Ref. TH 942

WEAK SELF-MASSES, CABIBBO ANGLE, AND BROKEN $SU_2 \times SU_2$

R. Gatto +)
CERN - Geneva

and

G. Sartori and M. Tonin
Istituto di Fisica dell'Università, Padova

A B S T R A C T

An excellent determination of the Cabibbo angle is obtained by relating it to strong interaction parameters through the requirement that weak self-masses be free of quadratic divergences. Speculations are presented as to a relation of both the angle and $SU_2 \times SU_2$ breaking to electromagnetism.

+) On leave of absence from Istituto di Fisica dell'Università, Firenze.

68/987/5 - TH 942
17 September 1968

In this note we shall show how an excellent determination of the Cabibbo angle [1] is obtained from the requirement that the second order matrix element for emission and reabsorption of the intermediate boson be finite. The insistence on a single charged Cabibbo current of universal form will lead us to a model where the non-vanishing of the angle, the small $SU_2 \times SU_2$ violation in the Lagrangian, and the non-vanishing pion mass are all related to electromagnetism [2].

We start from the matrix element

$$M = -i g_w^2 \int \frac{d^4 q}{q^2 - M_W^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2} \right) \int d^4 x \, e^{iqx} \Big[\langle \alpha | T(j_\mu(x) j_\nu^\dagger(0) + j_\mu^\dagger(x) j_\nu(0)) | \beta \rangle -$$

$$- \langle 0 | T(j_\mu(x) j_\nu^\dagger(0) + j_\mu^\dagger(x) j_\nu(0)) | 0 \rangle \Big] \qquad (1)$$

for a transition between the hadronic states α and β by emission and reabsorption of the intermediate charged boson W, coupled to the weak current j_μ. The quadratic divergent part in Eq. (1) can be separated by two partial integrations [3] and written in the form

$$\langle \alpha | M_2 | \beta \rangle - \langle 0 | M_2 | 0 \rangle$$

where

$$M_2 = i \left(\frac{g_w}{M_W} \right)^2 \int \frac{d^4 q}{q^2} \int d^4 x \, e^{iqx} \delta(x_0) \left\{ [D(x), j_0^\dagger(0)] - [D^\dagger(x), j_0(0)] \right\} \qquad (2)$$

with $D(x) = (\partial/\partial x_\mu) j_\mu(x)$. We assume a strong Hamiltonian $H = H_0 + \epsilon_0 u_0 + \epsilon_8 u_8$ (for the moment neglecting electromagnetism) where H_0 is invariant under chiral $SU_3 \times SU_3$, the scalars u_i $(i = 0, 1, \ldots, 8)$ together with corresponding pseudoscalars v_i belong to a $(3, \bar{3})$ and $(\bar{3}, 3)$

representation of $SU_3 \times SU_3$, and ϵ_0 and ϵ_8 are real parameters. It follows that the divergences of the vector and axial-vector unitary currents are linear combinations of the u_i and v_i with coefficients proportional to ϵ_0 and ϵ_8. For the weak current j_μ we assume, provisionally, a general form

$$j_\mu = (V_\mu^1 + iV_\mu^2)\cos\theta_V + (V_\mu^4 + iV_\mu^5)\sin\theta_V +$$
$$+ \beta\left[(A_\mu^1 + iA_\mu^2)\cos\theta_A + (A_\mu^4 + iA_\mu^5)\sin\theta_A\right]$$

For $\beta = 1$ and $\theta_V = \theta_A$ the current j_μ satisfies universality (i.e., the integrated fourth component of j_μ, j_μ^+, and of a neutral $j_\mu^{(0)}$ satisfies an SU_2 algebra [4]). We shall, for the moment, disregard the universality requirement, but return to it in the second part of this note. Inserting j_μ and the corresponding divergence D (as derived from the assumed weak current and Hamiltonian) into Eq. (2) one obtains that the quadratic divergent term M_2 is proportional to a linear combination of the following operators of our $(3,\bar{3})$ and $(\bar{3},3)$ representation : u_0, u_3, u_8, u_6 and v_7, with coefficients depending on ϵ_0, ϵ_8, θ_A, θ_V and β . However, as stressed by Bouchiat, Iliopoulos and Prentki [5], u_6 and v_7, being proportional to divergences of unitary currents, do not contribute between states of equal four-momentum [such as our states α and β in Eq. (1)]. We shall also ignore contributions from the unitary singlet u_0 (it contributes for instance an equal mass renormalization to each component of a unitary multiplet [6]). Thus, for a theory effectively free of quadratic divergences [7] it is enough to require in the expression for M_2 the vanishing of the coefficient in front of u_3 (no divergent $\Delta S = 0$, $\Delta T = 1$ amplitudes) and of that in front of u_8 (no divergent $\Delta S = 0$, $\Delta T = 0$, octet amplitudes). When explicitly written, the two conditions are seen to be equivalent to :

