take into account the large mass differences - especially for pseudoscalar mesons - only in phase space factors ? <sup>14)</sup> ;
- c) are the I spin and U spin invariances factorizable in the sense that SU(3) breaking terms depending on both I and U can be neglected as done for instance in Ref. 1) ?

These questions are obviously closely related to each other and they cannot be solved in a model independent way. Only experiments will decide if the assumptions made are reliable or not.

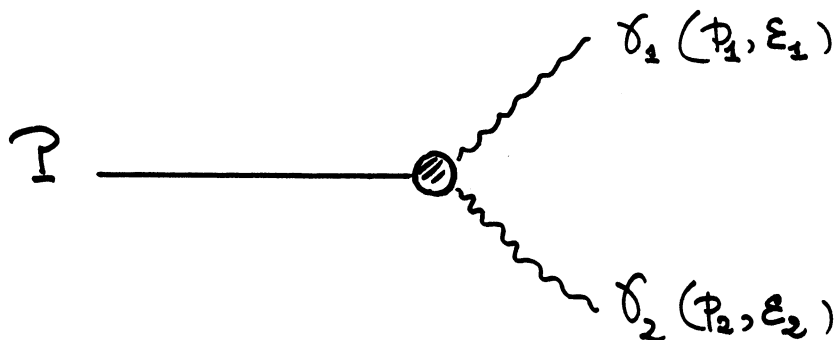
## II. - PSEUDOSCALAR MESONS

- 1) We study the  $2\gamma$  decay mode of the pseudoscalar mesons  $P = \pi^0, \eta^0$  and  $X^0$ .

From Lorentz invariance, space reflexion invariance and current conservation, the transition matrix element has the following structure

$$\langle \delta_1, \delta_2 | T | P \rangle = g_P \epsilon_{\mu\nu\rho\sigma} p_1^\mu \epsilon_1^\nu p_2^\rho \epsilon_2^\sigma \quad (1)$$

and the kinematics is indicated on Fig. 1.



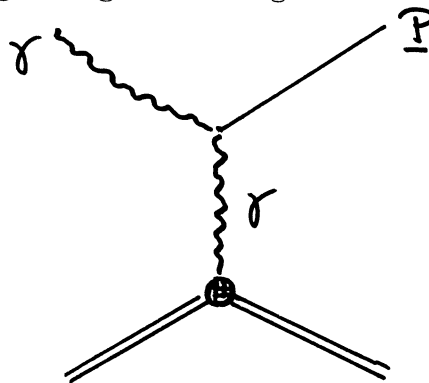
- Figure 1 -

The decay width is computed from (1) to be

$$\Gamma(P \Rightarrow \gamma_1 + \gamma_2) = \frac{\pi \alpha^2}{4} m_P^3 g_P^2$$

where  $\alpha$  is the fine structure constant  $\alpha = \frac{1}{137}$ .

The best way to measure the  $P \Rightarrow 2\gamma$  decay is to look at the coherent photoproduction of the  $P$  meson in the Coulomb field of an heavy nucleus, the so-called Primakoff effect <sup>15)</sup> represented by the one-photon exchange diagram of Fig. 2.



- Figure 2 -

In the laboratory frame the  $P$  meson is produced at an angle  $\theta$  and the angular distribution proportional to  $\theta / [\delta^2 + \theta^2]^2$  has a strong narrow peak near the forward direction, because of the smallness of  $\delta$  which tends to zero at high energy.

The  $\pi^0$  life-time and the  $\rho^0 \Rightarrow 2\gamma$  partial life-time have been measured by this method

$$\Gamma(\pi^0 \Rightarrow 2\gamma) = (11.7 \pm 1.2) \text{ eV} \quad 16)$$

$$\Gamma(\rho^0 \Rightarrow 2\gamma) = (1.01 \pm 0.23) \text{ keV} \quad 17)$$

2) The U spin invariance of electromagnetic interactions forbids the decay of the U spin vector  $P_8^1$  into two photons <sup>18)</sup> so that there are only two independent amplitudes describing the  $2\gamma$  decay of the P mesons, one  $g_8$  for the octuplet U spin scalar  $P_8^0$  and one  $g_1$  for the singlet U spin scalar  $P_1^0$ .

