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COMMENTS ON RADIATIVE K_{e3} DECAYS

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ABSTRACT

Calculations of the radiative K_{e3} branching ratios have been performed with cuts in the photon energy as well as in the electron photon angle as imposed by the experimental set-up. We found good agreement with preliminary experimental results.

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1. The radiative decay modes $K^0_{e3\gamma}$ and $K^\pm_{e3\gamma}$ (i.e., $K^0_L \to \pi^\pm e^\mp \nu \gamma$, and $K^\pm \to \pi^0 e^\pm \nu \gamma$) have been recently studied by Fearing, Fischbach and Smith $^{1)}$, with the help of soft photon theorems and current algebra techniques. This allows them to compute the "branching ratios"

$$R^{\overset{0}{\pm}}(E) = \Gamma(K^{\overset{0}{\pm}}_{e3\gamma}, E_\gamma > E) / \Gamma(K^{\overset{0}{\pm}}_{e3}), \tag{1}$$

where E_γ is the photon energy in the kaon rest frame. Thus they obtain the theoretical value

$$R^0_{TH}(E = 30 \text{ MeV}) = 2.39 \cdot 10^{-2}. \tag{2}$$

The analysis of heavy liquid tracks for both processes, $K^0_{e3\gamma}$ $^{2)}$ and $K^+_{e3\gamma}$ $^{3)}$, is in progress at CERN. A published preliminary result for the former gives

$$R^0_{EX}(E \cong 30 \text{ MeV}) = (0.75 \pm 0.40) \cdot 10^{-2}. \tag{3}$$

It has been remarked $^{1)}$ that the discrepancy between these values R^0_{TH} and R^0_{EX} suggests that either : a) "structure dependent terms are much more important than expected", or b) "in the experiment, some radiative events have been .... incorrectly classified as non-radiative ones". The main purpose of this note is to examine this second point.

2. We are concerned with the following experimental situation. To identify $K_{e3\gamma}$ decay events, heavy liquid is used as a photon detector around the decay point. Thus the energetic electrons of K_{e3} decay undergo necessarily a strong external bremsstrahlung on the heavy liquid nuclei. If along the electron track the points of this bremsstrahlung and of K_{e3} decay cannot be resolved, the event will be misinterpreted as $K_{e3\gamma}$ decay. This major confusion is avoided, by knowing that the external bremsstrahlung is produced with a very small angle between electron and photon. Let us call it $\Theta_{e\gamma}$ in the kaon rest system. The experimental criterion $^{2)}$ rejects as suspicious the candidates for

$K_{e3\gamma}$ decay events with $\Theta_{e\gamma} < \Theta \cong 30^\circ$. The theoretical question is to know how many truly $K_{e3\gamma}$ decay events are wasted by this rigorous criterion.

Our computation is independent of that in Ref. 1), but has been carried out in a similar way 4). The resulting photon spectra for several cuts Θ are shown in Figs. 1 and 2, for the case of $K^0_{e3\gamma}$ and $K^+_{e3\gamma}$ decays. For $\Theta = 0^\circ$, we obtain of course the same curves as in Ref. 1); but a small cut, $\Theta = 10^\circ$, produces a drastic reduction in this photon spectrum. It is a phenomenon related to the logarithmic divergence for vanishing electron mass 5).

By numerical integration of these different spectra, the "branching ratios"

$$R^{\overset{0}{+}}(E,\Theta) = \Gamma(K^{\overset{0}{+}}_{e3\gamma},\ E_\gamma > E,\ \Theta_{e\gamma} > \Theta) \,/\, \Gamma(K^{\overset{0}{+}}_{e3}) \tag{4}$$

can be computed. They are given in Tables I and II. It can be seen that the value corresponding to the experimental cuts,

$$R^{0}_{TH}(E = 30\ \mathrm{MeV},\ \Theta = 30^\circ) = 0.79 \cdot 10^{-2}, \tag{5}$$

is in complete agreement with the experimental value in Eq. (3).

3. Let us discuss next whether any cut Θ is really needed, and if it is, what this cut should be. For the former purpose, we have evaluated the external bremsstrahlung which electrons proceeding from K^+_{e3} decay undergo in a 2 cm path inside light freon. This path length corresponds roughly to the distance along which the bremsstrahlung and K_{e3} decay points cannot be resolved. In this computation, the effect of screening of the freon nuclei by their electronic shell must be estimated 6). The results for several cuts in E_γ and for the cases of no screening and of complete screening are given in Table III. By comparison with the first line in Table II, it can be concluded that both phenomena are at least of the same order, and some cut Θ is really needed 7).

