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SOFT AND REAL PIONS IN NUCLEI*)

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In the last few years some very interesting studies have been made of the interactions of pions of variable mass. As you may imagine, these investigations have been carried out by theoreticians, although it would of course be very interesting if the experimentalists also could vary the mass of the pion continuously.

As the mass of the pion is pushed down towards zero some very remarkable theorems emerge: one of them says that the elastic scattering of such pions at low energies becomes universal; it depends only on the isotopic spin of the target and not on its size, shape or structure. The universal formula that results is the following: the scattering length a is related by an isospin factor to a universal constant L which depends on the decay constant of the pion f_π

$$a = L\ 2\underset{\sim}{t}\cdot\underset{\sim}{T} \qquad L = \mu/8\pi f_\pi^2$$

This formula, that was derived by Weinberg and Tomozawa, has been applied to elementary particles and it works very well. But it is a universal expression and it should apply to any target, to nuclei and to bigger systems like a water melon or the moon. Now in these big systems we do not expect the scattering length to depend only on the number of protons and neutrons. For these systems we would rather expect the scattering to be a measure of the size. Already light nuclei show important deviations from this simple expression.

Of course the real mass is not zero and we should perhaps not be surprised by deviations. But the simple expression works for

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elementary particles and we may well wonder by what mechanism exactly the mass causes the deviations. Many arguments have been given: in particular, the many low-lying excited states in these systems have been blamed. There are millions of states of energy well below the pion mass and as we extrapolate to zero mass we encounter successively all these states. As this did not look harmless, a common reaction of elementary particle physicists has been to forget about nuclei.

But nuclear physicists are not allowed to forget about nuclei and it is a very interesting problem to find the right way to look at these soft pion theorems in order to apply them. The secret of this can be guessed at quite easily just by looking at the previous expression for the scattering length. This is proportional to the pion mass and as this mass goes to zero the amplitude, and therefore the threshold interaction, also goes to zero. If there is no interaction the pion is like a neutrino penetrating fully the scattering system. The scattering is then given by the Born approximation. Nobody is surprised that in the Born approximation the scattering is proportional to the number of constituents, even for the moon.

Why then should the Born approximation be good for the nucleon and bad for nuclei and the moon? In the nucleon case we have only one scale, the ratio μ/m of the pion to the nucleon mass, which is a small quantity. In nuclei we have other scales than the ratio of the masses:

- One is the ratio of the radial extent to the pion Compton wavelength μR. This is far from small and one therefore expects important size effects. This is indeed the case.
- Another scale characterizes the granular structure of the nucleus. The typical excitation energies representative of this structure ε_N are much smaller than the pion mass $\varepsilon_N/\mu << 1$. However, the scale associated with this structure is not the huge ratio $\mu/\varepsilon_N >> 1$, but instead $\mu d \approx 1$, where d is the correlation distance, essentially the internucleon distance. These excitations are then not disastrous, as they were thought to be.

Before giving the quantitative arguments to support these statements, I first want to remind you of the ingredients that enter into the composition of these soft pion theorems. They are essentially three:

i) the partially conserved axial current hypothesis, PCAC, which relates the divergence of the axial current to the pion field Φ: $\partial_\nu J_\nu^A = f_\pi \mu^2 \Phi$;

ii) the current algebra which provides us with a certain number of basic commutators which are used in the derivation of these low-energy theorems;

iii) the soft pion assumption $q_\lambda = 0$ which takes all the components of the pion four momentum equal to zero.

The question is now how to generalize the expressions obtained with this soft pion assumption to the real pion? How should we relate the physical amplitude to the universal constant L? This has been done in the nucleon case by de Alfaro-Fubini-Furlan-Rosetti [1,2] and we apply their method to the nuclear case [3].

We take the equal time commutator of two axial charges Q_A^+ and Q_A^-. Its expectation value between the two nuclear ground states (at rest) is from the algebra of currents proportional to the third component of the isospin.

$$\langle N | [Q_A^+ \, Q_A^-] | N \rangle = 2M \tau_3 \ .$$

This particular commutator will lead us to the isospin antisymmetric amplitude a^-, which describes isospin effect and charge exchange. The symmetric amplitude will be discussed later. This commutator is then expanded by introducing a complete set of states; and let us see which ones are of interest to us. They are:

1) The nuclear states alone. They correspond to the nucleonic degrees of freedom without pion. The axial charge has a negative parity and can carry only to the states of negative parity relative to the ground state. The ground state itself gives no contribution.

