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THE QUARK PARTON MODEL FOR DEEP INELASTIC LEPTON SCATTERING

M. Gourdin *)

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*) On leave of absence from the Laboratoire de Physique Théorique, Orsay.

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I. INTRODUCTION

After the lectures of Professors Preparata, Treiman, de Alfaro and Coleman, I am pretty sure that people here have a deep understanding of inelastic lepton scattering from a theoretical point of view, using the fashionable concepts of light cone behaviour of current commutators or of broken scale invariance.

The aim of these lectures is twofold. First, I hope to give, with the quark parton model, an elementary understanding of the previous sophisticated languages. Secondly, I shall try to be quantitative and to use the available experimental data as much as possible.

The kinematics of these reactions is a ten-year old story so that we only give a brief review of the main features in Section II in order to fix the notations.

The parton model has been imagined by Feynman [1)] to explain in a simple way the characteristic features of the high energy hadronic interactions. It has been applied by Bjorken [2)] and many others to deep inelastic lepton scattering. The language is introduced in Section III and for a more complete discussion of the basis, we refer to previous Erice lectures like those by Drell [3)].

We then focus our attention on the quark parton model first studied by Bjorken and Paschos [4)] in a particular form and generalized by Llewellyn Smith [5)] by the inclusion of gluons. A comparison of this model with experiment has been made in a previous paper [6)] where an additional technical assumption concerning the longitudinal momentum distribution for partons in quarks was made for simplicity.

The scaling functions are expressed in Section IV in terms of the quark and antiquark distribution functions in the nucleon. We then obtain three general relations between the scaling functions which are characteristic of the model. Proton and neutron as targets are then related imposing charge symmetry so that we can derive four independent relations between the scaling functions for proton and neutron beside the usual charge symmetry relations. It is interesting to remark that these four relations can be derived in the integrated form from current algebra in the gluon model [5)] showing the strong connexion between these two approaches as emphasized in Professor Treiman's lectures for abstracting the structure of current commutators on the light cone.

Section V is devoted to a quick study of the sum rules obtained integrating the scaling functions. Unfortunately, the experimental information is too poor to allow conclusions.

The consideration of the first moment of the scaling functions made in Section VI is more instructive. Using the positivity property of the distribution functions, we get from the actual experimental data on neutrino and electroproduction interesting results and predictions. In particular, it is shown that the quark parton model can accommodate experiment if and only if gluons are present in the nucleon.

Finally, in Section VII, we discuss the implications of internal symmetries and the way to derive more constraints on the scaling functions. The consequences of isotopic spin symmetry for nucleons are explicitly given.

II. KINEMATICS

Let me first review in a very schematic way some important kinematical results needed for our study of deep inelastic scattering.

1) Structure functions [7)]

In the one-photon exchange approximation for inelastic electron or muon scattering on an unpolarized target, the differential cross-section observing only the final lepton is described by two structure functions which are conveniently expressed in terms of the two total cross-sections for the scattering of the polarized electromagnetic current on the same target σ_T and σ_L where T means transverse and L longitudinal.

In the local Fermi theory of weak interaction or in a theory with intermediate boson, again only one unit of angular momentum is exchanged between the leptons and the hadrons. Therefore, the inelastic scattering of neutrinos or antineutrinos on an unpolarized target will be described by three structure functions when only the final lepton is detected and its mass neglected. These structure functions can be interpreted in terms of the three total cross-sections for the scattering of a weak current of helicity λ on the same target σ_λ where λ takes the values +1, 0, -1.

For electromagnetic interactions, the invariance with respect to space reflection implies the equality $\sigma_+ = \sigma_-$ so that we have only one transverse cross-section $2\sigma_T = \sigma_+ + \sigma_-$. For weak interactions of the V - A type, the difference $\sigma_+ - \sigma_-$ measures the vector (V) axial vector (A) interference.

2) Variables

With the notations as indicated on Fig. 1 where the waved line symbolizes the electromagnetic or the weak current, the momentum transfer at the lepton vertex is $q = k - k'$. We then define two independent scalar variables q^2 and W^2 where W is the effective mass of the undetected hadronic system

$$W^2 = -(p+q)^2$$

Another useful variable is the energy carried by the current in the laboratory system, ν, covariantly defined by

$$\nu M = - p \cdot q$$

and related to q^2 and W^2 by

$$W^2 = M^2 - q^2 + 2M\nu$$

where M is the target mass.

The three structure functions σ_λ are Lorentz scalar functions of the two scalar variables q^2 and W^2.

We shall also use the Mandelstam variable of the direct channel

$$s = -(k+p)^2$$

3) Elastic scattering

In the particular case of elastic scattering $W = M$ so that the W^2 dependence of the structure functions is simply given by the Dirac delta distribution $\delta(W^2 - M^2)$. In terms of the q^2, ν variables, this becomes $\delta(q^2 - 2M\nu)$.

When the current scatters elastically off a spin zero particle, it can have only a longitudinal polarization as a consequence of simple helicity arguments so that in this case $\sigma^{e\ell}_{\pm} = 0$. If now the target has spin 1/2, it can be easily checked that for pointlike elastic scattering in the high energy limit, the elastic longitudinal cross-section vanishes $\sigma^{e\ell}_{0} = 0$.

4) Scaling

It has been proposed by Bjorken [8] that the structure functions for large q^2 and W^2 scale in the sense

$$\lim_{\substack{q^2 \Rightarrow \infty \\ W^2 \Rightarrow \infty \\ \xi = \frac{q^2}{2M\nu}\ \text{fixed}}} \frac{1}{\pi} \nu M \sigma_\lambda(q^2, W^2) = F_\lambda(\xi) \tag{1}$$

Experiments for electroproduction [9] and neutrino production [10] support this conjecture.

5) High energy differential cross-sections

The high energy limit in the scaling region is defined by the condition : q^2, W^2, s tend to infinity so that their ratios two by two remain fixed. We then introduce a second scaling variable $\rho = E'/E$ where $E(E')$ is the ingoing (outgoing) lepton energy in the laboratory frame. The relation between E, ρ , ξ and the laboratory scattering angle θ is simply

$$\rho^{-1} = 1 + \frac{2E \sin^2 \frac{\theta}{2}}{M\xi}$$

The high energy limit in the scaling region of the differential cross-section is computed in the one-photon exchange approximation for electroproduction and in the local Fermi interaction for neutrino and antineutrino induced reactions

$$\frac{d^2\sigma^e}{d\rho\, d\xi} \Rightarrow \frac{4\pi\alpha^2}{s} \frac{1}{(1-\rho)^2\xi} \left[(\rho^2+1) F_T^e(\xi) + 2\rho F_L^e(\xi) \right] \tag{2}$$

$$\frac{d^2\sigma^\nu}{d\rho\, d\xi} \Rightarrow \frac{G^2}{2\pi} s\, \xi \left[\rho^2 F_+^\nu(\xi) + F_-^\nu(\xi) + 2\rho F_0^\nu(\xi) \right] \tag{3}$$

$$\frac{d^2\sigma^{\bar\nu}}{d\rho\, d\xi} \Rightarrow \frac{G^2}{2\pi} s\, \xi \left[\rho^2 F_-^{\bar\nu}(\xi) + F_+^{\bar\nu}(\xi) + 2\rho F_0^{\bar\nu}(\xi) \right] \tag{4}$$

As usual α is the fine structure constant and G the weak Fermi coupling constant.

6) High energy total cross sections

The total cross-sections are obtained integrating the differential ones in the region $0 \leq \xi \leq 1$, $0 \leq \rho \leq 1$.

