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MUON STORAGE RING - III

pplication to polarized μ - p scattering

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The proposed storage ring for ~ 1.4 GeV/c muons will be a good source of pure muons, and can be used for other experiments in addition to (g - 2). In particular, by introducing a hydrogen target into the ring one can study μ - p scattering. In such an experiment

(i) pion contamination is eliminated by π decay

(ii) one can use a small H_2 target with multiple traversals

(iii) the muons are polarized and precessing. At certain times the spin is transverse to the left, and at other times is transverse to the **right**. Therefore one can measure the asymmetry in the scattering of polarized muons with good cancellation of geometric errors.

Estimates for such an experiment are presented below.

1) Stored muon intensity

As it is no longer necessary to have the muons turn in an almost uniform field the ring would be reshimmed for such an experiment. In general one would introduce straight sections for the actual experiment (field free region), and could perhaps include quadrupoles or alternating gradients to increase the acceptance, and therefore the intensity. As an example, however, we consider a weak focussing ring with n = 0.25 and aperture 10 cm (vertical) x 30 cm (horizontal). (For these experiments the whole aperture of the magnets would be useful).

The basic formulae are the same as before*. But now

*) See "Calculations on Muon Storage Ring Project", December 20th, 1962 referred to as II.

$$\lambda_H = \rho(1-n)^{-\frac{1}{2}} = 1.16\rho$$

$$\lambda_V = \rho\, n^{-\frac{1}{2}} = 2\rho$$

Hence the beam misses the target vertically on alternate turns. On the 2nd turn the rotation in the horizontal phase plane is $\sim 120^\circ$, with the result that nearly all the beam misses horizontally, and goes on to make 6 turns. The time required is 380 ns, and during this interval 0.8 of the pions decay.

The typical $\pi - \mu$ decay angle is 11 mR (projected angle 8 mR) compared with a vertical acceptance of 8.5 mR and horizontal acceptance 43 mR. Taking a loss due to vertical decay angle of 0.7 and following through the usual reasoning one finds

$$N_\mu = 0.21 \frac{ab^3}{\rho^4} n^{\frac{1}{2}} (1-n)^{\frac{1}{2}} p_\pi \frac{d^2n}{d\Omega dp_\pi} \qquad (1)$$

Inserting $a = 5$, $b = 15$, $\rho = 300$ cm, $p_\pi = 1.35$ GeV/c and $\frac{d^2n}{d\Omega dp_\pi} = 2.3$ sterad^{-1} (GeV/c)$^{-1}$ we find for 10^{11} protons per pulse

$$N_\mu = 80{,}000\ \mu/\text{pulse}. \qquad (2)$$

For the usual $5 \cdot 10^{11}$ protons/pulse we get

$$4 \times 10^5\ \mu/\text{pulse} \qquad (3)$$

2) Scattering experiment

The cross-section for $\mu - p$ elastic scattering calculated from Rosenbluth formula for 1.4 GeV muons is plotted versus momentum transfer q in fig. 1. Hofstadter form factors have been included. One sees for 900 MeV/c momentum transfer,

$$\frac{d\sigma}{d\Omega} = 10^{-32}\ \text{cm}^2. \qquad (4)$$

The thickness of hydrogen that the beam can traverse is limited by energy loss rather than multiple scattering. The aperture available for muon storage is ~ 22 cm horizontally: $\Delta p/p = 22/300 =$ 7% total spread. Allowing an energy loss of 3.5% we find

$$\Delta E = 47 \text{ MeV}$$

corresponding to 11 g/cm^{-2} (5)
of hydrogen.

A typical scattering apparatus is shown in fig. 2. The target is 4 cm in diameter and 5 cm long. It introduces, on the average, into the pipe of area 220 cm^2 a mass of hydrogen of $20\pi \times .07 \div 220 = 20\ mg/cm^2$ per turn. Hence for 11 gm one requires 550 turns which takes 31.5 μs, or 1.1 μ- life times.

