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PRECISION DETERMINATION OF THE β-PARAMETER IN Λ-DECAY

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I. INTRODUCTION

The general theory of the experiment has been given in our previous proposal[1]. The polarization of the decay proton is specified by the parameters α, β and γ. In the co-ordinate system (fig. 1) in which the Λ is at rest, we choose the z axis to be along the Λ spin direction, the x axis such that the proton momentum lies in the x-z plane, and the y axis to form a right hand co-ordinate system. The angle Θ is the polar angle, i.e. the angle between the Λ spin and the proton momentum. In this system the components of the proton polarization are[2]:

$$P_x = (\alpha \sin\Theta + P_\Lambda(1-\gamma)\sin\Theta\cos\Theta)/(1+\alpha P_\Lambda \cos\Theta) \tag{1a}$$

$$P_y = \beta P_\Lambda \sin\Theta/(1+\alpha P_\Lambda \cos\Theta) \tag{1b}$$

$$P_z = (\gamma P_\Lambda + \alpha\cos\Theta + P_\Lambda(1-\gamma)\cos^2\Theta)/(1+\alpha P_\Lambda\cos\Theta). \tag{1c}$$

In the experiment we measure in the laboratory system the transverse proton polarization P_t parallel to the production plane:

$$P_t = P_y \cos X - P_x \sin X \tag{2}$$

in which X is the angle between the projections on the production plane of the proton momentum in the lambda rest system and in the laboratory. Since the proton momentum in the laboratory is close to the Λ line of flight, a good approximation to χ is the azimuthal angle between the x axis (in the system transcribed above) and the lambda line of flight. See fig. 1.

In terms of α, β and γ, the transverse polarization in the production plane is given by

$$P_t = (\beta\, P_\Lambda \cos\chi - [\alpha + P_\Lambda (1-\gamma)\cos\Theta]\sin\chi)\sin\Theta / (1 + \alpha P_\Lambda \cos\Theta) \quad (3)$$

In order to select a sample of events which are useful for determining β, it is necessary to choose those events with χ near 0^0 and Θ near 90^0. These are events in which the proton momentum is really coincident with the Λ line of flight in the Λ rest system. In order to eliminate a dependence on the accuracy of the known values of α and γ, it would also be desirable to choose events with a distribution in χ which is symmetric around 0°. Deviation from this distribution in the sample chosen by our triggering system introduces a calculable bias into the experiment, which is discussed below.

II. PRECISION IN β DETERMINATION

The determination of β from the azimuthal asymmetry of proton scatterings in a polarization analyser is achieved if each event with the proton scattering at an azimuth φ_s with respect to the Λ^0 production plane is counted with a weight

$$w_i = P^2(\Theta_s, E_s)\sin^2\varphi_s \cdot \left(\frac{dP_t}{d\beta}\right)^2 \quad (4)$$

In this formula $P(\Theta_s, E_s)$ is the analysing power of the analyser for scatterings at energy E_s and an angle Θ_s.

The resulting statistical error in β is

$$\delta\beta = (\Sigma\, w_i)^{-1/2} \quad (5)$$

in which the sum extends over lambda events containing useful proton scatters.

The expected error in an experiment in which N Λ-events are observed is

$$\delta \beta_{exp} = \left(N \cdot \left\langle \left(\frac{d P_t}{d \beta} \right)^2 \cdot A(E_p) \right\rangle_{Av} \right)^{-1/2} \qquad (6)$$

in which the average is taken over production and decay distribution of the observed Λ - events. $A(E_p)$ is the analysing power of the polarization analyser.

$$A(E_p) = \frac{N_0}{A} \int_0^{E_p} \frac{dE \cdot}{-dE/dx} \int_0^{\pi} 2\pi \, d(\cos \Theta) P^2(\Theta_s, E) \cdot \frac{1}{2} \frac{d\sigma}{d\omega}(\Theta_s, E) \qquad (7)$$

where N_0 is Avogadro's number, A the atomic weight, and dE/dx the energy loss per g/cm^2 of the analysing material, and $d\sigma/d\omega$ (Θ_s, E_s) the differential cross section for protons of energy E_s .

The expression

$$\left\langle \left(\frac{d P_t}{d \beta} \right)^2 A(E_p) \right\rangle_{Av}$$

in eq. (5) will be referred to as the figure of merit of the experiment.

We have calculated $A(E_p)$ using the polarization and differential cross section data for proton - carbon scattering compiled by Peterson.[5] An average value of 10 MeV proton energy loss due to inelastic interactions was assumed. This curve is shown in Fig. 2.

III. APPARATUS

The top view of the proposed set-up is shown in fig. 3. The set of carbon plate spark chambers, in which the transverse polarization of the proton will be analysed, will be contained in a slightly modified version of the University of Geneva polarimeter. The graphite plate spark chambers, most of which already exist, are made of plates 1 cm thick; the total thickness of carbon will be such to stop 400 MeV protons. The optical system of the Geneva polarimeter gives a direct top view and a side view via a 45° mirror. In practically all of the events accepted by the triggering system, the protc will remain in the polarimeter.

The polarimeter will also contain

a) the thin plate spark chamber to observe the incoming pion direction;

b) a thin-walled liquid hydrogen target. The diameter and length of the target are not critical, but the end of the hydrogen volume should be as close as possible to the spark chamber in which the lambda decay is observed. For the present study we have assumed this distance to be 3 cm and the target length to be 4 cm;

c) the $\bar{B}$ counter, assumed to be 1 cm thick;

d) the thin plate decay chamber, 10 cm thick;

e) the counters π and $\bar{\pi}$, the dimensions of the π detector are such that it subtends angles from 30° to 70° as seen from the exit of the $\bar{B}$ counter;

f) a proton telescope consisting of the counters P_1 and P_2 , and perhaps a Čerenkov counter.

IV. OPTIMIZATION OF THE EXPERIMENT

The experiment will use a system of triggering counters which will restrict the angles and energies of the pion and proton from the decay of the Λ^0 . The experiment must be optimized to achieve the best possible accuracy in the determination of β in a given amount of machine time.

The following general consideration can be made. We have assumed that the experiment will be performed in a beam of negative pions of momentu 1.03 GeV/c with a maximum intensity of $3\cdot10^5$ pions per PS burst. This is well within the performance of the q or m beams for a PS intensity of 10^{12} protons per second. We aim in the first instance to take one photograph per PS pulse. The optimization of the experiment aims at obtaining the best possible figure of merit for a triggering rate of 1 trigger per burst.

This optimization procedure uses a Monte Carlo programme on the CDC 6600 which generates random artificial events with the production and decay distributions of the Λ^0 , selects those which fit the geometric constraints of the triggering system, lists the relevant quantities and plots the corresponding histograms.

