EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN/ISR-TH/69-44

HOW DO ANGLE AND POSITION ERRORS AT INJECTION AFFECT BEAM SIZE AND BEAM DENSITY ?

by

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Geneva, July 1969

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1. INTRODUCTION

We work in the normalized (x, x') phase plane in which betatron oscillations with amplitude a are represented by a circle of radius a.

A centred beam of emittance πR^2 is represented by the area of a circle of radius R around the origin. A displaced beam with the same emittance is given by a circle of radius R with the centre at x = zR. $z \geq o$ is the injection error parameter. This is shown in Fig. 1. Obviously, an error in normalized angle is equivalent to an error in normalized displacement.

After filamentation has taken place betatron oscillations will occupy a circle of radius (z + 1) R.

We shall answer the following questions:

i) What is the density distribution inside this circle for several given density distributions at the beginning, and for a given injection error parameter z?

ii) What is the visible density distribution as seen by a detector which detects the spatial density only?

iii) What is the effect of injection errors on the luminosity of the ISR?

2. ANALYSIS

We use both polar co-ordinates (r, φ) and Cartesian co-ordinates (x, x') in the normalized phase plane.

Let the normalized density distribution of a perfect beam be called f(r). The beam with injection errors will then have a distribution function F(r, φ, z) which is given by:

$$F(r, \varphi, z) = f\left(\sqrt{r^2 + z^2 - 2zr \cos \varphi}\right)$$

Filamentation will smooth out the azimuthal variations of the density distribution function and yield a new distribution function $\bar{F}$:

$$\bar{F}(r, z) = \frac{1}{2\pi} \int_0^{2\pi} F(r, \varphi, z) \, d\varphi$$

It is also interesting to calculate how much of a mismatched filamented beam fits into a given radius. This is given by the integrated distribution function D:

$$D(r, z) = 2\pi \int_0^r y \, \bar{F}(y, z) \, dy$$

The "visible" distribution function G(x, z) is obtained by integrating $\bar{F}$ over all slopes x':

$$G(x, z) = \int_{-\infty}^{\infty} \bar{F} \left(\sqrt{x^2 + x'^2}, z\right) dx'$$

Finally, the luminosity of the ISR, assuming perfect overlap of the two colliding beams, is proportional to L(z):

$$L(z) = \int_{-\infty}^{+\infty} G^2(x, z) \, dx$$

3. NUMERICAL EXAMPLES

The functions defined above were calculated for uniform, parabolic and Gaussian density distributions in the injected beam. In all cases analytical formulae which are given below were used as far as possible. The remaining functions were computed by numerical integration. The accuracy in the integration routine GAUSS2*) is good and the relative error is less than 10^{-4}.

*) CERN 6600 PROGRAM LIBRARY D 103

3.1 Uniform beam

$$f(r) = \frac{1}{\pi} H(1 - r)$$

$r \leq 1 - z$ $\quad \bar{F}(r, z) = \frac{1}{\pi}$ for $z \leq 1$

$\bar{F}(r, z) = 0$ for $z > 1$

$1 - z < r \leq 1 + z$ $\quad \bar{F}(r, z) = \frac{\alpha}{\pi^2}$ where $\alpha = \cos^{-1}\left(\frac{r^2 + z^2 - 1}{2rz}\right)$

$r > 1 + z$ $\quad \bar{F}(r, z) = 0$

Let $A = \sqrt{(1 - z)^2 - x^2}$, $\quad B = \sqrt{(1 + z)^2 - x^2}$

$x \leq 1 - z$ $\quad G(x, z) = \frac{2}{\pi} A + 2 \int_A^B \bar{F}\left(\sqrt{x^2 + x'^2}, z\right) dx'$ $\quad z \leq 1$

$G(x, z) = 2 \int_A^B \bar{F}\left(\sqrt{x^2 + x'^2}, z\right) dx'$ $\quad z > 1$

$1 - z < x \leq 1 + z$ $\quad G(x, z) = 2 \int_0^B \bar{F}\left(\sqrt{x^2 + x'^2}, z\right) dx'$

$x > 1 + z$ $\quad G(x, z) = 0$

3.2 Parabolic beam

$r < |1 - z|$ $\quad \bar{F}(r, z) = \frac{2}{\pi}(1 - r^2 - z^2)$ $\quad z \leq 1$

$\bar{F}(r, z) = 0$ $\quad z > 1$

$|1 - z| < r \leq 1 + z$ $\quad \bar{F}(r, z) = \frac{2}{\pi^2}\left[\alpha(1 - r^2 - z^2) + 2rz \sin \alpha\right]$

$r > 1 + z$ $\quad \bar{F}(r, z) = 0$

α is the same as for the uniform beam.

