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CALCULATIONS ON MUON STORAGE RING PROJECT

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The general outline of the scheme has already been described in 4733/NP/ab of 22nd October, 1962, and remains valid. Here I give detailed calculations in general justifying previous statements, but usually giving a more exact numerical estimate than was available before.

A more detailed discussion is given of (i) measurement of the magnetic field, (ii) systematic errors in the polarization angle and (iii) the background of stable particles stored in the ring.

1.) Intensity of injected pions.

Assume a continuous weak focussing ring, mean radius ρ, vertical aperture 2a, horizontal aperture 2b. A gradient index n = 0.1 is assumed throughout.

The horizontal and vertical betatron oscillations are determined by the wavelengths

$$\lambda\!\!\!^{-}_H = \rho\ (1-n)^{-\frac{1}{2}} = 1.05\ \rho$$

$$\lambda\!\!\!^{-}_V = \rho\ n^{-\frac{1}{2}} = 3.17\ \rho$$

In one turn, the phase of the horizontal osciallation is retarded by 0.05 x 2π, while the phase of the vertical oscillation is advanced by 0.32 x 2π.

In the vertical plane assume that the target extends from the bottom of the ring up to the medium plane as indicated in fig. 1b. As an obstacle to circulating particles it therefore blocks the left hand semi circle in the phase diagram, fig. 1a. The optimum point in the target to illuminate with the injected beam is the double shaded region, the corresponding vertical acceptance angle being $\pm$ 0.5 $a/\lambda\!\!\!^{-}_V$. As the particles describe their vertical oscillations this strip rotates clockwise at 120° per turn. The positions after 1 and 2 turns are as indicated, all the beam missing the target, but after 3 turns the particles would all hit the target as far as the vertical motion is concerned: (we shall consider below whether they miss horizontally).

For $\rho = 3$ metres, $B = 15$ kG, stored momentum 1350 MeV/c, the revolution period is 63 ns and $\gamma_\pi = 9.7$ giving pion life time 250 ns, equivalent to 4 turns. So in 3 turns 0.52 of the pions decay.

We shall now show that on the third turn about 0.5 of the pions miss the target horizontally. The phase space diagram for horizontal oscillations is given in fig. 2. We assume the equilibrium orbit in the centre of the aperture, and the target at distance b_1 from the centre. The horizontal acceptance angle is $\pm (b^2 - {b_1}^2)/\lambda_H = \pm 0.7 \frac{b}{\lambda_H}$, if $b_1 = 0.5$ b. The bar representing the populated area in phase space rotates clockwise at the rate 18° per turn, or 54° in 3 turns, as shown in the upper part of fig. 2. The horizontal diagram is important only every 3 turns, when we have a near coincidence with the target vertically. On the third turn $\sim$ 0.5 of the particles miss horizontally, and then go on to make $\sim$ 17 turns before hitting the target: during this time practically all of them decay. This adds to the decaying pions the fraction x 0.5 (undecayed after 3 turns) x 0.5 (miss horizontally) = 0.25.

Note that this fraction depends on the position of the equilibrium orbit in the horizontal aperture, fig. 2. If the target were very thin the possible positions for the equilibrium orbit would occupy a horizontal distance b from coordinate $y = \frac{1}{2}(b_1 + b)$ to $\frac{1}{2}(b_1 - b)$. But when the equilibrium orbit is close to the target the chance of a collision horizontally at the critical third turn is high. We cover this by reducing the contribution of turns after the third by a factor 0.5, i.e. to 0.25 x 0.5 = 0.12.

Hence the overall fraction of π which decay in the ring is 0.5 + 0.12 = 0.62.

The momentum spread is

$$\Delta p_\pi = p_\pi \times b/\rho$$

The absolute number of these pions is obtained using the yield curves of Diddens et al for 25 Gev protons on beryllium. They show that the differential yield in the forward direction per interacting proton is $\frac{d^2n}{d\Omega dp_\pi} \sim 2.3$ sterad^{-1} (Gev/c)$^{-1}$

Hence the number of pions stored and decaying in the ring is

$$N_\pi = \underbrace{0.7\ \frac{2b}{\lambda_H}}_{\text{horizontal angle}} \times \underbrace{\frac{a}{\lambda_V}}_{\text{vertical angle}} \times \underbrace{\frac{b}{\rho} p_\pi}_{\Delta p_\pi} \times \underbrace{0.62}_{\text{fraction decaying}} \times \frac{d^2n}{d\Omega\, dp_\pi}$$

$$= 1.0\ \frac{ab^2\ n^{\frac{1}{2}}\ (1-n)^{\frac{1}{2}}}{\rho^3}\ p_\pi\ \frac{d^2n}{d\Omega\, dp_\pi} \qquad (1)$$

2.) Kinematics of $\pi - \mu$ decay

$$p_\mu = \gamma_\pi \left(p_\mu^* \cos\Theta^* + \beta_\pi\ E_\mu^* \right)$$

where starred quantities are measured in the pion rest frame, unstarred in the laboratory.

With $p_\mu^* = 30$ MeV/c, $E_\mu^* = 109$ MeV this gives

$$p_\mu = \gamma_\pi \times 139\ (0.79 + 0.21 \cos\Theta^*)$$

$$= p_\pi\ (0.79 + 0.21 \cos\Theta^*) \qquad (2)$$

while the fraction of decays in angular range $(0 - \Theta^*)$ is

$$\Delta I = \tfrac{1}{2}\ (1 - \cos\Theta^*) = \frac{1}{0.42} \frac{\Delta p}{p} = 2.6\ \frac{\Delta p}{p} \qquad (3)$$

The range of decay angles accepted is determined largely by the momentum acceptance of the ring. The horizontal phase diagram for a typical case is given in fig. 3. Neglecting at first the

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$\pi - \mu$ decay angle. We suppose that the π equilibrium orbit is at the centre, O, of the aperture. The μ equilibrium orbit can range from the inner edge A.

$$\frac{\Delta p_\mu}{p_\mu} = \frac{b}{\rho}$$

To be stored in orbits which do not intersect the target the muons must lie inside the dotted phase space circle while the pions are circulating in the annular region between B and C. The shaded overlap area indicates the region in which pions must decay to put a muon into storage. For the optimum case shaded area is 0.42 of the annulus. But the overlap area drops to zero as the muon equilibrium orbit approaches O or A, see fig. 3b. Typical behaviour of the overlap area is indicated in fig. 3c, from which it is reasonable to take as an average $\Delta p_\mu/p_\mu = 0.5\ b/\rho$ with ~ 0.2 of the pions giving a muon in the storage region of phase space.

