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Target normal spin asymmetry and charge asymmetry for ep ($e\mu$) elastic scattering and the crossed processes

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Abstract

Two kinds of asymmetry arise from the interference of the Born amplitude and the box-type amplitude corresponding to two virtual photons exchange, namely charge-odd and the one spin asymmetries. In case of unpolarized particles the charge-odd correlation is calculated. It can be measured in combination of electron proton and positron proton scattering experiments. The forward-backward asymmetry is the corresponding quantity which can be measured for the crossed processes. In the case of polarized proton the one-spin asymmetry for annihilation and scattering channels has been calculated. The additional structure function arising from the interference is explicitly given. Arguments, based on analyticity, allow to prove that the effect of proton form factors nearly compensates the contribution of inelastic intermediate states. This justifies the structureless proton assumption. The uncertainty due to this assumption is discussed.

PACS numbers:

I. INTRODUCTION

The two photon exchange (TPE) amplitude of electron(positron)-hadron elastic scattering $e + h \to e + h$ as well as the crossed processes $e^+ + e^- \to h + \bar{h}$ and $h + \bar{h} \to e^+ + e^-$ are the simplest quantum processes. It is known [1] that the expansion on the number of closed loops of Feynman diagrams, describing the corresponding contribution to the amplitudes of these processes is equivalent to the expansion on Plank constant $\hbar$. The interference between the Born and the TPE amplitudes gives rise to the simplest quantum effects such as charge asymmetry and single spin asymmetry, which can not be expressed in terms of classical probabilities- the moduli of amplitudes squared. This motivates the study of these observables, as a source of information on the quantum structure of hadrons. Contrary to the case of ordinary box type Feynman diagram written in frame of QED for the interaction of two different leptons, the electron-hadron scattering amplitude at the second order of QED coupling, contains the tensor corresponding to the virtual photon Compton scattering on hadron. This quantity is object of both experimental and theoretical attentions [2].

This tensor can be written as the sum of two terms, when only strong interaction contributions to Compton amplitude are taken into account. One of them (the elastic term) is the generalization of the Born term with the strong-interaction form factors at the vertices of the interaction of the virtual photons with the hadron. We suppose that the hadron before and after the interaction with the photons is the same. Another term (the inelastic term) corresponds to inelastic channels such as pions and nucleons and similar hadronic states which can be excited in the intermediate state, within the vertices of the virtual photon interactions.

However, these strong-interaction contributions considerably compensate each other in such a way that the TPE amplitude can be calculated for the case of structureless hadron i.e. at the QED level switching out the strong interactions. To prove this statement, let us note that the box type amplitude can be written in the form

$$A \sim \int \frac{d^2 k_\perp ds_2}{q_1^2 q_2^2} L_{\mu\nu} \Delta_{s_2} H^{\mu\nu}, \tag{1}$$

where $L_{\mu\nu}$, and $H_{\mu\nu}$ are the Compton tensors of the lepton and the hadron, $q_{1,2}$ are the four-momenta of the virtual photons, $s_2 = (q_1 + p)^2$ is the invariant mass squared of the hadron intermediate state and Δ_s represents the s-channel discontinuity of hadron Compton amplitude [3]. In the physical sheet, the Compton amplitude has a pole, corresponding to a single hadron state in the intermediate state and two cuts: the right one, corresponding to the inelastic states in s-channel and the left one, starting at $s_2 < -9M^2$ (M is the hadron mass). Using the dispersion representation of the Compton amplitude it was shown (see [3]) that the relation holds

$$A_{left} + A_{Born} = A_{elastic} + A_{inelastic}, \qquad (2)$$

where A_{Born} is the Born amplitude with the strong interactions swithched off. Omitting the left cut contribution A_{left}, (our estimate shows that it can be included in 10% error bar [3]), we see that the effects of the hadron form factors compensate the contributions from the inelastic channels. In this approach we cannot take into account the intermediate states due to the resonances such as $\Delta(1232)$, as they corresponds to the singularities (poles) situated on the second sheet of the s_2 plane.

Up to now we do not have a similar proof of such a cancellation for the annihilation channels, but it looks natural, considering the dispersion representation in the crossed channels.

Keeping in mind these arguments we can omit the formfactor as well as the intermediate states with more then one hadron and apply the results of QED calculations for the box amplitude. As for the Born amplitude, it must be considered in the usual form containing the strong form factors.

Our paper is organized as follows. In sections II-VII we consider the charge asymmetry, resulting as a difference of cross sections of e^+p and e^-p quasielastic scattering and the forward-backward asymmetry of $p\bar{p}$ creation in e^+e^- annihilation and in the conjugated process, the annihilation $p\bar{+}p \to e^+ + e^-$. In the section VI we discuss the consequences of the crossing relations for these processes, expressed in terms of new structure functions. Section VIII is devoted to the calculation of proton spin asymmetry arising from interference of Born and TPE amplitudes for the scattering and annihilation processes. The results of numerical analysis and some comments are given in conclusion. Appendices detail specific steps of the calculation and give explicit expression for the relevant integrals.

II. CHARGE ODD BACKWARD-FORWARD ASYMMETRIES

The two photon exchange amplitude in elastic electron-proton scattering was widely discussed in the literature in the past [4]. The interest in its possible contribution to electron hadron scattering was recently renewed in connection with precise data obtained at Jefferson Laboratory (JLab), as a possible explanation to the discrepancy observed among different experiments. For example, the presence of two photon exchange would essentially modify the straightforward relation existing among the differential cross section in elastic electron hadron scattering and the electromagnetic hadron form factors [5]. It would induce an angular dependence, which could be detected as a deviation from the linearity in the Rosenbluth plot. This was investigated in [6], where the results of two experiments on electron deuteron (ed) elastic scattering [7, 8] were reanalyzed, in order to find an explanation of the discrepancy in the unpolarized elastic cross section, at the same momentum transfer squared, Q^2, but at different angles and initial energies.

Recently, measurements of the electromagnetic proton form factors using the polarization transfer method [9] where the polarization of the scattered proton in the elastic electron proton (ep) scattering with longitudinally polarized electrons became possible, and gave surprising results [10]. These works showed that the electric and charge distributions in the proton are different, and do not follow a dipole behavior as a function of Q^2, as indicated by the unpolarized cross section measurements [11]. Both types of experiments (with polarized and unpolarized particles) should bring the same basic information on the proton structure. Moreover, in case of unpolarized particles some information about Compton scattering on proton can be extracted. Usually the contribution of two-photon exchanges amplitude is expressed as an additional form factor of the nucleon, in such a way that the matrix element of elastic $ep \to ep$ scattering

$$e(p_1) + P(p) \to e(p_1') + P(p') \tag{3}$$

can be written in the form [12]:

$$M^{(2)} = \frac{i\alpha^2}{t}\bar{u}(p_1')\gamma_\mu u(p_1) \times \bar{u}(p')\Big[F_1(s,t)\gamma_\mu - \frac{F_2(s,t)}{2M}\gamma_\mu\hat{q} + \frac{1}{t}F_3(s,t)(\hat{p}_1+\hat{p}_1')(p+p')_\mu\Big]u(p),$$

with $q = p_1 - p_1'$, $t = q^2$, $s = 2p_1p$.

The quantities $F_{1,2,3}$-are functions of two Mandelstam variables s, t and can be considered as a generalization of the Dirac and Pauli form factors of the nucleon, which are functions of Q^2 only. The matrix structure F_3 appears only in multi photon exchange amplitudes. We do not consider here the electron spin-flip amplitudes associated with $\gamma_\mu\gamma_5$ [12], which are absent in the electron mass zero limit, considered below. The two-photon exchange mechanism can give an information on strong interaction properties of proton, such as polarizability. Virtual Compton scattering on proton is described by six chiral amplitudes, which can be reduced to three by assuming helicity conservation.

The motivation of this paper is to obtain all possible information following from an exact QED approach, and apply it for electron-hadron processes. The results obtained below can be used as a comparison, when extracting the effects of strong interactions in the relevant experimental data, obtained in *ep*-elastic scattering at high luminosity facilities. Similarly, the results on the annihilation channel can be used as a reference for similar experiments at $c-\tau$ factories. The present results hold for energies in the GeV range, small when compared to Z, W boson masses.