For electroproduction, the total elastic cross-section is infinite by itself as a well-known consequence of the vanishing of the photon mass. A finite total inelastic cross-section can be computed but nothing really simple emerges [11].

For neutrino and antineutrino reactions, we make the strong assumption that the result is not too different from the one obtained by using the scaling forms (3) and (4) in the complete integration domain. We then deduce a linear rising with s of the total cross-sections

$$\sigma_{TOT}^{\nu,\bar{\nu}}(s) \Rightarrow \frac{G^2 s}{2\pi} \, \underline{A}^{\nu,\bar{\nu}}$$

where the constants A^{ν} and $A^{\bar{\nu}}$ are first moment distributions given by

$$\underline{A}^{\nu,\bar{\nu}} = \int_0^1 \xi \, \underline{A}^{\nu,\bar{\nu}}(\xi) \, d\xi$$

with as a result of the integration over ρ

$$\underline{A}^{\nu}(\xi) = \frac{1}{3} F_+^{\nu}(\xi) + F_-^{\nu}(\xi) + F_0^{\nu}(\xi) \qquad (5)$$

$$\underline{A}^{\bar{\nu}}(\xi) = \frac{1}{3} F_-^{\bar{\nu}}(\xi) + F_+^{\bar{\nu}}(\xi) + F_0^{\bar{\nu}}(\xi) \qquad (6)$$

III. THE PARTON MODEL

1) Because of the lack of time, we cannot discuss in detail the justification of the Feynman parton model and we refer to the original papers of Bjorken [2], Bjorken and Paschos [4], Llewellyn-Smith [5], Gross and Llewellyn Smith [12] and of many other authors not quoted here.

The hadrons are assumed to be composite systems of partons and in the $|\vec{p}| \to \infty$ frame of the hadrons, where the Lorentz invariant structure functions are computed, there are two fundamental principles from which the scaling emerges directly,

a. the structure functions in the scaling region are incoherent sums of the various parton contributions';

b. the interaction of the partons with the electromagnetic and the weak currents is pointlike in the scaling region.

2) In the hadron $|\vec{p}| \to \infty$ frame the transverse momentum of the partons is ignored and the parton α carries a fraction x_α of the hadron momentum $\vec{p}$ so that we get the approximate relation for the parton energy momentum vector $p_\alpha \simeq x_\alpha p$ with

$$0 \leq x_\alpha \leq 1 \qquad\qquad \sum_\alpha x_\alpha = 1 \tag{7}$$

The partons are almost free and elastic scattering on a pointlike parton is relevant. As seen in the previous Section, the parton α contribution carries a Dirac delta distribution with the argument $q^2 + 2p_\alpha \cdot q = q^2 - 2x_\alpha M \nu$ which relates x_α to the scaling variable ξ.

3) The probability to find in the hadron a configuration with N partons is called P_N. The conservation of probabilities implies

$$\sum_N P_N = 1 \tag{8}$$

Mean values will be used later referring to this distribution of states and for a quantity O which can depend on N we introduce the notation

$$\langle O \rangle = \sum_N{}' P_N \, O(N)$$

4) Let us define the N dimensional correlation function $f^N(x_1, x_2, \ldots x_N)$ describing the distribution of longitudinal momentum for the N parton configuration. The normalization condition is written taking into account the constraint (7)

$$\iint \cdots \int dx_1 dx_2 \ldots dx_N \, f^N(x_1, x_2, \ldots, x_N) \, \delta\left(\sum_\alpha x_\alpha - 1\right) = 1 \tag{9}$$

The density of probability for the parton α to have the longitudinal momentum $x\vec{p}$ in the N parton configuration is simply obtained integrating f^N over all variables but x_α

$$f_\alpha^N(x) = \iint \cdots \int dx_1\, dx_2 \cdots dx_N\, f^N(x_1, x_2 \cdots x_N)\, \delta\left(\sum_\beta x_\beta - 1\right) \delta(x_\alpha - x) \tag{10}$$

and from (9) we deduce the obvious normalization condition

$$\int_0^1 f_\alpha^N(x)\, dx = 1 \tag{11}$$

Another interesting property concerns the first moment of the momentum distribution given by

$$\bar{x}_\alpha = \int_0^1 x\, f_\alpha^N(x)\, dx \tag{12}$$

Using the definition (10) of $f^N(x)$ and the normalization condition (9) it is straightforward to prove the equality

$$\sum_\alpha \bar{x}_\alpha = 1 \tag{13}$$

5) With these concepts and definitions, the computation of the structure functions in the scaling limit is straightforward. In a model with only spin 0 and spin 1/2 partons the longitudinal scaling functions receive only spin 0 parton contributions and the transverse scaling functions only spin 1/2 parton contributions.

For electromagnetic interactions we get

$$2 F_T^e(\xi) = \sum_N P_N \sum_\alpha^{(1/2)} f_\alpha^N(\xi)\, Q_\alpha^2 \tag{14}$$

$$2 F_L^e(\xi) = \sum_N P_N \sum_\alpha^{(0)} f_\alpha^N(\xi)\, Q_\alpha^2 \tag{15}$$

where Q_α^2 is the value of the squared electric charge for the parton α .

For weak interactions, the Cabibbo theory with an angle θ_c is used in a chiral symmetric form in order to save the Adler sum rule for neutrino production. The results are

$$F_{\pm}^{\nu}(\xi) = \sum_N{}' P_N \sum_\alpha{}'^{(1/2)} f_\alpha^N(\xi)(1 \mp \varepsilon_\alpha)\left[\cos^2\theta_c\, I_\alpha^2 + \sin^2\theta_c\, V_\alpha^2\right] \tag{16}$$

$$F_{\pm}^{\bar\nu}(\xi) = \sum_N{}' P_N \sum_\alpha{}'^{(1/2)} f_\alpha^N(\xi)(1 \mp \varepsilon_\alpha)\left[\cos^2\theta_c\, \bar{I}_\alpha^2 + \sin^2\theta_c\, \bar{V}_\alpha^2\right] \tag{17}$$

$$F_{0}^{\nu}(\xi) = \sum_N{}' P_N \sum_\alpha{}'^{(0)} f_\alpha^N(\xi)\left[\cos^2\theta_c\, I_\alpha^2 + \sin^2\theta_c\, V_\alpha^2\right] \tag{18}$$

$$F_{0}^{\bar\nu}(\xi) = \sum_N{}' P_N \sum_\alpha{}'^{(0)} f_\alpha^N(\xi)\left[\cos^2\theta_c\, \bar{I}_\alpha^2 + \sin^2\theta_c\, \bar{V}_\alpha^2\right] \tag{19}$$

where $\varepsilon_\alpha = +1$ for partons, $\varepsilon_\alpha = -1$ for antipartons. The weak charges are defined by the mean values of I spin and V spin operator products

$$I_\alpha^2 = \langle\alpha|\, I^- I^+ \,|\alpha\rangle \qquad V_\alpha^2 = \langle\alpha|\, V^- V^+ \,|\alpha\rangle$$

$$\bar{I}_\alpha^2 = \langle\alpha|\, I^+ I^- \,|\alpha\rangle \qquad \bar{V}_\alpha^2 = \langle\alpha|\, V^+ V^- \,|\alpha\rangle$$

The position of these operators on the SU(3) root diagram is shown on Fig. 2.