Clearly this calculation should be refined by considering various positions for the π equilibrium orbit, and computing more exactly the overlap area for all possible μ equilibrium orbits. But the above estimates seem satisfactory as to order of magnitude.

With the typical $\Delta p_\mu/p_\mu$ found above we can now evaluate the decay angles and allow for this effect. For b = 5 cm

$$\Delta p_\mu/p_\mu = 0.5\ b/\rho = 1/120. \quad \text{Hence using (2),}$$

$$\theta^{*2}/2 = 1/24$$

$$\theta^* = 0.29 = 19^\circ$$

The maximum decay angle in the laboratory is found from the ratio of transverse to forward momentum

$$\theta = 30 \sin\theta^*/1350 = 7\ \text{mR}$$

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This is to be compared with the horizontal acceptance angle

$$\frac{b}{\lambda_H} = \frac{5}{1.05 \times 300} = 16 \text{ mR}$$

and the vertical acceptance angle $\frac{a}{\lambda_V} = \frac{5}{3.17 \times 300} = 5$ mR

To take account of the change in angle in the decay, the muon acceptance circle in phase space must be displaced vertically. In the horizontal plane the displacement is small relative to the scale of the diagram, and will in any case sometimes improve the situation. We assume no loss due to horizontal decay angle.

Vertically the worst case is shown in fig. 4 which gives an overlap of 0.14. As one expects all possibilities from this to a perfect overlap one will not be far wrong in assuming a factor 0.5 on the average.

With these factors and using (3) the trapping efficiency for a decaying pion to give a stored muon, permanently missing the target is

$$T = \underbrace{2.6 \times 0.5 \frac{b}{\rho}}_{\text{(solid angle in } \mu \text{ centre of mass)}} \times \underbrace{0.2}_{\text{(horizontal phase space)}} \times \underbrace{0.5}_{\text{(vertical decay angle)}} = 0.2\ \% \qquad (4)$$

3.) Stored muon Intensity

Combining with (1) we obtain the number of stored muons

$$N_\mu = 0.15 \frac{ab^3}{\rho^4} n^{\frac{1}{2}} (1-n)^{\frac{1}{2}} p_\pi \cdot \frac{d^2n}{d\Omega dp_\pi} \qquad (5)$$

Assuming 10^{11} protons per pulse incident on a target of lead 20 cm long, (1 interaction length), in which 0.7 of the protons interact, and with

$$\frac{d^2n}{d\Omega dp} = 2 \text{ sterad}^{-1} \ (\text{Gev/c})^{-1}$$

(Diddens et al)

$$a = b = 5 \text{ cm}$$
$$\rho = 300 \text{ cm}$$
$$p_\pi = 1.35 \text{ Gev/c}$$

we find

$$\left.\begin{aligned} N_\mu &= 1400\ \mu \text{ / pulse} \\ &= 70\ \mu/ \text{ r.f. burst} \end{aligned}\right\} \qquad (6)$$

4.) Trajectories of electrons from muon decay in the ring

To find out whether the decay electrons emerge from the magnet and if so in what direction, we have used a Monte-Carlo computer programme, based on the following logic.

The magnet aperture is 10 cm high x 30 cm wide. The position of the muon decay is vertically on the median plane and horizontally randomly varied within ± 5 cm of the centre. (This is the region we have assumed for particle storage to keep well away from the fringing field). The field is assumed to be uniform, the electron trajectory therefore being a circular arc inside the pole faces and a straight line outside. The muons are assumed to be travelling parallel to the centre line of the ring with negligible change of direction due to vertical and horizontal oscillations. The initial direction of the electron is then determined by the decay angle Θ and decay azimuth angle $\emptyset$, see fig. 5.

Consider an electron emitted in the μ-rest frame at angle Θ^* to the muon momentum, with momentum $p^*/p^*_{max} = x$. The spectral function[1] is

$$W(x)\ dx \sim x^2\ (1.5 - x)\ dx$$

1) C. Bouchiat and L. Michel, 1957, Phys. Rev. 106, 170

To generate this probability distribution of x from a random variable Z we need

$$dz \propto W(x)\ dx$$

Therefore $$z = \int_0^x W(x)\ dx = (2x^3 - x^4) \qquad (7)$$

This relation has been normalized so that the range Z = 0 - 1 gives x in the range 0 - 1 as required.

The procedure for finding x is therefore (i) choose a random number Z in range 0 - 1, (ii) solve (7) for x. This ensures that the values of x are distributed according to the theoretical spectral function.

From the electron momentum p^* in the μ-rest frame one computes the momentum in the laboratory as follows:

$$p^* = \tfrac{1}{2} m_\mu c \times x$$

Therefore, $$\gamma_e^* = \left\{\left(\frac{m_\mu c^2 x}{2m_e c^2}\right)^2 + 1\right\}^{\frac{1}{2}}$$

$$= \frac{m_\mu}{2m_e} \left\{ x^2 + \frac{4m_e^2}{m_\mu^2} \right\}^{\frac{1}{2}} \qquad (8)$$

If β_μ and γ_μ specify the motion of the muon in the laboratory, we have for the electron in the laboratory

$$\gamma_e = \gamma_\mu \gamma_e^* \left\{ 1 + \beta_\mu \beta_e^* \cos \Theta^* \right\}$$

so using (8), and putting $p_\mu = \gamma_\mu m_\mu$,

$$p_e = m_e \gamma_e = \tfrac{1}{2} p_\mu \left\{ x^2 + \frac{4m_e^2}{m_\mu^2} \right\}^{\frac{1}{2}} \left\{ 1 + \beta_\mu \beta_e^* \cos \Theta^* \right\} \qquad (9)$$

where $\beta_e^* = (1 - 1/\gamma_e^{*2})^{\frac{1}{2}}$

Note that the maximum electron momentum is almost exactly equal to p_μ.

The laboratory emission angle of the electron Θ is given by

$$\gamma_\mu \tan \Theta = \frac{\beta_e^* \sin \Theta^*}{\beta_\mu + \beta_e^* \cos \Theta^*} \qquad (10)$$

The values of $\cos \Theta^*$ in (9) and (10) are chosen at random in the range -1 to +1. (i.e. the electron emission is treated as isotropic at this stage, and because the muon spin is rotating this gives the average behaviour. The question of electron asymmetry is introduced later). The azimuth angle of the decay ϕ, see fig. 5, is random in the range 0 - π. Then the final emission angles of the electron are u inthe horizontal plane and v in the vertical plane as indicated in fig. 5 where

$$\tan u = \tan \Theta \sin \phi \qquad (11)$$

$$\tan v = \tan \Theta \cos \phi \cos u \qquad (12)$$

Equations (7) to (12) specify the electron momentum and emission angles in the laboratory, as a function of three random variables giving the spectral function and the zenith and azimuth angles in the μ-rest frame. We then compute by simple triangle tirgonometry whether the electron emerges on the outside or inside of the ring, and the angle Ψ of emergence relative to the local tangent to the ring. See fig. 6. The computer sorts the decays two ways into intervals of laboratory energy E and emergence angle Ψ. It remains to compute, for the particles falling in a particular energy-angle box, what is the average asymmetry coefficient, and what is the μ-spin angle corresponding to maximum intensity.