At first, we consider the process of creation $\mu^+\mu^-$ pairs in electron-positron annihilation:

$$e^+(p_+) + e^-(p_-) \to \mu^+(q_+) + \mu^-(q_-). \tag{4}$$

The cross-section in the Born approximation, can be written as:

$$\frac{d\sigma_B}{dO_{\mu_-}} = \frac{\alpha^2}{4s}\beta(2-\beta^2+\beta^2c^2), \tag{5}$$

with $s = (p_+ + p_-)^2 = 4E^2$, $\beta^2 = 1 - \frac{4m^2}{s}$, E-the electron beam energy in center of mass reference frame (implied for this process below), m, m_e-are the masses of muon and electron, $c = \cos\theta$, θ is the angle of μ_--meson emission to the electron beam direction.

The interference of the Born amplitude

$$M_B = \frac{i4\pi\alpha}{s}\bar{v}(p_+)\gamma_\mu u(p_-)\bar{u}(q_-)\gamma_\mu v(q_+),$$

with the box-type M_B one results in parity violating contributions to the differential cross section, i. e. the ones, changing the sign at $\theta \to \pi - \theta$, and as a consequence of charge-odd correlation we can construct:

$$A(\theta, \Delta E) = \frac{\mathrm{d}\sigma(\theta) - \mathrm{d}\sigma(\pi-\theta)}{\mathrm{d}\sigma_B(\theta)}. \tag{6}$$

Here we take into account as well the emission of an additional soft real photon with energy not exceeding some small value ΔE, so that $A(\theta, \Delta E)$ is free from the infrared singularities.

III. PROCESS $e^+ + e^- \to \mu^+ + \mu^-(\gamma)$

Part of the results presented here were previously derived in a paper with one of us (E. A. K.) in Ref. [13], and partially published in [14].

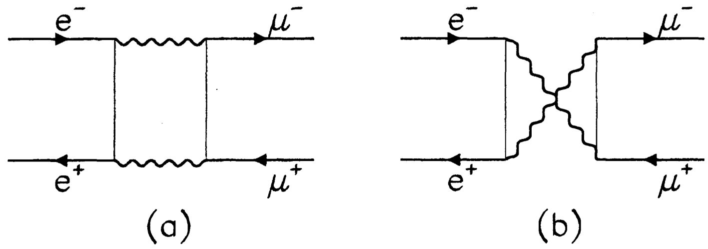

FIG. 1: Feynman diagrams for two-photon exchange in elastic *ep* scattering: box diagram (a) and crossed box diagram (b).

There are two box-type Feynman amplitudes (Fig.1). We calculate only one of them, the uncrossed diagram (Fig.1a) with matrix element

$$M_a = i\alpha^2 \int \frac{\mathrm{d}^4 k}{i\pi^2} \frac{\bar{u}(q_-) T v(q_+) \times \bar{v}(p_+) Z u(p_-)}{(\Delta)(Q)(P_+)(P_-)},$$
$$(\Delta) = (k-\Delta)^2 - m_e^2, \quad (Q) = (k-Q)^2 - m^2, \quad (P_\pm) = (k \mp P)^2 - \lambda^2, \tag{7}$$

with λ-"photon" mass and

$$T = \gamma_\alpha(\hat{k} - \hat{Q} + m)\gamma_\beta, \quad Z = \gamma_\beta(\hat{k} - \hat{\Delta})\gamma_\alpha,$$
$$\Delta = \frac{1}{2}(p_+ - p_-), \quad Q = \frac{1}{2}(q_+ - q_-), \quad P = \frac{1}{2}(p_+ + p_-). \tag{8}$$

We will systematically neglect the contributions of the type

$$\frac{m_e^2}{m^2}, \frac{m^2}{M_Z^2} \ll 1, \tag{9}$$

and we assume

$$m^2 = \frac{s}{4}(1-\beta^2) \sim s \sim -t \sim -u. \tag{10}$$

The explicit form of kinematical variables used below is:

$$\Delta^2 = -P^2 = -\frac{s}{4}, \quad Q^2 = -\frac{1}{4}s\beta^2, \quad \sigma = \Delta Q = \frac{1}{4}(u-t),$$
$$u = (p_- - q_+)^2 = -\frac{s}{4}(1+\beta^2+2\beta c), \quad t = (p_- - q_-)^2 = -\frac{s}{4}(1+\beta^2-2\beta c). \tag{11}$$

The contribution to the cross section of the amplitude arising from the crossed Feynman diagram (Fig.1b), M_b, can be obtained from M_a by the crossing relation

$$\frac{\mathrm{d}\sigma_a(s,t)}{\mathrm{d}\Omega_\mu} = -\frac{\mathrm{d}\sigma_b(s,u)}{\mathrm{d}\Omega_\mu}, \tag{12}$$

which has the form

$$\frac{\mathrm{d}\sigma_a(s,t)}{\mathrm{d}\Omega_\mu} = \frac{\beta\alpha^3}{2\pi s^2}\mathrm{Re}[R(s,t)], \tag{13}$$

with

$$R(s,t) = \int \frac{\mathrm{d}^4k}{i\pi^2}\frac{1}{(\Delta)(Q)(P_+)(P_-)}\frac{1}{4}Tr\left((\hat{q}_- + m)T(\hat{q}_+ - m)\gamma_\mu\right) \times \frac{1}{4}Tr\left(\hat{p}_+ Z\hat{p}_-\gamma_\mu\right). \tag{14}$$

The scalar, vector and tensor loop momentum integrals are defined as:

$$J; J_\mu; J_{\mu\nu} = \int \frac{\mathrm{d}^4k}{i\pi^2}\frac{1; k_\mu; k_\mu k_\nu}{(\Delta)(Q)(P_+)(P_-)} \tag{15}$$

Using symmetry properties, the vector and tensor integrals can be written as:

$$J_\mu = J_\Delta \cdot \Delta_\mu + J_Q \cdot Q_\mu, \tag{16}$$

$$J_{\mu\nu} = K_0 g_{\mu\nu} + K_P P_\mu P_\nu + K_Q Q^\mu Q^\nu + K_\Delta \Delta_\mu \Delta_\nu + K_x(Q_\mu\Delta_\nu + Q_\nu\Delta_\mu). \tag{17}$$

The quantity $R(s,t)$ can be expressed as a function of polynomials P_i as:

$$R = P_1 J + P_2 J_\Delta + P_3 J_Q + P_4 K_0 + P_5 K_\Delta + P_6 K_Q + P_7 K_P + P_8 K_x, \tag{18}$$

where the explicit form of polynomials is given in Appendix A. Using the explicit expression for the coefficients $J_\Delta, ..., K_x$ (See Appendix B) we obtain

$$\begin{aligned} R(s,t) &= 4(\sigma-\Delta^2)(2\sigma-m^2)F + 16(\sigma-\Delta^2)(\sigma^2+(\Delta^2)^2-m^2\Delta^2)J \\ &+4[(\Delta^2)^2 - 3\Delta^2\sigma + 2\sigma^2 - m^2\sigma]F_Q + 4[2(\Delta^2)^2 - 2\Delta^2\sigma + 2\sigma^2 - m^2\Delta^2]F_\Delta \\ &+4[(\Delta^2)^2 + \Delta^2\sigma + m^2\Delta^2]G_Q + 4[-(\Delta^2)^2 + \sigma^2 - 2m^2\Delta^2]H_Q, \end{aligned} \tag{19}$$

with the quantities $F \div H_Q$ given in Appendix B. Finally the charge-odd part of differential cross section has the form

$$\begin{aligned} \left(\frac{\mathrm{d}\sigma^{e\bar{e}}_{virt}(s,t)}{\mathrm{d}\Omega_\mu}\right)_{odd} &= -\frac{\alpha^3\beta}{2\pi s}\mathcal{D}^{ann}, \\ \mathcal{D}^{ann} &= \frac{1}{s}[R(s,t) - R(s,u)] = (2-\beta^2+\beta^2c^2)\ln\left(\frac{1+\beta c}{1-\beta c}\right)\ln\frac{s}{\lambda^2} + \mathcal{D}^{ann}_V \end{aligned} \tag{20}$$