2) The second important states are the rescattering states where the nucleus is left in its ground state and a pion is also present.

3) There can be inelastic excitation of a nucleus with a pion present.

There are also many other states which could contribute like the states with more than one meson. However, it is known from the study of the corresponding problem in the nucleus case that these states are unimportant and we will neglect them also in the nuclear case.

We proceed then by replacing the matrix element of the axial charge by that of the pion field using PCAC. And doing this we get the following relation

$$a^- = L + \text{corrections} \ .$$

The important point is that the threshold amplitude is taken for the real pions and not for the synthetical pions of zero mass, so that we have an expression for the corrections to the soft pion limit.

What are these corrections? They are what is left of the expansion of the commutator when we have extracted the physical threshold amplitude. I do not want to enter into the technical details but what remains is essentially the three contributions that we mentioned. Only a piece of the second one has been used explicitly in the threshold amplitude, the so-called semi-disconnected part.

I discuss now these corrections starting with the most important one -- the coherent rescattering. That is the nuclear size effect. If we keep only these terms we can write a dispersion relation which involves the matrix elements f_{0q} for the off-shell scattering of a pion of momentum zero to a momentum q

$$a^{-} = L + \frac{1}{2\pi^2} \int d^3q \frac{|f_{0q}|^2}{q^2\left(1 + \frac{q^2}{\mu^2}\right)^{3/2}} .$$

For simplicity we have not written the isospin indices in f_{0q}. Since we have to treat off-shell effects, it is rather natural to introduce an optical potential for the pion nucleus interaction. For potential scattering the corresponding dispersion relation can be written and it is very similar indeed. The amplitude a is the sum of the Born amplitude (the characteristic constant in a non-relativistic dispersion relation for potential scattering) and of an integral over off-shell matrix elements

$$a = a_{Born} + \frac{1}{2\pi^2} \int d^3q \frac{|f_{0q}|^2}{q^2} .$$

The integrals in the two relations have the same structure so that if we take the pion to be non-relativistic in the first one ($q \ll \mu$) they look precisely the same. [The convergence of the integrand is rapid enough in moderately light nuclei ($A \lesssim 30$) to take the pion non-relativistic.]

In view of the similarity of the two equations, we can then try to compare the Born term to the universal constant L. The deviation of the amplitude from the soft pion limit is then the distortion effect. This is of course approximate since we did not include the other contributions to the sum rule. But we can see how this interpretation stands the comparison with the experiments.

From π mesic atom data the charge exchange integral is found to have a volume integral so that

$$\frac{1}{2\pi} \int V\, d^3x = a_{Born} = 0.094\ \mu^{-1} .$$

In our optics this should be the universal constant $L = 0.09\ \mu^{-1}$. The test is quite successful. It is a sensitive test; it is not at all indifferent to interpret the soft pion value as the physical amplitude or as the Born value. These two quantities are very different. For a nucleon number A = 18 for instance, the amplitude $a^{(-)}$ is $0.047\ \mu^{-1}$, only one half of the Born value.

We have now to check that the remaining contributions to the sum rule do not destroy this beautiful agreement. The first group of intermediate states to consider are the excited states of the nucleus with no pion present. If the nucleons were at rest in the nucleus there would be no contribution, for an axial charge has no matrix elements between nucleons at rest. Because of the fermi motion there is an effect but it is a small one ($\sim$ 4%) [4]. This effect is associated with the scale μd.

However, this is not the whole story for these states because there exists also pion absorption. For excitation energies of the order of the pion mass, pion absorption occurs by a many-body process and this has not been included in the previous description. This effect is somewhat hard to estimate and although no good argument can be given for its detailed magnitude the absorption rate of pions in nuclei indicates that this effect is small, probably of a few per cent only.

The next group of states to be discussed are the inelastic excitations of the nucleus by the pion. They can be related to the correlations in the nucleus. They give two kinds of terms:

- First, the correlation that exists for the free nucleon, the self correlation. It is the rescattering effect on the nucleon itself and it increases the amplitude by 6 to 9%.