The mixing between the octuplet  $\eta_8$  and the singlet isoscalars is written as <sup>7)</sup>

$$\eta^0 = \cos\theta_P \eta_8 - \sin\theta_P \eta_1$$

$$X^0 = \cos\theta_P \eta_1 + \sin\theta_P \eta_8$$

and the physical coupling constants are given in Table I.

	$g_8$	$g_1$
$g_{\pi^0}$	$\frac{\sqrt{3}}{2}$	0
$g_{\eta^0}$	$\frac{1}{2} \cos\theta_P$	$-\sin\theta_P$
$g_{X^0}$	$\frac{1}{2} \sin\theta_P$	$\cos\theta_P$

- Table I -

3) Let us now analyze the experimental data. By convention we choose  $g_{\pi^0}$  to be positive and in the absence of  $\eta^0$ - $X^0$  mixing the relation  $g_{\eta^0} = \sqrt{3} g_{\pi^0}$  <sup>2)</sup> suggests to take also  $g_{\eta^0} > 0$ . From experiment we obtain

$$g_{\pi^0} = (0,337 \pm 0,017) \text{ GeV}^{-1}$$

$$g_{\eta^0} = (0,382 \pm 0,044) \text{ GeV}^{-1}$$

With the two values  $\theta_P^L$  and  $\theta_P^Q$  of the pseudoscalar mixing angle we compute the reduced matrix elements

$$g_8 = (0,389 \pm 0,019) \text{ GeV}^{-1}$$

$$g_{\frac{1}{2}}^L = -(0,51 \pm 0,12) \text{ GeV}^{-1}$$

$$g_{\frac{1}{2}}^Q = -(1,08 \pm 0,27) \text{ GeV}^{-1}$$

It is then straightforward to predict the  $X^0 \Rightarrow 2\gamma$  partial decay width

$$\Gamma^L (X^0 \Rightarrow 2\gamma) = (5,6 \begin{matrix} + 3,5 \\ - 2,7 \end{matrix}) \text{ keV}$$

$$\Gamma^Q (X^0 \Rightarrow 2\gamma) = (39 \begin{matrix} + 23 \\ - 18 \end{matrix}) \text{ keV}$$

The two values differ by almost one order of magnitude and an absolute measurement of the  $X^0 \Rightarrow 2\gamma$  decay mode using for instance the Primakoff effect would be of the greatest interest to clarify the situation concerning the  $\eta^0 - X^0$  mixing. It will provide a check of the present scheme where the mass differences between the  $\pi^0$ ,  $\eta^0$  and  $X^0$  mesons have been taken into account only through the phase space factor  $m_P^3$  (18). Obviously the U spin invariance can be applied in different ways for instance on the dimensionless quantity  $m_P g_P$  so that the phase space factor becomes simply  $m_P$ : such a possibility will imply a very large  $X^0 \Rightarrow 2\gamma$  width certainly in contradiction with experiment and the common sense.

### III. - VECTOR MESONS

1) Let us now consider the radiative decay of a vector meson V

$$V \Rightarrow P + \gamma$$

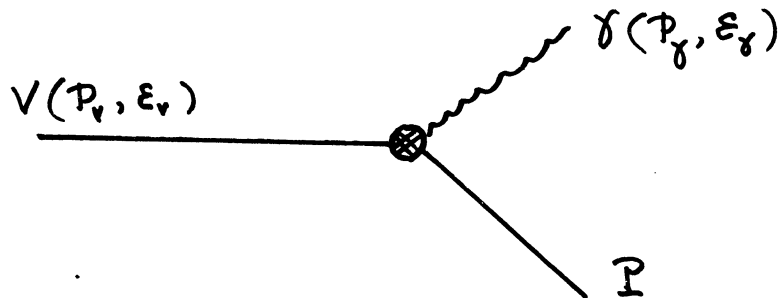
or, when  $m_P > m_V$  the radiative decay of a pseudoscalar meson P