Suppose the muon spin direction is specified by unit vector $\underline{\sigma}$. The intensity of electron emission in any direction can be written

$$I = 1 + \underline{A} \cdot \underline{\sigma}$$

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where $\underline{A}$ is the "asymmetry vector" of length equal to the asymmetry coefficient (a function of energy) and direction parallel to the electron emission Θ^* considered. The average intensity over n decays is therefore

$$\bar{I} = 1 + \frac{1}{n} \Sigma \underline{A}_r \cdot \underline{\sigma} = 1 + \bar{\underline{A}} \cdot \underline{\sigma}$$

where $\bar{\underline{A}} = \frac{1}{n} \Sigma \underline{A}_r$ is the vector average of the individual asymmetry vectors $\underline{A}_r$. Thus $|\bar{\underline{A}}|$ determines the mean asymmetry coefficient for any group of decays, and the condition $\underline{\sigma} \parallel \bar{\underline{A}}$ determines the spin direction $\underline{\sigma}$ which gives a maximum counting rate for this group.

In computing $\bar{\underline{A}}$ we sum up the horizontal projections of the individual $\underline{A}_r$, parallel and perpendicular to the muon velocity. The individual asymmetry coefficient $|\bar{A}_r|$ is a function of the electron energy in the μ-rest frame

$$A(x) = |\underline{A}| = \frac{2x - 1}{3 - 2x} \qquad (13)$$

The components of $\underline{A}$ in the horizontal plane are found from fig. 5.

$$A(x) \cos \Theta \text{ parallel to } p_\mu$$

and

$$A(x) \sin \Theta \cos \phi \text{ perpendicular to } p_\mu.$$

Thus we determine for the n decays going into a particular energy-angle compartment the average asymmetry coefficient

$$A = \frac{1}{n} \left[(\Sigma A(x) \cos \Theta)^2 + (\Sigma A(x) \sin \Theta \cos \phi)^2 \right]^{\frac{1}{2}} \qquad (14)$$

and the muon spin angle Θ_1^* for maximum intensity

$$\Theta_1^* = \tan^{-1} \left\{ \frac{\Sigma A(x) \sin \Theta \cos \phi}{\Sigma A(x) \cos \Theta} \right\} \qquad (15)$$

A typical set of results is given in Table 1, 2 and 3. Combining the appropriate boxes one obtains the results already given in 4733/NP/äb. Notably for electrons energy at $\Psi > 500$ mR, the electron collection efficiency is 13% with an average asymmetry coefficient of 0.19.

5.) Time required to measure precession frequency

We wish to have a statistical error on the muon precession frequency, as determined from the decay electron counts, of $\sim 1/5000$. Essentially we have to determine the phase of the wave at early times and at a known time interval later. The running time required is determined by the late time measurement where the counting rate is low. Statistically the optimum point for the late time is after 2 life times, (56 μs), but to reduce systematic errors it will be preferable to measure if possible at 110 μs on say the 25th precession cycle. What running time is required to determine the phase of this sine-wave to 1/200 of a cycle, or .03 radian?

If the sine wave is represented in amplitude and phase by the vector $\underline{V}$ it is readily shown that with N counts the vector error in $\underline{V}$ is

$$\underline{v} = \sqrt{\frac{2}{N}} \text{ equally in all directions.}$$

Hence the error in phase angle is

$$\sigma_{\phi} = \frac{1}{A} \cdot \sqrt{\frac{2}{N}} \qquad (16)$$

where A is the amplitude.

For the required accuracy we find

$$N = \frac{2}{A^2 x\ (.03)^2} = \frac{2}{(.19)^2\ (.03)^2} = 60{,}000$$

With 1400 μ stored per pulse, giving after 110 μs (4 life times)

$$1400 \times 0.02 \times 0.13 = 3.6 \text{ detected electrons}$$

per pulse we shall require about 20,000 pulses, i.e. 20 hours of normal PS operation, at 10^{11} per pulse

Note that this is a pessimistic calculation because we have taken no credit for the electrons detected earlier than 110 μs and these will contribute also to the determination of the precession frequency. In fact if we had worked at 55 μs (the optimum, 2 life times) equation (16) would have implied a running time of only 12 hours to reach the same statistical accuracy in the oscillation frequency.

6.) Errors in the angle measurement

The observed modulation of the electron counting rate is the resultant of many superposed sine waves which have slightly differing initial phases: (i) due to the spread in muon birth times, exponentially distributed with characteristic time 250 ns, giving a total phase spread of order $\frac{250}{4000}$ = 0.06 cycle; and (ii) due to spread in $\pi - \mu$ decay angle of $\pm \Theta^*$, that is $\pm 19^\circ$ (see p. 4), or ± 0.05 cycle. These effects reduce the effective polarization of the combined beam to $\sim \cos 20^\circ$, i.e. to 0.94; the amplitude of the modulation is reduced by this amount.

This effect by itself does not interfere with the measurement. An error could arise if the population of the combined beam changes with time in such a way as to change the mean initial phase. Suppose for example that left-hand decays lay an orbits which had a higher probability of being lost by collisions with the walls than right-hand decays. Then at late times only right-hand decays would survive, so the phases measured at early and at late times would correspond to a different sample of muons, and there would be an error.

To estimate this effect we have calculated the distribution of $\pi - \mu$ decay angles Θ^*. (See fig. 3c). We assume as before that the π equilibrium orbit is centred in the aperture and calculate, curve (1), the distribution of μ - equilibrium orbits. The corresponding values of Θ^* are given by curve (2). The mean value of Θ^* is found to be 14°, (0.25 radian). Hence, if a fraction of the muons are lost (other than by decay) and these are asymmetrically divided $(1 + \alpha)$ from right-hand $\pi - \mu$ decays and $(1 - \alpha)$ from left-hand $\pi - \mu$ decays, the error in initial polarization is

$$\Delta\Theta = 2\,\eta\alpha \times 0.25 = 0.5\,\eta\alpha \qquad (17)$$

For an error of 1/2000 in our measurement we require $\Delta\Theta \leqslant 0.025$, i.e. $\alpha\eta \leqslant 0.05$. The best way to ensure this is to build the ring very carefully so that less than 5% of the particles are lost between the early time measurement and the late time measurement, i.e. between the 100th and the 1750th turn. Because the particles are turning constantly in the same path there is no new perturbation of the orbit to be expected after the 100th turn, and we may anticipate that by then the population will have settled down into orbits that do not hit an obstacle: that is a loss of only 5% may be quite practical to achieve.

