# **EUROPEAN LABORATORY FOR PARTICLE PHYSICS**



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# SINGLE-BUNCH PARAMETERS PROPOSED FOR 500 GEV AND 1 TEV CLIC OPTIONS

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#### **Abstract**

Single-bunch parameters are updated for the 500 GeV (centre-of-mass) option in order to get a luminosity that is as high as possible, together with acceptable beam-strahlung effects. With the same requirements, single-bunch parameters are proposed for the 1 TeV option. They are all based on simulations of the beam behaviour in the main linac and of the beam-beam phenomena at collision. The expected single-bunch luminosity is in excess of  $10^{33}$  cm<sup>-2</sup> s<sup>-1</sup> for the 500 GeV option, and lower than  $10^{34}$  cm<sup>-2</sup> s<sup>-1</sup> for the 1 TeV option, where multibunch mode might be required.

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# Corrigendum

February 1995

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# 1 INTRODUCTION

The last list of parameters for a  $2 \times 250$  GeV CLIC collider [1] was established on the basis of the beam characteristics and emittances at the linac end, obtained by tracking, and on an optimization of the final focus parameters with the code RBEAM developped at CERN (L. Wood). These were based on a single bunch of  $6 \times 10^9$  going through the final focus acceptance. The rationals [2] were to fix  $\beta_y^*$  from the bunch length  $\sigma_z$ , deduce the nominal beam sizes  $\sigma_x^*$  and  $\sigma_y^*$  from the Oide limit, check aberration effects and keep the relative energy loss  $\delta_B$  low enough (~ 5%, because of the unavoidable presence of initial state radiation). The 'beam strahlung parameter' Y and the loss  $\delta_B$  (Appendix) were estimated to be 0.18 and 5.9% respectively [4], from the analytical expression and the code RBEAM existing at the time. Since then, the analytical expressions have changed [6, 7], the code found to be incorrect [3] and other values have been subsequently published in CLIC parameter lists [5], with a much higher  $\delta_B$ . This fact and the necessity to control beam-beam quantities at collision lead to a reevaluation and a search for better sets of CLIC beam parameters.

#### 2 STARTING POINT

The first thing to do was to re-evaluate the most relevant quantities, for the basic parameters of Ref. [1]. This was done by using the recent analytical description [6, 7] of beambeam phenomena, recalled in the Appendix. In parallel, independent numerical simulations were run by P. Chen using ABEL [8] and V. Telnov using his own code [9]. The values obtained are listed in Table 1.

Table 1
Beam-beam quantities with previous 2 × 250 GeV CLIC [1]

Analytical con	nputation (App.)
Luminosity with pinch	$2.2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$
$\sigma_{\rm z}$ to $\beta^*$ ratio, $A_{\rm x}/A_{\rm y}$	0.077/1.0
Disruption parameter D <sub>x</sub> /D <sub>y</sub>	1.34/15.3
Enhancement factor $H_{D_x}/H_{D_y}$	5.1/3.13
Nominal beam sizes $\sigma_x^* / \sigma_y^*$	90/8 nm
Beam sizes with pinch $\overline{\sigma}_x^* / \overline{\sigma}_y^*$	40/5.5 nm
Luminosity enhancement H <sub>D</sub>	3.3
Beamstrahlung parameter Y	0.35
Photon number ny	4.7
Relative energy loss $\delta_{\rm B}$	0.36
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Luminosity enhancement H <sub>D</sub> ,	3.0
Photon number ny	3.66
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Luminosity enhancement H <sub>D1</sub>	2.6
Relative energy loss $\delta_{\rm B}$	0.23
Luminosity fraction (> 98% c.m. energy) $\delta_L$	18%

Table 1 teaches us a certain number of things. The discrepancy between approximate formulae and numerical simulations is relatively large, probably because the beam is quasi-flat  $(R = \sigma_x^*/\sigma_y^* = 11)$ . The scaling laws are good only to  $\pm 10\%$ , or more in this case. Nevertheless, two independent simulations give very consistent results. The number of emitted photons per electron and the energy loss  $\delta_B$  (calculated to be between 20 and 23%) are too large with the old set of parameters, while the fraction of the luminosity spectrum contained within 2% of the centre-of-mass energy  $\delta_L$  is too low. Beam parameters must be found in order to improve this situation, and to get a  $\delta_B$  of around 5 to 7% and a  $\delta_L$  near to or larger than 50%.