$$P \Rightarrow V + \gamma$$

From Lorentz invariance, space reflexion invariance and current conservation, the transition matrix element has the following structure

$$\langle \delta, \mathbb{P} | T | V \rangle = g_{V\mathbb{P}\delta} \varepsilon_{\mu\nu\rho\sigma} p_\delta^\mu \varepsilon_\delta^\nu p_V^\rho \varepsilon_V^\sigma \quad (2)$$

and the kinematics is indicated on Fig. 3



- Figure 3 -

The decay widths are computed from (2) to be

$$\Gamma(V \Rightarrow \mathbb{P} + \delta) = \frac{\alpha}{24} g_{V\mathbb{P}\delta}^2 m_V^3 \left(1 - \frac{m_{\mathbb{P}}^2}{m_V^2}\right)^3 \quad \text{when } m_V > m_{\mathbb{P}}$$

$$\Gamma(\mathbb{P} \Rightarrow V + \delta) = \frac{\alpha}{8} g_{V\mathbb{P}\delta}^2 m_{\mathbb{P}}^3 \left(1 - \frac{m_V^2}{m_{\mathbb{P}}^2}\right)^3 \quad \text{when } m_{\mathbb{P}} > m_V$$

The radiative transition  $\omega \Rightarrow \pi^0 + \gamma$  has been known for a long time and represents an appreciable fraction of the  $\omega$  width. The world average value is <sup>19),20)</sup>

$$\Gamma(\omega \Rightarrow \pi^0 + \gamma) = (1,12 \pm 0,20) \text{ MeV}$$

Two rare decay modes of the  $\phi$  meson have been recently observed in an electron-positron colliding beam experiment at Orsay and the measured branching ratio are <sup>21)</sup>

$$B(\phi \Rightarrow \pi^0 + \gamma) = (0,19 \pm 0,08) 10^{-2}$$

$$B(\phi \Rightarrow \eta^0 + \gamma) = (1,92 \pm 0,54) 10^{-2}$$



Using, for the  $\phi$  meson total width, an average value of 4 MeV (20) we deduce

$$\Gamma(\phi \Rightarrow \pi^0 + \gamma) = (7,6 \pm 3,2) \text{ keV}$$

$$\Gamma(\phi \Rightarrow \eta^0 + \gamma) = (77 \pm 23) \text{ keV}$$

2) From U spin invariance alone there exist fixe reduced matrix elements one defines as  $A_{jk}^U$  for the transition  $V_j^U \Rightarrow P_k^U + \gamma$ . The U spin index takes the values  $U = 0, 1$  and two other indices  $j$  and  $k = 1, 8$  indicate respectively the vector meson and pseudoscalar meson SU(3) representations. Table II gives the coupling constants  $g_{VP\gamma}$  for the physical transitions in terms of these five reduced matrix elements.

If, in addition, we assume that the photon belongs to an adjoint SU(3) representation we have two constraints

$$A_{11}^0 = 0 \quad -A_{-88}^0 + A_{88}^0 = 0$$

the second one being due to particle-antiparticle conjugation invariance.

