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PROPOSED HIGH PRECISION G-2 EXPERIMENT

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The determination of the muon anomalous magnetic moment to ± 0.4% is close to the ultimate that can be achieved with the existing apparatus. But this should not be the end of the programme. The very fact that there is agreement with theory so far makes a higher precision desirable.

The experiment proposed here, using radically different techniques, seems capable of achieving at least ten times the present accuracy, i.e. a measurement to order 1/2000, or ± 0.04%.

I. Theoretical objectives

The muon is not the same as the electron. Quite apart from its different mass, we now know that in the weak interaction it is coupled to a different kind of neutrino. To clarify this question we must search for other differences: the magnetic moment, which carries information about the muon coupling to other fields, and its spatial structure if any, is a very sensitive probe. A discrepancy with the theoretical value, if established, would be an important experimental datum.

Note here that the charged intermediate vector boson can produce an effect of order 0.01% in (g-2), (dependent on boson magnetic moment and weak interaction cut-off). Also the modification of the photon propagator by coupling to ρ-meson gives a perturbation of the same order. Therefore the proposed experiment (to 0.04%) will approach the point at which we can say, any unknown interactions of the muon are so small that they can safely be neglected.

Recently it has been suggested that the photon may be a Regge pole. This hypothesis implies a different (g-2) value for the muon, and, as pointed out by Drell, the present experiment already limits seriously the possible slope of the photon Regge trajectory. If the theoretical value were confirmed with a further factor of 10 in accuracy, the Regge-pole hypothesis for the photon would be virtually eliminated.

This is only one possible way of modifying quantum electrodynamics, and it is more usual to express our knowledge in terms of a conventional Feynman cut-off parameter. So far, Q.E.D. appears to work up to $\Lambda \sim 1$ GeV/c and we accept the renormalization technique because of its success. This may not, however, be the ultimate solution to the divergence problems, and one may find discrepancies at higher momentum transfers. The proposed measurement would probe Q.E.D. up to $\Lambda \sim 3.3$ GeV/c.

Note that there are several experiments which will be testing Q.E.D. and muon structure during the next few years: e.g. electron colliding beam at 500 MeV (Stanford), muon wide angle pair production in hydrogen (Frascati), and μ - p scattering (CERN). The muon (g-2) will however retain its essential value; it will remain the only measurement on the muon which involves electrodynamics only and is not contaminated by strong interactions.

II. Outline of proposed experiment

To measure the anomalous moment $a \equiv \frac{1}{2}(g-2)$ one measures the precession angle ϑ of spin relative to momentum for muons which spend time t in magnetic field B,

$$\vartheta = a\,\omega_o B t$$

where ω_o a constant known to 0.002%. So far we have worked with about 1 cycle of precession measured to 1° accuracy with B = 16 kG, t = 4 μs. To improve the accuracy we must now increase the precession effect to see 10 or more cycles. This can be done either by increasing B to ~ 150 kG, (not considered practical at present) or by increasing t to ~ 40 μs. This second alternative is quite practical if we use relativistic muons (~ 1 GeV) whose lifetime will be dilated to ~ 25 μsec.

A detailed scheme for storing 1.35 GeV/c muons in a storage ring 6 metres in diameter, B = 15 kG, has been worked out and will now be described. It is not claimed at the present stage that this is the optimum design. The object is to show that the experiment is feasible, and to estimate the magnitude of the project.

III. Storage Ring

Because we must measure $\bar{B}$ very accurately a weak focusing continuous storage ring is chosen, see Fig. 1. At 15 kG and radius 300 cm the stored momentum is 1.35 GeV/c. A window 10 cm vertically × 30 cm horizontally is proposed, of which only the central 10 × 10 cm will be used to avoid fringing field effects. Focusing is provided by a radial gradient n = 0.1 - 0.25. The muon lifetime is 28 μs. A C-magnet open on the inside is used for reasons which will appear below.

Injection of Polarized muons

To produce muons one would normally need a long decay path for 1.4 GeV/c pions, and one would then have muons accupying a large volume in phase space which must be inflected into the ring. All these problems are avoided by bringing 25 GeV protons onto a target in the ring.