We will now show that, even if there is a loss, there is no reason to expect an asymmetric loss between left and right-hand decays. In fig. 7 we show a typical phase diagram the two dotted circles corresponding to the muon phase area for left and right-hand $\pi - \mu$ decays. It is seen that the distribution of muons in the available phase area is identical in the two cases. This is illustrated in the lower diagram which shows an oscillatory π giving rise to a muon μ_1 by left-hand decay. We can always find a corresponding muon μ_2 produced by right-hand decay which has exactly the same position in the magnet and oscillation amplitude. Therefore there should be no first order tendency to lose muons born by left-hand as opposed to right-hand decay or vice-versa.

Conceivably there may be second order effects due to small non-uniformities in the magnet which favour losses of particular muons, but it is hard to expect α larger than 0.1. In this case we can allow $\eta \sim 0.5$ (50% loss) and still achieve the desired accuracy. Thus the order of magnitude of the error to be expected seems to be small.

This is a problem for which it is hard to lay down the final solution in advance. If there is no loss, $\eta < 0.05$ the problem is solved. But if this condition is not satisfied a detailed study of the consistency of data taken under varying conditions may be necessary to convince ourselves that the effect is indeed small.

7.) Measurement of magnetic field.

The magnet can be measured by nuclear resonance which gives a detectable signal in fields with non-uniformity up to 0.05%/cm. As the gradient in the ring at $n = 0.1$ corresponds to 0.03%/cm we shall be able to measure all parts of the field by this method.

The quantity required is the average field seen by the muons, averaged that is in time for an individual muon, and averaged again over the ensemble of muon trajectories. The muons are confined to a region of 7.5 cm in radius. Fig. 8 shows the typical distribution of population to be expected*, together with two extreme cases of a triangular distribution. The difference in the centroid position for the two triangles is 2.5 cm corresponding to $2.5/3000 = \pm 4 \times 10^{-4}$ in mean field. It is likely that by pure calculation of the muon equilibrium orbits and oscillations on the lines indicated above we can improve this error by the factor 2 needed for an accuracy of $\pm 1/5000$. So this is a tractable problem.

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*Note: This distribution includes the effect of horizontal oscillations and therefore differs from the distribution of equilibrium orbits, Fig. 3c.

Nevertheless it would be very desirable to survey experimentally the muon population in the ring. As soon as one tries to count the muons at various points in the aperture one has to worry about the possible background of stable particles. If we work with μ^- these will be e^- and $\bar{p}$.

8.) Background of stable particles

In principle stable particles emerging from the primary target cannot be permanently stored in the ring, because they will hit the target again after ~ 20 turns. With a lead target 20 cm long the energy loss is ~ 250 MeV for $\bar{p}$, and e^- will be eliminated by bremsstrahlung, so one collision with the target is enough to eliminate both particles. To be permanently stored a stable particle must suffer a scattering in the storage volume, which changes its trajectory so that it in future misses the target. Such a scattering can be produced by (i) gas in the ring or (ii) the counter introduced to count muons.

To estimate the number of particles which will miss the target after 20 turns we have to calculate the broadening of the image due to scattering in a system with continuous focussing properties. The amplitude of oscillation at any instant can be calculated from the instantaneous angle Θ and displacement y of the particle

$$A^2 = \lambda^2 \Theta^2 + y^2$$

Hence the change in amplitude due to scattering in path length S is given by

$$\frac{d}{ds}(A^2) = \lambda^2 \frac{d}{ds}(\Theta^2) = \lambda^2 \frac{15^2}{(p\beta c)^2} \frac{S}{X_o}$$

where X_o is the radiation length in the material.
Thus the initially sharp edges of the image will be blurred by a gaussian tail of characteristic distance

$$\sigma = \left[\langle\Delta(A^2)\rangle\right]^{\frac{1}{2}} = \lambda \frac{15}{p\beta c} \left(\frac{S}{X_o}\right)^{\frac{1}{2}} \quad (18)$$

In the vertical plane the target will reach the median plane: then there is no orbit that does not intersect the target whatever the scattering. For the horizontal case insert in (18) λ = 315 cm S = 400 metres (20 turns) and X_o = 320 m corresponding to air at N.T.P. we find

$$\sigma = 4 \text{ cm} \quad \text{at N.T.P.}$$
$$= 0.1 \text{ cm} \quad \text{at } 0.5 \text{ mm pressure.}$$

The image of the target now has a gaussian edge of form $\exp(-y^2/2\sigma)$. The integrated intensity in the tail beyond m standard deviations is

$$(2\pi)^{\frac{1}{2}} \sigma \int_{y = m\sigma}^{\alpha} e^{-y^2/2\sigma} dy$$

$$\frac{(2\pi)^{\frac{1}{2}} \sigma}{2w} \quad \eta_m \qquad (19)$$

of the whole beam, where η_m is the usual probability of getting an error greater than $m\sigma$, and w is the initial width of the uniformly illuminated image.

Suppose now for example an initial illuminated width of 1 cm, with sharply defined edges, at the centre of a target 2 cm wide: (how to get this is described below). To miss the target the beam must be diffused 5 mm, i.e. 5σ if we have 0.5 mm pressure. According to (19) the fraction of the beam which misses the target is

$$(2\pi)^{\frac{1}{2}} \times 0.05 \times 6 \times 10^{-5} = 7 \times 10^{-6} \qquad (20)$$

At production the relative intensities of the stable particles to π^- are estimated as

$$\left.\begin{aligned} N(\bar{p}) / N(\pi^-) &\sim 10^{-3} \\ N(e^-) / N(\pi^-) &\sim 5.10^{-2} \end{aligned}\right\} \qquad (21)$$

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The first ratio is well established from data on anti-proton beams. For the second we note that on the average we expect after 1 radiation length in the target

$$N(e^- \text{ at } 1.4\text{ GeV/c}) \sim 2N(\pi^0 \text{ at } 5.6\text{ GeV/c}) \sim 2N(\pi^- \text{ at } 5.6\text{ GeV/c})$$

because the available energy is shared among 4 electrons, of which 2 are negatively changed.