To estimate the amount of the most convenient cut, the angular distribution of internal and external bremsstrahlungs has been computed $^{8)}$. The result is shown in Fig. 3, where the forward peaks of both phenomena are compared in logarithmic scale. The external bremsstrahlung is completely negligible outside the very forward peak. The cross-over of both curves take place around $\theta_{e\gamma} = 6.2^{\circ}$ and for a cut $\theta = 10^{\circ}$ the ratio between the two phenomena is around 0.12. It would be therefore worth while to measure as carefully as possible for each event the angle $\theta_{e\gamma}$, in order to reduce the size of its critical cut.

Let us conclude with a comment on alternative a). This alternative acknowledges the weakness of applying soft photon theorems $^{9)}$, when the photon by hypothesis is hard. Indeed, the Low amplitude supplies the exact terms of order k^{-1} and k^{0}, k being the photon four-momentum. Thus, by squaring the Low amplitude, we obtain the exact terms of order k^{-2} and k^{-1}, i.e., the Burnett and Kroll intensity, and we obtain also a part of the term of order k^{0}. The contribution of each one of these three terms to the spectrum of the photon in K^{0}_{e3} decay is shown in Fig. 4 $^{10)}$. This figure shows that the contributions of these three terms are very different in the Low low energy region, let us say, for $E_{\gamma} < 30$ MeV. But the quantity R^{0}_{TH} $(E = 30 \text{ MeV})$ in Eq. (2) is just the integration of this total spectrum for $E_{\gamma} > 30$ MeV. In this high energy region, the relative significance of the three terms is not so different, and for $E_{\gamma} > 90$ MeV it is in fact reversed. A crucial question is therefore the contribution of structure dependent terms of order k^{0} or higher, which in principle could be important. However, it has been shown by the authors of Ref. 1) that these terms, if estimated with current algebra assumptions, are indeed very small. The agreement between experiment and the calculation with the appropriate cuts support this.

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TABLE I Branching ratio $R^{o}_{TH}(E,\Theta)$, as defined in Eq. (4).

	E = 10 MeV	E = 20 MeV	E = 30 MeV	E = 40 MeV
$\Theta = 0^{o}$	$4.99\cdot10^{-2}$	$3.28\cdot10^{-2}$	$2.37\cdot10^{-2}$	$1.79\cdot10^{-2}$
$\Theta = 10^{o}$	2.51	1.69	1.25	0.96
$\Theta = 20^{o}$	1.92	1.30	0.96	0.74
$\Theta = 30^{o}$	1.57	1.06	0.79	0.61
$\Theta = 40^{o}$	1.31	0.89	0.67	0.51
$\Theta = 50^{o}$	1.11	0.76	0.57	0.44

TABLE II Branching ratio $R^{+}_{TH}(E,\Theta)$, as defined in Eq. (4)

	E = 10 MeV	E = 20 MeV	E = 30 MeV	E = 40 MeV
$\Theta = 0^{o}$	$4.24\cdot10^{-2}$	$2.80\cdot10^{-2}$	$2.03\cdot10^{-2}$	$1.53\cdot10^{-2}$
$\Theta = 10^{o}$	1.78	1.22	0.91	0.71
$\Theta = 20^{o}$	1.20	0.83	0.63	0.50
$\Theta = 30^{o}$	0.87	0.61	0.47	0.37
$\Theta = 40^{o}$	0.65	0.46	0.36	0.29
$\Theta = 50^{o}$	0.48	0.35	0.27	0.22

TABLE III External bremsstrahlung of K^{+}_{e3} electrons for 2 cm path in light freon, for different cuts, E, in the photon energy.