$$\begin{aligned} \mathcal{D}^{ann}_V &= \left(1-2\beta^2+\beta^2c^2\right)\left[\frac{1}{1+\beta^2+2\beta c}\left(\ln\frac{1+\beta c}{2} + \ln\frac{s}{m^2}\right)\right. \\ &- \left.\frac{1}{1+\beta^2-2\beta c}\left(\ln\frac{1-\beta c}{2} + \ln\frac{s}{m^2}\right)\right] \end{aligned}$$

$$
\begin{aligned}
&+ \beta c \left[\phi(\beta) \left(\frac{1}{2\beta^2} - 1 - \frac{\beta^2}{2} \right) - \frac{1}{\beta^2} \ln \frac{s}{m^2} - \frac{\pi^2}{6} + \frac{1}{2} \ln^2 \frac{s}{m^2} \right. \\
&- \left. \frac{1}{2} \ln^2 \frac{1-\beta c}{2} - \frac{1}{2} \ln^2 \frac{1+\beta c}{2} + \mathrm{Li}_2 \left(\frac{1+\beta^2+2\beta c}{2(1+\beta c)} \right) + \mathrm{Li}_2 \left(\frac{1+\beta^2-2\beta c}{2(1-\beta c)} \right) \right] \\
&- \frac{m^2}{s} \left[\ln^2 \frac{1-\beta c}{2} - \ln^2 \frac{1+\beta c}{2} + 2\mathrm{Li}_2 \left(\frac{1+\beta^2+2\beta c}{2(1+\beta c)} \right) - 2\mathrm{Li}_2 \left(\frac{1+\beta^2-2\beta c}{2(1-\beta c)} \right) \right],
\end{aligned}
$$

where $\phi(\beta) = sF_Q$, F_Q is given in Appendix B and

$$
\mathrm{Li}_2\,(z) = -\int_0^z \frac{\mathrm{d}x}{x} \ln(1-x) \tag{21}
$$

is the Spence function. The quantity $\mathcal{D}^{ann} - \mathcal{D}_V^{ann}$ suffer from infrared divergences, which will be compensated taking into account the soft photons contribution (see below).

IV. SCATTERING CHANNEL

Let us consider now the elastic electron muon scattering $e + \mu \to e + \mu$ which is the crossed process of (4). The Born cross section is the same for the scattering of electrons and positrons on the same target. Taking the experimental data from the scattering of electron and positron on the same target (muon or proton), one can measure the difference of the corresponding cross-sections which is sensitive to the interference of the one and two photon exchange amplitudes. For the case of proton target, in the Laboratory (Lab) frame, the differential cross section as a function of the energy of the initial electron, E and of the electron scattering angle, θ_e, was derived in Ref. [?]:

$$
\begin{aligned}
\frac{d\sigma^{ep}}{d\Omega} &= \frac{\alpha^2 \cos^2 \frac{\theta_e}{2}}{4E^2 \sin^4 \frac{\theta_e}{2}} \frac{1}{\rho} \left[\frac{F_e^2 + \tau F_m^2}{1+\tau} + 2\tau F_m^2 \tan^2 \frac{\theta_e}{2} \right], \\
\rho &= 1 + \frac{2E}{m} \sin^2 \frac{\theta_e}{2}, \quad \tau = \frac{-t}{4m^2} = \frac{E^2}{m^2 \rho} \sin^2 \frac{\theta_e}{2},
\end{aligned} \tag{22}
$$

and it is known as the Rosenbluth formula. The Sachs electric and magnetic proton form factors, F_e and F_m are related to the Pauli and Dirac form factors by $F_e = F_1 - \tau F_2, F_m = F_1 + F_2$. For the scattering on muon, one replaces $F_1 = 1, F_2 = 0$ and Eq. (22) becomes

$$
\frac{d\sigma_B^{e\mu}}{d\Omega} = \frac{\alpha^2 (s^2 + u^2 + 2tm^2)}{2m^2 \rho^2 t^2}, \quad s = 2p_1 p = 2mE, \quad t = -2p_1 p_1', \quad u = -2pp_1' = -\frac{s}{\rho}. \tag{23}
$$

The charge-odd contribution to the cross section of $e\mu-$elastic scattering is:

$$
\begin{aligned}
\left(\frac{d\sigma_{virt}^{e\mu}}{d\Omega_e} \right)_{odd} &= -\frac{\alpha^3}{2\pi m^2 \rho^2} Re\,(\mathcal{D}^{sc}), \\
\mathcal{D}^{sc} &= \frac{1}{t}[\mathcal{D}(s,t) - \mathcal{D}(u,t)] = \frac{2}{t^2}[s^2 + u^2 + 2tm^2] \ln \frac{-u}{s} \ln \frac{-t}{\lambda^2} + \mathcal{D}_{virt}^{sc}
\end{aligned} \tag{24}
$$

with

$$\mathcal{D}^{sc}_{virt} = \frac{s-u}{t}\left[\frac{1}{2}\ln^2\left(\frac{-t}{m^2}\right) - \frac{\tau}{1+\tau}\ln\left(\frac{-t}{m^2}\right) + m^2\bar{F}_Q\left(6\tau + 2 - \frac{2\tau^2}{1+\tau}\right)\right]$$
$$+ \frac{s}{t}\left[-\ln^2\frac{s}{-t} + \pi^2 + 2\mathrm{Li}_2\left(1+\frac{m^2}{s}\right)\right] - \frac{u}{t}\left[-\ln^2\frac{u}{t} + 2\mathrm{Li}_2\left(1+\frac{m^2}{u}\right)\right]$$
$$+ \frac{(1-2\tau)}{(-4\tau)}\left[2\ln\left(\frac{s}{-u}\right)\ln\left(\frac{-t}{m^2}\right) + \ln^2\left(\frac{-u}{m^2}\right) - \ln^2\left(\frac{s}{m^2}\right) + \pi^2\right.$$
$$\left. + 2\mathrm{Li}_2\left(1+\frac{m^2}{s}\right) - 2\mathrm{Li}_2\left(1+\frac{m^2}{u}\right)\right] + \left(2m^2 - \frac{su}{t}\right)\left[\frac{\ln\frac{s}{m^2}}{m^2+s} - \frac{\ln\frac{-u}{m^2}}{m^2+u}\right] \quad (25)$$

with the help of the following relation:

$$m^2\bar{F}_Q = -\frac{1}{4\sqrt{\tau(1+\tau)}}\left[\pi^2 + \ln(4\tau)\ln x + \mathrm{Li}_2\left(-2\sqrt{\tau x}\right) - \mathrm{Li}_2\left(\frac{2\sqrt{\tau}}{\sqrt{x}}\right)\right], \quad (26)$$

where

$$x = \frac{\sqrt{1+\tau}+\sqrt{\tau}}{\sqrt{1+\tau}-\sqrt{\tau}}.$$

V. SOFT PHOTON EMISSION

In this section the emission of soft real photons in the Lab reference frame for $e\mu$- scattering is calculated. Following Ref. [15], the odd part of cross section

$$\frac{d\sigma^{soft}}{d\sigma_0} = -\frac{\alpha}{4\pi^2}\cdot 2\int\frac{d^3k}{\omega}\left(\frac{p_1'}{p_1'k} - \frac{p_1}{p_1k}\right)\left(\frac{p'}{p'k} - \frac{p}{pk}\right)_{S_0,\ \omega<\Delta\varepsilon} \quad (27)$$

must be calculated in the special reference frame S_0, where the sum of the three-momenta of proton and of the recoil proton is zero $\vec{z} = \vec{k} + \vec{p}' = 0$. Really, in this frame, the on-mass shell condition of the scattered muon $\delta((z-k)^2 - m^2)$, $z = p_1 + p - p_1'$ does not depend on the direction of the emitted photon. The photon energy can be determined as the difference of the energy of the scattered electron and the corresponding value for the elastic case: the maximum value of the photon energy $\Delta\varepsilon$ in the S_0 frame is related with the energy of the scattered electron, detected in the Lab frame ΔE as (see [15], Appendix C),

$$\Delta\varepsilon = \rho\Delta E. \quad (28)$$

The calculation of the soft photon integral with $\omega\Delta\varepsilon$ can be performed using t'Hooft and M. Veltman approach (see [16], Section 7). We find

$$\frac{d\sigma^{soft}_{e\mu}}{d\Omega} = -\frac{\alpha^3}{\pi}\frac{(s^2+u^2+2tm^2)}{2m^2\rho^2t^2}\left\{2\ln\rho\ln\left[\frac{(2\rho\Delta E)^2}{\lambda^2 x}\right] + \mathcal{D}^{sc}_{soft}\right\},$$
$$\mathcal{D}^{sc}_{soft} = -2\mathrm{Li}_2\left(1-\frac{1}{\rho x}\right) + 2\mathrm{Li}_2\left(1-\frac{\rho}{x}\right),\ \rho = \frac{s}{-u}, \quad (29)$$