$x < |1 - z|$ $$G(x, z) = \frac{8}{3\pi} A (1 - x^2 - 2z^2 + z) + 2 \int_A^B \bar{F} \left(\sqrt{x^2 + x'^2}, z\right) dx' \qquad z \leq 1$$

$$G(x, z) = 2 \int_A^B \bar{F} \left(\sqrt{x^2 + x'^2}, z\right) dx' \qquad z > 1$$

$|1 - z| < x \leq 1 + z$ $$G(x, z) = 2 \int_o^B \bar{F} \left(\sqrt{x^2 + x'^2}, z\right) dx'$$

$x > 1 + z$ $$G(x, z) = o$$

3.3 Gaussian beam

$$f(r) = \frac{1}{\pi} e^{-r^2}$$

$$\bar{F}(r, z) = \frac{1}{\pi} I_o(2rz) e^{-r^2 - z^2} \quad *)$$

4. RESULTS AND CONCLUSIONS

Graphs of the distribution function for z = o and z = 0.5 are shown in Figs. 2 to 7. The luminosity is plotted as a function of z in Fig. 8. A programme for computing these functions (BEAMS) and tables of them are available to those interested. Already at z = 0.5 injection errors cause a noticeable loss of beam density in the middle of the beam and luminosity.

*) This formula was calculated earlier by C. Bovet.