The secondary particles emitted forward of momentum ~ 1.35 GeV/c will be temporarily stored in the ring, and one can show that with n ~ 0.25, as a result of vertical and horizontal oscillations, a large fraction will make ~ 8 turns before again hitting the target.* In this time 90% of the pions will decay. The circulating π intensity is

$$I_\pi = 0.7 \frac{ab^2 n^{1/2}(1-n)^{1/2}}{\rho^3} p_\pi \frac{d^2N}{d\Omega dp_\pi} \qquad (1)$$

where 2a = vertical window height
2b = horizontal window width
ρ = orbit radius
p_π = pion momentum
$\frac{d^2N}{d\Omega dp_\pi}$ = yield of pions from 25 GeV proton interaction, given by Diddens et al.

The numerical factor includes the probability of missing the target for 8 turns.

When the pions decay the muons emitted forward have nearly the same momentum; therefore they will also be stored. In fact owing to the small change of momentum and to the $\pi - \mu$ decay angle a fraction of the muons will be trapped into orbits which no longer intersect the target and remain permanently stored. The trapping efficiency is

$$T \sim 0.7 \times 0.2 \times 2.5\ b/\rho \qquad (2)$$

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* The n value is not critical

the first factor being a vertical loss due to the decay angle, the second a horizontal loss either in the walls or target due to decay angle and change of momentum, and the third factor being governed by $\Delta p/p$ for the ring.* $T \sim 0.5\%$ for the ring described.

The overall stored muon intensity

$$I_\mu = 0.4 \, \frac{ab^3 n^{1/2}((1 - n)^{1/2}}{\rho^4} \, p_\pi \, \frac{d^2\sigma}{d\Omega \, dp_\pi} \qquad (3)$$

For 10^{11} protons hitting a 16 cm thick copper target† this gives $I_\mu = 4000$ muons stored per pulse. After 150 μs, (35 cycles of g-2 precession), there will still be 20 muons in the ring.

Note that as only the very forward $\pi - \mu$ decays are accepted the muons will be 95% polarized.

IV. Measurement of spin angle, ϑ

One method of measuring the muon spin direction after storage for time t is to deflect the particles out of the ring, bring them to rest and observe the $\mu - e$ decay in a field free target. This appears possible, but it is not necessary. It turns out that we can get all the information we need by watching the muons decay in flight while they are still in the ring.

When the muon decays, the electron energy varies rapidly with decay angle ϕ in the rest frame.

$$E_{\text{electron}} = \gamma_\mu \, \gamma_e (1 + \beta_\mu \beta_e \cos \phi)$$

$$\sim 1.35 \times \tfrac{1}{2}(1 + \cos \phi).$$

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* The numerical factors in (2) depend only slowly on the values of n, a and b. They have been estimated for a ring of the proposed dimensions with $n = 0.25$.

† 1 interaction length.

Hence by sorting the decay electrons in energy we can preferentially select a particular ϕ. In fact this is done automatically by the magnetic field of the ring.

As $p_{\text{electron}} < p_{\mu}$, the electrons tend to emerge on the inside of the ring. Owing to the effect of the magnetic field the high energy particles ($\phi \sim 0$) come out almost tangentially while the low energy electrons ($\phi \sim 180°$) emerge at steep angles. To select a range of ϕ it is only necessary to count particles emerging from the ring in an appropriate range of angle.

The results of a Monte-Carlo calculation which includes the decay electron spectrum and the variation of asymmetry parameter with energy, are given in the Tables 1-3. 40% of the electrons emerge on the inside of the ring. One sees for example that by selecting exit angles greater than 500 mR one has a group of backward emitted electrons representing ~ 13% of the decays and with an overall asymmetry 0.18 (cf. 0.7 with 25% electron counting efficiency realized in cyclotron experiments). A typical coincidence counter assembly for selecting decay electrons emitted in the range of angle 30° - 80° is sketched in Fig.1. Absorbers and other devices to cut down background would of course be included.

The overall effect to be observed will be as follows. After injecting 25 GeV protons we await ~ 1 μsec for pions to decay, and then start to observe the decay electrons. One should then see an exponentially decaying counting rate following the 28 μsec lifetime of the muon, strongly modulated by the (g-2) precession, with period ~ 4.2 μs. The frequency of this modulation, of which we expect to see ~ 25 cycles, determines (g-2).

V. Timing

As there will be many cycles of precession it is not necessary to know exactly when the muons were born. There must however be a fixed time mark from which to initiate the electronic timing sequence.