6) <u>Adler sum rule</u> [13]

The Adler sum rule appears to be a direct consequence of the I spin and V spin commutation relations

$$[I^+, I^-] = 2I^3 \qquad [V^+, V^-] = 2V^3$$

In the scaling function language, it is simply written as

$$\int_0^1 \left\{ \left[F_+^{\bar\nu}(\xi) + F_-^{\bar\nu}(\xi) + 2F_0^{\bar\nu}(\xi)\right] - \left[F_+^{\nu}(\xi) + F_-^{\nu}(\xi) + 2F_0^{\nu}(\xi)\right]\right\} d\xi = 4\left[\cos^2\theta_c\, I^3 + \sin^2\theta_c\, V^3\right] \tag{20}$$

where the normalization conditions (8) and (11) have been explicitly used.

IV. THE QUARK PARTON MODEL

1) In all that follows we shall be interested in the specific model [4)] where the interacting partons are quarks $(j = 1,2,3)$ and antiquarks $(j = -1, -2, -3)$ labelled as shown on Fig. 3 where the two fundamental representations of SU(3) have been drawn. Neutral particles called gluons might be present for dynamical purpose and they have the only role to carry a fraction of the hadron momentum being neutral in all the other respects [5)].

2) Calling N_j the number of quarks or antiquarks of type j present in the N parton configuration, it is convenient to work with the following set of distribution functions defined by

$$D_j(\xi) = \sum_N P_N N_j f_j^N(\xi) \tag{21}$$

The main property of these D_j's is to be positive and we shall use extensively this feature in our analyses. The vanishing of a $D_j(\xi)$ implies the non existence of quark or antiquark of type j in the hadron.

From the normalization condition (11) and the definition of mean values we have a simple interpretation of the ξ integral of the distribution functions in terms of the mean number of type j quark or antiquark in the hadron

$$\int_0^1 D_j(\xi)\, d\xi = \langle N_j \rangle \tag{22}$$

The convergence of the integral (11) implies for all the functions $f_j^N(\xi)$ the property in the neighbourhood of $\xi = 0$

$$\lim_{\xi \to 0} \xi f_j^N(\xi) = 0$$

If there is only a finite number of partons the same property holds for the distribution $D_j(\xi)$ from its definition (21) and the integral (22) is convergent. The same argument cannot be used if the sum (21) contains an infinite number of terms and the integral (22) can be divergent.

3) The baryonic charge B, the electric charge Q and the hypercharge Y are additive quantum numbers. We know their values for quarks and antiquarks and the conservation of B, Q and Y implies constraints on the mean numbers of quarks and antiquarks

$$\langle N_1 - N_{-1} \rangle = B + Q \qquad \langle N_2 - N_{-2} \rangle = B + Y - Q \qquad \langle N_3 - N_{-3} \rangle = B - Y \qquad (23)$$

so that, a priori, only the mean number of antiquarks is free.

4) In the quark parton model, the longitudinal scaling functions are predicted to vanish as explained previously. In order to compute the transverse, scaling functions (14), (16) and (17) we must know the electromagnetic and weak charges for quarks and antiquarks. The results are given in the Table.

α	1	2	3	-1	-2	-3
Q^2_α	4/9	1/9	1/9	4/9	1/9	1/9
I^2_α	0	1	0	1	0	0
$\bar{I}^2_\alpha$	1	0	0	0	1	0
V^2_α	0	0	1	1	0	0
$\bar{V}^2_\alpha$	1	0	0	0	0	0

It is convenient for the weak scaling functions to separate the contributions coming from the $\Delta Y = 0$ and $|\Delta Y| = 1$ transitions

$$F_\pm^{\nu,\bar\nu}(\xi) = \cos^2\theta_c \, G_\pm^{\nu,\bar\nu}(\xi) + \sin^2\theta_c \, H_\pm^{\nu,\bar\nu}(\xi)$$

The scaling functions for an unspecified target are then given by the following expressions

$$2F_T^e(\xi) = \frac{4}{9}\left[D_1(\xi) + D_{-1}(\xi)\right] + \frac{1}{9}\left[D_2(\xi) + D_{-2}(\xi) + D_3(\xi) + D_{-3}(\xi)\right] \qquad (24)$$

$$G_{+}^{\nu}(\xi) = 2\,D_{-1}(\xi) \qquad\qquad H_{+}^{\nu}(\xi) = 2\,D_{-1}(\xi)$$
$$G_{-}^{\nu}(\xi) = 2\,D_{2}(\xi) \qquad\qquad H_{-}^{\nu}(\xi) = 2\,D_{3}(\xi)$$
$$G_{+}^{\bar{\nu}}(\xi) = 2\,D_{-2}(\xi) \qquad\qquad H_{+}^{\bar{\nu}}(\xi) = 2\,D_{-3}(\xi) \qquad (25)$$
$$G_{-}^{\bar{\nu}}(\xi) = 2\,D_{1}(\xi) \qquad\qquad H_{-}^{\bar{\nu}}(\xi) = 2\,D_{1}(\xi)$$

From electroproduction, neutrino and antineutrino processes, one can measure nine structure functions. The number of different types of quarks and antiquarks being six we have to our disposal only six distribution functions $D_j(\xi)$ so that the quark parton model predicts <u>three relations</u> that one can, for instance, write as

$$H_{+}^{\nu}(\xi) = G_{+}^{\nu}(\xi) \qquad\qquad H_{-}^{\bar{\nu}}(\xi) = G_{-}^{\bar{\nu}}(\xi)$$
$$(26)$$
$$2F_{T}^{e}(\xi) = \frac{2}{9}\left[G_{+}^{\nu}(\xi) + G_{-}^{\bar{\nu}}(\xi)\right] + \frac{1}{18}\left[G_{-}^{\nu}(\xi) + G_{+}^{\bar{\nu}}(\xi) + H_{-}^{\nu}(\xi) + H_{+}^{\bar{\nu}}(\xi)\right]$$

These relations are strict tests of the most general quark parton model.

5) When the target is a nucleon, we use <u>charge symmetry</u> to relate the proton and neutron distributions as follows

$$D_{\pm 1}^{n}(\xi) = D_{\pm 2}^{p}(\xi) \qquad D_{\pm 2}^{n}(\xi) = D_{\pm 1}^{p}(\xi) \qquad D_{\pm 3}^{n}(\xi) = D_{\pm 3}^{p}(\xi)$$

All the scaling functions on a neutron target are known from the scaling functions on a proton target. As an illustration, let us give some example of such relations

a. <u>for $\Delta Y = 0$ weak reactions</u>

$$G_{\pm}^{\nu n}(\xi) = G_{\pm}^{\bar{\nu} p}(\xi) \qquad\qquad G_{\pm}^{\bar{\nu} n}(\xi) = G_{\pm}^{\nu p}(\xi) \qquad (27)$$

b. <u>for $|\Delta Y| = 1$ weak reactions</u>

$$H_-^{\nu n}(\xi) = H_-^{\nu p}(\xi) \qquad\qquad H_+^{\bar\nu n}(\xi) = H_+^{\bar\nu p}(\xi) \tag{28}$$

c. <u>between electromagnetic and weak scaling functions</u>

$$\begin{aligned} F_T^{ep}(\xi)+F_T^{en}(\xi) &= \frac{5}{36}\left[G_+^{\nu p}(\xi)+G_-^{\nu p}(\xi)+G_+^{\bar\nu p}(\xi)+G_-^{\bar\nu p}(\xi)\right]+\frac{1}{18}\left[H_+^{\bar\nu p}(\xi)+H_-^{\nu p}(\xi)\right] \\ F_T^{ep}(\xi)-F_T^{en}(\xi) &= \frac{1}{12}\left[G_+^{\nu p}(\xi)-G_-^{\nu p}(\xi)-G_+^{\bar\nu p}(\xi)+G_-^{\bar\nu p}(\xi)\right] \end{aligned} \tag{29}$$

All other possible equalities [5] are linear combinations of (26), (27), (28) and (29).