3) Let us now analyze the experimental data. We choose the phase conventions which will best agree later with the vector meson dominance model

$$g_{\omega\pi^0\gamma} > 0 \quad g_{\phi\pi^0\gamma} < 0 \quad g_{\phi\eta^0\gamma} < 0$$

From experiment we compute the three coupling constants

$$g_{\omega\pi^0\gamma} = (2,89 \pm 0,26) \text{ GeV}^{-1}$$

$$g_{\phi\pi^0\gamma} = -(0,16 \pm 0,02) \text{ GeV}^{-1}$$

$$g_{\phi\eta^0\gamma} = -(0,82 \pm 0,12) \text{ GeV}^{-1}$$

	$\frac{1}{4}(A_{88}^0 - A_{88}^1)$	$\frac{1}{2} A_{18}^0$	$\frac{1}{2} A_{81}^0$	$\frac{1}{2}(A_{88}^0 + A_{88}^1)$	$A_{11}^0$
$\rho \Rightarrow \pi \delta$	1	0	0	1	0
$\rho \Rightarrow 2\delta$	$\sqrt{3} \text{Cos}\theta_P$	0	$-\sqrt{3} \text{Sin}\theta_P$	0	0
$\chi \Rightarrow \rho \delta$	$\sqrt{3} \text{Sin}\theta_P$	0	$\sqrt{3} \text{Cos}\theta_P$	0	0
$\omega \Rightarrow \pi \delta$	$\sqrt{3} \text{Sin}\theta_V$	$\sqrt{3} \text{Cos}\theta_V$	0	0	0
$\omega \Rightarrow 2\delta$	$-\text{Sin}\theta_V \text{Cos}\theta_P$	$\text{Cos}\theta_V \text{Cos}\theta_P$	$-\text{Sin}\theta_V \text{Sin}\theta_P$	$\text{Sin}\theta_V \text{Cos}\theta_P$	$-\text{Cos}\theta_V \text{Sin}\theta_P$
$\chi \Rightarrow \omega \delta$	$-\text{Sin}\theta_V \text{Sin}\theta_P$	$\text{Cos}\theta_V \text{Sin}\theta_P$	$\text{Sin}\theta_V \text{Cos}\theta_P$	$\text{Sin}\theta_V \text{Sin}\theta_P$	$-\text{Cos}\theta_V \text{Cos}\theta_P$
$\rho \Rightarrow \pi \delta$	$\sqrt{3} \text{Cos}\theta_V$	$-\sqrt{3} \text{Sin}\theta_V$	0	0	0
$\rho \Rightarrow 2\delta$	$-\text{Cos}\theta_V \text{Cos}\theta_P$	$-\text{Sin}\theta_V \text{Cos}\theta_P$	$-\text{Cos}\theta_V \text{Sin}\theta_P$	$\text{Cos}\theta_V \text{Cos}\theta_P$	$\text{Sin}\theta_V \text{Sin}\theta_P$
$\rho \Rightarrow \chi \delta$	$-\text{Cos}\theta_V \text{Sin}\theta_P$	$-\text{Sin}\theta_V \text{Sin}\theta_P$	$\text{Cos}\theta_V \text{Cos}\theta_P$	$\text{Cos}\theta_V \text{Sin}\theta_P$	$-\text{Sin}\theta_V \text{Cos}\theta_P$
$K^{*\pm} \Rightarrow K^{\pm} \delta$	1	0	0	1	0
$K^{*0} \Rightarrow K^0 \delta$	-2	0	0	1	0

-- Table II --

We assume the photon to be the U spin scalar of an octuplet. The two last columns of Table II disappear and we have only three independent parameters

$$X = A_{88}^0 \quad Y = A_{18}^0 \quad Z = A_{81}^0$$

With the two possibilities for the mixings we compute the reduced matrix elements

$$\begin{aligned} X^L &= (1,89 \pm 0,18) \text{GeV}^{-1} & \left(\frac{Y}{X}\right)^L &= 1,45 \pm 0,05 & \left(\frac{Z}{X}\right)^L &= -(2,13 \pm 0,50) \\ X^Q &= (2,01 \pm 0,19) \text{GeV}^{-1} & \left(\frac{Y}{X}\right)^Q &= 1,33 \pm 0,24 & \left(\frac{Z}{X}\right)^Q &= -(5,88 \pm 1,22) \end{aligned}$$