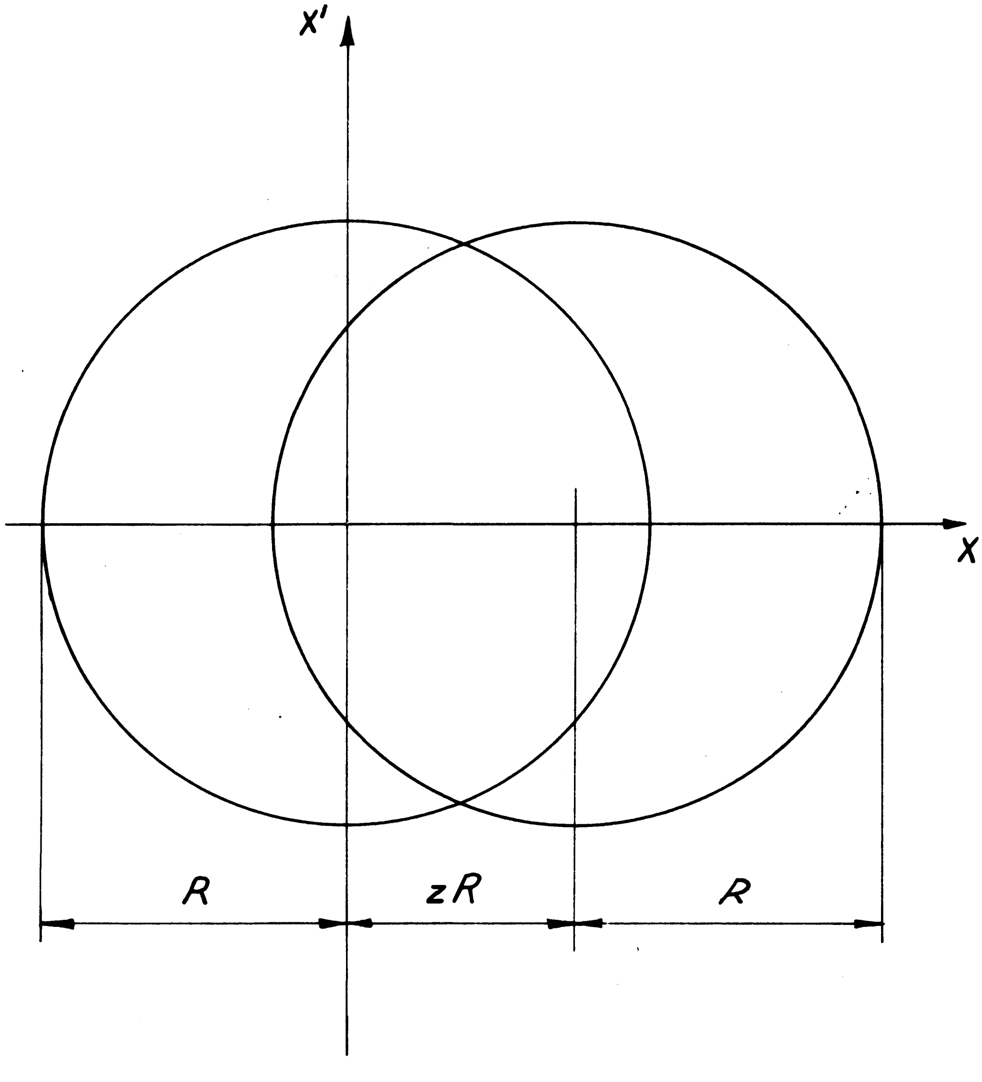

Fig. 1

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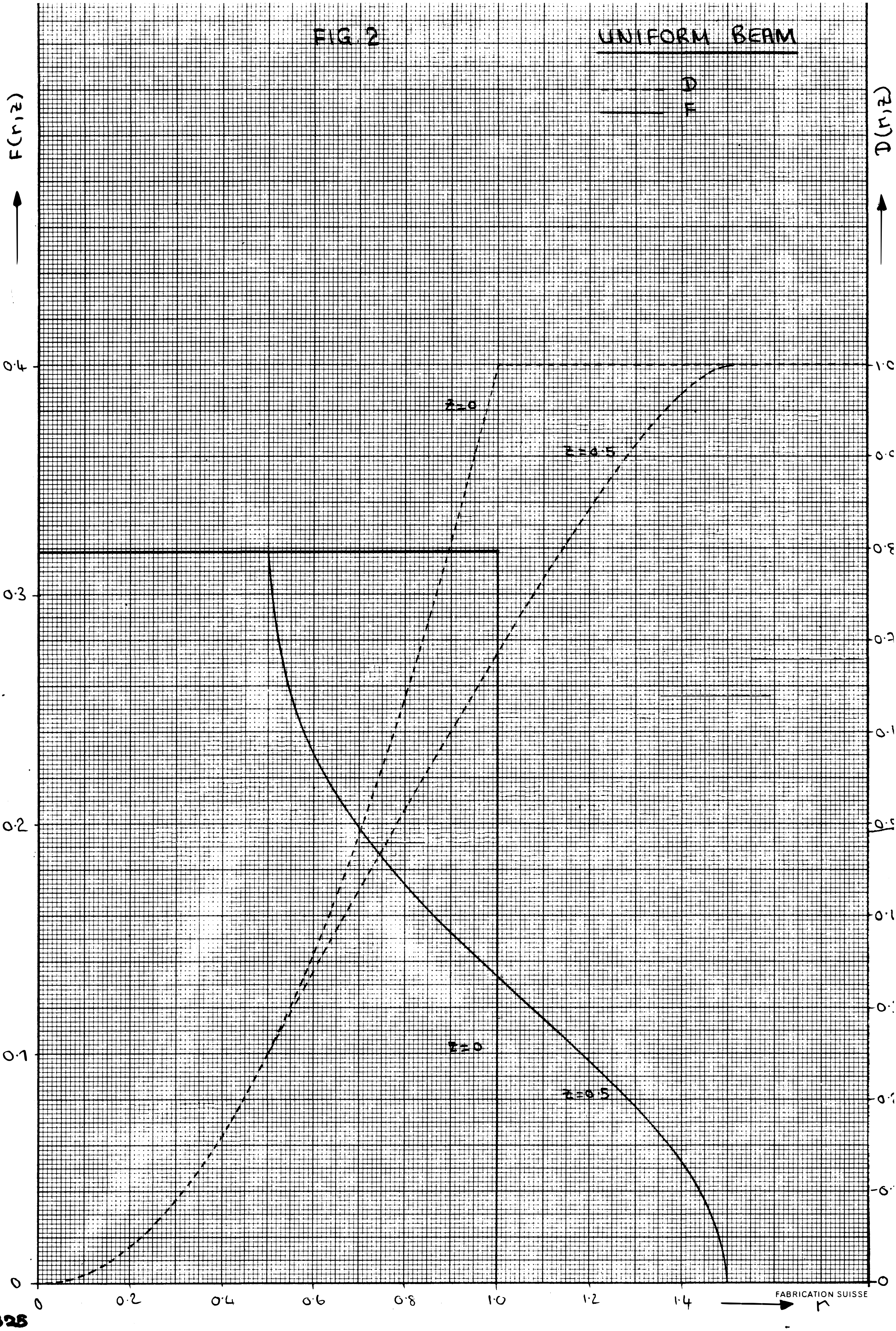
FIG. 2
UNIFORM BEAM
D
F
F(r,z)
D(r,z)
z=0
z=0.5
z=0
z=0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
r
1.0
0.8
0
FABRICATION SUISSE

