Archives

Ref.TH.1140-CERN

CONFIRMATION OF A NEW THEORETICAL VALUE FOR THE LAMB SHIFT

B.E. Lautrup, A. Peterman and E. de Rafael
CERN - Geneva

A B S T R A C T

We have performed a recalculation of the contribution to the Lamb shift from two fourth order vertex graphs. We agree with a recent calculation by Appelquist and Brodsky [1].

Ref.TH.1140-CERN
12 March 1970

There has been a recent change in the theoretical value for the Lamb shift. A new calculation [1] of the fourth order contribution to the slope of the Dirac form factor of the electron disagrees with the previous results [2],[3]. The new value for the slope, which contributes to order $\alpha^2(Z\alpha)^4mc^2$ to the Lamb shift, when added to the other radiative corrections [4] gives a $2S_{\frac{1}{2}} - 2P_{\frac{1}{2}}$ separation in hydrogen and deuterium, which is in excellent agreement with the results of recent experiments [5].

The contribution to Lamb shift is

$$\Delta E = \delta_{\ell 0} \frac{4(Z\alpha)^4 mc^2}{n^3} \sigma$$

where the quantity σ is defined as

$$\sigma = m^2 \left. \frac{\partial F_1(q^2)}{\partial q^2} \right|_{q^2=0}$$

It is important here to remark that we use a timelike metric.

The difference between the previous calculations [2],[3] and the recent one by Appelquist and Brodsky [1] (A.B.) lies first of all in an over-all change of sign for all terms, and secondly in a numerical disagreement between the non-infra-red contributions from the graphs a and b in Fig. 1 (the "corner graphs"). The authors of Ref. 2) (W.B.K.) calculated the infra-red divergent terms and part of the convergent terms analytically and estimated the remainder numerically, while Soto [3] performed the whole calculation analytically. The two results agree with each other (for the corner graphs) and the quoted value is

$$\sigma = \left(\frac{\alpha}{\pi}\right)^2 \left\{ -\frac{1}{12}\log^2\lambda^{-2} + \frac{1}{72}\log\lambda^{-2} + 2.432 \right\}$$

The new value by A.B. is the result of a completely numerical computation. They find (for the corner graphs)

$$\sigma = \left(\frac{\alpha}{\pi}\right)^2 \left\{ \frac{1}{12} \log^2 \lambda^{-2} - \frac{1}{72} \log \lambda^{-2} - 1.91 \pm 0.02 \right\}$$

In view of the crucial nature of this discrepancy we have considered it worth while to perform yet another evaluation of this quantity.

We reduced the graphs in Figs. 1a and 1b to parametric form by hand, inserting the parametrized vertex part, and afterwards parametrizing the outer loop by standard methods. In this way we obtained a division of σ into three parts

$$\sigma = \sigma_1 + \sigma_2 + \sigma_3$$

where σ_1 stems from the renormalization counter terms in the insertion and is easily calculable analytically

$$\sigma_1 = \left(\frac{\alpha}{\pi}\right)^2 \left\{ \frac{1}{6} \log^2 \lambda^{-2} - \frac{13}{24} \log \lambda^{-2} + \frac{5}{16} \right\}$$

The remainder $(\sigma_2 + \sigma_3)$ is originally a five-dimensional integral but a judicious choice of parameters allows one integration to be performed trivially. This integral was then divided into two parts, σ_2 and σ_3 of which σ_2 was infra-red divergent, but comparatively simple, and σ_3 was infra-red convergent, but extremely complicated. It was possible to extract the infra-red divergence analytically from σ_2 with the result

$$\sigma_2 = \left(\frac{\alpha}{\pi}\right)^2 \left\{ -\frac{1}{12} \log^2 \lambda^{-2} + \frac{19}{36} \log \lambda^{-2} + C_2 \right\}$$

where C_2 is constant for $\lambda \to 0$. By adding σ_1 and σ_2 we obtain the infra-red divergent parts given by A.B. The contribution σ_3 was evaluated numerically without trouble and the result is

$$\sigma_3 = \left(\frac{\alpha}{\pi}\right)^2 \left\{ -0.176 \pm 0.003 \right\}$$

The only remaining quantity to be determined is C_2. As it is difficult to keep track of the convergent parts left out during the extraction of the infra-red part we determined C_2 by calculating the full integral $\sigma_2(\lambda)$ numerically as a function of λ and afterwards cancelling out the divergent terms. Precisely, we fitted the quantity

$$C(\lambda) = \sigma_2(\lambda) - \left(\frac{\alpha}{\pi}\right)^2 \left\{-\frac{1}{12}\log^2\lambda^{-2} + \frac{19}{36}\log\lambda^{-2}\right\} + \\ + \left(\frac{\alpha}{\pi}\right)^2 \left\{\frac{5}{16} - 0.18\right\}$$

to the form

$$C(\lambda) = C + d_1\lambda\log^2\lambda^{-2} + d_2\lambda\log\lambda^{-2} + d_3\lambda \\ + d_4\lambda^2\log^2\lambda^{-2} + d_5\lambda^2\log\lambda^{-2} + d_6\lambda^2$$