- A second correction arises by the pair correlations. This is the effective field correction in a multiple scattering theory. There is an absence of matter around one particle which produces a distortion of the pion wave. Its effect is opposite to that of the self correlation (they are complementary since they correspond to the absence or presence of nuclear matter). Its value depends on the nucleus. In the region $10 < A < 20$ and for isospin ½ nuclei the pair correlation nearly compensates the self correlation effect so that no net effect results from the correlations.

We summarize now the main steps which have improved the approximations starting from the soft pion limit to reach the physical amplitude. Only the rescattering states are included and we have indicated the pion wave which corresponds to every step.

soft pion	+	coherent rescattering	+	inelastic excitations	$\longrightarrow$	physical amplitude
$\downarrow$		$\downarrow$		$\downarrow$		
incident wave (Born approximation)		average wave in the medium		effective wave at the scatterer		

Can something similar be done for the isospin independent part of the interaction which is responsible for the bulk of the energy shifts in π mesic atoms? In principle it can, but instead of considering the commutator of two axial charges we have to take the commutator of an axial charge with the divergence of an axial current. This is not as well defined an object as the previous one. It is not given by the fundamental identities of the current algebra and its introduction necessitates other hypotheses.

However, we know from the nucleon case that this commutator produces a small scattering length ($\frac{1}{2}|a_n + a_p| < 0.02\ \mu^{-1}$) and indeed, theoretically, we expect this quantity to be of order μ^2. There is then no term in the expansion of first order in the pion mass. In order to have a consistent expansion it is essential to include all the terms of order μ^2, in particular the correlation effects. The soft pion limit plus the inelastic excitations gives a scattering length

$$a^+ \propto \frac{a_n + a_p}{2} - \frac{(a_n + a_p)^2 + 2(a_n - a_p)^2}{4} \left\langle \frac{1}{r} \right\rangle_{corr}$$

where $\langle 1/r \rangle_{corr}$ is the average value of $1/r$ over the correlation function. In a multiple scattering theory the normal parameter of the expansion is the single scattering amplitude and there is no stringent reason why the second term should be included. Here instead the parameter of the expansion is the pion mass. To be consistent the second term has to be included since both are of order μ^2. In fact it even dominates the first one (it accounts for about 70% of the potential).

The coherent rescattering represents, as previously, the distortion of the pion wave.

The interpretation that we propose for the soft pion limit as a Born approximation is not restricted to the elastic scattering. Another example is the radiative capture of the pion $\pi^- + A \rightarrow \gamma + B$.

In the soft pion limit the amplitude is proportional to the matrix element $\langle A|J_A^\mu \varepsilon_\mu|B\rangle$, of the product of the axial current with the photon polarization vector. This relates this process to the axial part of β decay and μ capture. However, the finite pion mass introduces deviations from this expression. A similar method as before shows that the most important deviation is the distortion of the pion wave and that the soft pion limit is essentially the

Born approximation for the interaction [5]. This gives the effective Hamiltonian for this transition $\mathcal{H}_{eff} = (e/2f_\pi)J_A^\mu \varepsilon_\mu$. The inelastic excitation here also represent the effective field correction.

To conclude my talk I want to summarize what we have learned. First we have a reasonable answer to a problem interesting in itself: what do the soft pion theorems become in nuclei? What is the meaning of the soft pion limit? The fact that it is such a simple thing as the Born approximation for the interaction should rehabilitate the nucleus in the eyes of the high-energy physicists. The nucleus is not such a complicated thing that these theories cannot be handled and we can make the link with the multiple scattering theory and the impulse approximation.

These methods also provide the effective Hamiltonian for the interaction, independently of the impulse approximation.

Finally, we learn something about the applicability of PCAC to nuclei. The rapid variation of nuclear matrix elements of the axial currents due to anomalous thresholds has cast doubt about this applicability. It is certainly true that the soft pion limits are not to be interpreted as the ultimate predictions for the physical amplitudes. These deviate appreciably from that limit and these deviations reflect the variations of the matrix element of the axial current. But these deviations have a calculable physical interpretation.

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