This will be provided by the injected proton beam, 10 ns long if we eject from the PS only one of the 20 circulating bunches. As the time scale to be measured is $\sim 50 - 100\ \mu$sec the precision of 10 nsec is already sufficient, but no doubt the time mark will be more precise than this.

Using only $^1/_{20}$ of the PS intensity at a time, the number of stored muons will be 200, out of which 30 decay electrons should be recorded. The electronics should ideally record digitally the time of detection of each of these particles, and store in a memory.

After say 5 msec one can then call for another bunch of protons from the PS, and repeat the measurement 20 times per PS pulse. To do this one requires an ejection system for the PS that will eject the 20 circulating bunches one at a time on demand. While this not planned at present, discussions with Messrs. Kuiper and Plass suggest that it is possible without serious difficulty.

One certainly should not inject into the ring all 20 bunches at a time, as the 2 μsec required will smear out the (g-2) precession, and reduce the amplitude of the modulation to half.

VI. Measurement of mean magnetic field, $\bar{B}$.

A serious effort will be necessary to have a stable field, uniform in azimuth. Stability to $\sim 10^{-4}$ has already been achieved with the (g-2) magnet at CERN, but for this experiment one should stabilize relative to a nuclear resonance probe as reference. This technique has already been used at Columbia and Berkeley.

The uniformity of the field is probably best realized by having $\sim$ 20 small correction windings distributed round the upper and lower poles, each of which can be separately excited. In this way one can "shim with current" to smooth local irregularities and correct the median plane.

On considering the effect on the orbit of harmonic azimuthal perturbations of order m and relative amplitude a_m one finds that the sideways perturbation of the orbit has amplitude.

$$b_m = \frac{a_m \rho}{m^2 + n - 1}$$

The only serious case is the first harmonic perturbation (m = 1). For 1 cm perturbation of the orbit one requires $a_1 < n/300 = 1/3000$ for n = 0.1. There should be no difficulty in reaching this accuracy with the method proposed.

Having surveyed the field with nuclear resonance it remains to determine the average position of the stored muons in the field. This is probably the most difficult part of the experiment. It is not possible to probe with mechanical flaps, (a) because the particles are penetrating and (b) because we may kill the counts by cutting off the pion parents and this does not give information about the muon orbits.

Instead one must use a scintillator to probe the orbits, and only interrogate this counter after the first few μsec are over and the pions are dead. A possible snag here is that there will be some protons and some electrons stored in the ring, and these will also give counts. However in the absence of gas scattering one expects all protons and electrons to be killed after ~ 10 - 50 turns by interacting with the target. So when there is no counter in the ring there are no protons. Unfortunately the counter itself is a scatterer, and this will cause some protons to be stored.

Estimating the effects with the aid of data of Diddens et al. on the π^+/p ratio at 1-2 GeV/c, one finds that with a pressure of 2.5 μ in the ring the initial μ^+/p ratio would be ~ 5. A counter 1 mm thick and 5 × 5 mm in section would by itself scatter a similar number of protons into storage. The overall electron production by π^0 in the target can be kept low by using high Z material, which absorbs the cascade rapidly. In this case one expects fewer electrons than protons.

Therefore the background of electrons and protons does not seem a serious problem.

Note that one can eliminate the protons by running the experiment on μ^-. (As we do not stop the muons there are no depolarization problems and we can measure μ^- as well as μ^+).

Note that this background arises only when measurements are made inside the ring to find out where the muons are. One does not expect particles scattered out of the ring (momentum transfer $\gtrsim 0.6$ GeV/c) to an important background for the decay electron counting.

Because of the difficulty in finding where the muons are in the ring it may pay to reduce the field gradient value to say $n = 0.1$, thus sacrificing intensity by a factor ~ 5, but reducing the precision with which the muon orbits must be known.

VII. Statistics, accuracy and running time

With the intensity given above for $n = 0.25$ we shall inject 200 muons per PS bunch. After 100 μs [25 cycles of (g-2) precession] there will be 4 left, giving 0.6 detected decay electrons per bunch in this time range.

To measure the phase of the precession to 1° with asymmetry A, the number of counts needed is

$$N = \frac{2 \times 57^2}{A^2} = 150{,}000$$

for $A = 0.2$

This would give an accuracy of 1:7500 in the (g-2) frequency, and takes ~ 250,000 PS bunches.

If we use only 1 bunch per PS cycle (1000/hr) the measurement would take 250 hrs. But as proposed, using every bunch (20,000/hr) the measurement takes only 12 hours.