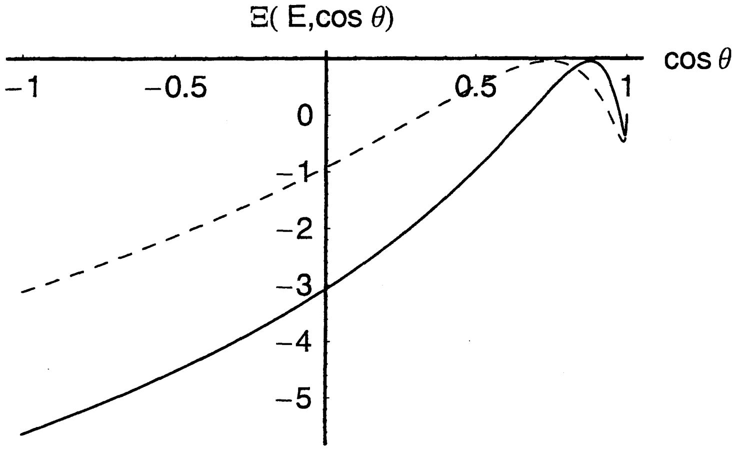

FIG. 2: $\Xi(s, \cos\theta)$ for $E = 5m$ (dashed line) and $E = 10m$, m is muon mass.

which is in agreement with Ref. [15].

The sum $(\mathrm{d}\sigma_{e\mu}^{virt} + \mathrm{d}\sigma_{e\mu}^{soft})_{odd}$ has the form

$$\begin{aligned} \left(\frac{\mathrm{d}\sigma_{e\mu}^{virt}}{\mathrm{d}\Omega_e} + \frac{\mathrm{d}\sigma_{e\mu}^{soft}}{\mathrm{d}\Omega_e}\right)_{odd} &= \frac{\alpha^3}{2\pi m^2\rho^2}\frac{(s^2+u^2+2tm^2)}{t^2}\left[-2\ln\rho\ln\frac{(2\rho\Delta E)^2}{-tx} + \Xi\right], \\ \Xi &= \Re\left[-\frac{t^2\mathcal{D}_{virt}^{sc}}{s^2+u^2+2tm^2} - \mathcal{D}_{soft}^{sc}\right]. \end{aligned} \tag{30}$$

and it is independent from the photon mass λ.

The function Ξ is shown in Fig. 2 as a function of $\cos\theta_e$ for given E/m . This result can be applied to the process $e\bar{e} \to p\bar{p}(\gamma)$, for the case of structureless proton, by changing the mass $m \to m_p$ in Eq. (30).

The ratio Δ between the difference and the sum (corresponding to the Born cross section) of the cross sections for $e^{\pm}p$ scattering is:

$$\Delta = \frac{\mathrm{d}\sigma^{e^-p\to e^-p(\gamma)} - \mathrm{d}\sigma^{e^+p\to e^+p(\gamma)}}{\mathrm{d}\sigma^{e^+p\to e^+p(\gamma)} + \mathrm{d}\sigma^{e^-p\to e^-p(\gamma)}} = \frac{\alpha}{\pi}\left[\Xi - 2\ln\rho\ln\frac{(2\rho\Delta E)^2}{-tx}\right]. \tag{31}$$

The odd contributions to the differential cross section for the process $e^+ + e^- \to p + \bar{p}$, due to soft photon emission, has the form:

$$(\mathrm{d}\sigma_{soft}^{e^+e^-\to\mu^+\mu^-(\gamma)})_{odd} = \mathrm{d}\sigma_0\left(-\frac{\alpha}{4\pi^2}\right)2\int\frac{\mathrm{d}^3k}{\omega}\left(-\frac{p_-}{p_-k} + \frac{p_+}{p_+k}\right)\left(\frac{q_+}{q_+k} - \frac{q_-}{q_-k}\right)_{S_0,\omega<\Delta\varepsilon}. \tag{32}$$

Again, the integration must be performed in the special frame S^0, where $\bar{p}_+ + \bar{p}_- - \bar{q}_+ = \bar{q}_- + \bar{k} = 0$. In this frame we have

$$(q_- + k)^2 - m^2 = 2(E_- + \omega)\omega \approx 2m\omega = (p_+ + p_- - q_+)^2 - m^2 = 4E(E - \varepsilon_+),$$

$$E - \varepsilon_+ = \frac{m}{2E}\Delta\varepsilon. \tag{33}$$

In the elastic case $E - \varepsilon_+^{el} = 0$ and the photon energy in the Lab system is

$$\Delta E = \varepsilon_+^{el} - \varepsilon_+ = \frac{m}{2E}\Delta\varepsilon. \tag{34}$$

The t'Hooft Veltman procedure for soft photon emission contribution leads to:

$$\frac{d\sigma_{ann}^{soft}}{d\Omega} = \frac{d\sigma_0}{d\Omega}.\frac{2\alpha}{\pi}\left[\ln\left(\frac{4E\Delta E}{m\lambda}\right)^2 \ln\frac{1+\beta c}{1-\beta c} + \mathcal{D}_S^{ann}\right] \tag{35}$$

with

$$\begin{aligned}\mathcal{D}_s^{ann} &= \frac{1}{2}\mathrm{Li}_2\left(\frac{-2\beta(1+c)}{(1-\beta)(1-\beta c)}\right) + \frac{1}{2}\mathrm{Li}_2\left(\frac{2\beta(1-c)}{(1+\beta)(1-\beta c)}\right)\\ &-\frac{1}{2}\mathrm{Li}_2\left(\frac{-2\beta(1-c)}{(1-\beta)(1+\beta c)}\right) - \frac{1}{2}\mathrm{Li}_2\left(\frac{2\beta(1+c)}{(1+\beta)(1+\beta c)}\right).\end{aligned} \tag{36}$$

The total contribution (virtual and soft) is free from infrared singularities and has the form

$$\begin{aligned}\frac{d\sigma_{ann}}{d\Omega} &= \frac{\alpha^3\beta}{2\pi s}(2-\beta^2+\beta^2c^2)\Upsilon, \quad \Upsilon = 2\ln\frac{1+\beta c}{1-\beta c}\ln\left(\frac{2\Delta E}{m}\right) + \Phi(s,\cos\theta),\\ \Phi(s,\cos\theta) &= \mathcal{D}_S^{ann} - \frac{\mathcal{D}_V^{ann}}{2-\beta^2+\beta^2c^2}.\end{aligned} \tag{37}$$

The quantity $\Phi(s,\cos\theta)$ is presented in Fig. 3.