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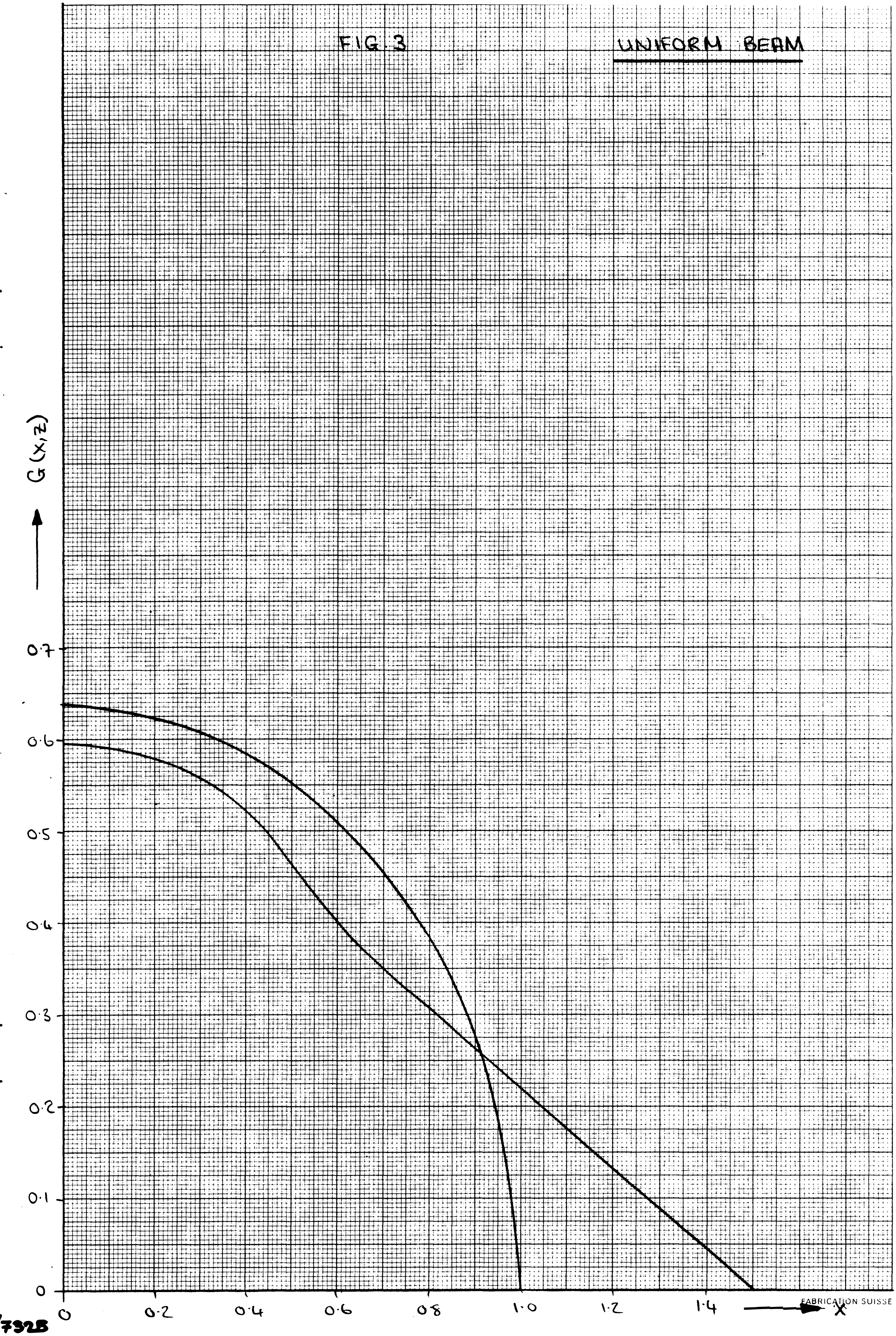
FIG. 3
UNIFORM BEAM
G (x, z)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
x
FABRICATION SUISSE