If we now reduce n to 0.1 to obtain more precision on the field one could do such a measurement in ~ 50 hours.

The above figures are given as an example to show that we have a factor in hand. This provides on the one hand a factor of safety, but on the other hand the possibility of sacrificing intensity to obtain even greater accuracy.

Clearly one will also need running time for preliminary tests and for probing the muon orbits (measurement of $\bar{B}$), but there is no reason why this should be excessive.

Overall one aims in the first instance to measure $\bar{B}$, and ϑ each to 1/5000, thus obtaining a to 1/2000.

VIII. Work, time, cost and staff estimates.

It should be emphasized that there are many alternative ways of constructing the ring magnets, so that present estimates are essentially very preliminary.

As sketched in Fig. 1 the magnet is made in 10 identical sections butted together, each section with its own winding. Such a construction is economic and straightforward, and would allow the use of the separate sections as bending magnets after the experiment is finished. The total magnet would weigh ~ 150 tons, and would probably cost 1.5 to 2M SwFr. One envisages a few months to finalize the design, and ~ 18 months for manufacture andassembly which means that one could be running the experiment early in 1965.

The magnet must be sited near the PS in a position which allows the ejected proton beam to be brought to it. This could be either in the South Hall, or in the East Area. The latter would appear preferable to avoid interference with the neutrino projects, in which case an ejected beam in the East Area with beam stopper should be engineered by 1965.

Installation costs for the magnet, including vacuum tank and support, and the apparatus need for measuring the magnetic field, and recording the time distribution of decay electrons could well cost ~ 1 M. SwFr. Thus the total project cost will be ~ 2-3 M. SwFr. which would however be spread out over the years 1963-5.

As to the staff requirements one would hope to find quite soon an experienced magnet engineer who could devote a large fraction of his time to the design of the ring. At later stages a group of 2-3 engineers, 3-4 physicists, an electronics expert plus suitable technicians could handle the problems effectively, in collaboration of course, with other staff. Fortunately, quite apart from the PS itself, there already exists at CERN a corporate body of experience and knowledge which would be invaluable for the project.

IX. Summary

A (g-2) experiment on the lines indicated seem feasible. Technical problems to be worked out include:

i) magnet design;

ii) stabilization with reference to nuclear resonance;

iii) accurate field measurement;

iv) digital timing of many concurrent times of flight and data storage;

v) detection system for decay electrons;

vi) methods for probing the muon orbits;

vii) ejected proton beam gymnastics

Project cost ~ 3 M. SwFr.

Time scale ~ 3 years.

At the end of this one would hope to have a (g-2) value accurate to 1/2000, plus 10-20 bending magnets which can put to use in other experiments.

X. Acknowledgement

This project has been discussed at various stages with Messrs. Johnsen, Kuiper, Ramm, Schoch and Zilverschoon, who have made a number of valuable comments and suggestions.

TABLE I

Number of electrons emitted per interval of energy and angle

electron energy: 0 .2 .4 .6 .8 1.0 × 1.35 GeV

angle between tangent and emerging electron trajectory (rad)

angle between tangent and emerging electron trajectory (rad)	0 – .2	.2 – .4	.4 – .6	.6 – .8	.8 – 1.0 × 1.35 GeV	
0 – 0.25	4	-	37	326	293	group (1)
0.25 – 0.5	8	192	382	55	-	reject these
0.5 – 0.75	83	175	-	-	-	group (2)
0.75 – 1.0	131	-	-	-	-	
1.0 – 1.25	85	-	-	-	-	
1.25 – 1.5	58	-	-	-	-	

Total decays	4000
Total emitted	843 = 46%
(1) Total emitted > 500 mR	532 = 13%
(2) Total emitted < 250 mR	660 = 16%

TABLE 2

Mean asymmetries for corresponding boxes

0.43	-	0.04	0.15	0.26
0.43	0.12	0.03	0.09	-
0.20	0.19	-	-	-
0.25	-	-	-	-
0.32	-	-	-	-
0.34	-	-	-	-

(1) $\bar{A}$ for angles $>$ 500 mR = 0.19

(2) $\bar{A}$ " " $<$ 250 mR = 0.19

TABLE 3

Mean decay angle in degrees in muon centre of mass for corresponding boxes

(0 = forward decay)
(+ = towards outside of ring)