In fig. 3 we plot the angles of the scattered μ and recoil proton versus momentum transfer. With detectors arranged symmetrically and each embracing the angular range $30^\circ - 60^\circ$ one covers a range of q from 660 - 1060 MeV/c and the solid angle $d\Omega =$ $(\cos \Theta_1 - \cos \Theta_2)\ d\phi$.

As we can detect μ on either side, $d\phi$ can easily be $\sim\pi$. Therefore

$$d\Omega = 0.366 \times \pi = 1.15 \quad (16)$$

Before calculating the rate of observed events one must correct the stored muon intensity (i) for decay before the scattering observation is started and (ii) for decay during the time we are sensitive to scatters. Let us allow 100 turns, (6 μs) for stable particles to be absorbed and pions to decay (24 mean lives): The muon intensity dropsto 0.79. After 31.1 μs the intensity has dropped to 0.33: therefore the mean intensity during the observations is ~ 0.55 (7)

Combining statements (3) to (7) one finds for the number of observed events per pulse

$$\begin{aligned} N &= 4.10^5 \times 0.55 \times 11 \times 6 \times 10^{23} \times 10^{-32} \times 1.15 \\ &= 0.015 \text{ per pulse} \\ &= 0.45 \text{ events/minute.} \end{aligned} \quad (8)$$

This is a rather substantial rate, but some caution is necessary.

(i) Not all these events will be at 900 MeV/c momentum transfer, because we have used a solid angle which covers 660 - 1060 MeV/c. For ± 10% in momentum transfer the number of events would be reduced by a factor ~ 2.

(ii) For measurement of the asymmetry in the scattering only the moments when the muons are transversely polarized will be useful. This in effect reduces the rate by a factor 2. However in this case there is less need to define the momentum transfer closely. Taking an effective rate 0.25/minute we shall measure the asymmetry to ± 5% in 1600 min, say 3 hours, or to ±1.8% in a day.

In the scattering apparatus sketched in fig. 2 the two spark chambers (SC) serve to define the angles of μ and p. The μ is required to count in No. 1, penetrate 40 cm of lead, and count again in No. 2. On the other side the proton counts in No. 1' and is stopped before reading No. 2'.

Note that this triggering signature rejects events due to stable particles stored in the ring. If the scattered particle is e^- or $\bar{p}$ it cannot reach counter No. 2.

The final check that the scattering is μ - p and not $\bar{p}$ - p is provided by the angle measurements. In fig. 4 we plot the opening angle for events of the two types, and we see that they can be clearly distinguished.

3) Background of stable particles in the ring

The analysis is similar to that given earlier but the scattering is now due to the H_2 target. With 20 mg of H_2 per turn we find for the spread in the image after 6 turns (see equation (18) of II)

$$\sigma = \lambda\!\!\!^{-} \; \frac{15}{p\beta c} \left(\frac{S}{X_o} \right)^{\frac{1}{2}}$$

$$= 1.16 \times 300 \times .01 \times \left(\frac{1}{4000}\right)^{\frac{1}{2}}$$

$$= .05 \text{ cm}$$

5490/p

Hence with the same target arrangement as proposed in II, the $\bar{p}$ contamination should be less than 10^{-6} and electron contamination less than 10^{-4}. These levels are completely satisfactory. The proposed counter system should eliminate any residual $\bar{p}$ and e^- events rather effectively, and if necessary we could also use water Cherenkov counters in the scattered beam to throw out the anti-protons.

4) Theoretical aspects

An asymmetry in the scattering of polarized muons cannot arise from one photon exchange. Its presence would therefore be a measure of the importance of 2 photon exchange in the scattering process, and would be an important check on the approximation used in deriving form factors from e - p and μ - p scattering.

Very little is known about 2 photon exchange scattering, but it seems reasonable to expect asymmetries ~ 5 - 10%. The main point of the measurement would be to study this process.

5) Conclusion

Use of the storage ring for μ - p scattering seems to be quite practical, and would give information on the polarization properties of the scattering which probably cannot be obtained in the near future by other means.

The author would like to thank Dr. Fries for the use of his tables of the μ - p scattering cross-section and kinematics, and Dr. J. C. Sens for calculations of μ - p and $\bar{p}$ - p kinematics.