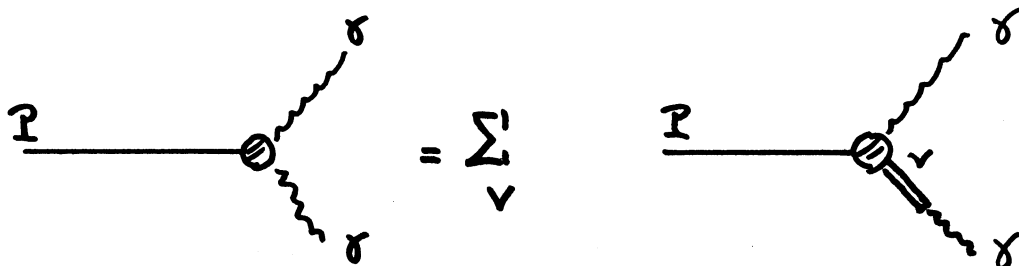
A more restrictive nonet symmetry formulated for instance in a quark model predicts <sup>7)</sup>

$$\frac{Y}{X} = \sqrt{2} \quad \frac{Z}{X} = -\sqrt{2}$$

The ratios obtained with a linear mass mixing are compatible with these values.

#### IV. - VECTOR MESON DOMINANCE

1) The P- $\delta$ - $\delta$  and P-V- $\delta$  vertices are related by the vector meson dominance as shown on Fig. 4



- Figure 4 -

The photon is assumed to belong to an octuplet so that only the U spin scalar combination of the octuplet  $V_8^0$  is coupled to the photon. We obtain the general relation

$$g_J = \sum_V A_{8J}^0 \frac{m_V^2}{W_V(0)} \frac{1}{f_{V_8^0}} \quad J = 1, 8$$

where the sum  $\sum_V$  extends over all possible vector mesons. The inverse vector meson propagator  $[W_V(S) - S]$  has been extrapolated at  $S = 0$  for a real photon where in the zero width approximation it simply reduces to  $m_V^2$ .

In the restricted vector meson dominance where only one nonet is present, using Clebsch-Gordan coefficients and taking into account phenomenologically finite width corrections <sup>22)</sup> we obtain the simple proportionality

$$g_J = \frac{2}{\sqrt{3}} \frac{1}{f_{\eta\pi\pi}} A_{8J}^0 \quad J = 1, 8$$

We disregard in what follows contributions due to new vector mesons like the  $\rho'$  - if it exists - and we discuss only the restricted vector meson dominance model.

2) The ratio  $Z/X$  is independent of the numerical value of the photon-vector meson coupling constant and in the two first lines of Table III we compare the VMD prediction

$$\left( \frac{Z}{X} \right)_{VMD} = \frac{g_1}{g_8}$$

with the results of the analysis made in the previous sections. In the two last lines of the same Table, we compute the quantity  $X$  which is known from  $\pi^0$  data only using the value  $f_{\rho\pi\pi} = 5.52$  which corresponds to a total  $\rho$  meson width of 127 MeV, the  $\rho$  meson mass being taken as  $m_\rho = 776$  MeV.

	VMD	Experiment
$(\frac{Z}{X})^L$	$-1.31 \pm 0.30$	$-2.13 \pm 0.50$
$(\frac{Z}{X})^Q$	$-2.77 \pm 0.70$	$-5.88 \pm 1.22$
$\frac{X^L}{(\text{GeV}^{-1})}$	$1.860 \pm 0.093$	$1.896 \pm 0,186$
$\frac{X^Q}{(\text{GeV}^{-1})}$	$1.860 \pm 0.093$	$2.018 \pm 0.196$

- Table III -

The quantitative agreement for X is good inside the errors and it can be considered as a success of the VMD model as formulated originally for meson radiative decays by Gell-Mann, Sharp and Wagner <sup>23)</sup>. The ratio Z/X is very sensitive to the mixing angles and a reasonable agreement can be reached with a linear mass mixing.