173	-	15	-20	-7
-177	-142	-91	-35	-
-178	-158	-	-	-
-161	-	-	-	-
-172	-	-	-	-
-174	-	-	-	-

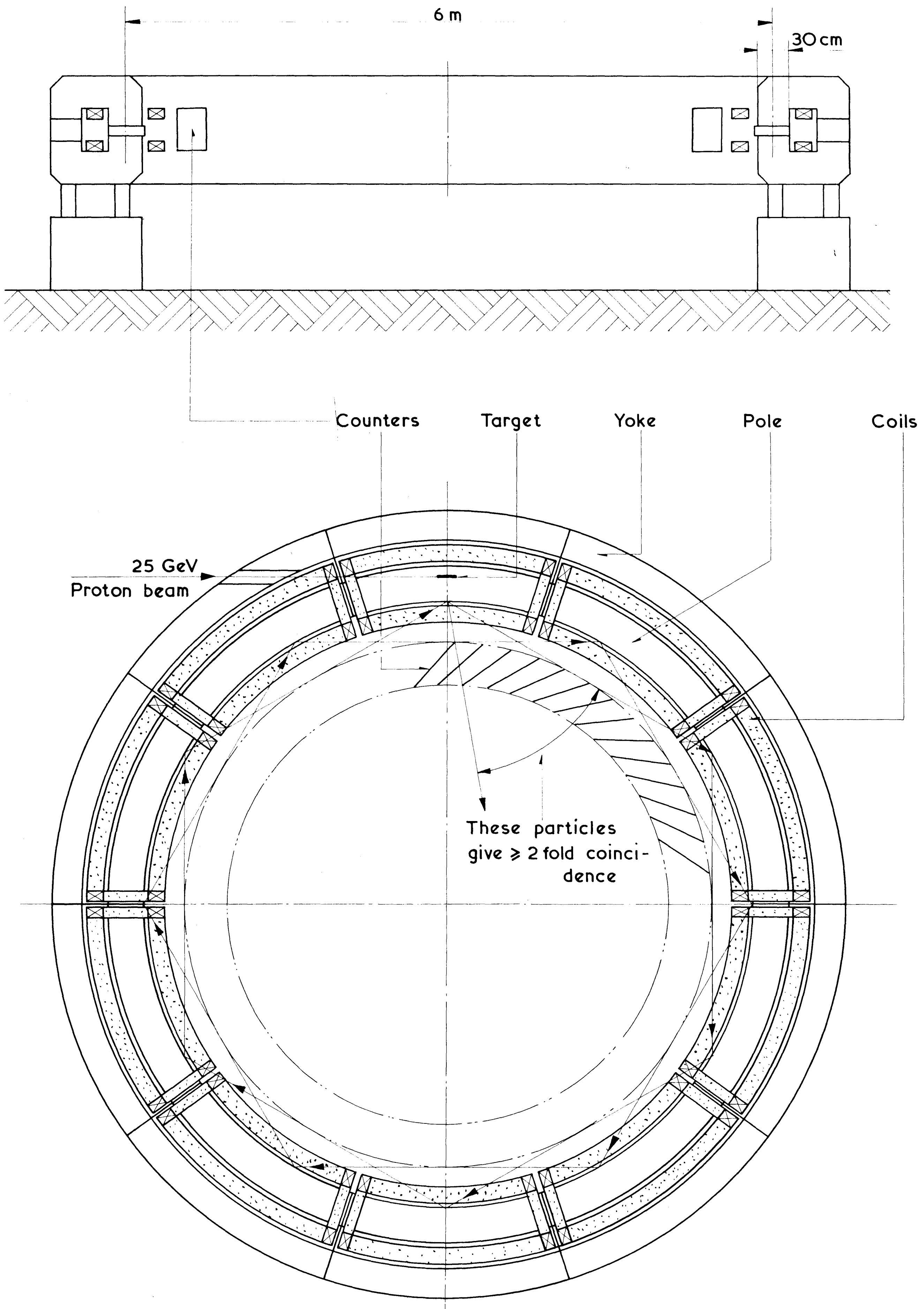
(1) Mean decay angle for > 500 mR group = -165°

(2) " " " " < 250 mR " = -12°

Counting separately groups (1) and (2) and comparing to generate an asymmetry value at each time gives overall

$$A = 0.17$$

with electron counting efficiency 29%.



MUON STORAGE RING.