The relevant asymmetry can be constructed from (6)

$$A = \frac{4\alpha}{\pi}\Upsilon. \tag{38}$$

VI. CROSSING SYMMETRY

In this section we formally consider the relations between the kinematical variables in the scattering and in the annihilation channel, $e^+ + e^- \to p + \bar{p}$. The reduced form of the differential elastic ep scattering cross section, commonly used, is defined as $\sigma_{red} = \tau F_m^2 + \varepsilon F_e^2$ and it is related to the differential cross section by:

$$\frac{d\sigma}{d\Omega} = \sigma_M\sigma_{red},\ \sigma_M = \frac{\alpha^2\cos^2\frac{\theta_e}{2}}{4E^2\sin^4\frac{\theta}{2}}\frac{1}{\rho\varepsilon(1+\tau)},\ \varepsilon = \frac{1}{1+2(1+\tau)\tan^2\frac{\theta_e}{2}}. \tag{39}$$

where ε is the polarization of the virtual photon, and varies from $\varepsilon = 0$, for $\theta_e = \pi$ to $\varepsilon = 1$, for $\theta_e = 0$.

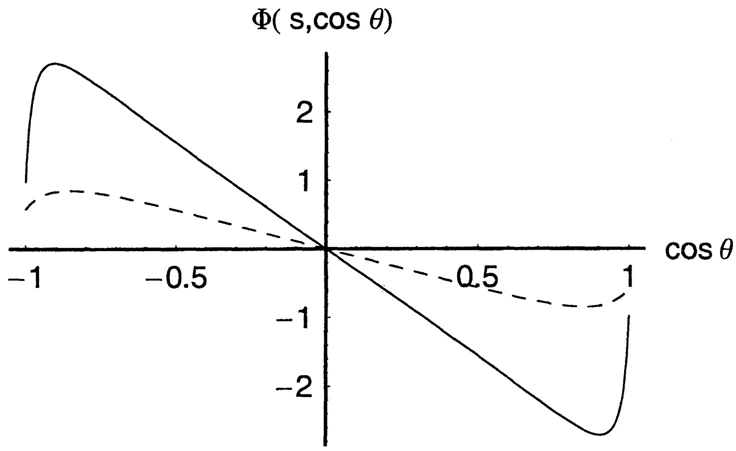

FIG. 3: $\Phi(s,\cos\theta)$, for $s=10m^2$ (dashed line) and $s=20m^2$, m is muon mass.

The crossing relation between the scattering channel $e+p\to e+p$ and the annihilation channel $e^+ + e^- \to p+\bar p$ consists in replacing the variables of the scattering channel $s = 2p_1p = 2Em$ and $Q^2=-t$ according to

$$m^2+s\to t=-2E^2(1-\beta c),\ Q^2\to -s=-4E^2,\ cos\theta=\cos\widehat{\vec p_-\vec q_-}, \tag{40}$$

where θ is the angle of the antiproton with respect to the incident electron, in the center of mass system (CMS).

The following relation holds, for the annihilation channel:

$$\cos^2\theta=\frac{(t-u)^2}{s(s-4M^2)},\ s+t+u=2m^2. \tag{41}$$

On the other hand, in the scattering channel, one has:

$$\frac{1+\varepsilon}{1-\varepsilon}=\frac{\cot^2\frac{\theta_e}{2}}{1+\tau}+1=\frac{(s-u)^2}{Q^2(Q^2+4M^2)},\ Q^2=s+u. \tag{42}$$

Therefore one proves the validity of the crossing relation:

$$\cos\theta=\sqrt{\frac{1+\varepsilon}{1-\varepsilon}}\equiv y, \tag{43}$$

based on the analytical continuation from the annihilation channel to the scattering one. This relation was derived in Refs. [6, 17]. Using this relation and the property of the 2γ contribution to the annihilation cross-section $\left(\frac{d\sigma}{d\Omega}(\theta)\right)_{2\gamma}=-\left(\frac{d\sigma}{d\Omega}(\pi-\theta)\right)_{2\gamma}$ i.e. $\left(\frac{d\sigma}{d\Omega}(\theta)\right)_{2\gamma}=\cos\theta f(\cos^2\theta,s)$ the authors of Ref. [17] built the ansatz for 2γ contribution to ep-elastic scattering

$$\frac{d\Delta\sigma}{d\Omega_e}(e^-p\to e^-p) = yf(y^2,Q^2);\;\; f(y^2,Q^2) = c_0(Q^2)+y^2c_1(Q^2)+y^4c_2(Q^2)+\dots \quad (44)$$

This property follows from the change of the sign of the contribution for virtual and real photon emission when the ($s \leftrightarrow u$) transformation is applied (see Eqs. (25,29,30) and relation (42)).

This form of the contribution of the $1\gamma\otimes 2\gamma$ interference to the differential cross section derives explicitly from C-invariance and crossing symmetry of electromagnetic interactions and excludes any linear function of ε for a possible parameterization of such contribution.

Let us note that not only the elastic channel must be taken into account: the interference of the amplitudes corresponding to the emission of a photon by electron and by proton must be considered, too.

Evidently, the relations derived above are valid for the considered processes with electrons and muons.

VII. DERIVATION OF THE ADDITIONAL STRUCTURE: ANNIHILATION CHANNEL

Let us start from the following form of the matrix element for the process $e^+(p_+)+e^-(p_-)\to\mu^+(q_+)+\mu^-(q_-)$ in presence of 2γ exchange:

$$M_2 = \frac{i\alpha^2}{s}\bar v(p_+)\gamma_\mu u(p_-)\times \bar u(q_-)\left(G_1\gamma_\mu - \frac{G_2}{m}\gamma_\mu\hat P + 4\frac{1}{s}G_3\hat\Delta Q_\mu\right)v(q_+), \quad (45)$$

where the amplitudes G_1-G_3 are complex functions of the two kinematical variables s, and t.

To calculate the structure G_3 from the 2γ amplitude (see Eq. (7)), both Feynman diagrams, (Figs. 1)a and 1)b must be taken into account. Similarly to Section II, only one of them can be calculated explicitly (the uncrossed one), whereas the other can be obtainedfrom this one by appropriate replacements.

To extract the structure G_3 we multiply Eq. 45 subsequently by

$$\begin{aligned}&\bar u(p_-)\gamma_\lambda v(p_+)\times\bar v(q_+)\gamma_\lambda u(q_-),\\ &\bar u(p_-)\hat Q v(p_+)\times\bar v(q_+)u(q_-),\\ &\bar u(p_-)\hat Q v(p_+)\times\bar v(q_+)\hat\Delta u(q_-),\end{aligned} \quad (46)$$

and perform the summation on fermions spin states.

Solving the algebraical set of equations we find

$$
\begin{aligned}
G_1^a &= \frac{1}{\beta^4 \sin^4\theta}\left\{(8B^a + A^a\beta^2\sin^2\theta)(1-\beta^2\cos^2\theta) - 4C^a\beta\cos\theta\left[2-\beta^2(1+\cos^2\theta)\right]\right\},\\
G_2^a &= \frac{1}{\beta^4 \sin^4\theta}\left\{\beta(1-\beta^2)(A^a\beta\sin^2\theta - 8C^a\cos\theta) + 4B^a\left[2-\beta^2(1+\cos^2\theta)\right]\right\} \qquad (47)\\
G_3^a &= \frac{1}{\beta^3 \sin^4\theta}\left[-A^a\beta^2\sin^2\theta\cos\theta - 8B^a\cos\theta + 4\beta C^a(1+\cos^2\theta)\right],
\end{aligned}
$$

with

$$
\begin{aligned}
A^a &= \int \frac{d^4k}{i\pi^2}\frac{1}{(\Delta)(Q)(P_+)(P_-)}\frac{1}{s}Tr(\hat{p}_+ Z\hat{p}_-\gamma_\lambda) \times \frac{1}{4}Tr\left[(q_- + m)T(\hat{q}_+ - m)\gamma_\lambda\right],\\
B^a &= \int \frac{d^4k}{i\pi^2}\frac{1}{(\Delta)(Q)(P_+)(P_-)}\frac{m}{s^2}Tr(\hat{p}_+ Z\hat{p}_-\hat{Q}) \times \frac{1}{4}Tr\left[(\hat{q}_- + m)T(\hat{q}_+ - m)\right], \qquad (48)\\
C^a &= \int \frac{d^4k}{i\pi^2}\frac{1}{(\Delta)(Q)(P_+)(P_-)}\frac{1}{s^2}Tr(\hat{p}_+ Z\hat{p}_-\hat{Q}) \times \frac{1}{4}Tr\left[(\hat{q}_- + m)T(\hat{q}_+ - m)\hat{\Delta}\right].
\end{aligned}
$$