As seen from eq.(6) the figure of merit involves two factors the sensitivity $\frac{dP_t}{d\beta}$ of the measured polarization to β and the analysing power $A(E_p)$ of the polarimeter. To have the first factor large, one prefers, according to eq.(3) Λ decays with $\vartheta \approx 90^\circ$, $\chi = 0^\circ$. These are events in which the proton is emitted parallel or antiparallel to the Λ line of flight. In the first case, the proton has in the laboratory preferentially a high energy, in the second case, it is low. Fig. 2 shows that the analysing power of the carbon polarimeter increases rapidly with energy. One therefore prefers to select the events with forward emission of the proton.

These general arguments have led us to consider in more detail the following two triggering criteria:

Case I : A forward going pion with a short projected range of

$$0.1 < R_\pi < 2 \text{ g/cm}^2 \qquad \text{(Ia)}$$

or

$$0.1 < R_\pi < 1 \text{ g/cm}^2 \qquad \text{(Ib)} .$$

The corresponding low π energy will leave most of the Λ energy to the proton. Fig. 4 shows that the requirement of low π energy selects forward going protons in the Λ^0 system almost independently of Λ momentum.

The original proposal used a triggering system in which the pion went backwards in the laboratory. This system was found to have the disadvantages of favouring Λ^0 produced with low energy and therefore a low proton energy, which makes the polarization analysis difficult. We are grateful to Dr. C. Rubbia for clarifying this point. In the present system, we trigger on π of large but forward angles. This has the additional advantage that a hydrogen target can be used, with a gain in Λ polarization. We have used the results of Crawford[4)] , that the polarization of Λ^0 produced by 1.03 GeV/c π^- is close to 100% over most of the angular distribution.

For this reason the calculations have been done assuming a polarization of 100% for all values of the production angles. This, however, is an approximation which will be corrected in later calculations. The polarization of the Λ^0 must vanish from symmetry considerations at $\Theta_\Lambda^* = 0^\circ$ and 180°. The polarization data of Steinberger[3]) at 1.12 GeV/c incident pion energy indicates that the polarization is small and negative for values of Θ_Λ^* less than 60°. This leads us to prefer a triggering system which accepts a wide range of production angles.

Case II. A minimum range for the proton

$$R_p > 60 \text{ g/cm}^2$$

No range requirement for the pion.

These two cases represent the two extremes for this type of triggering. Intermediate cases will be studied in the future.

Out of a total of 8600 artificial Λ events produced in the target, 1613 would decay after the anticoincidence counter in the charged mode. The number satisfying the criteria Ia, Ib and II are 101, 44 and 74 .

Figure 5a shows the cos Θ_Λ^* distribution for all the decays and for the 3 selection criteria. The selection of forward Θ_Λ^* by criterium II is prominent, and is an argument against, because of the uncertainty of the Λ polarization in this region, as discussed above.

Figure 5b gives the corresponding distributions in $\beta \cdot \frac{dP_t}{d\beta}$. It is clear that all three criteria select a sub-sample with high sensitivity to β .

Figure 5c shows the distribution of the figure of merit as defined in section II. The selection criteria are quite equivalent with an average figure of merit of 0.83×10^{-2} for case Ia, 1.12×10^{-2} for case Ib, and 1.17×10^{-2} for case II.

V. BACKGROUND AND TRIGGER RATE

In the Cronin - Overseth experiment, with a very simple triggering system, one picture out of three contained a lambda decay. With our more sophisticated and selective triggering system essentially every picture should be of this kind. As long as the trigger rate is the main limitation on the number of pictures taken in the experiment, this will allow a factor 3 more events to be collected.

The main sources of background triggers are the following:

a) charge exchange reactions in which one of the decay gammarays materialises in the pion counter and the neutron gives a recoil proton through the proton telescope. With reasonable assumptions about conversion probabilities we arrive at an upper limit of 0.5 triggers per 10^6 incoming pions of such events. Without the Čerenkov counter, we would also have the same order of rate from events where a gammaray shower triggers the proton telescope.

b) Neutron stars in the last mm of the anticoincidence counter $\bar{\bar{B}}$ or in the thin foil spark chambers sending one particle into the π counter and a proton into the proton telescope. The number of neutrons interacting is about 10^{-6} per pion. Only a small fraction of these reactions produce tracks of sufficient length to trigger the counters, for example by the reaction

$$n + {n \atop p} \rightarrow \pi^- + p + {n \atop p}$$, which

at a typical neutron energy of 300 MeV would have a cross section of only 3% geometric. We therefore estimate that this effect gives a rate of less than one trigger per 10^7 pions.

c) Associated production with K_1^0 in a charged decay mode triggering the set-up. These events will be a small fraction of the events triggered by the lambda, because of the lower probability for charged decay modes and the different decay kinematics. Furthermore the Čerenkov counter in the proton telescope will reduce these background triggers even more.

d) Accidental coincidences of different types. The most important contribution comes from an accidental coincidence of a count in the pion counter with a true coincidence in the remaining counters. For a beam rate of 10^6 particles/sec this accidental rate is much less than the rates from backgrounds a) and c). This is partly due to the fact that only photomultiplier noise and general room background pulses in the pion counter will contribute to the accidental rate because of the anti-coincidence counter in the incoming beam.

Some of these estimates of the background are supported by the result of a background test run in the q beam using a counter set-up similar to that envisaged in the experiment. The trigger rate found in these runs was of the order of $3 \cdot 10^{-6}$ per incoming pion. In particular the contribution from pion charge exchange was found to be small since the addition of a lead converter in front of the proton telescope did not increase the trigger rate. The result further supports our prediction that accidental rates will not contribute appreciably to the background.

The triggering rate for the experiment has been estimated from the Monte Carlo results. We assume an incident pion beam of intensity $3 \cdot 10^5$/burst, which corresponds to an instantaneous rate of 10^6 pions/sec. Using the known lambda production cross section, the rate of events coming from lambdas produced in the liquid hydrogen is 0.79/burst. We assume that we will have approximately the same number of triggers coming from lambdas produced in the end wall of the hydrogen target and the first 1 mm of the $\bar{B}$ counter, which are useful events with lower Λ^0 polarization. The total trigger rate, therefore, including a possible 20% contribution from background events is:

$$\text{trigger rate} \simeq 2.0 \text{ events/burst.}$$

This number has been calculated for the selection criterion Ib. The rate can be adjusted through the use of a different range requirement of the pion, or by introducing a minimum proton range requirement (case II), or both. The exact trigger requirements remain to be optimized as discussed above.

VI. BIASES AND PRECISION OF RESULT

The term

$$[\alpha + P_\Lambda(1-\gamma)\cos\vartheta]\sin\chi$$

in the expression (3) for the measured transverse polarisation P_t constitutes a bias in the determination of β unless the determination is made with an apparatus which does not discriminate between the two signs of the angle χ, between the proton direction in the Λ^0 system and the laboratory system. This was approximately true for the very loose triggering system and wide polarimeter of CO, at least to an extent such that the bias was within the wide statistical errors of their experiment. The more restrictive triggering systems we propose to use will introduce an asymmetry in the χ distribution, as shown by the result of the Monte Carlo calculation, which gives the weighted distribution in tg χ of fig.6 and fig. 7 for case Ia and case II. The average of tg χ is 0.2 for case Ia. The second term in (3) would then with $\alpha = 0.62$ and for a effective Λ^0 polarization of 0.8 give a contribution of 0.16 compared to the expected theoretical value of β . cos X of 0.08. It is thus clear that this bias has to be properly treated if we aim at an accurate value for β .