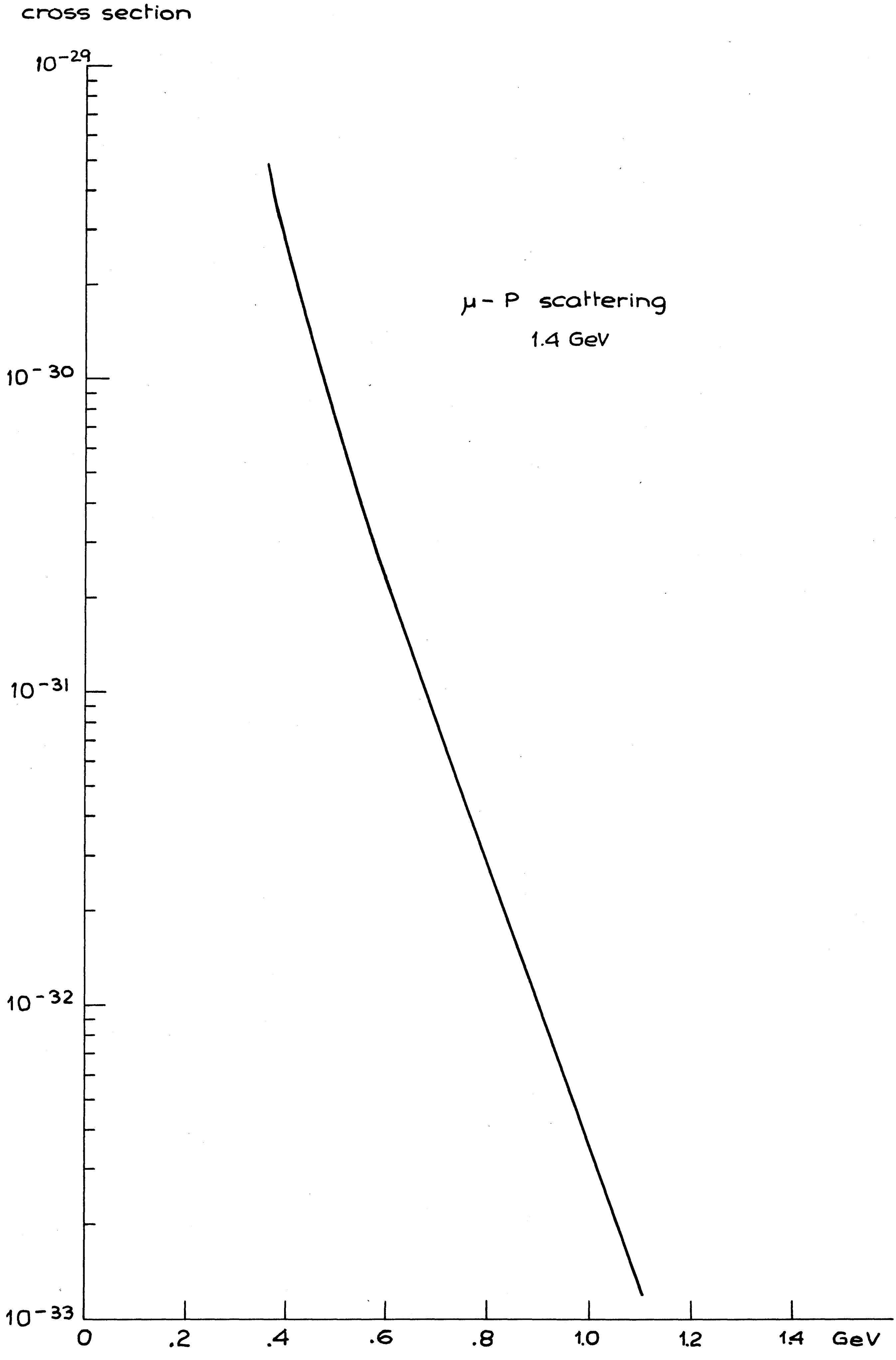