The explicit value for G_3^a is:

$$
\begin{aligned}
G_3^a &= \frac{2s}{\beta^3(1-c^2)^2}\left\{\frac{1}{2}G_Q(1-c^2)\beta^3(1-\beta c)\right.\\
&+ \frac{1}{2}H_Q\beta^2(1-c^2)\left[c(-3+5\beta^2) - \beta - \beta c^2\right]\\
&+ F_\Delta c\left[1 - 4\beta^2 + 2\beta^4 + c^2\beta^2(3-4\beta^2) - 2\beta c(1-2\beta^2)\right] \qquad (49)\\
&+ F_Q\beta\left[-c^2 + \beta c\left(-\frac{1}{2} - 4\beta^2c^2 + \frac{5}{2}c^2\right) + \beta^2\left(-\frac{1}{2} + 2\beta^2c^2 + \frac{3}{2}c^2\right)\right]\\
&- 2Js\beta^2c(1-c^2)(1-\beta^2)(1-\beta c)\\
&+ \left. Fc\left[1 + \beta^2c^2 - 2\beta^4 - 4\beta^4c^2 + \beta c(-3 + 4\beta^2 + 2\beta^4 + \beta^2c^2)\right]\right\}.
\end{aligned}
$$

The contributions from the crossed Feynman diagram can be obtained from Eqs. 49 by:

$$
(A^b, B^b, C^b)_{crossed} = -[A^a, B^a, C^a(\cos\theta \to -\cos\theta)]_{uncrossed}. \qquad (50)
$$

As one can see, in the quantities G_1, G_2, and G_3 infrared divergencies are present.

VIII. PROTON SPIN ASYMMETRY

The targed spin asymmetry for processes $e^+ + e^- \to p + \bar{p}$, $p + \bar{p} \to e^+ + e^-$ (in CMS frame) is defined as

$$
\frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = (\vec{a}\vec{n})R_n, \qquad (51)
$$

where $\vec{a}$ is the proton polarization vector, $\vec{n} = (\vec{q}_- \times \vec{p}_-) / |\vec{q}_- \times \vec{p}_-|$ is the unit vector normal to the scattering plane, $d\sigma^\uparrow$ is the cross section of processes with proton polarization vector $\vec{a}$, $d\sigma^\downarrow$ is the cross section of processes with proton polarization vector $-\vec{a}$. Thus the denominator of the left hand side in Eq. (51) is the unpolarized cross section of process $e^+e^- \to p + \bar{p}$ which is well-known [18]:

$$\frac{d\sigma^{e^+e^- \to p\bar{p}}}{d\Omega} = \frac{\alpha^2 \beta}{4s}\left[\left(1+\cos^2\theta\right)|G_M|^2 + \left(1-\beta^2\right)|G_E|^2 \sin^2\theta\right], \tag{52}$$

with $\beta = \sqrt{1 - \frac{4M^2}{s}}$ is the velocity of proton in c.m. frame, s is the total energy square and θ is the angle between vectors $\vec{q}_-$ and $\vec{p}_-$.

The difference of cross sections in (51) is the s-channel discontinuity of interference of the Born-amplitude with TPE amplitude

$$d\sigma^\uparrow - d\sigma^\downarrow \sim Re \sum \left(A^+_{elastic} \cdot A_{TPE} + A_{elastic} \cdot A^+_{TPE}\right). \tag{53}$$

Using the density matrix of final proton $u(p)\bar{u}(p) = (\hat{p} + M)(1 - \gamma_5 \hat{a})$ we get

$$\begin{aligned} & Re \sum \left(A^+_{elastic} \cdot A_{TPE} + A_{elastic} \cdot A^+_{TPE}\right) = 32 \frac{(4\pi\alpha)^3 (2\pi i)^2}{s\pi^2} Re(Y), \\ & Y = F_1(s) \int \frac{dk}{i\pi^2} \frac{1}{(\Delta)(Q)(+)(-)} \times \frac{1}{4} Tr\left[\hat{p}_1 \gamma^\alpha \hat{p}_1' \gamma^\mu \left(\hat{k} - \hat{\Delta}\right)\gamma^\nu\right] \times \\ & \times \frac{1}{4} Tr\left[(\hat{p} - M)(-\gamma_5 \hat{a})\gamma_\alpha (\hat{p}' + M)\gamma_\nu \left(\hat{k} - \hat{Q} + M\right)\gamma_\mu\right]. \end{aligned} \tag{54}$$

Performing the loop-momenta integration we can express the right hand side of Eq. (54) in terms of basic integrals (see Appendix XI)

$$Re(Y) = 4M(a, \Delta, Q, P) Im\left(F_Q - G_Q + H_Q\right) Re(F_1(s)), \tag{55}$$

where $(a, \Delta, Q, P) \equiv \varepsilon^{\mu\nu\rho\sigma} a_\mu \Delta_\nu Q_\rho P_\sigma = (\sqrt{s}/2)^3 (\vec{a}\vec{n}) \beta \sin\theta$. Using the expressions listed in Appendix XI we have:

$$Im\left(F_Q - G_Q + H_Q\right) = \frac{\pi}{s}\psi(\beta) = \frac{\pi}{s\beta^2}\left(\frac{1-\beta^2}{\beta}\ln\frac{1+\beta}{1-\beta} - 2\right). \tag{56}$$

We would like to note that Pauli formfactor F_2 doesn't contribute to the one-spin asymmetry.

Thus, after standard algebra, the following expression for spin asymmetry can be obtained (for processes $e^+e^- \to \vec{p}\,\bar{p}$, $\vec{p}\,\bar{p} \to e^+e^-$):

$$R_n = 2\alpha \frac{M}{\sqrt{s}} \frac{Re\left(G_E + \frac{s}{4M^2} G_M\right)}{1 + \frac{s}{4M^2}} \frac{\beta\psi(\beta)\sin\theta}{(1+\cos^2\theta)|G_M|^2 + (1-\beta^2)|G_E|^2 \sin^2\theta}. \tag{57}$$

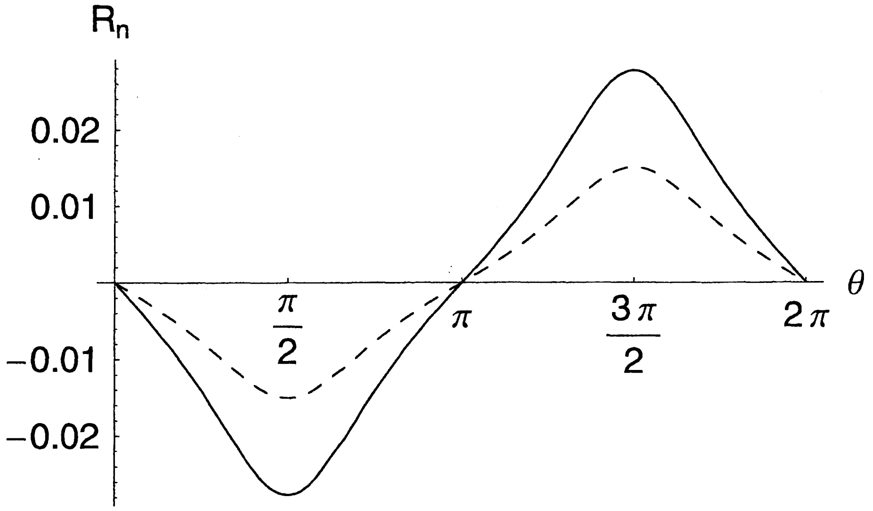

FIG. 4: Asymmetry R_n for the case of dipole proton formfactors for energies $s = 4.1$ Gev^2 (dashed line) and $s = 4.3$ Gev^2 (solid line).