Two methods offer themselves to resolve this problem. If α and P_Λ are accurately known we can correct for the influence on P_t using the measured distribution over χ, without loss of accuracy. With the present limits of error on α and assuming a 10% uncertainty in P_Λ this correction would give a systematic error of about 0.02 in β , of the same order as the statistical error. However, it may be possible to determine α and P_Λ more accurately in the course of the experiment, so that this systematic error is made negligible.

The other method applies if α and P_Λ are unknown or badly known and consists in removing the asymmetry of the experimental data with respect to χ by calculating separately P_t for events within symmetrically situated bins in the χ distribution and to take the average between the values for these bins. This method would not introduce a systematic error but would increase the statistical error. It can be shown that the statistical error is not substantially increased (by more than 15%) if the value of the asymmetry is less than 3, as seems indeed to be the case.

More refined methods can certainly be devised which optimize the accuracy of the final result also in the intermediate case with finite, but non-zero errors in the values α and P_Λ .

We therefore feel confident that this bias will not appreciably influence the accuracy of the final result. More detailed studies of the effect will, however, be made using artificial events.

The precision of measurement of the ranges and angles involved in the reconstruction of the decay will influence the accuracy of the final result, via the angles ϑ and χ in eq.(3) which are needed for the interpretation. Preliminary estimates indicate that these errors will be small compared to the statistical errors. However, a more detailed study will be undertaken.

VII. CONCLUSIONS

We find that we can get an event rate of at least one per burst, leading to 300,000 pictures in 60 shifts at 2.3 sec repetition rate. Our estimated background event rate is low and we should have 200,000 good Λ events. With a figure of merit of $0.8 \cdot 10^{-2}$ per event and an average Λ polarization of 0.7, we arrive at a statistical error $\Delta\beta = 0.04$, compared to the present value of ref. 2, (Cronin-Overseth), $\Delta\beta = \pm 0.24$, which will be improved by a factor of 2 in their present experiment. As discussed, systematic errors should be small or could be corrected for.

Manual scanning of 300,000 pictures and measuring of about 20,000 scattering events would require about 20 girl·months. We intend, in collaboration with a group from the University of Lund, Sweden, to use a Vidicon on-line to the CDC 6600 for the immediate analysis of the results, with simultaneous photography, to be analysed with Luciole or HPD.

Testing time-invariance to this accuracy may require use of dispersion theory to interpolate the $\pi-N$ phase shifts to the relevant energy or possibly more accurate $\pi^- p$ scattering experiments.

In addition to the production run at the PS, we will need a calibration of the polarimeter at the SC as an extension of the calibration run foreseen by the Geneva group. We will also need 3 weeks of parasiting in the final beam for setting up and testing of the equipment. We would be ready for this in November-December 1965.

REFERENCES

1) J.K. Bienlein, W. Cleland, G. Conforto, H.-J. Gerber and M. Veltman, Proposal for an experimental test time reversal invariance in lambda decay (CERN-NP Internal report, 5 April, 1965) (Unpublished).

2) J.W. Cronin and O.E. Overseth, Phys.Rev. 129, 1795 (1965).

3) J. Steinberger, 1958 CERN Conference on High-Energy Physics, p. 147.

4) F.S. Crawford et al., 1962 CERN Conference on High-Energy Physics, p. 270.

5) Z. Peterson, Lawrence Radiation Laboratory Report UCRL 10622 (unpublished).

FIGURE CAPTIONS

Fig. 1 : Diagram which illustrates the co-ordinate system described in the text. The system is in the lambda rest system, with the z axis along the lambda spin. The x axis is chosen such that the proton momentum lies in the x - z plane. The dotted line is the projection of the laboratory momentum projected on the production plane, which is close to the direction of the lambda line of flight.

Fig. 2 : Proton analyzability $A(E_p)$ versus the proton kinetic energy. This function is differential scattering times the square of the analysing power intergrated over the scattering angles and integrated over the energy range 0 to E_p.

Fig. 3 : Diagram of the apparatus. The top view of the polarimeter of the University of Geneva is shown. The dotted lines indicate the cross section in the side view. The trigger requirement is ($B, \bar{B}, \pi, \bar{\pi}, P_1, P_2, \check{C}$). All the counters are plastic scintillators except $\check{C}$ which is a lucite Čerenkov counter.

Fig. 4 : Kinetic energy of the pion E_π versus the angle Φ between the proton momentum and the lambda momentum in the centre of mass system of the lambda. Curves are drawn for different lambda laboratory momenta.

Fig. 5 : Results of the Monte Carlo calculation for three triggering requirements and for a complete sample of events. For each quantity, four distributions are made:

1) A complete sample (shown at bottom). This distribution is similar to that in the experiment of CO. The vertical scale for these histograms are reduced by the factor shown in the lower left hand corner.

2) The events in which the pion enters the pion detector with the projected range ($R_\pi^{Proj} = R_\pi \cos \Theta_\pi$, in which R_π is the range of the pion in carbon and Θ_π is the laboratory angle with respect to the beam line) is between 0.1 and 2.0 g/cm^2 (Case Ia).

3) The events in which the pion enters the pion detector with the projected range between 0.1 and 1.0 g/cm^2 (Case Ib).

4) The events in which the projected range of the proton is greater than 60 g/cm^2 of carbon (shown at top). (Case II).
The following quantities are plotted in each set of histograms with the same horizontal scale.

Fig. 5a : The cosine of the emission angle of the Λ^0 in the production centre of mass system. In the generation of the events, the known production angular distribution was used.

Fig. 5b : The quantity $\beta \left(\frac{dP_t}{d\beta}\right)$ described in the text. In the calculation P_Λ was set equal to unity and the theoretical value of β_{th} = - 0.08 was used.

Fig. 5c: The figure of merit for each event $\left(\frac{dP_t}{d\beta}\right)^2 A(E_p)$, which is described in the text.

Fig. 6 : Weighted distribution of tan χ for the events selected by a maximum pion range of 2 g/cm^2 (Case Ib). The weight for each event is the quantity $\left(\frac{dP_t}{d\beta}\right)^2 A(E_p)$ as described in the text.

Fig. 7 : Weighted distribution of tan χ for the events selected by a minimum proton range of 60 g/cm^2 (Case II).