V. SUM RULES

1) The integration over ξ of the scaling functions involves the integration over ξ of the distributions $D_j(\xi)$. From Eq. (22) we get linear combinations of the mean values of the number of quarks and antiquarks in the hadron.

As a trivial consequence of the B, Q and Y charge conservation we obtain two sum rules using the constraints (23)

$$\int_0^1 \left[F_-^{\bar\nu}(\xi)-F_+^{\nu}(\xi)\right] d\xi = 2(B+Q) \tag{30}$$

$$\int_0^1 \left[F_-^{\nu}(\xi)-F_+^{\bar\nu}(\xi)\right] d\xi = 2(B+Y-Q)\cos^2\theta_c + 2(B-Y)\sin^2\theta_c \tag{31}$$

The difference between (30) and (31) is the Adler sum rule originally derived from the current algebra of time components and shown to be valid in all parton models with chiral symmetry in Eq. (20). The sum of (30) and (31) is the Gross-Llewellyn Smith sum rule which was derived from a quark model current algebra of space components [12].

2) Let us specialize to a nucleon target. As pointed out in the previous section charge symmetry relates the neutron and proton distributions so that it is sufficient to discuss the proton ones.

In all quark parton models we have the obvious lower bounds

$$\langle N_1^p \rangle \geq 2 \qquad \langle N_2^p \rangle \geq 1 \qquad \langle N_j^p \rangle \geq 0 \text{ for } j = 3,-1,-2,-3$$

which in terms of scaling functions can be written as

$$\int_0^1 F_-^{\bar{\nu}p}(\xi)\,d\xi \geq 4 \qquad \int_0^1 F_-^{\bar{\nu}n}(\xi)\,d\xi \geq 2 \tag{32}$$

$$\int_0^1 F_-^{\nu p}(\xi)\,d\xi \geq 2\cos^2\theta_c \qquad \int_0^1 F_-^{\nu n}(\xi)\,d\xi \geq 4\cos^2\theta_c \tag{33}$$

$$\int_0^1 2F_T^{ep}(\xi)\,d\xi \geq 1 \qquad \int_0^1 2F_T^{en}(\xi)\,d\xi \geq \frac{2}{3} \tag{34}$$

Obviously, these inequalities make useful sense <u>if and only if the integrals converge.</u> Assuming such a convergence we are able to derive non trivial bounds for the mean value of the total number of quarks and antiquarks N_q in the nucleon. The result using electroproduction scaling functions is simply

$$\frac{9}{5}\int_0^1 \left[2F_T^{ep}(\xi)+2F_T^{en}(\xi)\right]d\xi \leq \langle N_q \rangle \leq \frac{9}{2}\int_0^1 \left[2F_T^{ep}(\xi)+2F_T^{en}(\xi)-1\right]d\xi \tag{35}$$

3) The experimental situation at the time of the Kiev Conference (1970) was the following [9]

$$\int_{\frac{1}{12}}^1 2F_T^{ep}(\xi)\,d\xi = 0,58 + \text{errors}$$

$$\int_{\frac{1}{12}}^1 2F_T^{en}(\xi)\,d\xi = 0,45 + \text{errors}$$

The lower bounds (34) were not yet satisfied and if the integrals converge they will converge slowly so that we need more data before to reach a sensible conclusion.

Now from experiment, it is not clear if the quantity $\xi F_T^{ep}(\xi)$ and $\xi F_T^{en}(\xi)$ are constant or zero when ξ tends to zero. Again, more experimental information at lower values of ξ can resolve this important problem.

VI. POSITIVITY CONSTRAINTS

1) In this section we study the first moment of the quark and antiquark distribution functions defined by

$$d_j = \int_0^1 \xi\, D_j(\xi)\, d\xi \tag{36}$$

The integrals (36) are expected to be convergent so that we are in a more comfortable position to make useful statements.

The first moment relation (13) is written in the quark parton model language as

$$\sum_j' d_j + \sum_{\text{gluons}} \bar{x}_\alpha = 1$$

It is then convenient to introduce a parameter ε measuring in some sense the amount of gluons in the hadron

$$\sum_{\text{gluons}} \bar{x}_\alpha = \varepsilon \qquad\qquad \sum_j' d_j = 1 - \varepsilon \tag{37}$$

From the positivity of all the first moments of distributions, we deduce the allowed range of variation of

$$0 \leqslant \varepsilon < 1 \tag{38}$$

In particular $\varepsilon = 0$ corresponds to a hadron made only of quarks and antiquarks and $\varepsilon \neq 0$ implies the existence of gluons in this model.

2) Let us define the electroproduction integral

$$I^e = \int_0^1 \xi\, 2\bar{F}_T^{\,e}(\xi)\, d\xi \tag{39}$$

which from Eqs. (24) and (37) can be written as

$$\bar{I}^e = \frac{1}{9}(1-\varepsilon) + \frac{1}{3}(d_1 + d_{-1}) \tag{40}$$

For neutrino and antineutrino reactions, the interesting quantities are the constants A^{ν} and $A^{\bar{\nu}}$ which govern the linear rising of the total cross-sections in the local Fermi interaction. From Eqs. (5), (6) and (25) we get

$$A^{\nu} = \frac{2}{3} d_{-1} + 2\left[\cos^2\theta_c\, d_2 + \sin^2\theta_c\, d_3\right] \tag{41}$$

$$A^{\bar{\nu}} = \frac{2}{3} d_1 + 2\left[\cos^2\theta_c\, d_{-2} + \sin^2\theta_c\, d_{-3}\right] \tag{42}$$

The separation between strangeness conserving and strangeness changing transitions is achieved by putting

$$A^{\nu,\bar{\nu}} = \cos^2\theta_c\, B^{\nu,\bar{\nu}} + \sin^2\theta_c\, C^{\nu,\bar{\nu}} \tag{43}$$

3) We first study the electroproduction on nucleon. Using charge symmetry we deduce the following inequality from positivity

$$\frac{2}{9}(1-\varepsilon) < I^{ep} + I^{en} \leq \frac{5}{9}(1-\varepsilon) \tag{44}$$

The equality at the upper limit holds when no strange quarks and antiquarks are present in the nucleon.

Equations (44) and (38) imply the existence of an absolute bound

$$0 < I^{ep} + I^{en} \leq \frac{5}{9} \tag{45}$$

and limits on ε when the sum $I^{ep} + I^{en}$ is known from experiment

$$0 \leq \varepsilon \leq 1 - \frac{9}{5}\left(I^{ep} + I^{en}\right) \tag{46}$$

The experimental situation is the following [9]

$$\int_{\frac{1}{12}}^{1} \xi\, 2F_T^{ep}(\xi)\, d\xi = 0{,}14 + \text{errors}$$

$$\int_{\frac{1}{12}}^{1} \xi\, 2F_T^{en}(\xi)\, d\xi = 0{,}10 + \text{errors}$$

The evaluation of the rest of the integrals $\int_0^{1/12}$ leads to the following estimates with errors of the order of 20%

$$I^{ep} \simeq 0{,}16 \pm 0{,}03$$

$$I^{en} \simeq 0{,}12 \pm 0{,}025$$

and for the sum $I^{ep} + I^{en}$ of interest here

$$I^{ep} + I^{en} \simeq 0{,}28 \pm 0{,}04 \qquad (47)$$

This result satisfies the upper bound (45), it implies an upper limit for ε which in the one standard deviation limit is

$$\varepsilon < 0{,}57 \qquad (48)$$

and it leads to the value $\varepsilon = 0{,}50 \pm 0{,}07$ when there are no strange quarks or antiquarks in the nucleon.