	E = 10 MeV	E = 20 MeV	E = 30 MeV	E = 40 MeV
No screening	$2.85\cdot10^{-1}$	$1.80\cdot10^{-1}$	$1.27\cdot10^{-1}$	$0.95\cdot10^{-1}$
Complete screening	$2.69\cdot10^{-2}$	$1.82\cdot10^{-2}$	$1.36\cdot10^{-2}$	$1.05\cdot10^{-2}$

REFERENCES

1) E. Fischbach and J. Smith, Phys.Rev. 184, 1645 (1969); 1D, 957E (1970);

H.W. Fearing, E. Fischbach and J. Smith, Phys.Rev.Letters 24, 189 (1970); and Stony Brook preprint (1970).

2) G.R. Evans, M. Golden, J. Muir and K.J. Peach - Edinburgh; and I.A. Budagov, H.W.K. Hopkins, W. Krenz, F.A. Nezrick and R.G. Worthington - CERN, Phys.Rev.Letters 23, 427 (1969);

also private communications of H.W.K. Hopkins and K.J. Peach. They propose the value $E = 30$ MeV as reasonable, the materialization of softer photons being irrecognizable, and the value $\Theta = 30^o$.

3) D. Cundy, private communication.

4) The input amplitude is supplied by the Low expression, applied to the usual V-A non-radiative decay. Only one form factor, f_+, is significant. Its momentum transfer dependence can be neglected (to use the value $\lambda_+ = 0$ or $\lambda_+ = 0.03$ changes scarcely by 1% the $K_{e3\gamma}$ photon spectrum, when normalized to the K_{e3} decay). The trace of this amplitude squared has been covariantly integrated over pion and neutrino four-momenta. The resulting expression has been integrated analytically over the electron energy and numerically over the invariant mass of the pion and neutrino. Computation of traces, substitutions of four-vectors and of expressions corresponding to analytical integrations are conveniently performed by the symbolic manipulation program SCHOONSHIP, which has been written by M. Veltman for the CERN CDC 6500 and 6600 computers. As algebraic output a two-variable function has been obtained which occupies 64 and 13 punched cards for the cases of $K^o_{e3\gamma}$ and $K^+_{e3\gamma}$ respectively. It is ready to be used in a numerical FORTRAN program, which can display on line the curves of the different photon spectra.

5) Cf., T. Kinoshita, J.Math.Phys. 3, 650 (1962).

6) W. Heitler, "The Quantum Theory of Radiation", Oxford, Clarendon Press, 3rd edition (1965), pp.248-249.

Heitler's formulae need only to be integrated in E_γ and weighted with the electron spectrum of K_{e3} decay. The computation is easier for the K^+ case, kaon and nuclei being relatively at rest.

7) In the case of K_L^o decay in vacuum, this cut could be theoretically suppressed.

8) For the external bremsstrahlung, four successive integrations are required of the energy and angular distribution given by W. Heitler, loc.cit, p.244.
Similar angular integrations have been computed by
P.V.C. Hough, Phys.Rev. 74, 80 (1948),
but unfortunately Hough is interested in the angular distribution related to the incoming electron, which, in our case, is by hypothesis undetected.
The shape of this very forward peak, between 0.5^o and 0^o, is completely masked by the screening effect. For the case of internal bremsstrahlung, it can be computed well. Its maximum of 34.5 is at 0.22^o and falls down until 31.9 at 0^o.

9) F.E. Low, Phys.Rev. 110, 974 (1958);
S.L. Adler and Y. Dothan, Phys.Rev. 151, 1267 (1966);
T.H. Burnett and N.M. Kroll, Phys.Rev.Letters 20, 86 (1968);
J.S. Bell and R.P. Van Royen, Nuovo Cimento 60A, 62 (1969).

10) The computation of these photon spectra is a simple variant of the computation described in Ref. 6), by merely changing the input amplitude.

FIGURE CAPTIONS

Figure 1 : Photon spectrum for $K^{o}_{e3\gamma}$ decay. The effect of introducing different cuts in the angle, $\theta_{e\gamma}$, between electron and photon in the kaon rest system, is shown.

Figure 2 : Photon spectrum for $K^{+}_{e3\gamma}$ decay, with the same cuts in $\theta_{e\gamma}$ as in Fig. 2.

Figure 3 : Angular distribution of the $K^{+}_{e3\gamma}$ decay, compared with that of the external bremsstrahlung of K^{+}_{e3} electrons for 2 cm path in light freon. $\theta_{e\gamma}$ is the angle between electron and photon in the kaon and nuclei rest system. The assumed cut in the photon energy is $E = 30$ MeV.