3) In a systematic study of the vector meson dominance model we must consider the complete chain of vertices

$$P-\delta-\delta \quad P-V-\delta \quad P-V-\gamma$$

involving non-radiative transition like for instance  $\omega \Rightarrow \pi^+ \pi^- \pi^0$  or  $\phi \Rightarrow \rho + \pi$ . The problem in its generality has been studied by many authors <sup>23),24)</sup> and more or less sophisticated models of symmetry breaking have been proposed to solve it. Let us just here emphasize three points :

- a) for some relations direct contact terms have been introduced in addition to the vector meson pole contributions <sup>25)</sup> with a limited success as far as the comparison between theory and experiment is concerned ;

- b) the corrections due to the finite width of the vector mesons are not negligible (especially for  $\rho$  and  $\phi$ ) and they can be computed only in a model dependent way involving an explicit form of the vector meson propagators ;
- c) the three types of vertices are compared with off-mass shell vector mesons so that an extrapolation is needed for the coupling constants which are defined for on-mass shell vector mesons ; such an extrapolation is generally assumed to affect only little the value of these coupling constants but this statement is convenient for practical purpose but certainly not exact.
- 4) We compute the various radiative decay widths of mesons choosing as input the experimental results concerning the  $\pi^0$  meson

$$\Gamma(\pi^0 \Rightarrow 2\gamma) \quad \Gamma(\omega \Rightarrow \pi^0 + \gamma) \quad \Gamma(\phi \Rightarrow \pi^0 + \gamma)$$

and making an empirical compromise between the values of the ratio  $Z/X$  obtained in Table III :

- a) linear mass mixing (LM)

$$\frac{Z}{X} = \frac{g_1}{g_2} = -1.62$$

- b) quadratic mass mixing (QM)

$$\frac{Z}{X} = \frac{g_1}{g_2} = -3.94$$

The results are given in Table IV and compared with the available experimental data. Let us remark that the  $\rho$  meson width is also predicted by the VMD model for  $\pi^0$  data only.

The best way to learn something about the  $\rho^0 - X^0$  mixing is to have absolute measurements of the  $X^0$  radiative decay widths for which the theoretical predictions can differ by one order of magnitude going from  $\theta_P^I = 23.4^\circ$  to  $\theta_P^Q = 10.1^\circ$ .

	LM	QM	Experiment
$\Gamma (\rho^0 \Rightarrow \pi^0 \gamma)$	114 KeV	126 KeV	< 250 KeV
$\Gamma (\omega \Rightarrow \pi^0 \gamma)$	1.12 MeV	1.12 MeV	input
$\Gamma (\phi \Rightarrow \pi^0 \gamma)$	7.6 KeV	7.6 KeV	input
$\Gamma (\pi^0 \Rightarrow 2 \gamma)$	11.7 eV	11.7 eV	input
$\Gamma (\rho^0 \Rightarrow \eta \gamma)$	108 KeV	141 KeV	-
$\Gamma (\omega \Rightarrow \eta \gamma)$	13.7 KeV	12.7 KeV	< 180 KeV
$\Gamma (\phi \Rightarrow \eta \gamma)$	93 KeV	122 KeV	(77 ± 23) KeV
$\Gamma (\eta \Rightarrow 2 \gamma)$	1.14 KeV	1.68 KeV	(1.01 ± 0.23) KeV
$\Gamma (X \Rightarrow \rho^0 \gamma)$	111 KeV	1.49 MeV	see below
$\Gamma (X \Rightarrow \omega \gamma)$	12.3 KeV	175 KeV	-
$\Gamma (\phi \Rightarrow X \gamma)$	1.5 KeV	5.4 KeV	-
$\Gamma (X \Rightarrow 2 \gamma)$	9.2 KeV	87 KeV	see below
$\Gamma (K^{*\pm} \Rightarrow K^\pm \gamma)$	65 KeV	72 KeV	-
$\Gamma (K^{*0} \Rightarrow K^0 \gamma)$	259 KeV	288 KeV	-
$\Gamma (\rho \Rightarrow 2 \pi)$	132 MeV	150 MeV	(125 ± 15) MeV