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FIG. 4

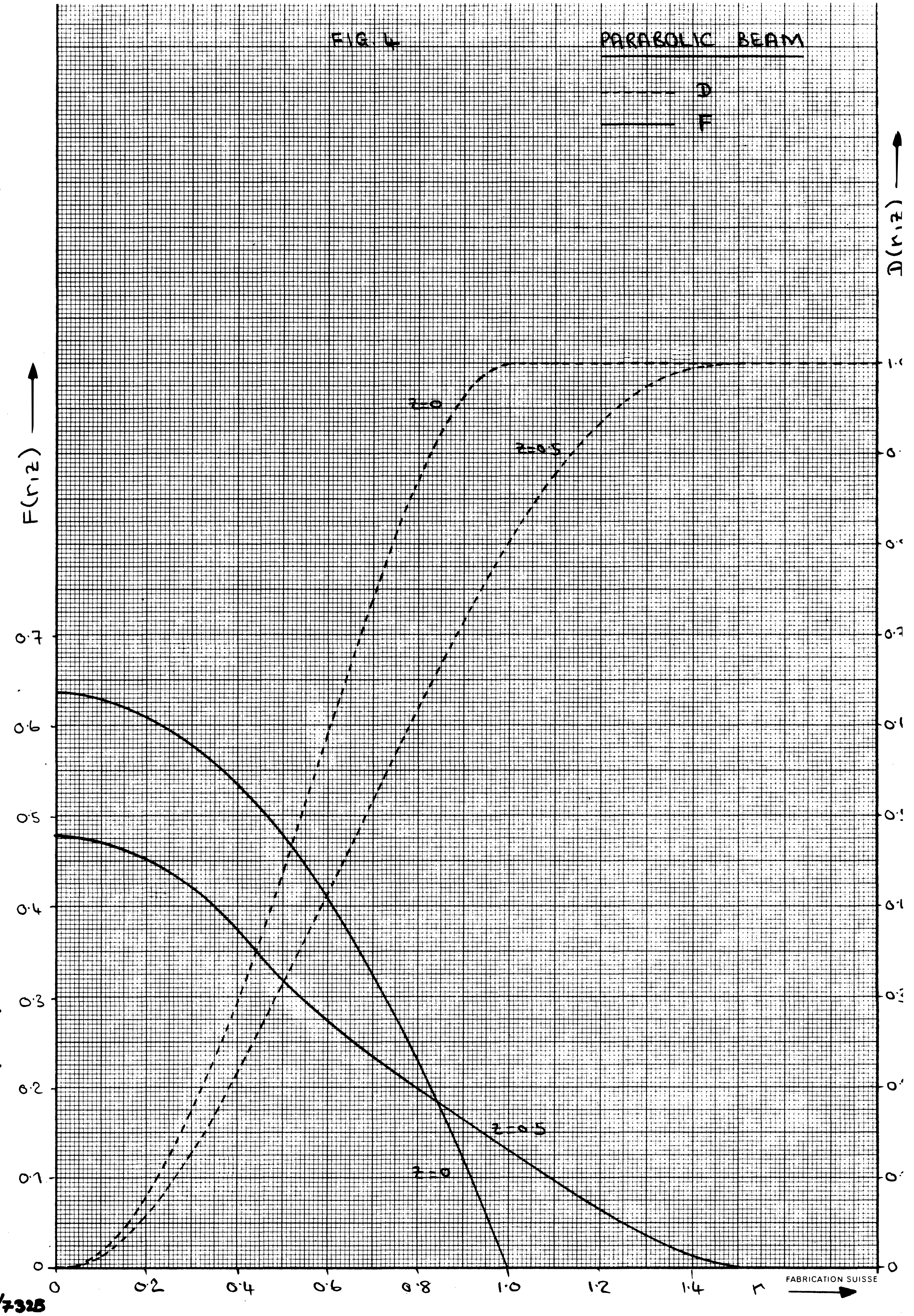
PARABOLIC BEAM
D
F
F(r,z)
D(r,z)
z=0
z=0.5
z=0.5
z=0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
r
FABRICATION SUISSE