The yield curves of Diddens et al show an appreciable drop in π yield in going from 1.4 to 5.6 GeV/c. Hence it is plausible to assume that after one radiation length of target $N(e^-)/\ N(\pi^-) \sim 1$. To reduce the overall ratio we propose to use lead as target material. Then one expects that electrons produced in the first layers of the target will be slowed down by bremsstrahlung and will not contribute; in effect only the π^0 production in the last ~ 2 radiation lengths should contribute, i.e. the last 1 cm of target out of a total length of 20 cm. This gives the estimate in (21) above.

Combining (20), (21) and (4) we obtain the ratios of stored stable particles to stored muons at 0.5 mm pressure

$$\left.\begin{aligned} \frac{N(\bar{\rho})}{N(\mu)} &= \frac{7.10^{-6} \times 10^{-3}}{2 \times 10^{-3}} = 4.10^{-6} \\ \frac{N(e^-)}{N(\mu)} &= \frac{7.10^{-6} \times 2.10^{-2}}{2 \times 10^{-3}} = 10^{-4} \end{aligned}\right\} \qquad (22)$$

Thus in the case considered the stable particles should not interfere with attempts to survey the muon distribution in the ring.

The postulated illuminated target width of 1 cm at the centre of a 2 cm wide block of material can be achieved by separating the two functions of the target (i) to produce particles and (ii) to absorb them. The production target is only 1 cm wide: the absorbing block 2 cm wide is separate and is mounted about 1.5 metre back in azimuth where it is not struck by the primary beam, its centre being at the same radius as the centre of the production target. (Note that this will not alter appreciably the muon intensity estimates presented in section (3) above).

It remains to inquire whether the counter introduced to survey the muon distribution will itself scatter an appreciable number of stable particles into storage orbits. For a scintillator 5 x 5 x 2 mm the mass introduced per turn is $0.2/300 = 0.7 \times 10^{-3}$ g/cm^2 equivalent to 0.2 mm air pressure. Hence one expects a scattering effect smaller than that estimated above, and the number of stable particles stored will again be negligible.

On the other hand the muon counting rate in such a scintillator will be ~ 5 counts/turn or 80 Mc/s. Expressed another way, each muon will count several times, on the average once every 300 turns. So intensity is no problem.

There will of course be an overall check on the background level by measuring after say 300 μs when all muons will have decayed.

TABLE 1

NUMBER ⟶ ENERGY

ANGLE ↓

1	0	0	0	0	0	0	0	0	124
3	0	0	0	0	0	0	0	82	341
3	0	0	0	0	0	0	63	461	24
1	0	0	0	0	0	99	479	115	0
3	0	0	0	0	112	450	155	0	0
1	0	0	0	75	325	201	0	0	0
2	0	0	10	287	242	4	0	0	0
2	0	0	151	275	30	0	0	0	0
5	0	23	224	97	0	0	0	0	0
3	0	122	224	8	0	0	0	0	0
7	2	149	88	0	0	0	0	0	0
3	33	191	17	0	0	0	0	0	0
10	61	111	0	0	0	0	0	0	0
8	80	56	0	0	0	0	0	0	0
9	101	22	0	0	0	0	0	0	0
13	103	3	0	0	0	0	0	0	0
13	82	0	0	0	0	0	0	0	0
14	69	0	0	0	0	0	0	0	0
29	59	0	0	0	0	0	0	0	0
35	43	0	0	0	0	0	0	0	0
46	31	0	0	0	0	0	0	0	0
39	26	0	0	0	0	0	0	0	0
52	16	0	0	0	0	0	0	0	0
52	4	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0

TABLE 2

ASYMMETRY COEFFICIENT X 1000									
56	0	0	0	0	0	0	0	0	259
294	0	0	0	0	0	0	0	245	296
534	0	0	0	0	0	0	219	222	238
112	0	0	0	0	0	119	163	223	0
467	0	0	0	0	99	109	166	0	0
111	0	0	0	24	35	107	0	0	0
662	0	0	205	59	64	121	0	0	0
54	0	0	93	55	40	0	0	0	0
459	0	186	128	120	0	0	0	0	0
288	0	172	117	269	0	0	0	0	0
397	311	167	141	0	0	0	0	0	0
440	231	167	111	0	0	0	0	0	0
256	237	182	0	0	0	0	0	0	0
81	219	224	0	0	0	0	0	0	0
427	179	198	0	0	0	0	0	0	0
201	220	397	0	0	0	0	0	0	0
126	276	0	0	0	0	0	0	0	0
322	269	0	0	0	0	0	0	0	0
232	235	0	0	0	0	0	0	0	0
296	281	0	0	0	0	0	0	0	0
318	259	0	0	0	0	0	0	0	0
216	295	0	0	0	0	0	0	0	0
250	225	0	0	0	0	0	0	0	0
306	111	0	0	0	0	0	0	0	0
334	0	0	0	0	0	0	0	0	0
324	0	0	0	0	0	0	0	0	0
328	0	0	0	0	0	0	0	0	0
345	0	0	0	0	0	0	0	0	0
361	0	0	0	0	0	0	0	0	0
297	0	0	0	0	0	0	0	0	0
341	0	0	0	0	0	0	0	0	0