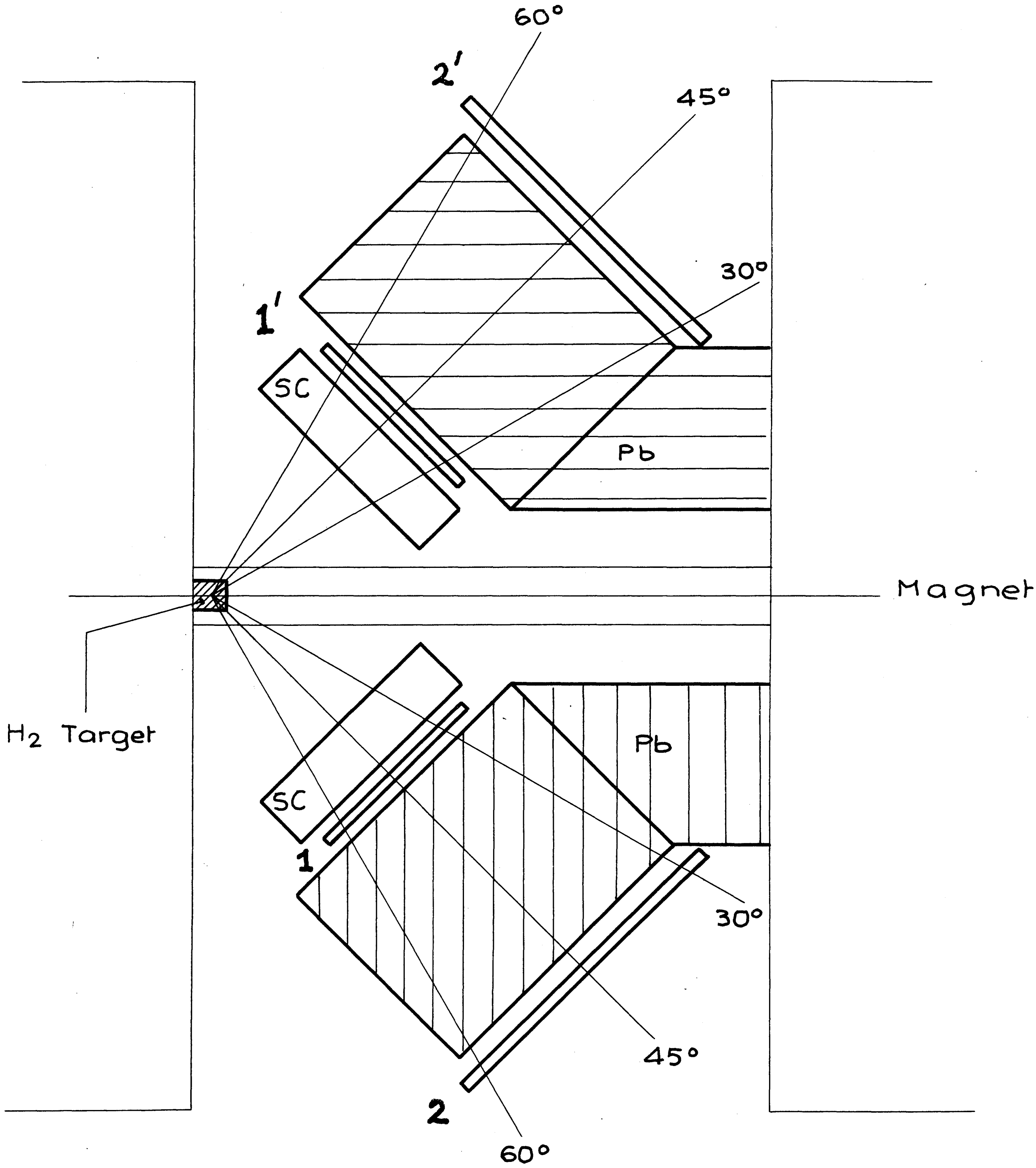