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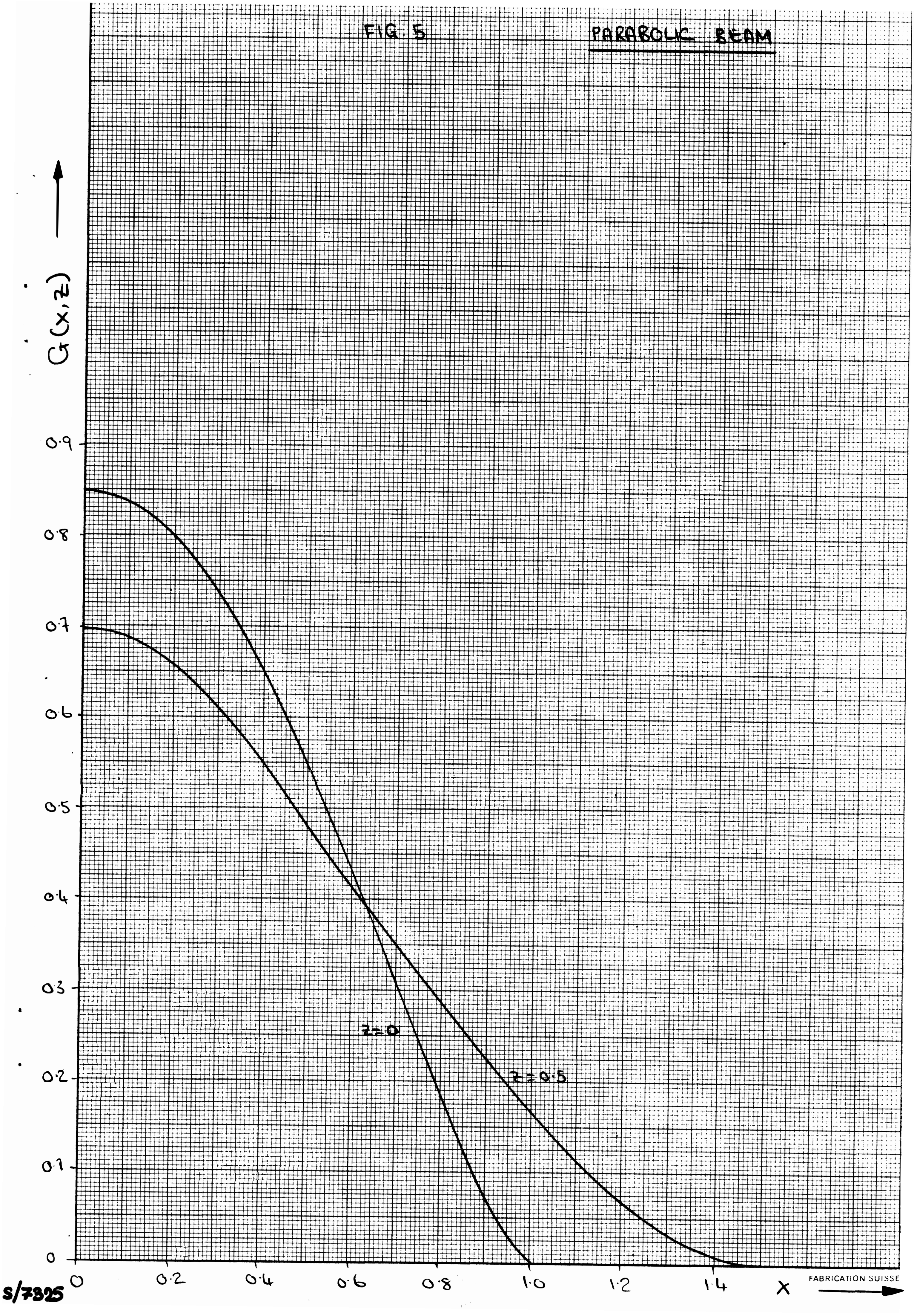
FIG 5