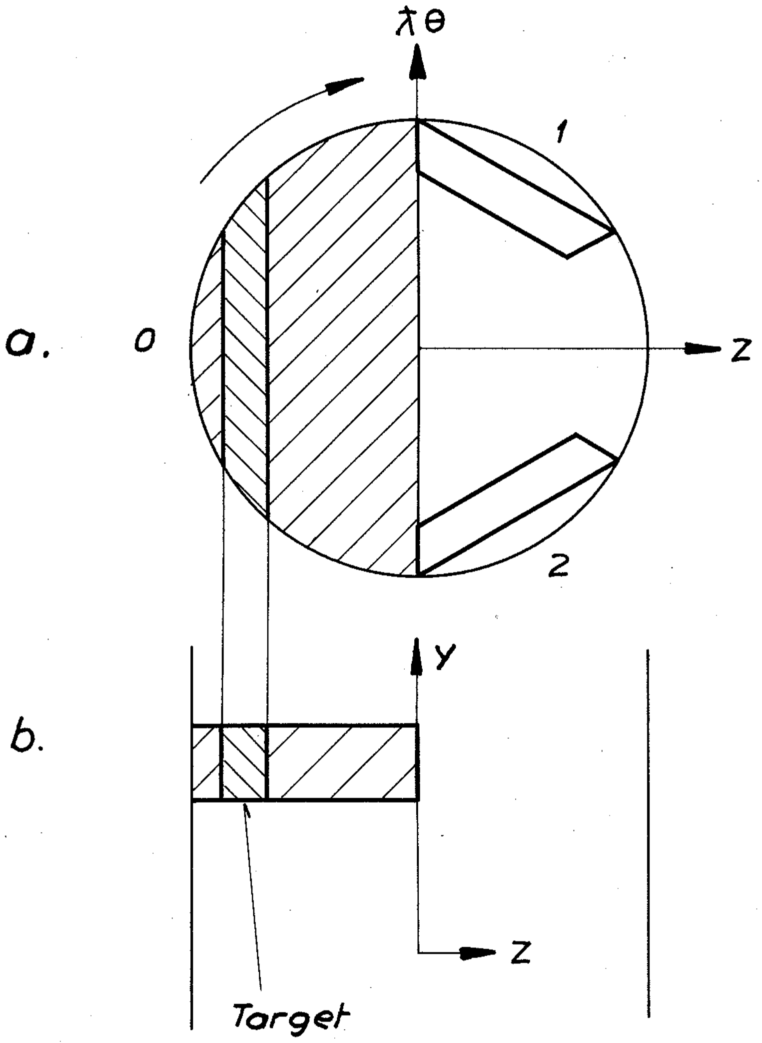


Fig.1 VERTICAL

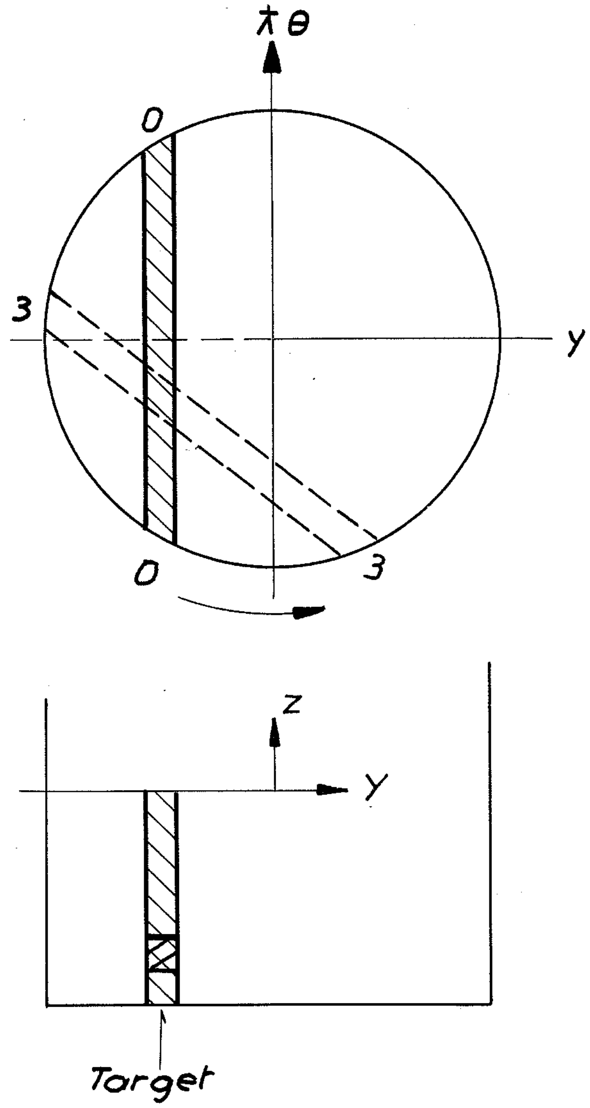


Fig.2 HORIZONTAL

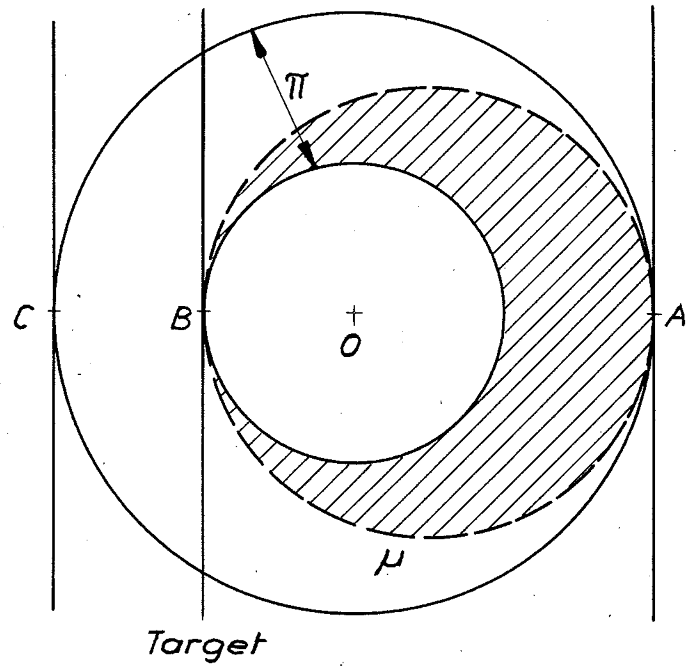
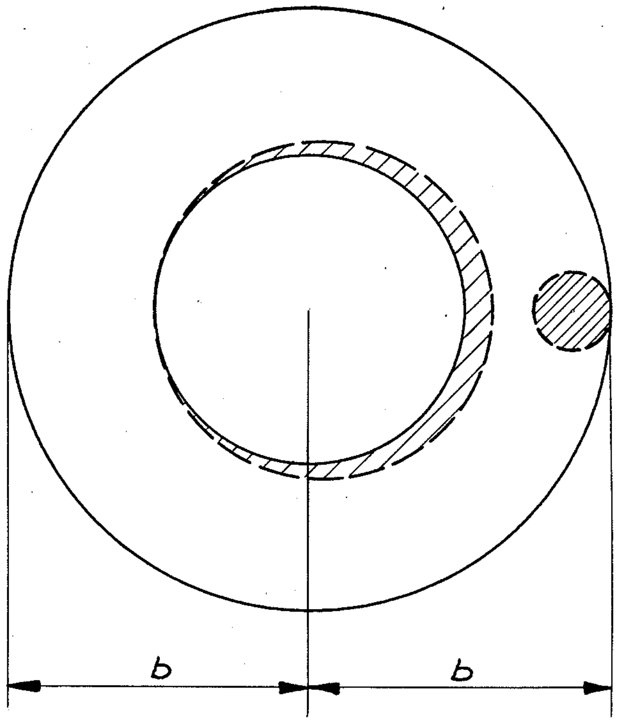