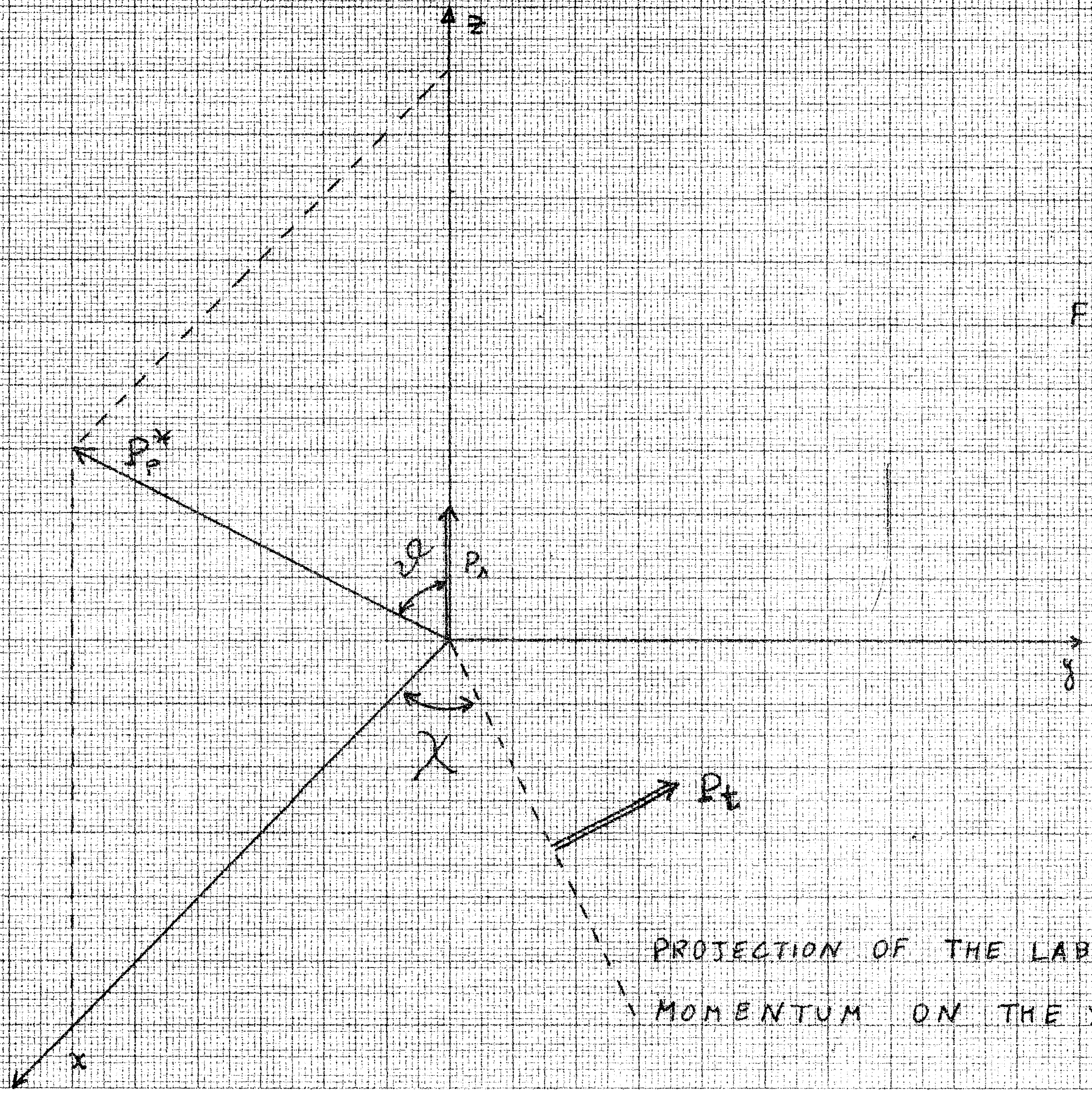


FIG. 1.

PROJECTION OF THE LABORATORY PROTON MOMENTUM ON THE XY PLANE (PROD. PLANE)