4) The same type of analysis can be carried out for neutrino and antineutrino reactions. We only sketch some results consequences of <u>positivity</u> :

$$B^{\nu p} + B^{\nu n} \leqslant 2(1-\varepsilon) \qquad C^{\nu p} + C^{\nu n} \leqslant 4(1-\varepsilon) \qquad (49)$$

$$B^{\bar{\nu} p} + B^{\bar{\nu} n} \leqslant 2(1-\varepsilon) \qquad C^{\bar{\nu} p} + C^{\bar{\nu} n} \leqslant 4(1-\varepsilon) \qquad (50)$$

$$B^{\nu p} + B^{\nu n} + B^{\bar{\nu} p} + B^{\bar{\nu} n} \leqslant \frac{8}{3}(1-\varepsilon) \qquad (51)$$

$$\frac{2}{3}(1-\varepsilon) \leqslant C^{\nu p} + C^{\nu n} + C^{\bar{\nu} p} + C^{\bar{\nu} n} \leqslant 4(1-\varepsilon) \qquad (52)$$

When strange quarks and antiquarks are absent in the nucleon, we get two interesting equalities

$$B^{\nu p} + B^{\nu n} + B^{\bar{\nu} p} + B^{\bar{\nu} n} = \frac{8}{3}(1-\varepsilon) \tag{53}$$

$$C^{\nu p} + C^{\nu n} + C^{\bar{\nu} p} + C^{\bar{\nu} n} = \frac{2}{3}(1-\varepsilon) \tag{54}$$

Absolute bounds are simply derived by putting $\varepsilon = 0$ in the upper limits of the inequalities (49) - (52). Limits on ε are obtained from the knowledge of the constants B and C as for instance

$$\varepsilon \leq 1 - \frac{B^{\nu p} + B^{\nu n}}{2} \tag{55}$$

We can use our information (48) on ε deduced from electroproduction data to obtain a lower limit on the total strangeness changing transitions using the first part of the inequality (52)

$$C^{\nu p} + C^{\nu n} + C^{\bar{\nu} p} + C^{\bar{\nu} n} > 0{,}28 \tag{56}$$

5) An experiment performed at CERN in a propane bubble chamber [10] gives some indication for a linear rising with energy of the total cross-sections. The results are

a. A(ν propane per nucleon) = 0,52 ± 0,13 ;

b. $A^{\nu n}/A^{\nu p} = 1{,}8 \pm 0{,}3$;

c. no strangeness changing events observed.

Taking into account the particular structure of the propane in protons and neutrons, we obtain the experimental figure

$$B^{\nu p} + B^{\nu n} = 1{,}15 \pm 0{,}29 \tag{57}$$

This result satisfies the absolute upper bound of 2 and it implies from (55) an upper limit on ε which, in the one standard deviation limit, is

$$\varepsilon < 0{,}57 \tag{58}$$

e.g., the same as that deduced from electroproduction (48).

6) We now combine the neutrino and electroproduction integrals in order to obtain the maximum of information from experiment.

As an immediate consequence of the first general relation (29) between structure functions we get the simple inequality

$$B^{\nu p} + B^{\nu n} \leqslant \frac{18}{5}\left(I^{ep} + I^{en}\right) \tag{59}$$

From the electroproduction data (47) $I^{ep} + I^{en} < 0{,}32$ so that the inequality (59) implies

$$B^{\nu p} + B^{\nu n} \leqslant 1{,}15$$

The experimental result (57) shows that neutrino and electroproduction data are consistent with the quark parton model constraint (59).

If we study in more details the relation (59) using Eqs. (40), (41), (42) and the requirement of positivity for the d_j's we easily prove the more elaborate inequality

$$\frac{18}{5}\left(I^{ep} + I^{en}\right) - \left(B^{\nu p} + B^{\nu n}\right) \geqslant \frac{4}{5}\left(d_3^p + d_{-3}^p\right)$$

On the other hand the electroproduction sum is computed from (40) to be

$$I^{ep} + I^{en} = \frac{5}{9}(1-\varepsilon) - \frac{1}{3}\left(d_3^p + d_{-3}^p\right)$$

and we can deduce a double inequality leading to an improved lower limit for ε

$$1-\frac{9}{2}\left(I^{ep}+I^{en}\right)+\frac{3}{4}\left(B^{\nu p}+B^{\nu n}\right)\leq \varepsilon \leq 1-\frac{9}{5}\left(I^{ep}+I^{en}\right) \qquad (60)$$

With the experimental data inserted in the one standard deviation limit, we obtain a limited range for

$$0{,}32 < \varepsilon < 0{,}57 \qquad (61)$$

so that <u>gluons must be present in the nucleon</u> to make the quark parton model consistent with experiment.

7) Some interesting consequences can be derived from this result. Starting from the equalities

$$B^{\nu p}+B^{\nu n}+B^{\bar{\nu} p}+B^{\bar{\nu} n}=8\left(I^{ep}+I^{en}\right)-\frac{16}{9}(1-\varepsilon)$$
$$C^{\nu p}+C^{\nu n}+C^{\bar{\nu} p}+C^{\bar{\nu} n}=\frac{56}{9}(1-\varepsilon)-10\left(I^{ep}+I^{en}\right)$$

and using for ε the double inequality (60) we deduce

$$\frac{4}{3}\left(B^{\nu p}+B^{\nu n}\right)\leq B^{\nu p}+B^{\nu n}+B^{\bar{\nu} p}+B^{\bar{\nu} n}\leq \frac{24}{5}\left(I^{ep}+I^{en}\right)$$
$$\frac{6}{5}\left(I^{ep}+I^{en}\right)\leq C^{\nu p}+C^{\nu n}+C^{\bar{\nu} p}+C^{\bar{\nu} n}\leq 18\left(I^{ep}+I^{en}\right)-\frac{14}{3}\left(B^{\nu p}+B^{\nu n}\right)$$

Using again the experimental data for neutrino and electroproduction, we make the following predictions in the one standard deviation limit

$$1{,}12 < B^{\nu p}+B^{\nu n}+B^{\bar{\nu} p}+B^{\bar{\nu} n} < 1{,}54 \qquad (62)$$
$$0{,}28 < C^{\nu p}+C^{\nu n}+C^{\bar{\nu} p}+C^{\bar{\nu} n} < 1{,}27 \qquad (63)$$

Limits can also be obtained for the ratio of antineutrino to neutrino transitions conserving strangeness, using again the positivity of the d_j's and the lower experimental limit for the neutrino data

$$0,33 \leq \frac{B^{\bar{\nu}p}+B^{\bar{\nu}n}}{B^{\nu p}+B^{\nu n}} < 0,83 \tag{64}$$

In Fig. 4 we give a graphical description of the situation concerning the strangeness concerving transition. For the strangeness changing part, we must keep in mind the general relation

$$\underline{I}^{ep}+I^{en} = \frac{7}{36}\left(B^{\nu p}+B^{\nu n}+B^{\bar{\nu}p}+B^{\bar{\nu}n}\right)+\frac{1}{18}\left(C^{\nu p}+C^{\nu n}+C^{\bar{\nu}p}+C^{\bar{\nu}n}\right)$$

already given in a differential form in Eq. (29).