$$\sin^2\theta_V / \sin^2\theta_A = \beta^2 \left(1 + \tfrac{2}{3}\rho\right) \qquad (3)$$

$$tg^2\theta_A = \rho / (3 + 2\rho) \qquad (4)$$

where we have introduced the useful parameter $\varrho = -(1+\sqrt{2}\,\epsilon_0/\epsilon_8)$. Note the two limits : $\varrho = 0$ implies that H is invariant under chiral $SU_2 \times SU_2$; $\varrho = \infty$ implies SU_3 invariance. From Eqs. (3) and (4) eliminating ϱ one gets

$$\frac{\sin^2\theta_A}{\sin^2\theta_V} = \beta^{-2}\,(1 - 2\,tg^2\theta_A)$$

Typically, for $\beta = 1$, one has $\theta_V - \theta_A = tg^3\,\theta_A = 10^{-2}$; for $\theta_V = \theta_A$, $\beta = 0.94$. In any case the required violation of universality turns out to be very small. However, for $\beta = 1$ and $\theta_V = \theta_A \equiv \theta$ one has $\varrho = 0$ i.e., exact $SU_2 \times SU_2$, and $\theta = 0$. The implication of a limit with $\theta = 0$ and exact $SU_2 \times SU_2$ is rather remarkable and we shall have to come back to this point. To calculate θ_A from Eq. (4), we resort to the work of Glashow and Weinberg [8] which allows for a determination of ϱ from Ward identities on two-point functions. Defining [8] $Z_i^{\frac{1}{2}} = \langle 0|u_i(0)|i\rangle$ one can write Eq. (4) in the form

$$tg^2\theta_A = \frac{1}{2}\,\frac{F_\pi\,\mu_\pi^2\,Z_\pi^{-\frac{1}{2}}}{F_K\,\mu_K^2\,Z_K^{-\frac{1}{2}}} \tag{5}$$

where F_π, F_K are the usual meson decay constants and μ_π, μ_K are the masses of π and K. Equation (5) already suggests one relevant feature : the proportionality of θ_A to the ratio of pion to kaon mass. But we can go further. From the work of Glashow and Weinberg [8] we can calculate ϱ in terms of the parameters $\beta_i = \mu_i^2\,F_i^2$ ($i = \pi$, K and $\varkappa$, the latter being the assumed scalar strange meson [8]). One has

$$\varrho = \frac{3}{4}\,(\beta_K - \beta_\pi - \beta_\varkappa \pm \sqrt{\Delta}\,)/\,\beta_\varkappa$$

where

$$\Delta = \beta_K^2 + \beta_\pi^2 + \beta_\varkappa^2 - 2\beta_K\beta_\pi - 2\beta_K\beta_\varkappa - 2\beta_\varkappa\beta_\pi$$

For the plus sign, when $\beta_\varkappa \to 0$, one has $\varrho \to \infty$ (i.e., SU_3 invariant Lagrangian) ; for the minus, when $\beta_\pi \to 0$ one has $\varrho = 0$ (i.e., $SU_2 \times SU_2$). Choosing the sign minus and with the values of the parameters β_K, $\beta_\varkappa$ and

β_π as given by Glashow and Weinberg [8] one obtains $\theta_A = 0.27$. Furthermore the solution is found to be remarkably stable against variation of the parameters. Note also that, since ρ is very small, one can approximate Eq. (4) as $tg^2\,\theta_A \cong \rho/3$.

We have seen that the requirement of absence of quadratic divergence leads to an excellent determination of the Cabibbo angle, provided one is willing to add a small term violating universality in the weak current. We have restricted our discussion to a theory with only charged currents ; theories with neutral currents can also be studied similarly, but they will not be discussed here (also in general they do not satisfy the requisite of universality [9]). A theory with only a charged universal Cabibbo current is generally considered as the most appealing possibility. On the other hand, the smallness of the universal addition required here suggests the conjecture that such an addition may have appeared because of having neglected electromagnetism. In the rest of this note we shall concentrate on analyzing such a possibility. To this purpose let us extend our model by including a term $\epsilon_3 u_3$ in the Hamiltonian. Our Hamiltonian is now $H = H_0 + \epsilon_0 u_0 + \epsilon_8 u_8 + \epsilon_3 u_3$. By adding a breaking in the direction "3" in SU_3 space we have included the relevant geometric feature brought about by electromagnetism. In terms of mathematical quarks, $\epsilon_0 u_0$, $\epsilon_8 u_8$, $\epsilon_3 u_3$ are mass terms and, in particular, $\epsilon_3 u_3$ makes the mass of the p and n quark different. The parameter ϵ_3 is clearly of the order e^2. We now assume a universal current (i.e., $\theta_A = \theta_V$ and $\beta = 1$ in the expression for j_μ) and derive, with the new Hamiltonian, the coefficients in front of u_3 and u_8 in the expression for M_2 (we call them b_3 and b_8 respectively). From the two equations $b_3 = 0$ and $b_8 = 0$ one obtains [10] :

$$tg^2\,\theta = \frac{\rho}{3+\rho} + \left[1 + \left(\frac{\rho}{3+\rho}\right)^2\right]^{\frac{1}{2}} - 1 \qquad (6)$$

$$\frac{\sqrt{3}\,\epsilon_3}{\epsilon_8} = \frac{\rho^2}{3+\rho + \left(\rho^2 + (3+\rho)^2\right)^{\frac{1}{2}}} \qquad (7)$$