PARABOLIC BEAM



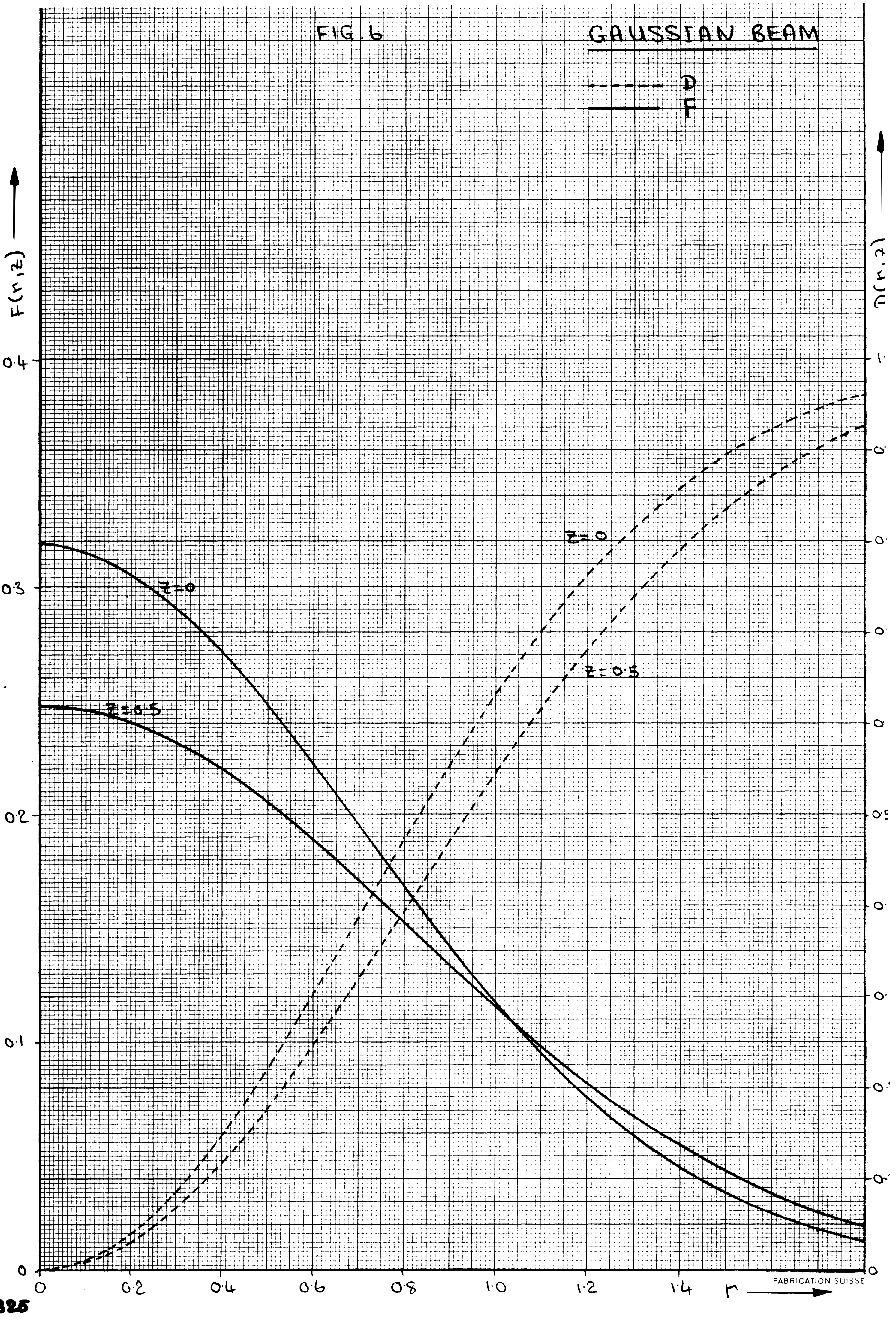


FIG. 6
GAUSSIAN BEAM
D
F
F(r,z)
D(r,z)
z=0
z=0.5
z=0
z=0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
r
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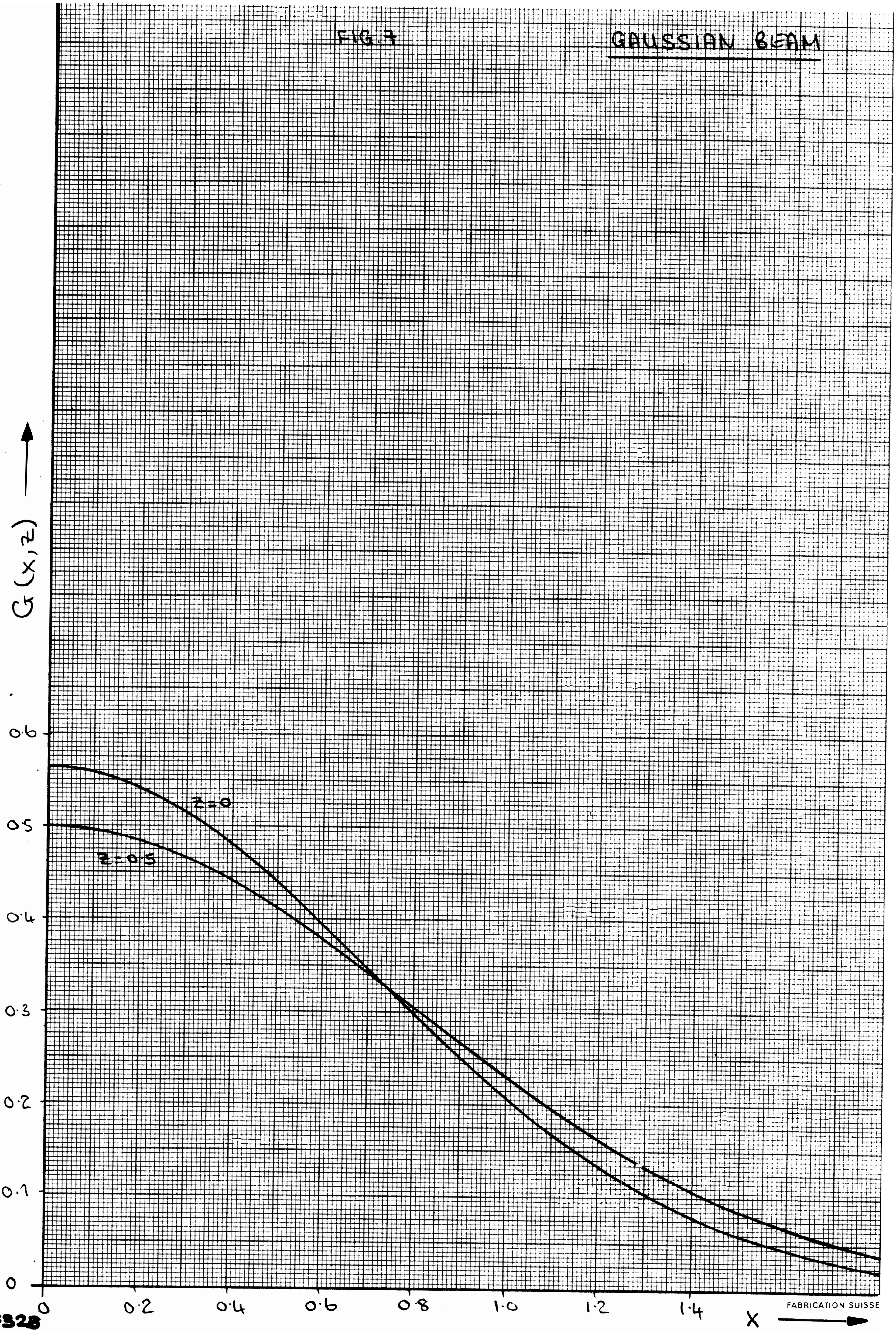
FIG. 7