Figure 4 : Photon spectrum for $K^{o}_{e3\gamma}$ decay. Contributions of the three terms in the squared Low amplitude are given. They are labelled by their order in the photon four-momentum k.

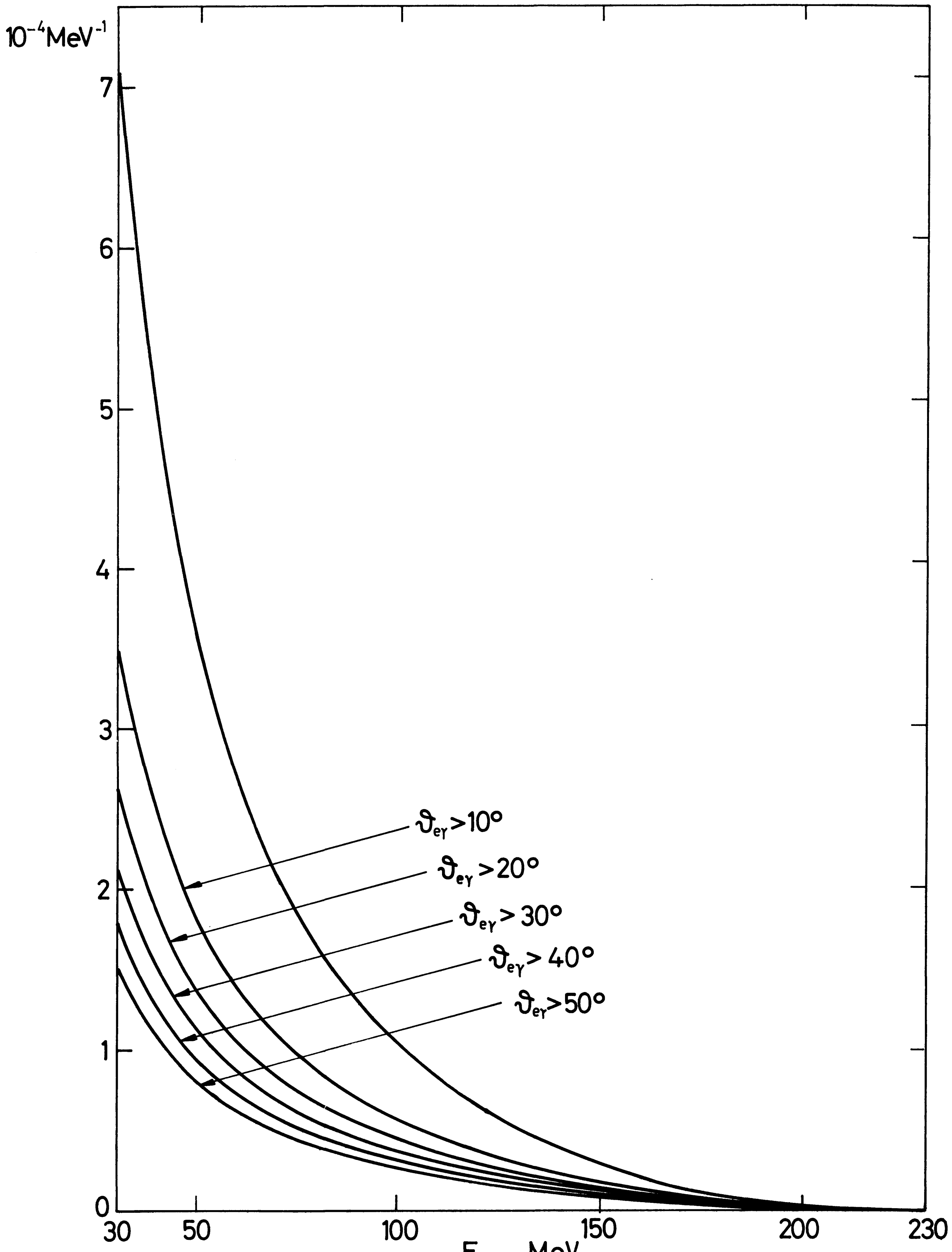
$10^{-4}\,\mathrm{MeV}^{-1}$
7
6
5
4
3
2
1
0
30
50
100
150
200
230
E_γ MeV
$\vartheta_{e\gamma} > 10°$
$\vartheta_{e\gamma} > 20°$
$\vartheta_{e\gamma} > 30°$
$\vartheta_{e\gamma} > 40°$
$\vartheta_{e\gamma} > 50°$