- Table IV -

We have only experimental indications on branching ratios and using <sup>19)</sup>

$$B(X^0 \Rightarrow \pi^+ \pi^- \gamma) = 30\%$$

we can predict from Table IV the  $X^0$  meson total width and the  $X^0 \Rightarrow 2\gamma$  branching ratio. This has been done in Table V inserting a finite  $\rho$  width correction of

	LM	QM	Experiment
$\Gamma_{X^0}$	440 KeV	5.8 MeV	< 4 MeV
$B(X^0 \Rightarrow 2\gamma)$	2.1 %	1.5 %	$(1.8 \pm 0.5) \%^{26)}$

- Table V -

#### V. - CONCLUDING REMARKS

1) A decisive improvement in the measurement of radiative decay of mesons is needed to hope some progress in the understanding of the broken  $SU(3)$  symmetry applied to electromagnetic interactions. The two sets of data for  $P \Rightarrow 2\gamma$  and  $V \Rightarrow P + \gamma$  ;  $P \Rightarrow V + \gamma$  must be considered separately as done here in order to test first the consistency of the scheme and to learn something about the iso-scalar meson mixings. This last point is very crucial. After that a relation between the vertices  $P\gamma\gamma$ ,  $PV\gamma$  and  $PVV$  can be attempted on the basis of a restricted or an extended vector meson dominance model which seems to work now in the 10% or 15% limit.



2) All the theoretical ways of describing the radiative decays of mesons have not been discussed here even if they are a priori equally reasonable. For instance other signs for the coupling constants  $g_{\rho^0\pi^0\gamma}$ ,  $g_{\omega\rho^0\gamma}$ ,  $g_{\phi\pi^0\gamma}$  are possible and the U spin and SU(3) relation can be applied on different quantities where mass factors have been included. The quantitative results will be different but for a scheme in agreement with experiment the qualitative feature will be the same and the most naïve approach we have presented here gives a consistent description of the available data and only new and more accurate experiments can make a selection between the various models.

3) A nice way to obtain information about the radiative decay of mesons is to use the Primakoff effect. Information on the transitions

$$\rho^\pm \Rightarrow \pi^\pm + \gamma$$

$$K^{*\pm} \Rightarrow K^\pm + \gamma$$

$$X^0 \Rightarrow 2\gamma$$

can be obtained from high energy coherent production processes on a nucleus or a nucleon

$$\pi^\pm + A \Rightarrow \rho^\pm + A$$

$$K^\pm + A \Rightarrow K^{*\pm} + A$$

$$\gamma + A \Rightarrow X^0 + A$$

Two experimental procedures can be used :

- a) detection of the produced meson near the forward direction as done for  $\pi^0$  and  $\eta^0$  photoproduction <sup>15)</sup> ;
- b) detection of the recoil nucleus at very small kinetic energies in all the allowed directions <sup>27),28)</sup>.

A discussion of these problems cannot be done here and we just mention this very clean experimental approach.

#### ACKNOWLEDGEMENTS

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Summer School in Physics (1970).
- 12) Because of the small  $\omega - \rho$  mass difference the comparison of  
 $\rho \Rightarrow e^+e^-$  and  $\omega \Rightarrow e^+e^-$  partial decay widths is essen-  
tially free of mass breaking ambiguities. The theoretical  
expression is

$$R = \frac{\Gamma(\omega \rightarrow e^+e^-)}{\Gamma(\rho \rightarrow e^+e^-)} \approx \frac{1}{3} \sin^2 \theta_V$$