π
C
B
O
A
μ
Target

FIG. 3a


b
b

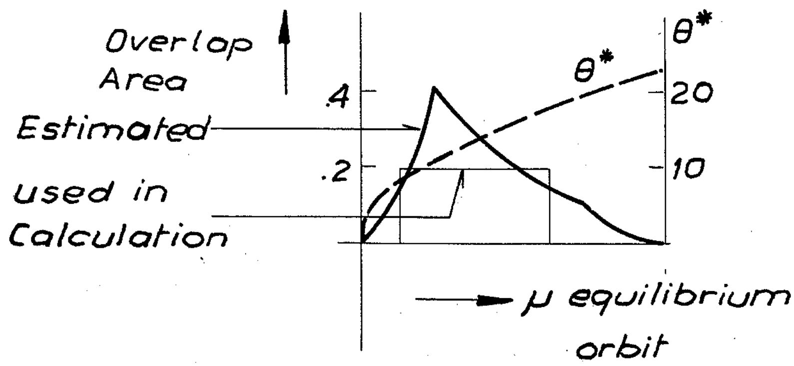

Overlap
Area
Estimated
used in
Calculation
.4
.2
θ*
θ*
20
10
μ equilibrium
orbit

FIG. 3c

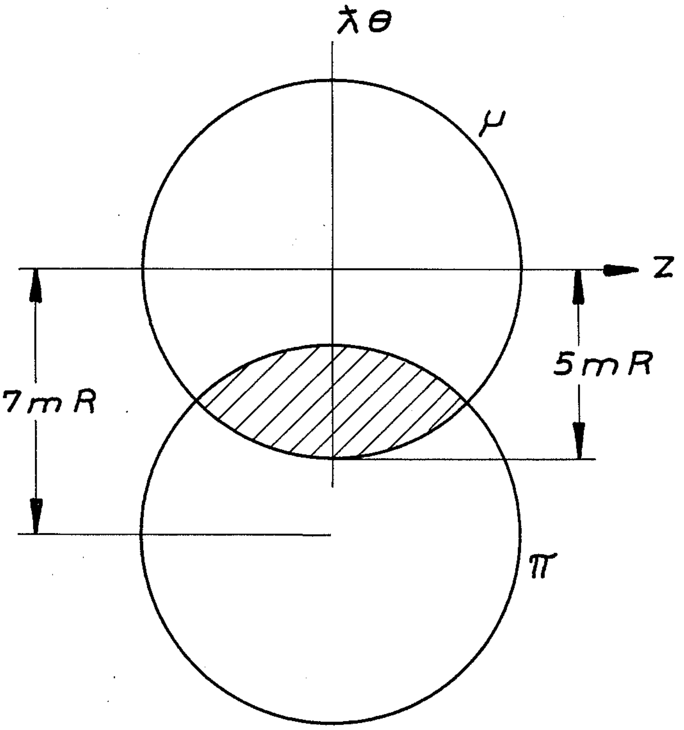


Fig. 4 Vertical decay angle — worst case