# 3 SEARCH FOR PARAMETER OPTIMIZATION

Starting from simplified formulae for L and  $\delta_B$  (Appendix), it is easy to indicate three possibilities for decreasing  $\delta_B$  and their incidence on L:

$$L \sim \frac{N_b^2}{\sigma_x^* \sigma_y^*} \qquad \delta_B \sim \frac{N_b^2}{\sigma_z^* \sigma_x^{*2}}. \tag{1}$$

Combining these two relations and eliminating  $\sigma_x^*$  and  $\sigma_z$  in favour of the vertical emittance  $\varepsilon_y$ , one can write

$$L \sim N_{\rm b} \sqrt{\frac{\delta_{\rm B}}{\varepsilon_{\rm y}}} \,. \tag{2}$$

- 1. The number of particles per bunch can be reduced, but this gives a corresponding reduction in luminosity, which cannot be recommended.
- 2. The bunch length  $\sigma_z$  can be increased, which does not change L directly, but may boost wake-field effects and then raise  $\sigma_y^*$ . However, with CLIC parameters, the scaling laws show that the dependence of  $\delta_B$  on  $\sigma_z$  is low (Fig. 1). To gain significantly,  $\sigma_z$  would need to be increased by a prohibitive factor of three or four. It must be noted that the apparent gain on  $\delta_B$  for small  $\sigma_z$  is not welcome for physics and luminosity distribution. The amplitude dN/dE of the photon spectrum decreases with the energy and is truncated at the electron energy. When  $\sigma_z$  gets smaller, both  $\gamma$ -spectrum and L-distribution spread more and the resulting  $\delta_B$  decreases in an undesirable way (luminosity peak is wider).

The conclusion is therefore that the bunch length  $\sigma_z$  can be selected independently of the arguments based on beam-beam phenomena, but in order to minimize the energy spread in the main linac (bunch shaping included) [10].

3. The horizontal beam size of  $\sigma_x^*$  can be enlarged, with the advantage that, according to Eq. (1), L decreases less rapidly than  $\delta_B$ . Moreover, for constant  $\delta_B$ , it is possible to adjust independently the intensity  $N_b$  and the dimension  $\sigma_x^*$ , as can be seen by using the scaling laws (Fig. 2). The resulting luminosity varies, however, proportionally to  $N_b$  and depends nonlinearly on  $\varepsilon_y$ . Our search for the optimum values for these two parameters has been guided by formula (2). Keeping  $\delta_B$  constant and near the desired value,  $N_b$  has to be pushed up until the maximum of the quantity  $N_b/\sqrt{\varepsilon_y}$  has been passed and found by beam tracking in the main linac [11]. Limited investigations (with a simple one-to-few trajectory correction) gave a curve (Fig. 3) with a maximum

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The first thing to do was to re-evaluate the most relevant quantities, for the basic parameters of Ref. [1]. This was done by using the recent analytical description [6, 7] of beambeam phenomena, recalled in the Appendix. In parallel, independent numerical simulations were run by P. Chen using ABEL [8] and V. Telnov using his own code [9]. The values obtained are listed in Table 1.

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around  $N_b = 8 \times 10^9$  (beyond this value, the emittance begins to blow up significantly). By comparison with the case  $N_b = 6 \times 10^9$ , for which the target value of  $\gamma \epsilon_y$  is  $2 \times 10^{-7}$  rad m [11, 12], the value at  $8 \times 10^9$  is approximately 30% larger, making it possible to boost L by  $\sim 17\%$ .

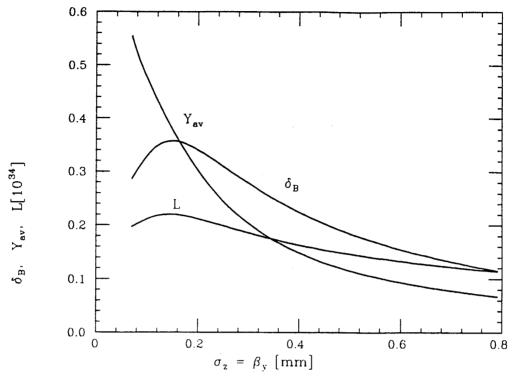


Fig. 1 Variation of the beamstrahlung parameter, relative energy loss and luminosity, with the bunch length.