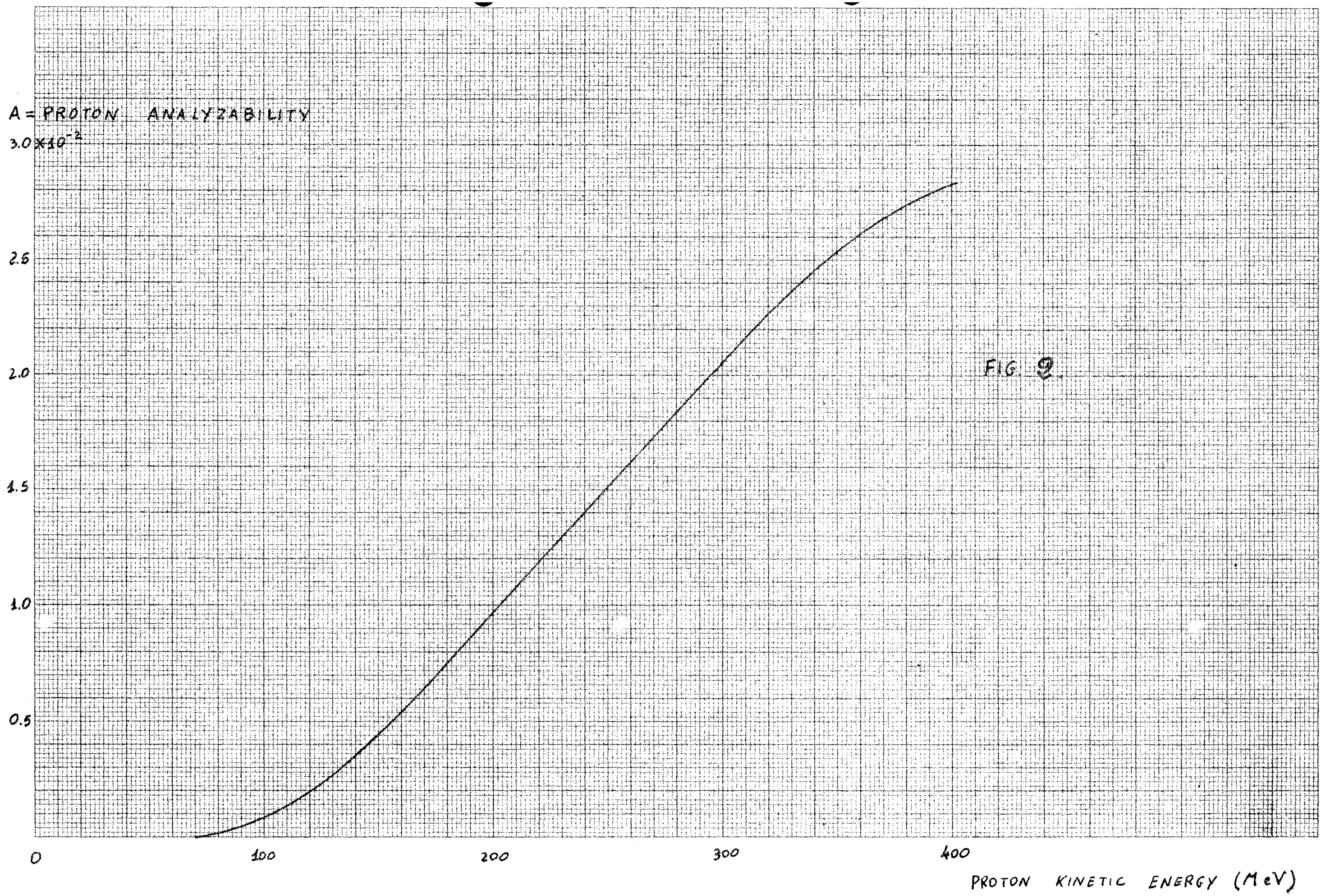

A = PROTON ANALYZABILITY
3.0×10⁻²
2.5
2.0
1.5
1.0
0.5
0
100
200
300
400
PROTON KINETIC ENERGY (MeV)
FABRICATION SUISSE

FIG. 9

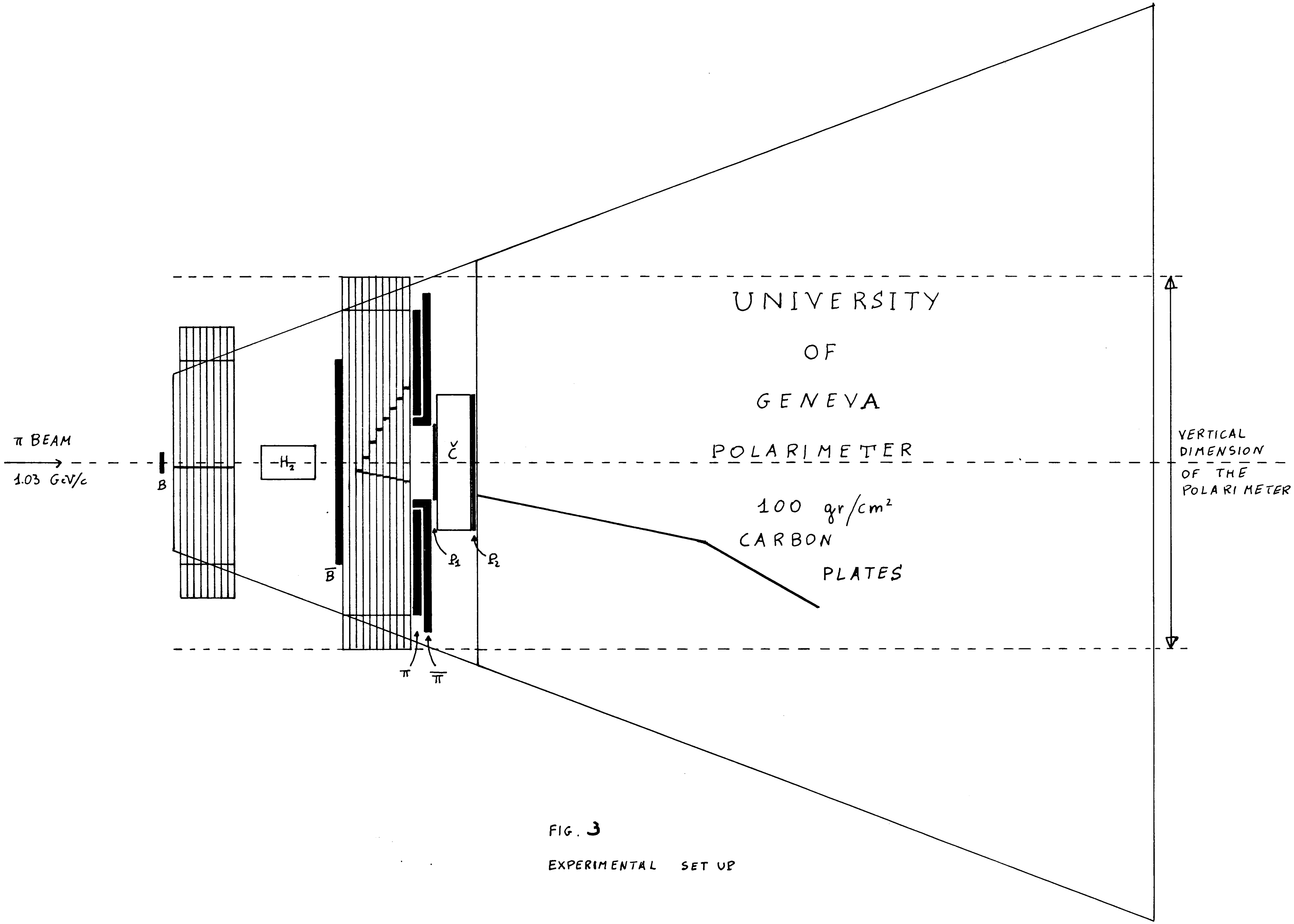
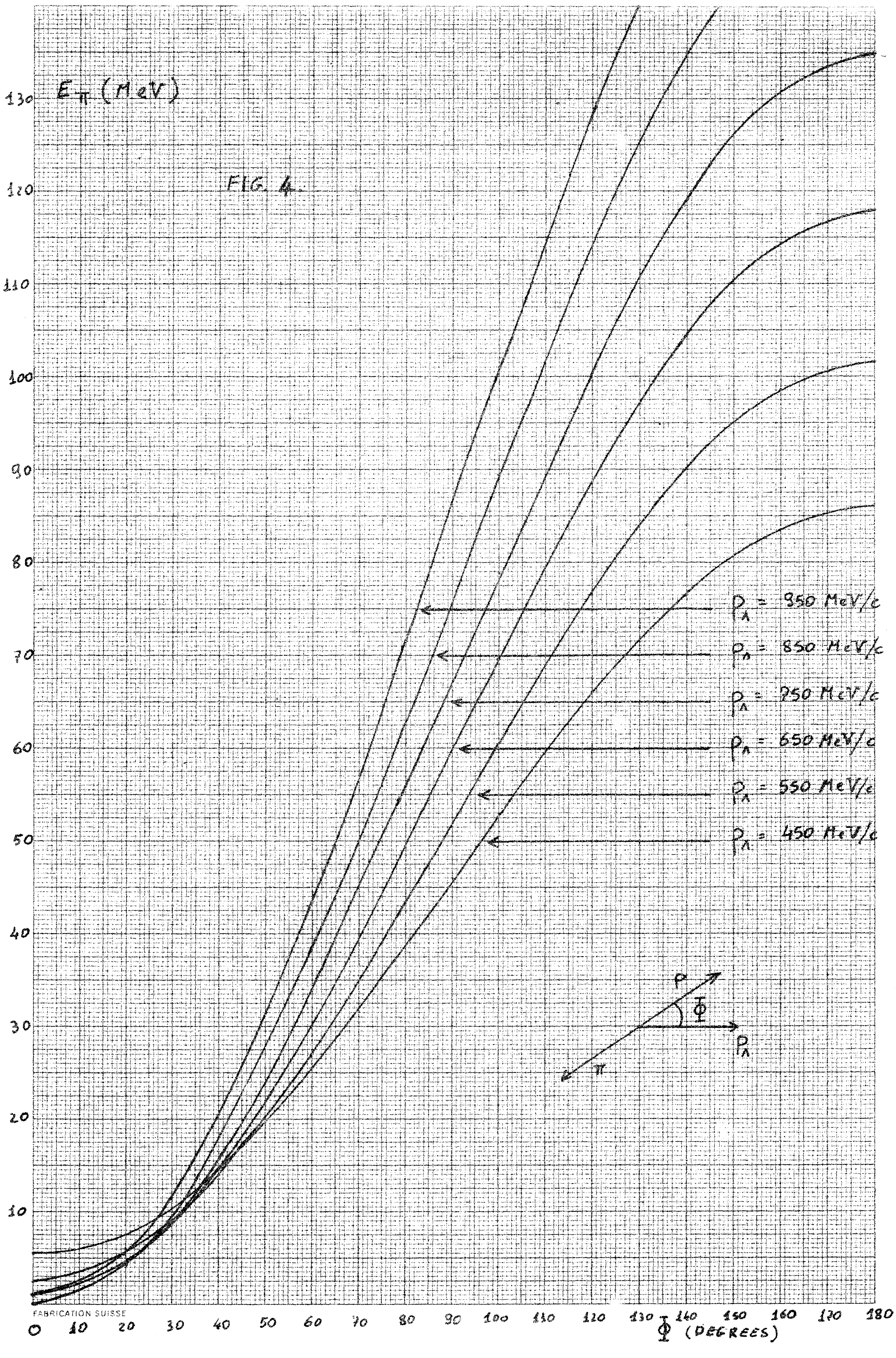