8) Up to now, we only considered the proton-neutron sums of structure functions in order to use at their maximum the positivity requirements. The proton-neutron differences can also be studied and there is some experimental information on them. Unfortunately, my allowed time is too short for such a discussion and we only recall the three general relation characteristic of the quark parton model (27), (28) and (29) which, in this language, are written as

$$C^{\nu p}-C^{\nu n} = \frac{1}{8}\left(B^{\nu n}-B^{\nu p}\right)-\frac{3}{8}\left(B^{\bar{\nu}p}-B^{\bar{\nu}n}\right)$$
$$C^{\bar{\nu}p}-C^{\bar{\nu}n} = \frac{3}{8}\left(B^{\nu n}-B^{\nu p}\right)-\frac{1}{8}\left(B^{\bar{\nu}p}-B^{\bar{\nu}n}\right)$$
$$\underline{I}^{ep}-\underline{I}^{en} = \frac{1}{4}\left(B^{\nu n}-B^{\nu p}+B^{\bar{\nu}n}-B^{\bar{\nu}p}\right)$$

VII. SYMMETRIES

1) We consider a forward scattering amplitude with different U(3) indices for the currents and for the hadrons.

$$\gamma^{l_1 m_1}+\alpha_1 \Rightarrow \gamma^{l_2 m_2}+\alpha_2$$

In the quark-antiquark representation, the system ℓm is associated to one of the nine $U(3)$ generators.

As previously, we use the Cabibbo theory for weak interactions and scaling functions generalizing (24) and (25) are simply given by

$$F_{\pm}^{\ell_2 m_2,\alpha_2;\ell_1 m_1,\alpha_1}(\xi) = \sum_{k,j} D_{kj}^{\alpha_2\alpha_1}(\xi)\left(1 \mp \xi_j g_A\right) < k \mid \overline{F}^{m_2 \ell_2} F^{\ell_1 m_1} \mid j > \tag{65}$$

where $g_A = 0$ for electromagnetic transitions and $g_A = +1$ in a chiral symmetry of weak interactions.

2) The F's are the infinitesimal generators of the $U(3)$ Lie algebra and in the two three dimensional fundamental representations of quarks and antiquarks, they have the explicit matrix forms [14)]

$$F^{\ell m} = |3,\ell\rangle\langle 3,m| \qquad \text{for} \quad D(1,0) \Rightarrow 3$$

$$F^{\ell m} = -|\bar{3}m\rangle\langle\bar{3}\ell| \qquad \text{for} \quad D(0,1) \Rightarrow \bar{3}$$

Substituting now in Eq. (65), we exhibit the quark and antiquarks contributions to the scaling functions

$$F_{\pm}^{\ell_2 m_2,\alpha_2;\ell_1 m_1,\alpha_1}(\xi) = \delta_{\ell_2\ell_1}(1 \mp g_A)\, D_{m_2 m_1}^{\alpha_2\alpha_1}(\xi) + \delta_{m_2 m_1}(1 \pm g_A)\, D_{-\ell_2 -\ell_1}^{\alpha_2\alpha_1}(\xi) \tag{66}$$

For instance, the weak currents previously studied correspond to the following set of indices

$$\text{neutrino induced reactions} \begin{cases} \text{I spin} & \ell_1 = \ell_2 = 1 \quad m_1 = m_2 = 2 \\ \text{V spin} & \ell_1 = \ell_2 = 1 \quad m_1 = m_2 = 3 \end{cases}$$

$$\text{antineutrino induced reactions} \begin{cases} \text{I spin} & \ell_1 = \ell_2 = 2 \quad m_1 = m_2 = 1 \\ \text{V spin} & \ell_1 = \ell_2 = 3 \quad m_1 = m_2 = 1 \end{cases}$$

For the currents of the Cartan sub-algebra of SU(3), we must impose the zero trace condition associated with the baryonic current

$$F_{\pm}^{B\alpha_2;B\alpha_1}(\xi) = \frac{1}{9}\sum_j{}' \left\{(1\mp g_A)D_{jj}^{\alpha_2\alpha_1}(\xi) + (1\pm g_A)D_{-j-j}^{\alpha_2\alpha_1}(\xi)\right\} \tag{67}$$

and, for instance, we recover a formula like (24) for the electromagnetic current

$$F_{\pm}^{Q\alpha_2;Q\alpha_1}(\xi) = F_{\pm}^{B\alpha_2;B\alpha_1}(\xi) + \frac{1}{3}\left\{(1\mp g_A)D_{11}^{\alpha_2\alpha_1}(\xi) + (1\pm g_A)D_{-1-1}^{\alpha_2\alpha_1}(\xi)\right\} \tag{68}$$

3) The quantities $D_{kj}{}^{2\;1}(\xi)$ are matrices in the space of the irreducible representation $D(\lambda_1,\lambda_2)$ for the hadrons with indices α_1 and α_2. Using the well-known decomposition in SU(3)

$$D(1,0)\otimes D(0,1) = D(0,0)\oplus D(1,1)$$

We apply the Wigner-Eckart theorem in a straightforward way and we define six reduced distributions, three associated with quarks

$$D_{m_2m_1}^{\alpha_2\alpha_1}(\xi) = \delta_{\alpha_2\alpha_1}\delta_{m_2m_1}D(\xi) + \langle\alpha_2|\Omega_{m_2m_1}^{8A}|\alpha_1\rangle D_A(\xi) + \langle\alpha_2|\Omega_{m_2m_1}^{8S}|\alpha_1\rangle D_S(\xi) \tag{69}$$

and three associated with antiquarks

$$D_{-\ell_2-\ell_1}^{\alpha_2\alpha_1}(\xi) = \delta_{\alpha_2\alpha_1}\delta_{\ell_2\ell_1}\overline{D}(\xi) + \langle\alpha_2|\Omega_{\ell_1\ell_2}^{8A}|\alpha_1\rangle \overline{D}_A(\xi) + \langle\alpha_2|\Omega_{\ell_1\ell_2}^{8S}|\alpha_1\rangle \overline{D}_S(\xi) \tag{70}$$

The skew symmetric isometry Ω_{rs}^{8A} is the generator itself F_{rs}. The symmetric isometry Ω_{rs}^{8s} involves the product of generators

$$\Omega_{rs}^{8S} = \sum_n \left(F_{rn}F_{ns} + F_{ns}F_{rn}\right) - \frac{2}{3}\delta_{rs}\,\infty$$

where the Casimir operator $x = \sum_{mn} F_{mn} F_{nm}$ is given for the irreducible $D(\lambda_1, \lambda_2)$ representation by

$$x = \frac{2}{3}(\lambda_1^2 + \lambda_2^2 + \lambda_1 \lambda_2) + 2(\lambda_1 + \lambda_2)$$

In the trivial case where the hadron belongs to a singlet representation of SU(3) we have only two reduced matrix elements

$$D_{m_2 m_1}(\xi) = \delta_{m_2 m_1} D(\xi)$$
$$\bar{D}_{-\ell_2 -\ell_1}(\xi) = \delta_{\ell_2 \ell_1} \bar{D}(\xi)$$

as expected all the SU(3) directions being equivalent. If now the hadron belongs to an irreducible representation $D(\lambda_1, \lambda_2)$ with $\lambda_1 \lambda_2 = 0$ the symmetric isometry vanishes and we only have four independent reduced distributions.