For small ρ, from Eq. (6), one has $tg^2\,\theta \cong \rho/3$, as before. We thus obtain a good value of θ [more precisely from Eq. (6), $\theta = 0.29$], but this time the current j_μ is of universal form. In addition we have Eq. (7). Numerically, from Eq. (7) one obtains $\epsilon_3/\epsilon_8 = 0.88(1/137)$, showing that the order of magnitude involved is not in disagreement with our conjecture. Equation (7) poses a problem of interpretation. Let us discuss two limiting cases : small ϵ_3 and large ϵ_3 (small or large as compared to ϵ_8). For small ϵ_3, Eq. (7) gives a small $\rho = (6\sqrt{3}\,\epsilon_3/\epsilon_8)^{\frac{1}{2}}$, i.e., the Hamiltonian is in such a case almost symmetrical with respect to chiral $SU_2 \times SU_2$ (this does not imply of course that also the solutions are close to $SU_2 \times SU_2$). In the limit of large ϵ_3 one has instead large ρ, i.e., a Hamiltonian which is almost SU_3 invariant. As we have seen, both the experimental value of θ and the determination of ρ à la Glashow and Weinberg, suggest that the right limit is that of small ϵ_3. It is interesting to summarize what happens near this limit :

i) the Hamiltonian is almost $SU_2 \times SU_2$ invariant [11], the deviation from symmetry being measured by the parameters ρ, which takes the value $\rho \cong (6\sqrt{3}\,\epsilon_3/\epsilon_8)^{\frac{1}{2}}$ (since $\epsilon_3 \propto e^2$ one has $\rho \propto \sqrt{e}$) ;

ii) the Cabibbo angle is small and given by $tg^2\,\theta = (2\epsilon_3/\sqrt{3}\,\epsilon_8)^{\frac{1}{2}}$ (therefore $\theta \propto \sqrt{e}$) ;

iii) the ratio $(F_\pi\,\mu_\pi^2\,Z_\pi^{-\frac{1}{2}})/(F_K\,\mu_K^2\,Z_K^{-\frac{1}{2}}) \cong \rho/3$.

A very suggestive picture has thus emerged : the deviations from $SU_2 \times SU_2$ in the Hamiltonian (and consequent non-vanishing pion mass) and the non-vanishing of the Cabibbo angle are both related to electromagnetism [12]. The relevant equations are numerically well satisfied.

One of us (R.G.) is indebted for discussions and remarks to Professors S. Glashow, T.D. Lee, Y. Nambu and S. Treiman. He would also like to thank Professors J. Prentki, W. Thirring and L. Van Hove for kind hospitality at CERN.

R E F E R E N C E S

1) N. Cabibbo - Phys.Rev.Letters 10, 531 (1963).

2) In a previous investigation on the same subject (Preprint, University of Padua), we had based our discussion on a Cabibbo current with $\theta_A \neq \theta_V$ neglecting electromagnetism.

3) The assumptions are :

i) Schwinger terms are c numbers ;

ii) the Bjorken limit (J.D. Bjorken - Phys.Rev. 148, 1467, 1966) can be applied to the Fourier transform of $T\left[D(x)D^+(0)\right]$.

4) M. Gell-Mann - Proceedings of the 1960 International Conference on High Energy Physics, University of Rochester, p. 508 (1960).

5) C. Bouchiat, J. Iliopoulos and J. Prentki - Preprint CERN TH 908 (1968).

6) We take from experiment that the solution of the field equations has to be approximately SU_3 symmetric (possibly through a mechanism of spontaneous breaking).

7) Our attitude is the following : we assume that finiteness has to be required (at least for the lowest order) before summing the weak perturbative expansion. In this attitude we may differ from other authors (e.g., M. Gell-Mann, M.L. Goldberger, N.M. Kroll and F.E. Low, to be published). Within our attitude and with the assumptions we make, particularly on the Hamiltonian, our result for the Cabibbo angle is a necessary condition for finiteness. Concerning the logarithmic divergences, which may be present in M, Eq. (1), a possibility is that the same "dynamical" mechanism which will assure finite e.m. mass differences (e.g., T.D. Lee,- Phys.Rev. 168, 1714, 1968 ; S. Ciccariello, M. Tonin and G. Sartori - Nuovo Cimento 55A, 1847, 1968) will make those divergences disappear also in our