With the two previous mixing angles  $\theta_V^L$  and  $\theta_V^Q$  the predictions are

$$R_{\text{linear}} = 0,124 \qquad R_{\text{quadratic}} = 0,139$$

both in excellent agreement with the experimental value deduced from the Orsay data <sup>10)</sup> :

$$R_{\text{experiment}} = 0,135 \pm 0,027$$

- 13) For instance if  $SU(3)$  symmetry holds for the quantities  $m_V^\alpha / f_V$  where  $\alpha$  is an integer and  $f_V^{-1}$  the dimensionless photon-vector meson coupling constant it must be also applied to the quantity  $g_{VP\gamma} / m_V$  where  $g_{VP\gamma}$  is defined in Eq. (2) as the vector meson pseudoscalar meson photon coupling constant.
- 14) As an alternative way one can break  $SU(3)$  symmetry in the coupling constants but the predictive power of the theory is highly depressed except in some specific models.
- See, for instance :
- A. Esteve, J. León and A. Tiemblo - Nuovo Cimento Letters N1, 30 (1971) ;
- A. Esteve, J. Navarro and A. Tiemblo - Preprint Madrid (1971) ;
- J. Julve, J. León and F.J. de Urries - Preprint Madrid (1971).
- 15) H. Primakoff - Phys.Rev. 81, 899 (1951).;
- A. Halprin, C.M. Andersen and H. Primakoff - Phys.Rev. 152, 1295 (1966).
- 16) G. Bellettini et al. - Nuovo Cimento 66A, 243 (1970).
- This result is in disagreement with the previous world average value of  $(7.2 \pm 1.2)$  eV. Nevertheless we think this measurement to be more reliable.

- 17) C. Bemporad et al. - Phys.Letters 25B, 380 (1967).  
A confirmation of this number by an independent experiment is necessary before taking it as certain.
- 18) See the papers of footnote 14), for a different type of breaking using a quark model.
- 19) Particle Data Group (1971).
- 20) This value assumes the decay  $\omega^0 \Rightarrow$  neutrals to be strongly dominated by the  $\pi^0 \gamma$  state and in particular the  $\pi^0 \pi^0 \delta$  mode is quoted in 19) to be less than 1% of the total. A possible larger value has been reported at this conference by M.N. Kreisler for the  $\pi^0 \pi^0 \delta$  mode and this will imply a reduction of the  $\omega \Rightarrow \pi^0 \delta$  decay width.
- 21) P. Benaksas et al. - Preprint Orsay LAL 1240 (1970) ;  
G. Cosme - Contribution to this Conference.
- 22) The restricted VMD model applied to the  $\pi$  meson electromagnetic form factor gives the normalization constraint
- $$\frac{m_\rho^2}{W_\rho(0)} \frac{f_{\rho\pi\pi}}{f_\rho} = 1$$
- where the factor  $m_\rho^2/W_\rho(0)$  can be interpreted as a finite width correction when the vertex functions are assumed to be constant. See Ref. 11).
- 23) M. Gell-Mann, D. Sharp and W. Wagner - Phys.Rev.Letters 8, 261 (1962).
- 24) M. Jacob - Preprint CERN Th. 846 (1967) ;  
E. Cremmer - Nuclear Phys. B14, 52 (1969) ;  
A. Baracca and A. Bramon - Nuovo Cimento 51A, 873 (1967) ;  
69A, 613 (1970) ;  
D. Greenberg - Preprint Pittsburgh (1969).
- 25) W. Alles - Nuovo Cimento Letters 3, 163 (1970) ; 4, 137 (1970) ;  
G.J. Gounaris - Phys.Rev. D1, 1436 (1970) ; D2, 2734 (1970).
- 26) D. Bollini et al. - Nuovo Cimento 58, 289 (1968) ;  
M. Basile - Contribution to this Conference.

- 27) L. Stodolsky - Phys.Rev.Letters 26, 404 (1971).
- 28) M. Gourdin - Preprint CERN Th. 1307 (1971).