Fig. 1

SIS/R/5436



Elevation 1:10

FIG. 2

SIS/R/5437

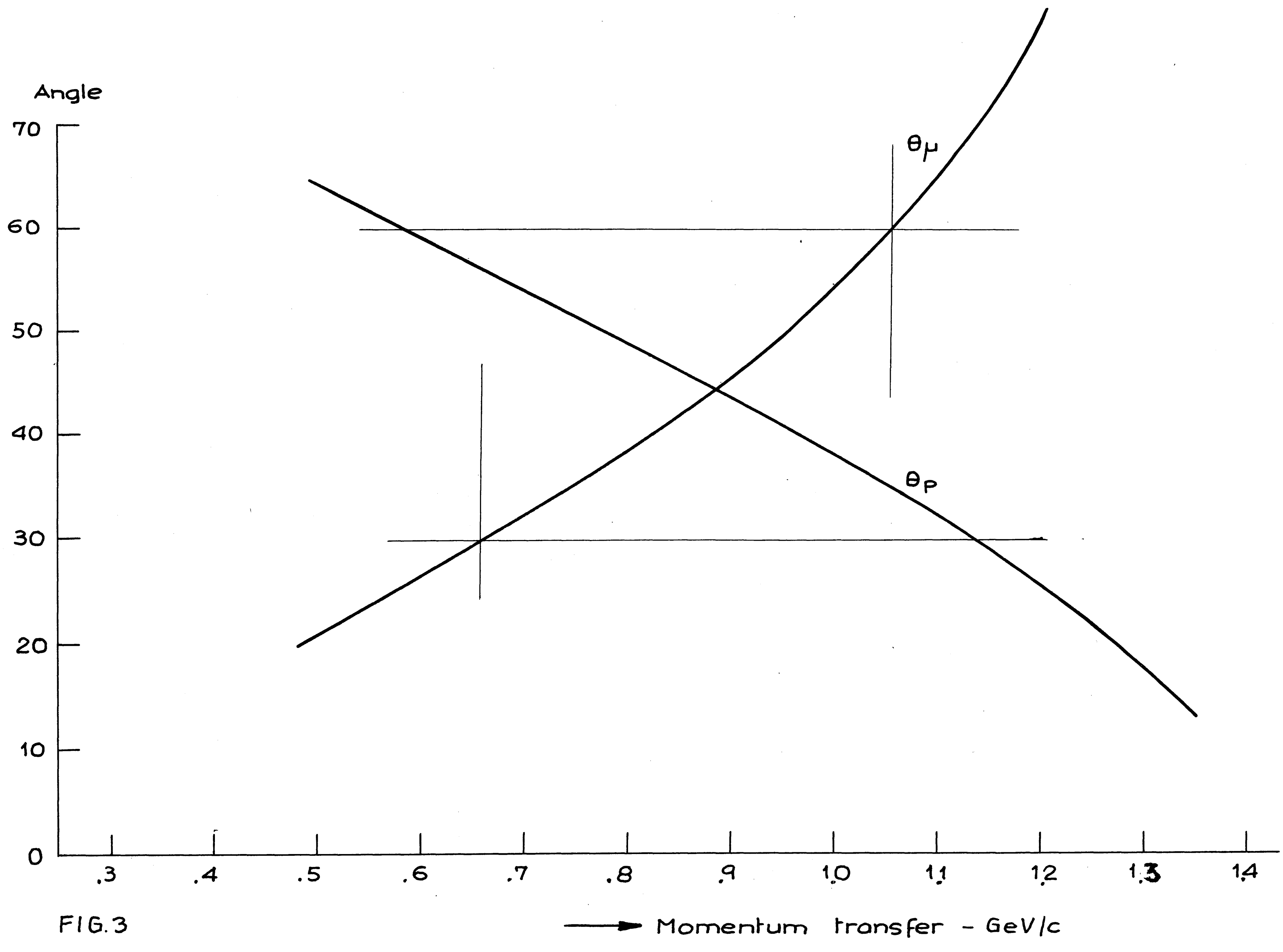

Angle
70
60
50
40
30
20
10
0
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
θ_μ
θ_P
Momentum transfer - GeV/c