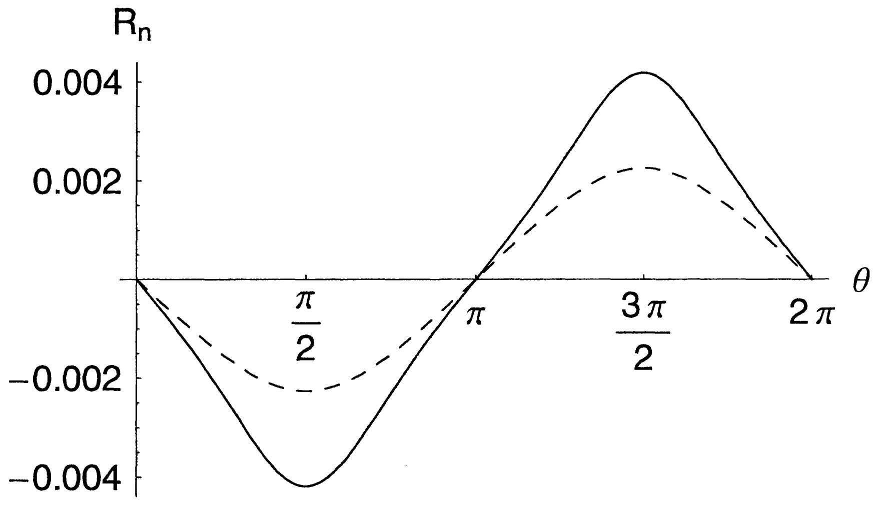

FIG. 5: Asymmetry R_n for the case of structureless proton for energies $s = 5$ Gev^2 (dashed line) and $s = 15$ Gev^2 (solid line).

and it is shown in Fig. 4, as a function of θ at several values of s, for the case of dipole form factors $G_M = \mu_p G_E = \mu_p[1 + s/(0.71 \ \ \mathrm{GeV}^2)]^{-2}$, $\mu_p = 2.79$.

Such considerations apply to the scattering channel when the initial protons is polarized.

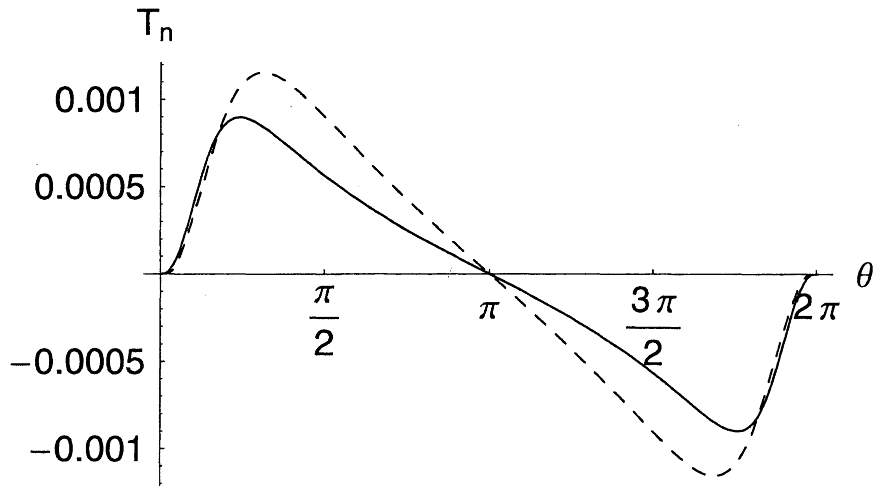

FIG. 6: Asymmetry T_n (with dipole proton form factors) for $E = 5$ Gev (dashed line) and $E = 10$ Gev (solid line).

Similarly to (56) one finds

$$Im_s\left(\bar{F}_Q - \bar{G}_Q + \bar{H}_Q\right) = -\frac{\pi}{s+M^2}. \tag{58}$$

(note that the s-channel imaginary part vanishes for the crossed photon diagram amplitude). The contribution of the polarization vector appears in the same combination

$$(a, \Delta, Q, P) = \frac{1}{2}(a, p, p_1, q) = \frac{ME^2}{2\rho}\sin\theta(\vec{a}\vec{n}). \tag{59}$$

The single spin asymmetry for the process $e^- + \vec{p} \to e^- + p$ (the initial proton is polarized) has the form:

$$\frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} = (\vec{a}\vec{n})T_n \tag{60}$$

with

$$T_n = \frac{\alpha}{2M^2}\frac{s^2}{s+M^2}\left(F_e + \tau F_m\right)\frac{\epsilon}{\rho\sigma_{red}}\sin\theta\tan^2\frac{\theta}{2}. \tag{61}$$

For the case of dipole formfactors $F_M = \mu_p F_E = \mu_p[1 - t/(0.71\ \mathrm{GeV}^2)]^{-2}$, this quantity is given on Fig. 6 as a function of θ, for two values of s.

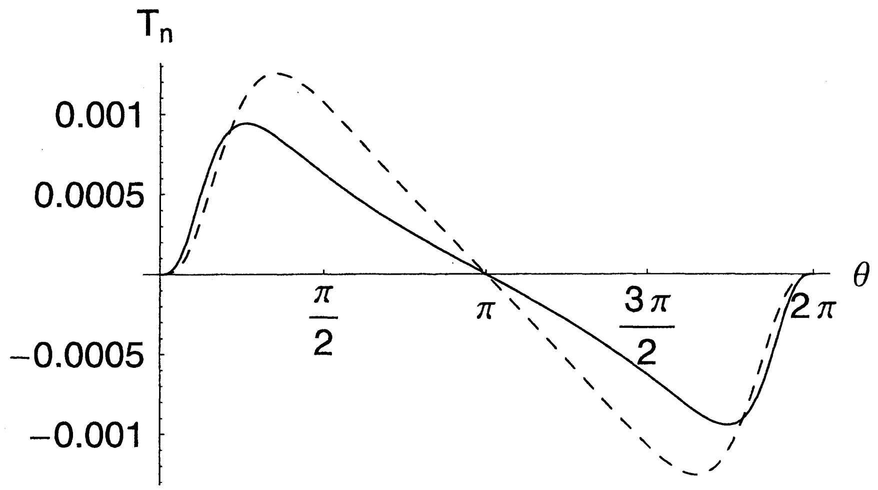

FIG. 7: Asymmetry T_n in case of structureless proton for $E = 5$ Gev (dashed line) and $E = 10$ Gev (solid line).

IX. CONCLUSIONS

We calculated QED radiative corrections to the differential cross-section of the processes $e^+ + e^- \to \mu^+ + \mu^-(\gamma)$, $e^\pm + \mu \to e^\pm + \mu(\gamma)$, arising from the interference between the Born and the box-type amplitudes. The relevant part of soft photon emission contribution, which eliminates the infrared singularities, was also considered.

Angular asymmetry, charge asymmetry as well as target spin asymmetry were calculated. These quantities are free from infrared and electron mass singularities. Numerical applications show that these observables are large enough to be measured.

Arguments based on analytical properties of the virtual Compton scattering amplitude allow to apply the results for the TPE amplitude obtained in frames of QED to hadrons reactions, with reasonable accuracy. The present calculation is therefore justified in the scattering channel and should be considered as a model in the annihilation channel. However, such extrapolation seems natural in case of small transfer momentum and high energy kinematics.

At large transfer momenta, the Born cross section decreases steeply, assuming dipole behavior of the form factors. Therefore, the square box type amplitude issued from TPE has to be taken into account, in the estimation of one-spin asymmetries.

The charge-asymmetry properties of radiative corrections in the annihilation channel induce non-trivial terms in the cross section due to crossing symmetry relations.

The parametrization (45) for the contribution to the matrix element arising from box-type diagrams in terms of three additional functions $G_i(s,t)$, $i=1,2,3$ suffers from infrared divergencies.

The results obtained here, for the processes $e^{\pm}p \to e^{\pm}(p\gamma)$ are particularly interesting in view of the experiments planned at Novosibirsk [19] and at JLab [20] as well as $e^+ + e^- \to N\bar{N}(\gamma)$, which can be investigated at Frascati [21] and Bejing [22].