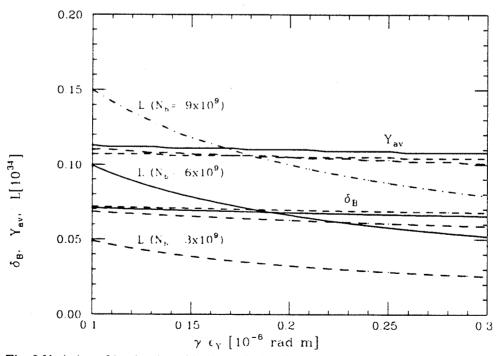


Fig. 2 Variation of luminosity with the vertical emittance at almost constant energy loss, but different bunch population.

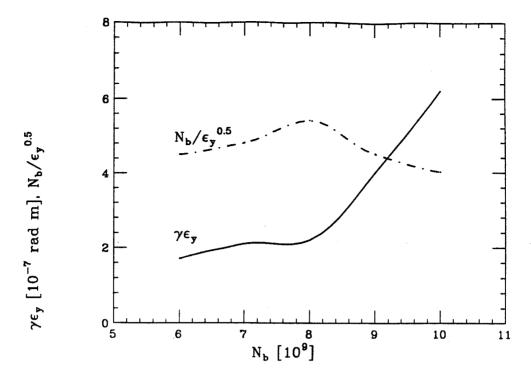


Fig. 3 Approximate variation of N<sub>b</sub> /  $\sqrt{\epsilon_V}$  with the bunch population.

The vertical emittance  $\gamma \epsilon_{\rm V}$  is obviously a very critical parameter. The quoted target value of  $\gamma \epsilon_v = 2 \times 10^{-7}$  rad m has been obtained by tracking the beam in a 250 GeV linac, with oneto-few trajectory corrections, different energy scaling [12], wake-free or dispersion-free algorithms that allows coping with 10 µm r.m.s. alignment tolerances for cavities, and position monitors [13]. The question that one may ask is whether better results are still possible by using even more elaborate algorithms. Two methods of correction had not yet been considered for CLIC up to this point. The first consists of having non-dispersive bumps in limited regions of the linac in order to position the beam off-centre there and globally compensate for the wakefield perturbations [14]. This has been recently studied [15] and found hardly applicable to CLIC, because the local strong wake-fields destroy the required characteristics of the nondispersive bumps. The second method exploits the beam-based alignment technique for correction, presented elsewhere [16]. In CLIC, this would mean measuring and then correcting, on top of the trajectory itself, a trajectory difference obtained with nominal bunch intensity (full wake-fields) on one side and with strongly reduced intensity (factor 10, i.e. almost no wakes) on the other [17]. Assuming that the pickup resolution can be maintained in a large range of beam intensity by improving the electronics, first simulated results look promising. If they are confirmed, the value of  $\gamma \epsilon_v$  at 250 GeV could perhaps be reduced by a factor of ~ 2 for unchanged alignment tolerances.

# 4 PROPOSED PARAMETERS AT 500 GEV C.M.

On the basis of the preceding, we can now propose single-bunch parameters for 500 GeV in the centre of mass. They imply a certain number of assumptions briefly listed as follows

- The population per bunch can be increased to  $8 \times 10^9$  before the vertical emittance breaks up.