4) The nucleon belongs to an octuplet and we now study this case in more detail. We use for α_1 and α_2 the same notations as for the currents with two indices

$$|\alpha_1\rangle \Rightarrow |a_1 b_1\rangle \qquad\qquad |\alpha_2\rangle \Rightarrow |a_2 b_2\rangle$$

The adjoint representation of the Lie algebra is given by [14)]

$$F_{rs} = \sum_n' \left\{ |rn\rangle\langle sn| - |ns\rangle\langle nr| \right\}$$

so that it is straightforward to compute the matrix elements of the skew symmetric and symmetric isometries between two octuplet states.

The consequences of SU(3) symmetry have been studied by Nachtmann [15)] in the form of positivity conditions. Because of the reduction of products of representations in SU(3)

$$D(1,1) \otimes D(1,0) = D(2,1) \oplus D(0,2) \oplus D(1,0)$$
$$D(1,1) \otimes D(0,1) = D(1,2) \oplus D(2,0) \oplus D(0,1)$$

he obtains three relations for the quark distributions and three relations for the antiquark distributions.

5) Let us consider here the more simple case of isotopic spin symmetry applied to the nucleon doublet. With $b_1 = b_2 = 3$, $a_1 = 1, 2$, $a_2 = 1, 2$ we get from (69) and (70)

$$D^{a_2 3; a_1 3}_{m_2 m_1} = \delta_{a_2 a_1}\delta_{m_2 m_1}\left[(D - 2D_S) - \delta_{m_1 3}(D_A - 3D_S)\right] + \delta_{m_1 a_1}\delta_{m_2 a_2}(D_A + 3D_S) \qquad (71)$$

$$D^{a_2 3; a_1 3}_{-\ell_2 -\ell_1} = \delta_{a_2 a_1}\delta_{\ell_2 \ell_1}\left[(\bar{D} - 2\bar{D}_S) - \delta_{\ell_1 3}(\bar{D}_A - 3\bar{D}_S)\right] + \delta_{\ell_2 a_1}\delta_{\ell_1 a_2}(\bar{D}_A + 3\bar{D}_S) \qquad (72)$$

where the ξ dependence has been dropped for simplicity.

Imposing only isotopic spin invariance, we have a decoupling between strange and non-strange quarks or antiquarks. The strange quark and antiquark parts are written from (71) and (72) as

$$D^{a_2 3; a_1 3}_{3\,3} = \delta_{a_2 a_1}(D - D_A + D_S)$$

$$D^{a_2 3; a_1 3}_{-3\,-3} = \delta_{a_2 a_1}(\bar{D} - \bar{D}_A + \bar{D}_S)$$

and we obtain the two first positivity conditions

$$D(\xi) - D_A(\xi) + D_S(\xi) \geq 0 \qquad (73)$$

$$\bar{D}(\xi) - \bar{D}_A(\xi) + \bar{D}_S(\xi) \geq 0 \qquad (74)$$

For the non-strange part the 4×4 matrix (71) for quarks and the 4×4 matrix (72) for antiquarks are reducible following the well-known property of the SU(2) representations

$$D(1/2) \otimes D(1/2) = D(0) \oplus D(1)$$

We then get in both cases two positivity conditions and a straightforward calculation gives

$$D(\xi)+2D_A(\xi)+4D_S(\xi)\geqslant 0 \tag{75}$$

$$D(\xi)-2D_S(\xi)\geqslant 0 \tag{76}$$

$$\bar{D}(\xi)-\bar{D}_A(\xi)-5\bar{D}_S(\xi)\geqslant 0 \tag{77}$$

$$\bar{D}(\xi)+\bar{D}_A(\xi)+\bar{D}_S(\xi)\geqslant 0 \tag{78}$$

6) The proton distributions $D_j^p(\xi)$ introduced in Section IV are linear combinations of D, D_A, D_S, $\bar{D}$, $\bar{D}_A$ and $\bar{D}_S$. The Clebsch-Gordan algebra is simple and the result is

$$\begin{aligned}
D_1^p(\xi) &= D(\xi)+D_A(\xi)+D_S(\xi)\\
D_{-1}^p(\xi) &= \bar{D}(\xi)+\bar{D}_A(\xi)+\bar{D}_S(\xi)\\
D_2^p(\xi) &= D(\xi)-2D_S(\xi)\\
D_{-2}^p(\xi) &= \bar{D}(\xi)-2\bar{D}_S(\xi)\\
D_3^p(\xi) &= D(\xi)-D_A(\xi)+D_S(\xi)\\
D_{-3}^p(\xi) &= \bar{D}(\xi)-\bar{D}_A(\xi)+\bar{D}_S(\xi)
\end{aligned}$$

The positivity constraints (73) - (78) due to isotopic spin symmetry are translated in the language of proton distribution as

$$D_3^p(\xi)\geqslant 0 \qquad\qquad D_{-3}^p(\xi)\geqslant 0 \tag{79}$$

$$D_2^p(\xi) \geq 0 \qquad D_{-1}^p(\xi) \geq 0 \tag{80}$$

$$2 D_1^p(\xi) \geq D_2^p(\xi) \tag{81}$$

$$2 D_{-2}^p(\xi) \geq D_{-1}^p(\xi) \tag{82}$$

The inequalities (81) and (82) are the only non trivial constraints, the others being the natural positivity. In the language of neutrino and antineutrino scaling functions (24) we obtain

$$2 \; G_{\pm}^{\bar{\nu}p}(\xi) \geq G_{\pm}^{\nu p}(\xi) \tag{83}$$

or, equivalently, using charge symmetry

$$2 \; G_{\pm}^{\nu n}(\xi) \geq G_{\pm}^{\nu p}(\xi) \tag{84}$$

CONCLUDING REMARKS

1) We have studied some aspects of the quark parton model for electroproduction, neutrino and antineutrino induced reactions. The scaling functions are described in terms of six distribution functions associated with the six types of quarks and antiquarks. The target being a nucleon, we use charge symmetry to relate the distribution functions for proton and neutron and we finally get four relations between structure functions characteristic of the quark parton model.

After integration of the scaling functions over ξ , we get sum rules measuring the mean number of each type of quarks in the hadron. The experimental situation for electroproduction is not complete enough to allow a clear conclusion.

The study of the first moment of the scaling functions gives a nice way to compare the model with experiment. We can check the consistency of the electron and neutrino data with the constraints of the model and we prove the existence of a sizeable amount of gluons in the nucleon using theoretical considerations based on the positivity of the distribution functions.

Finally, constraints due to symmetries are explicitly given in the particular case of isotopic spin invariance.

2) A simple way to remember the experimental situation is to consider a very simple model of the nucleon consisting of three quarks plus an arbitrary number of gluons. We have only two distribution functions $D_1^p(\xi)$ and $D_2^p(\xi)$ which can be determined from electroproduction data. The value of ε is found to be $\varepsilon = 0{,}50 \pm$ $\pm\, 0{,}07$.

The neutrino and antineutrino scaling functions are then completely known from the electroproduction ones. The weak current is left-handed, antiquarks being absent and the result $< N_q > = 3$ implies the low ξ behaviour of the scaling functions

$$\lim_{\xi \to 0} \xi\, D_{1,2}^{p}(\xi) = 0$$

The coefficients B and C for the total neutrino and antineutrino cross-sections are predicted from the electroproduction integrals I^{ep} and I^{en}. Using the experimental data we obtain

$$B^{\nu p} = 3 B^{\bar{\nu} n} = \frac{6}{5}\left(4 I^{en} - I^{ep}\right) = 0{,}38 \pm 0{,}12$$

$$B^{\nu n} = 3 B^{\bar{\nu} p} = \frac{6}{5}\left(4 I^{ep} - I^{en}\right) = 0{,}62 \pm 0{,}14$$

$$C^{\nu p} = C^{\nu n} = 0 \qquad C^{\bar{\nu} p} = B^{\bar{\nu} p} \qquad C^{\bar{\nu} n} = B^{\bar{\nu} n}$$

For the propane experiment, we deduce the following features

a. $A(\nu$ propane per nucleon$) = 0{,}45 \pm 0{,}10$;

b. $A^{\nu n}/A^{\nu p} = 1{,}63 \pm 0{,}10$;

c. no strangeness changing events predicted for neutrino induced events in the scaling region.