FIG.3

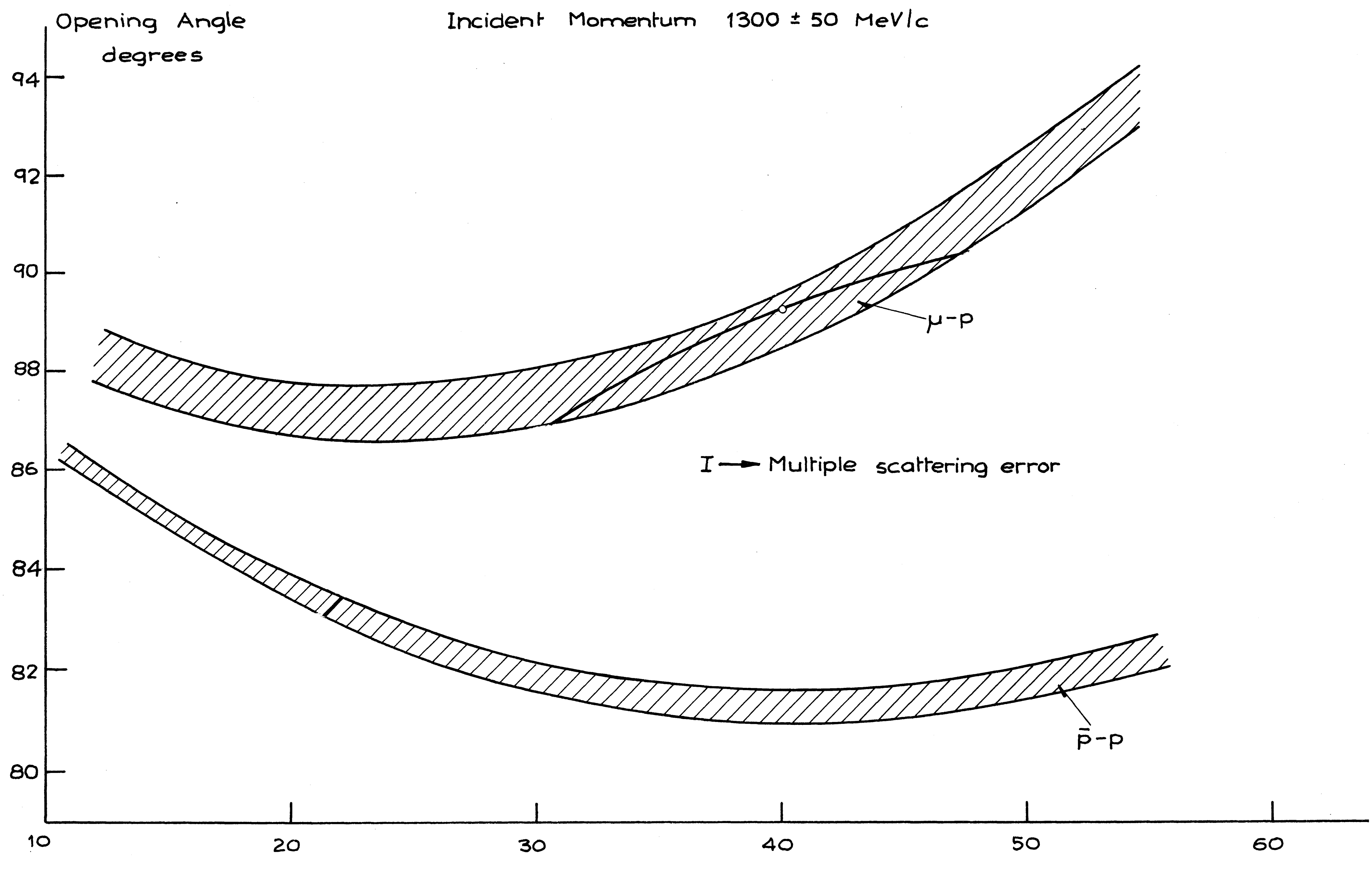

Opening Angle
degrees
Incident Momentum 1300 ± 50 MeV/c
94
92
90
88
86
84
82
80
μ-p
I → Multiple scattering error
p̄-p
10
20
30
40
50
60
Scattering Angle

FIG. 4