FIG. 3

EXPERIMENTAL SET UP



E_π (MeV)
FIG. 4.
p_Λ = 950 MeV/c
p_Λ = 850 MeV/c
p_Λ = 750 MeV/c
p_Λ = 650 MeV/c
p_Λ = 550 MeV/c
p_Λ = 450 MeV/c
p
Φ
p_Λ
π
FABRICATION SUISSE
Φ (DEGREES)

Cos θ_Λ^*

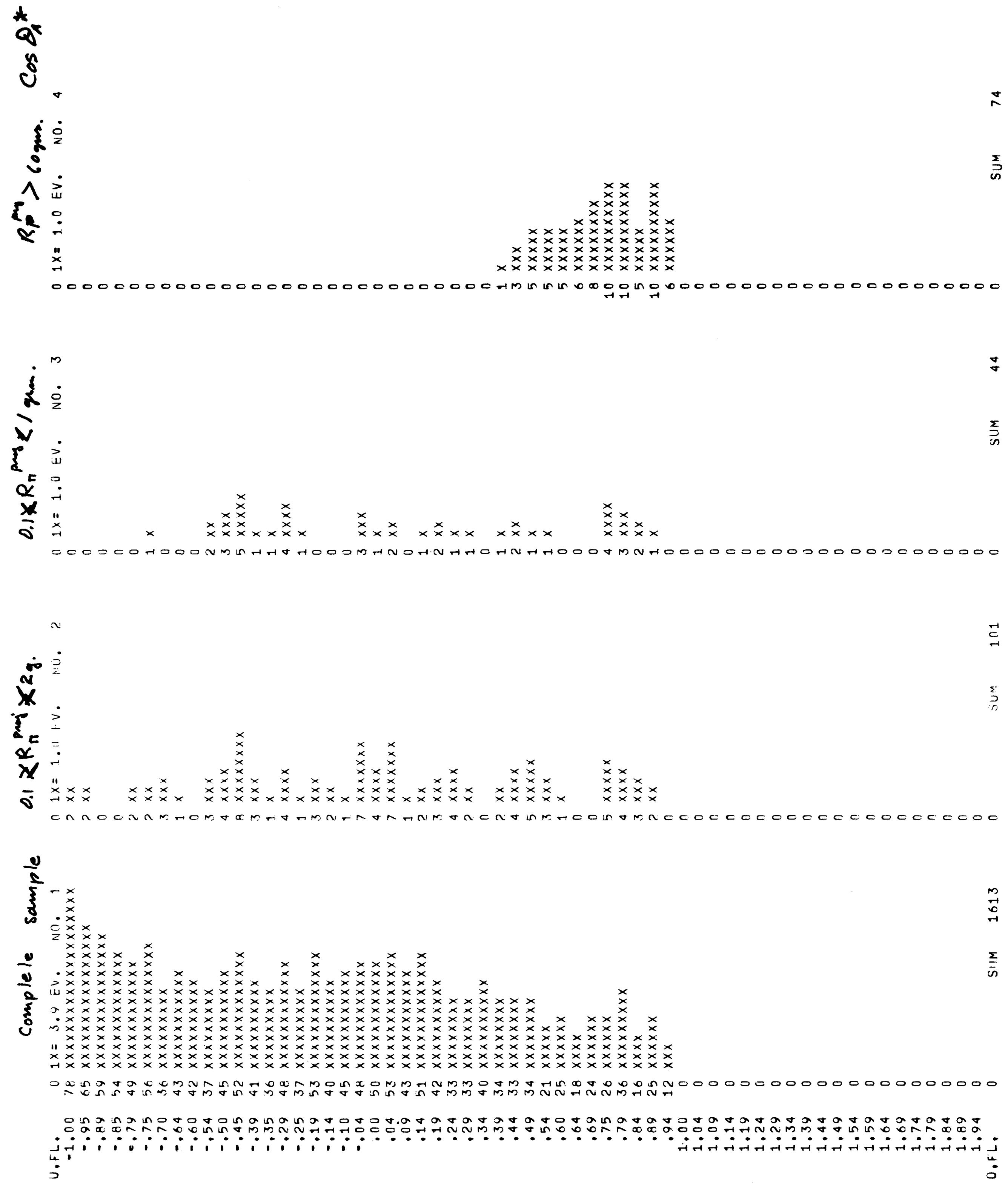


Fig. 5a

SIS/R/10872

$\beta \frac{dP_t}{d\beta}$

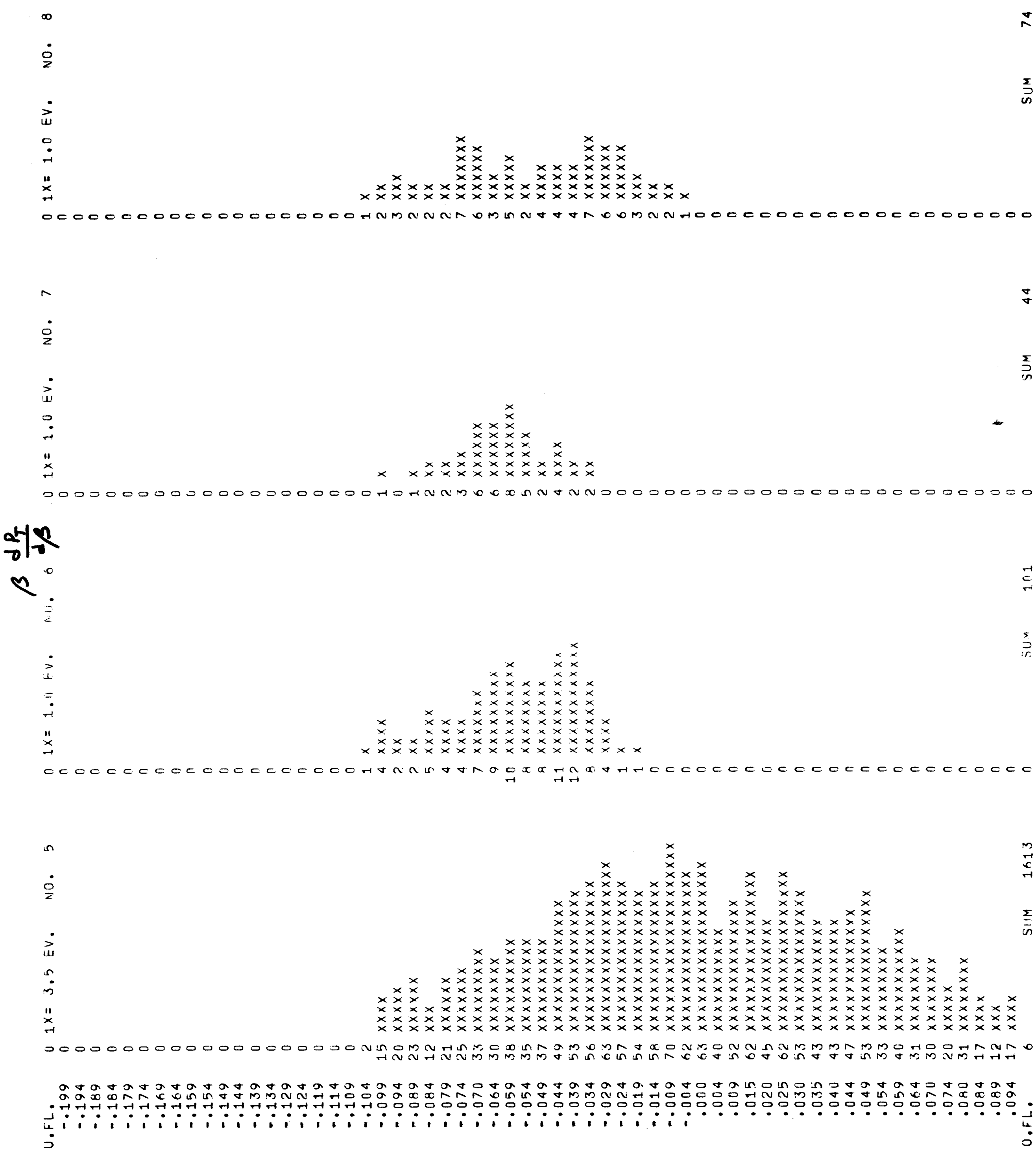


Fig 5b

Figure of Merit