The agreement between theory and experiment is very impressive, even in this very crude picture of the nucleon.

3) The same quark parton model can be used to study the polarization effects in high energy inelastic lepton scattering. We get scaling when the nucleon polarization is made parallel or antiparallel to the longitudinal of the incident lepton.

We consider only inelastic electron scattering polarization. The polarization scaling function is the mean value of the operator $\sigma_z Q^2$ in the quark parton model sense as before the transverse scaling function was the mean value of the operator Q^2 as obtained in Eq. (14). The Bjorken sum rule for polarization [16] is derived as a consequence of the model as previously the Adler sum rule and the Gross-Llewellyn Smith sum rule.

The most interesting result [17] is that the polarization effects are predicted to be one order of magnitude larger with a proton target than with a neutron target and this feature is characteristic of the quark parton model.

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FIGURE CAPTIONS

Figure 1 :

Inclusive reaction $\ell + p \to \ell' +$ anything.

Figure 2 :

Charged weak currents and the SU(3) root diagram.

Figure 3 :

The two SU(3) fundamental representations and the quark and antiquark weights.

Figure 4 :

The coefficients B in the ν, $\bar{\nu}$ plane.

a The big triangle is the region allowed by positivity of the distribution functions.

b The oblique lines correspond to the inequality (59) with the electroproduction data (47) taken with errors.

c The vertical lines are the neutrino-propane data (57).

d The dashed triangle is the resulting region compatible with experiments and positivity.

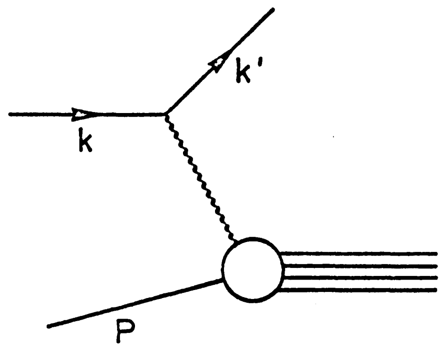

FIG.1

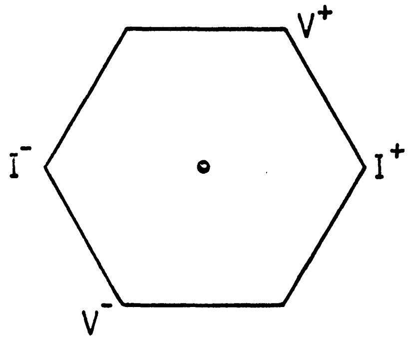

FIG. 2

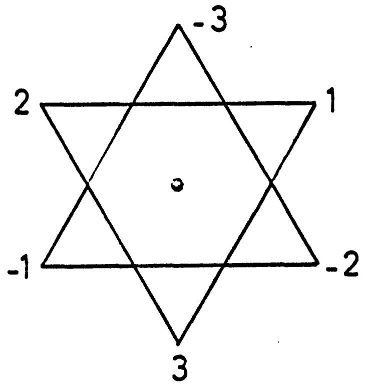

FIG. 3

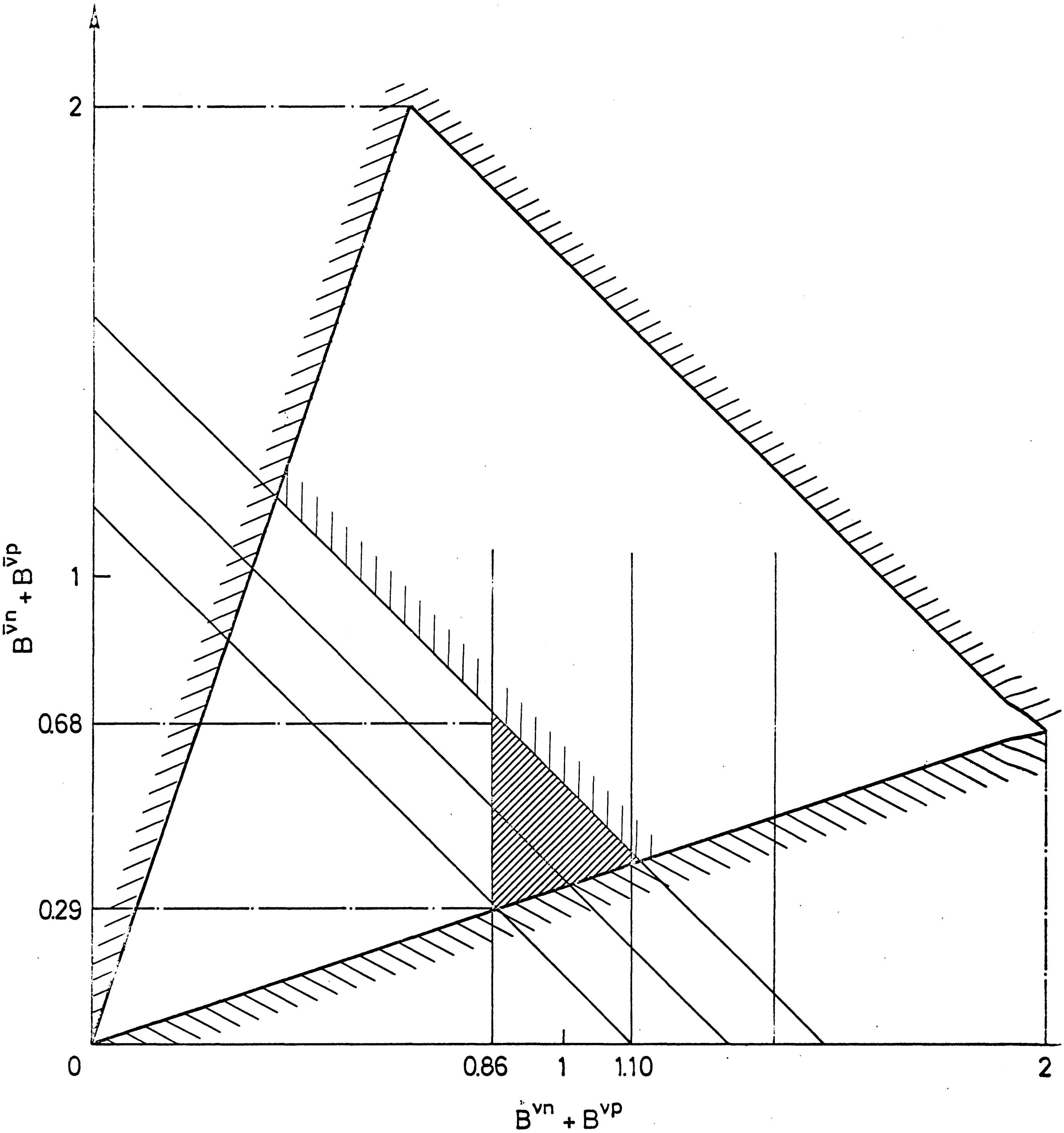

FIG.4