FIG.1

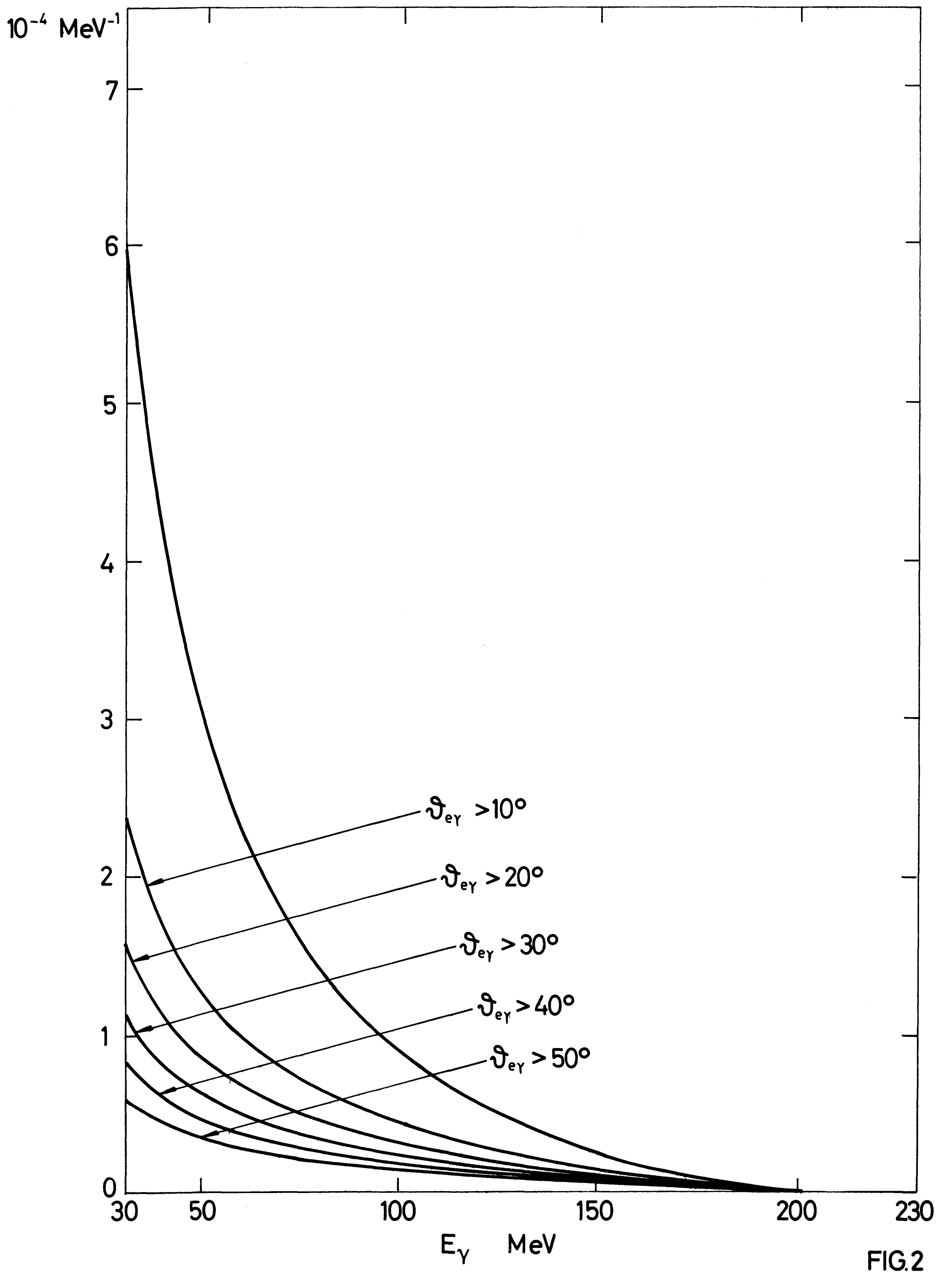
10⁻⁴ MeV⁻¹
7
6
5
4
3
2
1
0
30
50
100
150
200
230
E_γ MeV
ϑ_eγ >10°
ϑ_eγ >20°
ϑ_eγ >30°
ϑ_eγ >40°
ϑ_eγ >50°