This quantity will approach the non-infra-red divergent term, C, in σ for $\lambda \to 0$. Our result was (see also Fig. 2)

$$C = -1.95 \pm 0.05$$

$$d_1 = 0.6 \pm 1.1 \qquad d_4 = -6 \pm 9$$
$$d_2 = -11 \pm 22 \qquad d_5 = -17 \pm 34$$
$$d_3 = 73 \pm 128 \qquad d_6 = -77 \pm 134$$

This is in excellent agreement with the result of A.B. We remark that the uncertainty in C is somewhat larger than the uncertainty quoted by A.B. (± 0.02) mainly because we fitted with a large number of background terms. We included these in order to explore the possible influence of other background terms than those taken by A.B. in the determination of C. As can be seen from Fig. 2, $C(\lambda)$ is roughly constant over four orders of magnitude in λ^2.

We should like to mention that this calculation is independent of the calculation of A.B. as far as the analytical evaluation goes. Unfortunately, it was necessary to use the same integration subroutine for the numerical integration as was used by A.B. We attempted first to use standard Gaussian methods, but they turned out to be reliable only for $\lambda^{+2} \geq 10^{-3}$ and did furthermore not give any idea about the uncertainty on the result. A program written by G. Sheppey which was used by A.B. and which is described in Ref. 6), gave reliable results as far down as $\lambda^2 \gtrsim 10^{-9}$, and furthermore yielded an estimate of the uncertainty in the values. It has been thoroughly tested by the authors of Ref. 6) and the mere constancy of $C(\lambda)$ for small λ amplifies the confidence we have in its use.

Finally, we agree with A.B. in the over-all sign difference. One of us (B.E.L.) has furthermore checked the contribution from the vacuum polarization term (Fig. 1c) analytically with the result.

$$\sigma_{vacpol} = \left(\frac{\alpha}{\pi}\right)^2 \left\{ \frac{77}{864}\pi^2 - \frac{1099}{1296} \right\}$$

which has the opposite sign of the Soto value [3].

ACKNOWLEDGEMENTS

The authors are grateful to Dr. Stanley J. Brodsky for communication of the results of Ref. 1) prior to publication.

REFERENCES

1) - Th. Appelquist and S.J. Brodsky - SLAC Preprint (1970).

2) - J. Weneser, R. Bersohn and N.M. Kroll - Phys.Rev. 91, 1257 (1953).

3) - M.F. Soto, Jr. - Phys.Rev.Letters 17, 1153 (1966).

4) - G.W. Erickson and D.R. Yennie - Ann.Phys.(N.Y.) 35, 217 (1965) ; ibid. 35, 447 (1965).

5) - For a review, see :

S.J. Brodsky - "Status of Quantum Electrodynamics", Proceedings of the 4th International Symposium on Electron and Photon Interactions at High Energies, published by the Daresbury Nuclear Physics Laboratory (1969).

6) - J. Aldins, S.J. Brodsky, A.J. Dufner and T. Kinoshita - SLAC Preprint (1970).