$A(\varepsilon_p)\left(\frac{dP_t}{d\beta}\right)^2$

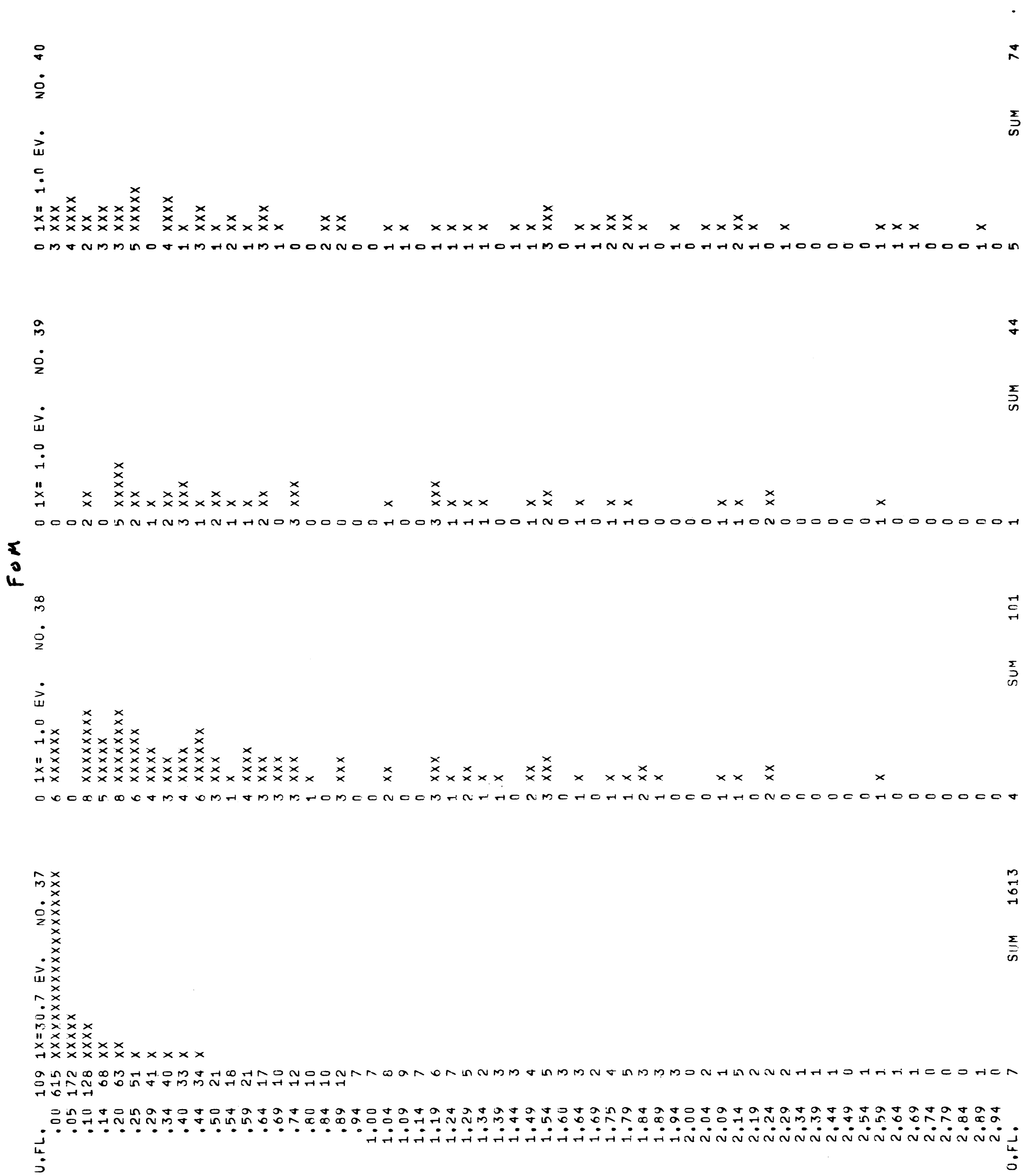


Fig 5c

SIS/R/10869

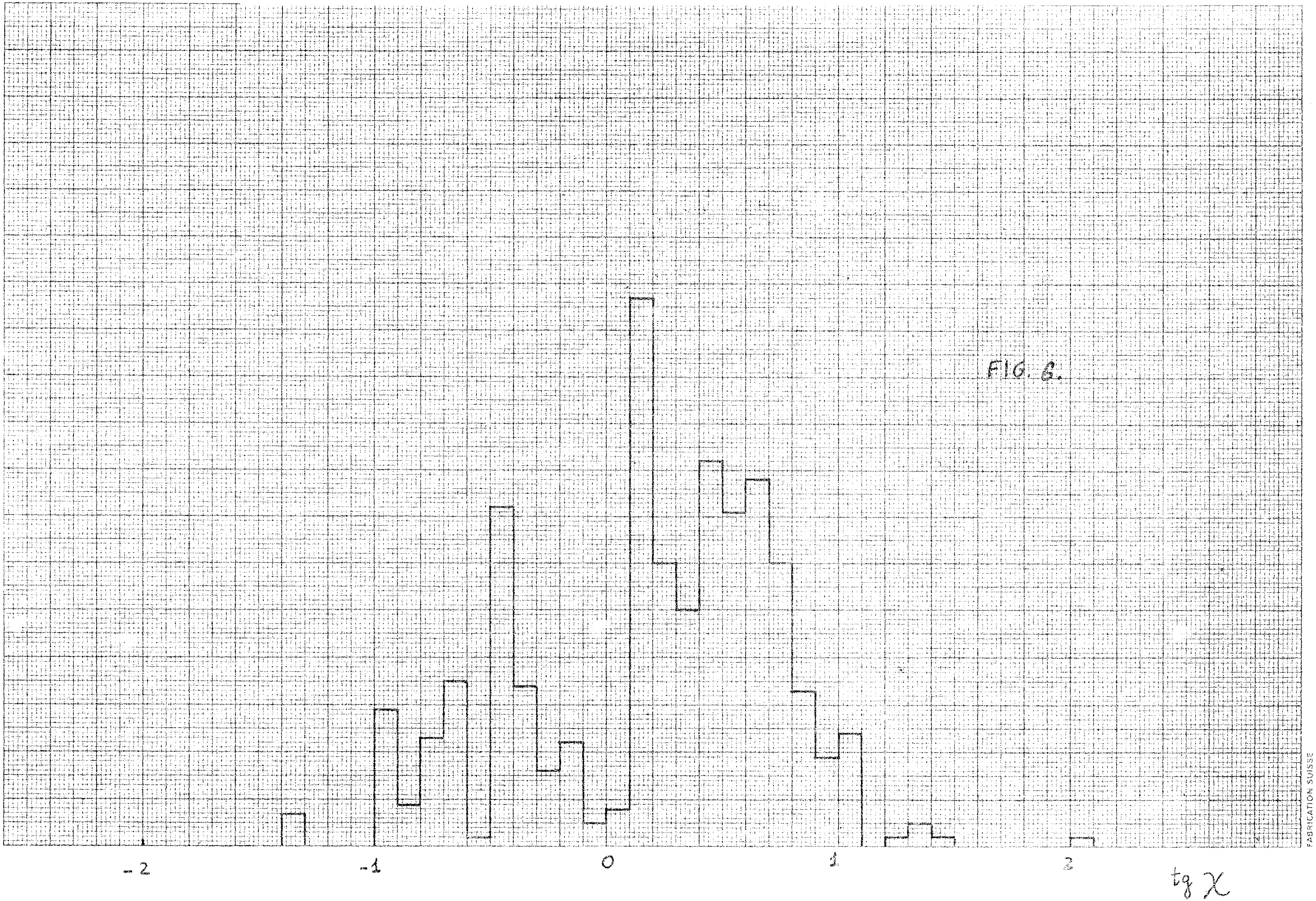


FIG. 6.

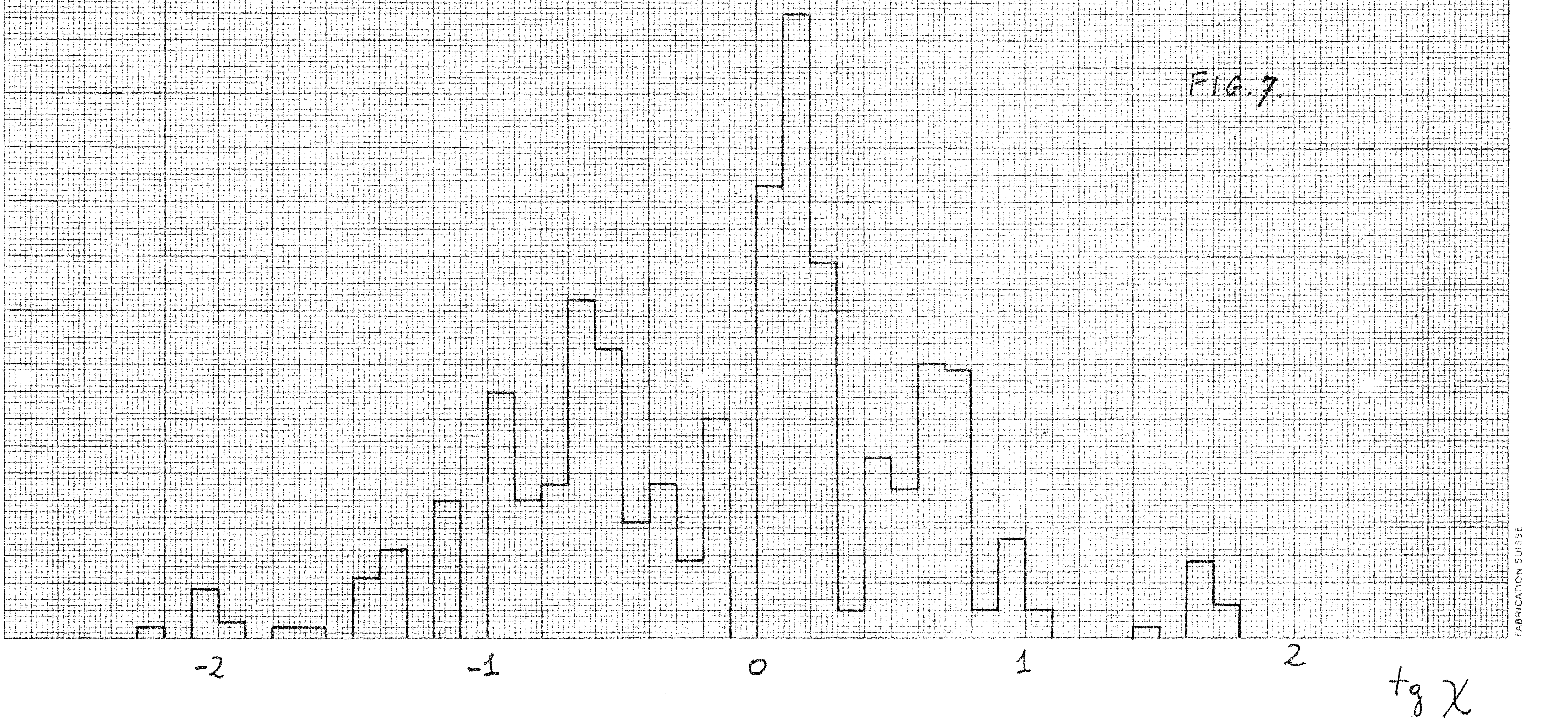


FIG. 7.