X. APPENDIX A: TRACE CALCULATION.

The explicit expressions for the polynomials P_i are:

$$\begin{aligned}
P_1 &= 8\{-(\Delta^2)^3 - \Delta^2\sigma^2 + 2\sigma^3 + [(\Delta^2)^2 - 2\Delta^2\sigma]m^2\},\\
P_2 &= 16[(\Delta^2)^2\sigma - \sigma^3 + \Delta^2\sigma m^2],\\
P_3 &= 8(\{2(\Delta^2)^2\sigma - 2\sigma^3 + m^2[(\Delta^2)^2 + 2\Delta^2\sigma - \sigma^2] + \Delta^2 m^4\},\\
P_4 &= 8[5(\Delta^2)^2 - 6\Delta^2\sigma + 5\sigma^2 - 5\Delta^2 m^2],\\
P_5 &= 8[(\Delta^2)^3 - 2(\Delta^2)^2\sigma + \Delta^2(\sigma^2 - m^2\Delta^2)],\\
P_6 &= 8\{(\Delta^2)^3 - 2(\Delta^2)^2\sigma + \Delta^2\sigma^2 + m^2[-(\Delta^2)^2 - 2\Delta^2\sigma + 2\sigma^2] - 2\Delta^2 m^4\},\\
P_7 &= 8[-(\Delta^2)^3 + 2(\Delta^2)^2\sigma - \Delta^2\sigma^2 + (\Delta^2)^2 m^2],\\
P_8 &= 8\{-(\Delta^2)^3 + (\Delta^2)^2\sigma - 3\Delta^2\sigma^2 + 3\sigma^3 - m^2[(\Delta^2)^2 + 3\Delta^2\sigma]\}.
\end{aligned} \tag{62}$$

XI. APPENDIX B: USEFUL INTEGRALS.

In the calculation of ep scattering we use the following set of scalar integrals with three and four denominators [13].

$$\begin{aligned}
F_\Delta &= \frac{-i}{\pi^2}\int \frac{d^4k}{(\Delta)(P_+)(P_-)} = \frac{1}{s}\left[\frac{\pi^2}{6} + \frac{1}{2}\ln^2\frac{s}{m_e^2}\right],\\
F_Q &= \frac{-i}{\pi^2}\int \frac{d^4k}{(Q)(P_+)(P_-)}\\
&= \frac{1}{s\beta}\left[\frac{1}{2}\ln^2\frac{1-\beta}{2} - \frac{1}{2}\ln^2\frac{1+\beta}{2} + \mathrm{Li}_2\left(\frac{1+\beta}{2}\right) - \mathrm{Li}_2\left(\frac{1-\beta}{2}\right)\right],
\end{aligned}$$

$$
\begin{aligned}
H &= \frac{-i}{\pi^2}\int \frac{\mathrm{d}^4k}{(\Delta)(Q)(P_+)} = G = \frac{-i}{\pi^2}\int \frac{\mathrm{d}^4k}{(\Delta)(Q)(P_-)} = -\frac{1}{2(m^2-t)}\left[\ln^2\frac{m^2-t}{m^2}\right.\\
&+ \left.\left(2\ln\frac{m^2-t}{m^2}+\ln\frac{m^2}{m_e^2}\right)\ln\frac{m^2}{\lambda^2} - \frac{1}{2}\ln^2\frac{m^2}{m_e^2} - 2\mathrm{Li}_2\left(-\frac{t}{m^2-t}\right)\right],\\
F &= \frac{1}{2}sJ - G = -\frac{1}{2(m^2-t)}\left[\left(2\ln\frac{m^2-t}{m^2}+\ln\frac{m^2}{m_e^2}\right)\ln\frac{s}{m^2}\right.\\
&- \left.\ln^2\frac{m^2-t}{m^2} + \frac{1}{2}\ln^2\frac{m^2}{m_e^2} + 2\mathrm{Li}_2\left(-\frac{t}{m^2-t}\right)\right],\\
J &= \frac{-i}{\pi^2}\int \frac{\mathrm{d}^4k}{(\Delta)(Q)(P_+)(P_-)} = -\frac{1}{s(m^2-t)}\left[\left(2\ln\frac{m^2-t}{m^2}+\ln\frac{m^2}{m_e^2}\right)\ln\frac{s}{\lambda^2}\right].
\end{aligned}
\tag{63}
$$

The terms proportional to m_e^2/s, m_e^2/m_μ^2 were neglected. Notations follow Ref. (8).

The vector integrals with three denominators are:

$$
\begin{aligned}
&\frac{1}{i\pi}\int \frac{k^\mu \mathrm{d}^4k}{(\Delta)(Q)(P_+)} = H_P P^\mu + H_\Delta \Delta^\mu + H_Q Q^\mu, \quad H_Q = \frac{1}{t}\ln\frac{m^2-t}{m^2},\\
&H_\Delta = \frac{1}{m^2-t}\left(-\ln\frac{m^2}{m_e^2} - \frac{m^2+t}{t}\ln\frac{m^2-t}{m^2}\right),\\
&H_P = H + \frac{1}{m^2-t}\left(\ln\frac{m^2}{m_e^2} + 2\ln\frac{m^2-t}{m^2}\right),\\
&\frac{1}{i\pi}\int \frac{k^\mu \mathrm{d}^4k}{(\Delta)(P_+)(P_-)} = G_\Delta \Delta^\mu, \quad G_\Delta = \frac{1}{s}\left(-2\ln\frac{s}{m_e^2} + \frac{1}{2}\ln^2\frac{s}{m_e^2} + \frac{\pi^2}{6}\right),\\
&\frac{1}{i\pi}\int \frac{k^\mu \mathrm{d}^4k}{(Q)(P_+)(P_-)} = G_Q Q^\mu, \quad G_Q = \frac{1}{s-4m^2}\left(-2\ln\frac{s}{m^2} + sF_Q\right).
\end{aligned}
\tag{64}
$$

Four denominator vector and tensor integrals were defined in (15). The relevant coefficients are:

$$
\begin{aligned}
J_\Delta &= \frac{1}{2d}\Big[(F+F_\Delta)\sigma - Q^2(F+F_Q)\Big],\\
J_Q &= \frac{1}{2d}\Big[(F+F_Q)\sigma - \Delta^2(F+F_\Delta)\Big], \quad F = \frac{1}{2}sJ - G, \quad d = \Delta^2Q^2 - \sigma^2.\\
K_0 &= -\frac{1}{2\sigma}\Big[\sigma(F - G + H_P + H_\Delta + H_Q) + H_\Delta(\sigma - \Delta^2) - H_Q(\sigma - Q^2)\\
&\quad +2P^2J_\Delta(\Delta^2 - 2\sigma) + \Delta^2G_\Delta - Q^2G_Q - 2p^2Q^2J_Q\Big],\\
K_\Delta &= -\frac{1}{2\sigma d}\Big[Q^2\sigma(G - F - H_P - 3H_\Delta + 6P^2J_\Delta)\\
&\quad +(\Delta^2Q^2+\sigma^2)(H_\Delta - 2P^2J_\Delta - G_\Delta) - Q^4(H_Q - 2P^2J_Q - G_Q)\Big],\\
K_P &= \frac{1}{2P^2\sigma}\Big[2\sigma(H_\Delta - 2P^2J_\Delta + H_p + \frac{1}{2}F - \frac{1}{2}G) + Q^2(H_Q - 2P^2J_Q - G_Q)\\
&\quad -\Delta^2(H_\Delta - 2P^2J_\Delta - G_\Delta)\Big],
\end{aligned}
\tag{65}
$$

$$K_Q = -\frac{1}{2\sigma d}\Big[-\Delta^2\sigma A_P + 2\Delta^4 A_\Delta + (\sigma^2 - 2\Delta^2 Q^2)A_Q\Big],$$
$$K_x = -\frac{1}{2d}\left(\sigma A_P + Q^2 A_Q - 2\Delta^2 A_\Delta\right),$$

where we used

$$A_\Delta = H_\Delta + 2\Delta^2 J_\Delta - G_\Delta,\ A_Q = H_Q + 2\Delta^2 J_Q - G_Q,\ A_P = F - G + H_P + 3H_\Delta + 6\Delta^2 J_\Delta. \quad (66)$$

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