- The normalized vertical emittance can probably be improved by a factor of ~ 2 at constant intensity using additional corrections, but will blow up by some 30 to 50% when  $N_b$  is raised to  $8 \times 10^9$ . This gives an altogether new target value of about  $1.5 \times 10^{-7}$  rad m for  $\gamma \epsilon_v$ .
- The bunch length  $\sigma_z$  is set to minimize the energy spread in the linac (0.2 mm at N<sub>b</sub> =  $8 \times 10^9$ ) and  $\beta_y^*$  is taken as equal to 0.9  $\sigma_z$  (i.e  $\beta_y^*$  = 0.18 mm), to reduce the hourglass effect as much as possible.
- The ratio of nominal beam sizes is set equal at  $\sim$  33, in order to get an energy loss  $\delta_B$  of below 7% and to make sure that the integrated luminosity in the last two percentage points below the c.m. energy exceeds  $\sim$  50% of the total luminosity (important in the search for a mass resonance, since it improves the signal-to-noise ratio and reduces the duration of the experiment).
- The normalised horizontal emittance has been relaxed to  $3 \times 10^{-6}$  rad m, since tracking shows that a larger  $\gamma \varepsilon_x$  helps the minimization of  $\gamma \varepsilon_y$  through a fine adjustment of the microwave quadrupoles. This choice implies that  $\beta_x^* = 10$  mm.
- The repetition rate is adjusted to give a maximum RF consumption of 100 MW.

The resulting beam parameters are summarized in Table 2, together with the expected performance deduced either from the scaling laws of the Appendix or from numerical simulations of the collisions.

### 5 PROPOSED PARAMETERS AT 1 TEV C.M.

Similarly, we are in the position to propose single-bunch parameters for 1 TeV in the centre of mass, following the same logic as in the 500 GeV case. Many assumptions are unchanged, excepting however the transverse emittances; they are therefore listed below:

- The bunch population  $N_b$ , the bunch length  $\sigma_z$  and the  $\beta$ -functions at the collision point have the same values as before (Section 4) since the arguments remain unchanged.
- Since the linac is now twice as long, the vertical emittance (normalized) is allowed to grow a little more and reach a value of  $2 \times 10^{-7}$  rad m (to be checked by tracking). As  $\gamma$  is twice as high though, the vertical beam size becomes 6 nm.
- Keeping the same beam-size ratio of 33 and  $\beta_x^*$  of 10 mm gives a horizontal-beam dimension of 200 nm. This then corresponds to a normalized horizontal emittance of  $3.9 \times 10^{-6}$  rad m, which equally allows a larger blow up.
- The repetition rate is maintained at the same value, accepting that the power required now doubles to about 200 MW.

The resulting set of beam parameters is summarized in Table 3, together with the expected performance, which has been estimated in two different ways: the first estimate comes from the formulae recalled in the Appendix and giving convenient scaling laws, while the second is obtained from numerical simulations of the beam-beam collisions.

#### 6 CONCLUSIONS

The present paper gives an update of the single-bunch beam parameters for CLIC at 500 GeV (Table 2) and 1 TeV (Table 3) in the centre-of-mass, and gives a brief summary of a work still in progress. This proposal aims at reasonable beam-beam parameters with a single-bunch luminosity as high as possible. Tables 2 and 3 simultaneously give performance estimates obtained from the approximate formulae recalled in the Appendix and from two independent numerical simulations done with different codes [8, 9]. This shows the rather good agreement between the different calculations, although checking the results through actual simulations once the parameters are fixed remains important.

Table 2 indicates that, for the 500 GeV CLIC, the single-bunch option makes it possible to get a luminosity in excess of  $10^{33}$  cm<sup>-2</sup> s<sup>-1</sup> and to control the beam-beam phenomena. By contrast, for the 1 TeV CLIC, Table 3 shows that the single bunch option does not permit the reaching of a luminosity of  $10^{34}$  cm<sup>-2</sup> s<sup>-1</sup>. Therefore, multibunch mode must still be considered for the expansion in energy. However, it must be noted that because of the long range wake fields, the parameters of each individual bunch in a train will have to be optimized again and will likely differ from those of Table 3.

# Acknowledgements

The authors are very grateful to P. Chen and V. Telnov who introduced the physics of the collisions and kindly ran the simulation codes.

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#### APPENDIX

According to the formalism elaborated in Refs. [6, 7], the beam-beam phenomena in linear colliders can be described approximately by algebraic formulae which are partly deduced from numerical simulations. These formulae give good results in the asymptotic cases of either a round or flat beam (with a large and fixed horizontal dimension) but are less reliable in the intermediate case of quasi-flat beams. In spite of their limitation, they offer the advantage of providing simple scaling laws that become very useful in the search for optimized beam parameters, though verification by simulations are eventually essential. Let us recall here the relations we have used in the course of the work reported in this note.