FIG.2

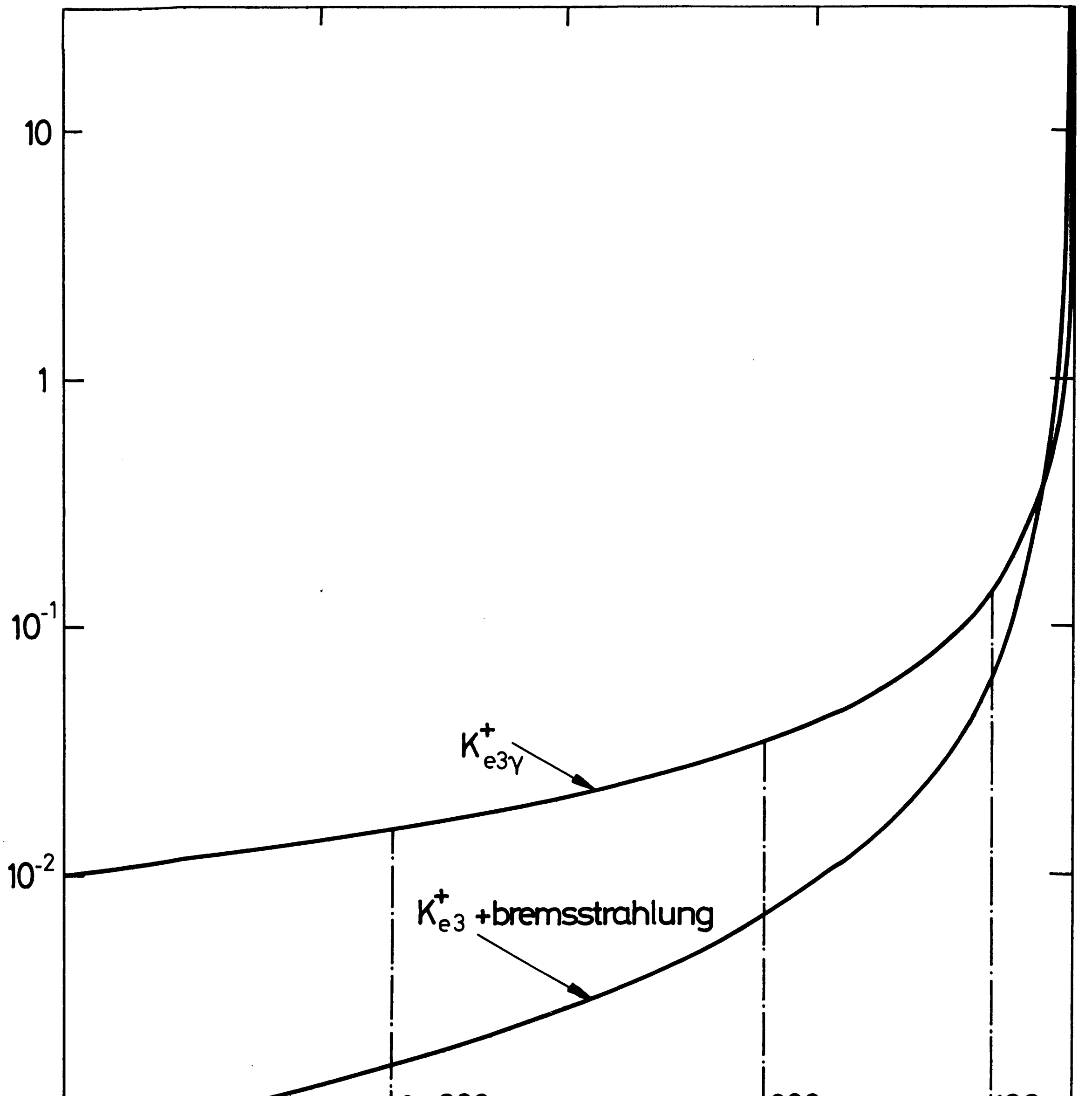
10
1
10^{-1}
10^{-2}
10^{-3}
$K^+_{e3\gamma}$
K^+_{e3} +bremsstrahlung
$\vartheta=30°$
20°
10°
0.80
0.85
0.90
0.95
1.00
$\cos\vartheta_{e\gamma}$