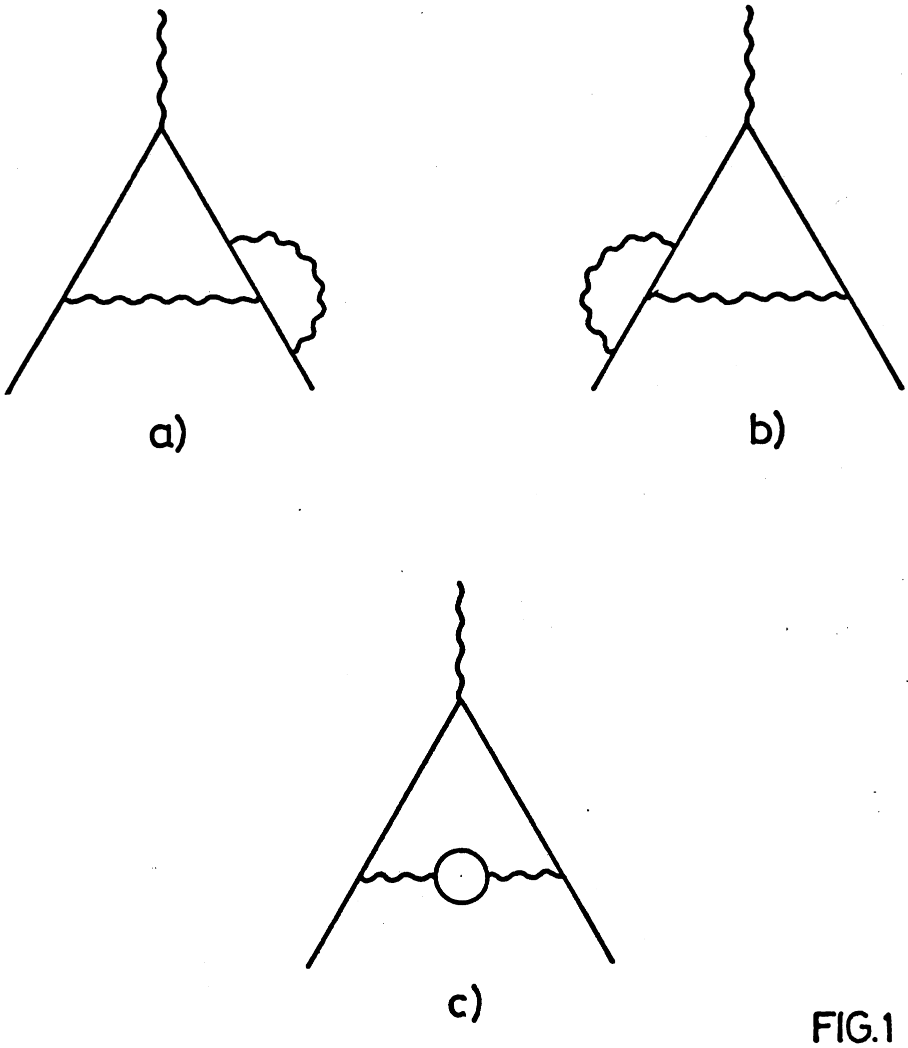

FIG.1

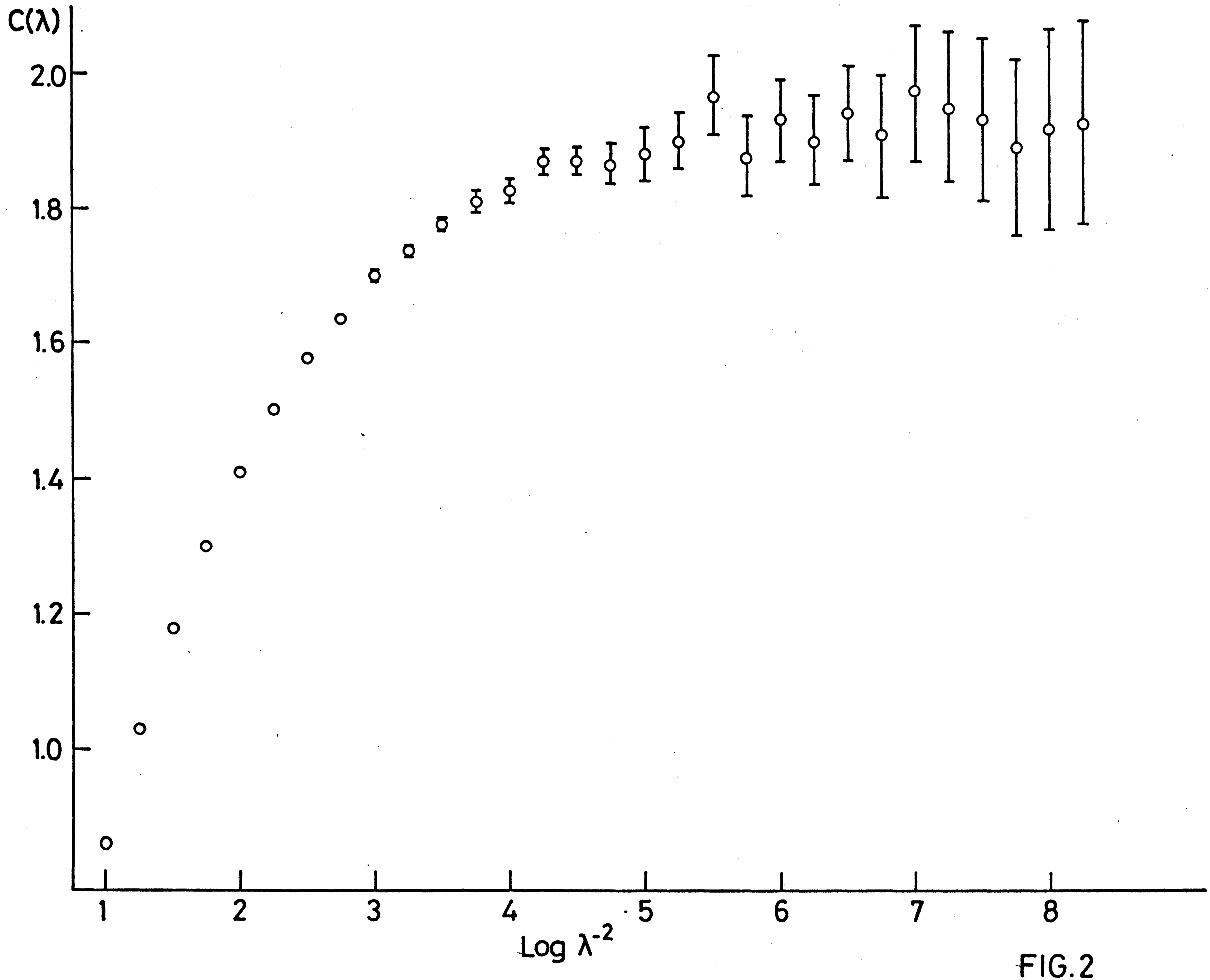

FIG. 2