The single bunch luminosity is given by the basic relation,

$$L = \frac{N_b^2}{4\pi} \frac{f_{\text{rep}}}{\overline{\sigma}_x^* \overline{\sigma}_y^*}, \tag{A1}$$

where  $N_b$  is the number of particles per bunch and  $f_{rep}$  the repetition rate (for  $k_b$  bunches per pulse, then L is simply multiplied by  $k_b$ ). In order to properly describe the beam-beam phenomena, the disruption effect must be included in both transverse dimensions. Therefore, in Eq. (A1), the nominal beam sizes have been replaced by effective beam sizes, defined by

$$\overline{\sigma}_{x}^{*} = \frac{\sigma_{x}^{*}}{(H_{D_{x}})^{1/2}} \qquad \overline{\sigma}_{x}^{*} = \frac{\sigma_{y}^{*}}{(H_{D_{y}})^{f(R)}},$$
 (A2)

where the exponent f(R) depends on the nominal beam aspect ratio  $R = \sigma_x^* / \sigma_y^*$  at the interaction point. The analytic derivation of the H<sub>D</sub> factors is very difficult and their behaviour comes from simulations [4]

$$H_{D} = 1 + D^{1/4} \left( \frac{D^{3}}{1 + D^{3}} \right) \left[ \ell n(\sqrt{D} + 1) + 2\ell n \left( \frac{0.8}{A} \right) \right], \tag{A3}$$

where D is written for either  $D_x$  or  $D_y$ , the 'disruption parameters' in the two transverse directions

$$D_{x,y} = \frac{2r_e N_b \sigma_z}{\gamma \sigma_{x,y}^* (\sigma_x^* + \sigma_y^*)}, \qquad (A4)$$

and A is the ratio of the bunch length  $\sigma_z$  to the  $\beta$ -function in either plane, at the interaction point

$$A_{x,y} = \frac{\sigma_z}{\beta_{x,y}^*} \,. \tag{A5}$$

The last term in (A3) takes into account the hourglass effect or variation of  $\beta_{x,y}$  along the bunch.

Computer simulation using ABEL [8] gave the variation of  $H_D$  with R and the following approximation for the exponent f(R)

$$f(R) = \frac{1 + 2R^3}{6R^3} \,. \tag{A6}$$

The combination of all these relations gives the total factor for the luminosity enhancement due to disruption

$$H_{D_t} = (H_{D_x})^{1/2} (H_{D_y})^{f(R)}.$$
 (A7)

As well as the single-bunch luminosity being of obvious importance, it is necessary to get quantitative information about the adverse effects of bunch deformation and beam-beam

phenomena. We have retained three basic quantites for this purpose. The first is the beamstrahlung parameter Y proportional to the fractional critical energy of the photons emitted in the collision [6]:

$$Y = \frac{5}{6} \frac{r_e^2 \gamma N_b}{\alpha \sigma_z (\overline{\sigma}_x^* + \overline{\sigma}_y^*)}.$$
 (A8)

The other two are the average number of emitted photons  $n_{\gamma}$  per electron and the relative energy loss  $\delta_B$  due to Beamstrahlung [6],

$$\begin{split} n_{\gamma} & \equiv 2.54 \; \frac{\alpha \sigma_z \; \Upsilon}{\lambda_e \gamma} \; \frac{1}{(1 + \Upsilon^{2/3})^{1/2}} \\ \delta_{\rm B} & \equiv \langle -\frac{\Delta E}{E} \rangle \cong 1.24 \; \frac{\alpha \sigma_z \Upsilon^2}{\lambda_e \gamma} \frac{1}{[1 + (1.5\Upsilon)^{2/3}]^2} \; , \end{split} \tag{A9}$$

which both depend on the parameter Y and some physics constants ( $r_e = 2.8179 \times 10^{-15}$  m,  $\lambda_e = 3.8616 \times 10^{-13}$  m and  $\alpha = 7.2993 \times 10^{-3}$ ).

Another important beam characteristic is the energy spectrum of the electrons that results from beamstrahlung. It is difficult to get an analytical expression of it, because of the possibility of multiphoton emission, and therefore simulations will be required. They give the differential luminosity spectrum, which extends toward low energies and has a peak near the centre-of-mass energy that should be as narrow as possible.