FIG. 3

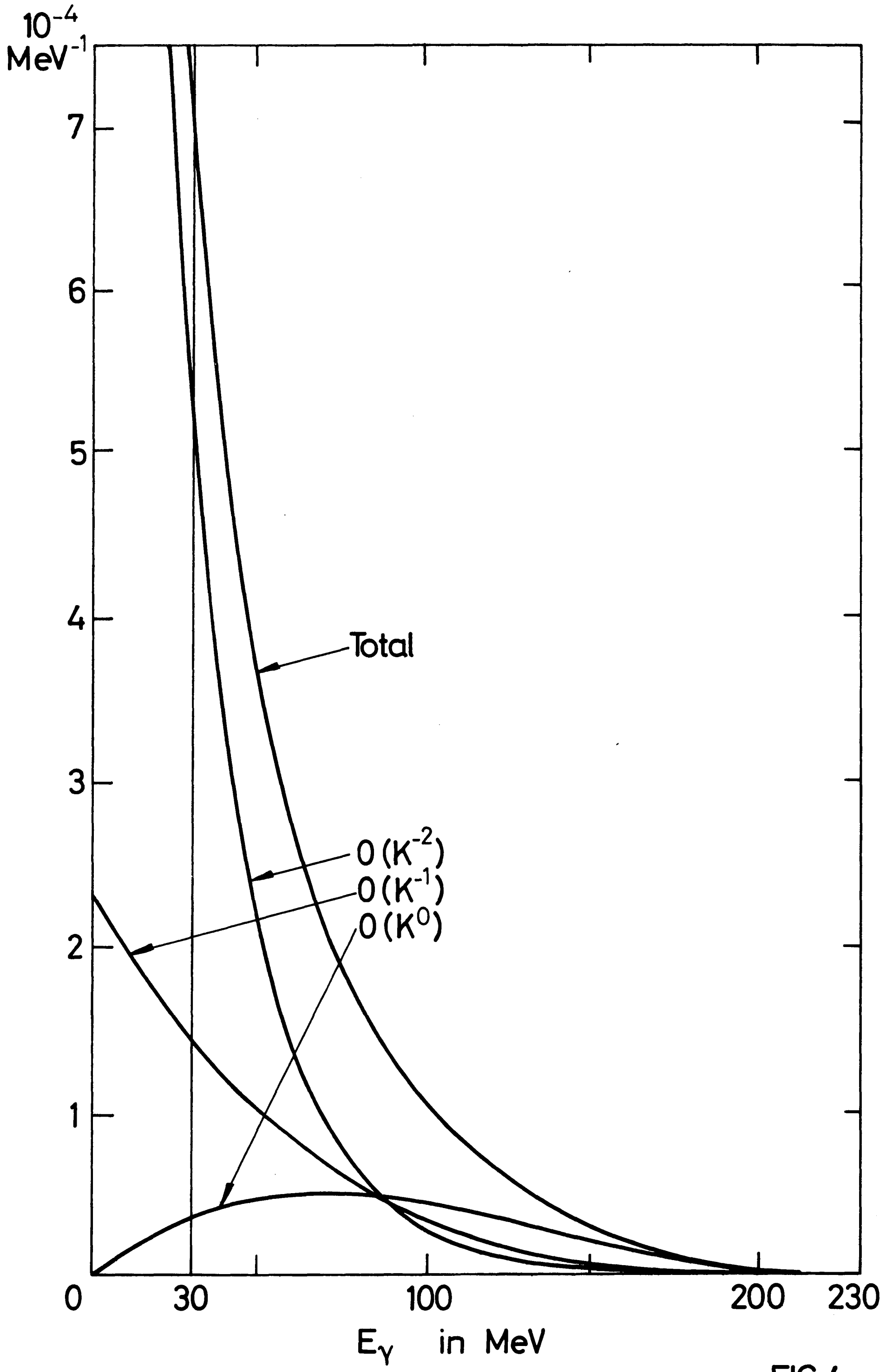
10^{-4} MeV^{-1}
7
6
5
4
3
2
1
0
30
100
200
230
E_γ in MeV
Total
$0(K^{-2})$
$0(K^{-1})$
$0(K^{0})$

FIG. 4