GAUSSIAN BEAM



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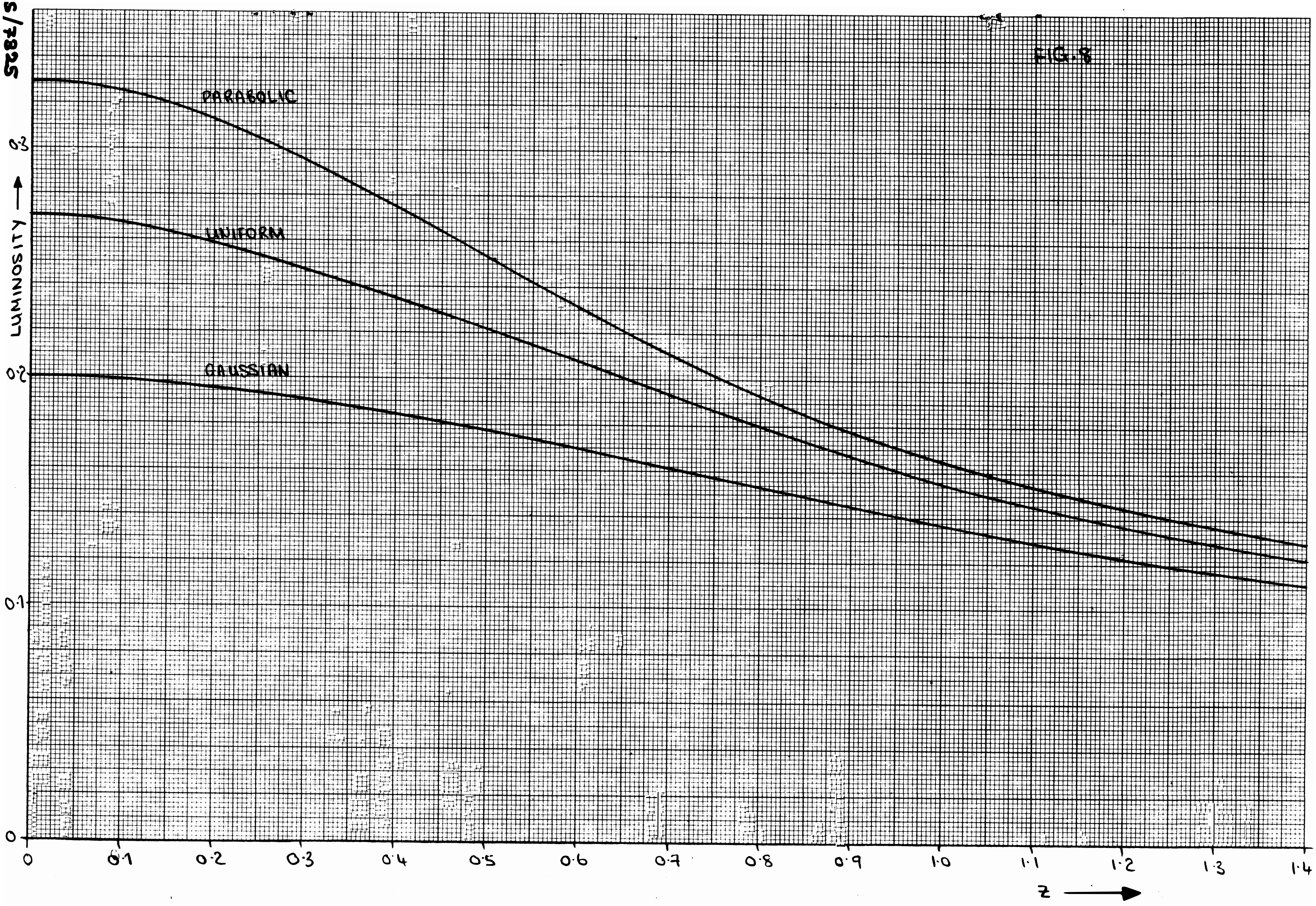
FABRICATION SUISSE

FIG. 8



PARABOLIC
UNIFORM
GAUSSIAN
LUMINOSITY →
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
1·1
1·2
1·3
1·4
Z →