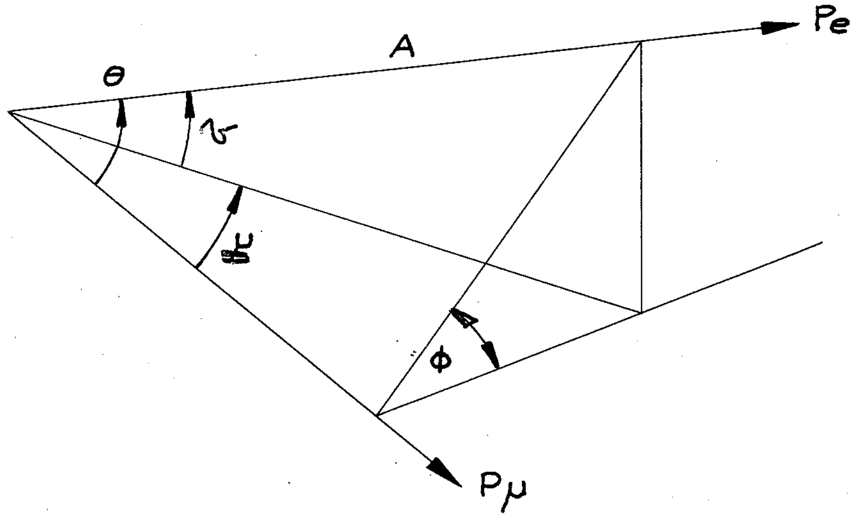


Fig. 5

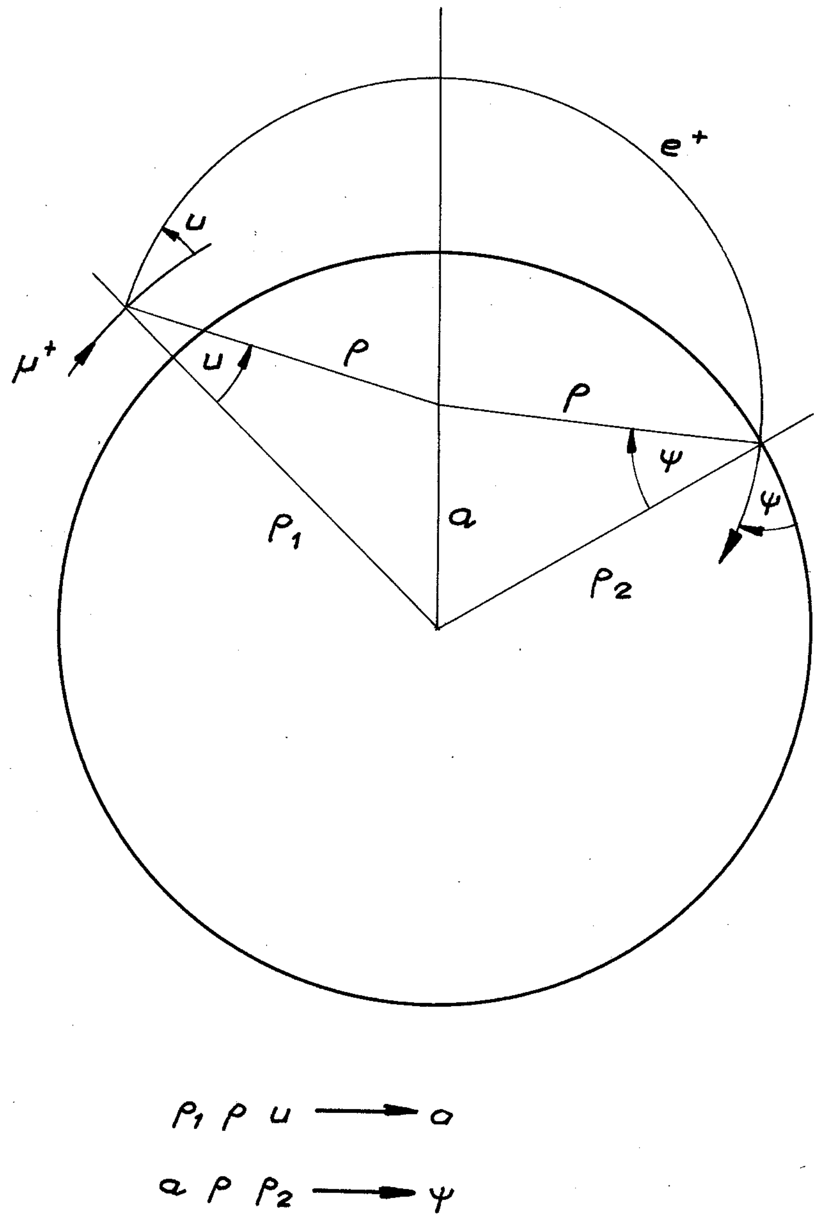


FIG. 6

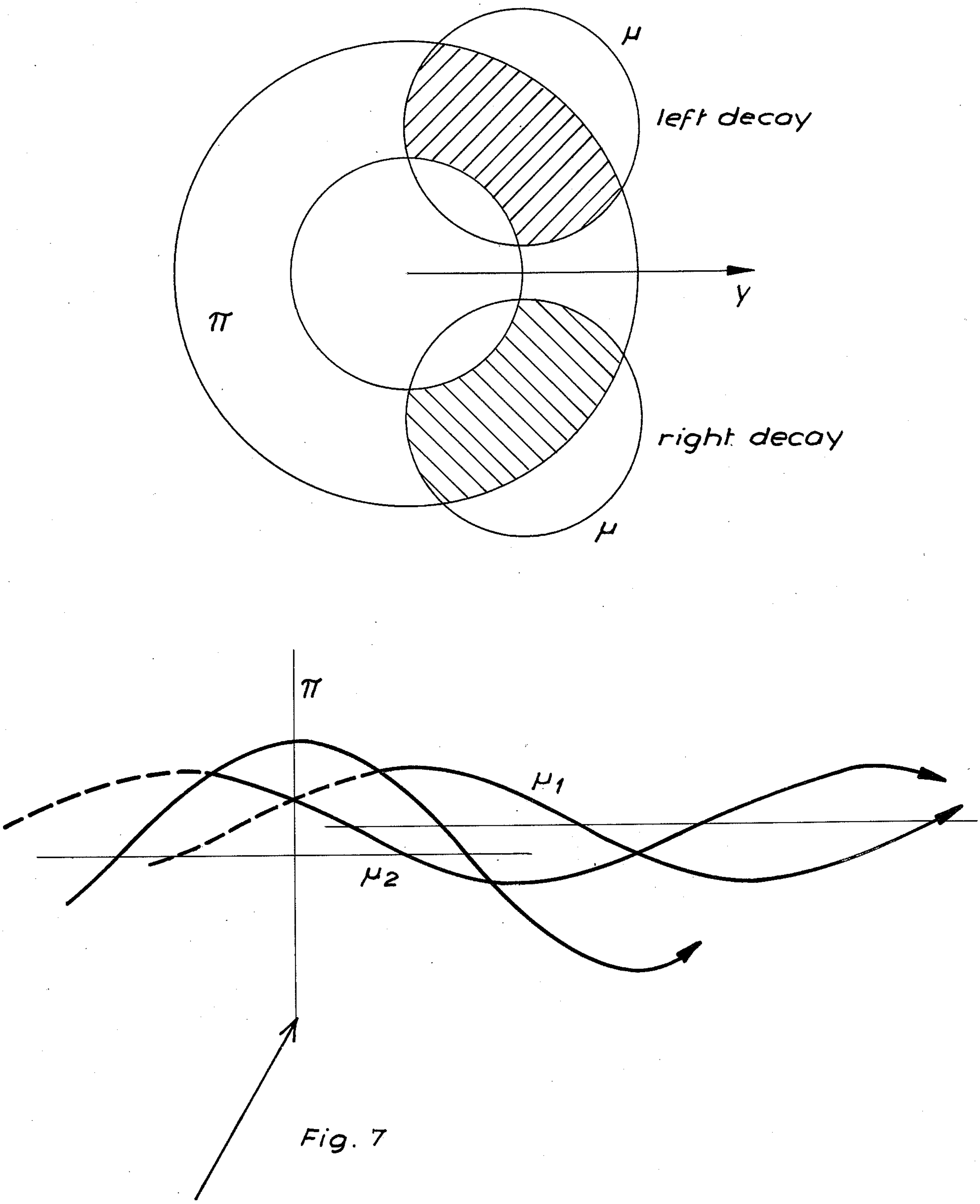


Fig. 7

REFLECTION SYMMETRY ABOUT THIS LINE.

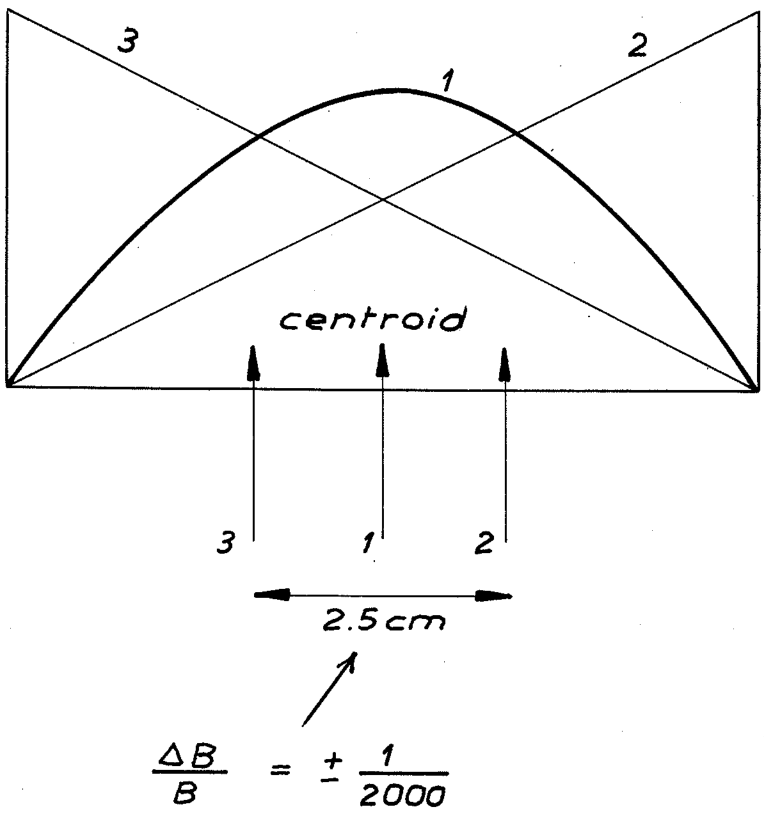


Fig. 8 MUON DISTRIBUTION