COMPARISON OF $K^{\pm}P$ DISPERSION RELATIONS WITH EXPERIMENT

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A B S T R A C T

Once subtracted forward angle dispersion relations are used to predict the real parts of the $K^{\pm}P$ scattering amplitudes as a function of energy, using previously derived values of the NYK coupling constants. These predictions are compared with experiment.

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Recently we have evaluated [1] the NYK coupling constants using unsubtracted forward dispersion relations for the K-N system. As described in I, the essential data that was used in this calculation was the $K^{\pm}N$ total cross-sections at all available energies and the real part of the $K^{\pm}N$ forward scattering amplitudes at very low energies [2]. In this note we want to present the predictions of the once-subtracted dispersion relations for the real parts of the $K^{\pm}p$ forward scattering amplitudes at higher energy and compare them with experiment. A recent work by N. Zovko [3] has questioned the consistency of the low energy $K^{\pm}N$ scattering data with the dispersion relations. We find that such a discrepancy does not exist if one includes, in a consistent manner, $(K^-p, \bar{K}^0n)$ mass difference effects in the Dalitz-Tuan formalism [4] for the low energy K^-p scattering.

Since the publication of I, more precise $K^{\pm}N$ total cross-sections between 1.0 and 2.45 GeV/c have become available [5]. We have redone the calculations of I including these data and find the same results for the NYK coupling constants namely $g^2_{p\Lambda K} = 4.8 \pm 1.0$, $g^2_{p\Sigma^0 K} \lesssim 3.2$.

The once subtracted dispersion relations can be written in the form [6]

$$D_{\pm}(\omega) = \frac{1}{2}\left[D_-(m_K) + D_+(m_K)\right] \mp \frac{\omega}{2m_K}\left[D_-(m_K) - D_+(m_K)\right] +$$
$$+ \frac{k^2}{4\pi^2}\int_{m_K}^{\infty} \frac{\omega'(\sigma_- + \sigma_+) \mp \omega(\sigma_- - \sigma_+)}{k'(\omega'^2 - \omega^2)}\,d\omega' +$$
$$+ \frac{k^2}{\pi}\int_{\omega_{\Lambda\pi}}^{m_K} \frac{(\omega' \mp \omega)A_-}{k'^2(\omega'^2 - \omega^2)}\,d\omega' + \sum_Y \frac{k^2}{(\omega_Y \pm \omega)} \cdot \frac{X(Y)}{\omega_Y^2 - m_K^2} \qquad (1)$$

where $D_{\pm}$ and $A_{\pm}$ are, respectively, the real and imaginary parts of the $(K^{\pm}, p)$ scattering amplitude in the laboratory system,

$$\omega_Y = \frac{m_Y^2 - m_p^2 - m_K^2}{2m_p} \quad \text{and} \quad X(Y) = g^2_{pYK} \frac{(m_Y - m_p)^2 - m_K^2}{4 m_p m_Y},$$

$Y = \Lambda$, and Σ^0, g^2_{pYK} is the renormalized, rationalized pseudoscalar coupling constant, ω and k are the total energy and the momentum of the K meson in the laboratory system respectively. As in I, A_- in the unphysical region is assumed to have its S wave part given by the Dalitz-Tuan formalism and a p wave part only in $I = 1$ coming from a $Y_1^*(1385)$ pole term, which can be written in the form $k^2/(\omega_Y \pm \omega)\,|Z_p(Y_1^*)|$.

The numerical evaluation of $D_{\pm}(\omega)$ is insensitive to exactly how the $I = 1$ pole term is divided between the Σ^0 pole and the Y_1^* pole as long as Eq. (3') of I is used. For definiteness we have assumed that $g^2_{p\Sigma^0 K} = \frac{1}{27} g^2_{p\Lambda K}$ corresponding to an f/d ratio of $\frac{2}{3}$ [7].

Above 20 GeV/c, we used a Regge pole model extrapolation for the total cross-sections. We used the parameters kindly provided by C. Foggart and R.J.N. Phillips. The results for $D_{\pm}(\omega)$ for $\lesssim 3.5$ GeV/c are not sensitive to the exact form of the particular Regge fit used in the asymptotic region, since the total asymptotic contribution is less than the errors. We have not explored the sensitivity of $D_{\pm}(\omega)$ for higher momenta, to the fit used in the asymptotic region, so that the errors quoted for the contributions from the asymptotic region are only the statistical ones associated with this particular model.

The evaluation of the principal value integrals was carried out using a straight line interpolation between the most accurate experimental points.

$\underline{K^+p}$:

The calculated values of $\alpha_+(\omega) = D_+(\omega)/A_+(\omega)$ for $k \gtrsim 0.7$ GeV/c, are presented in Fig. 1, together with all known experimental values [8] of α_+. Above 1.2 GeV/c only the magnitudes of α_+ have been measured, but they have been plotted as negative quantities in Fig. 1. The values of $D_+(\omega)$ have been calculated using both the low energy K^-p parameters of solution I of Kim [9] and of solution II of Sakitt et al. [10] (the K^-p parameters of Kittel et al. [11] are essentially the same as those of Sakitt et al.). In fact we find that these different solutions yield values for $D_+(\omega)$ that differ by a small fraction of the calculated error. Therefore only the points of Kim are plotted. This insensitivity of $D_+(\omega)$ to the low energy K^-p solutions is a consequence of our use of the unsubtracted dispersion relation in I to derive the Λ and $(\Sigma^0 + Y_1^*)$ pole term contribution : that is we use different pole term contributions for Kim and for Sakitt parameters as given in Eq. (3') of I. The agreement between dispersion theory predictions and experiment for $D_+(\omega)$ and $\alpha_+(\omega)$ is reasonably good except in the highest momentum region ($k \gtrsim 10$ GeV/c). Similar but not identical calculations for $D_+(\omega)$ have been made recently by N.M. Queen [12] above 1 GeV/c. The general trend of his results for $D_+(\omega)$ is the same as ours except that when he uses our value of the NYK coupling constants obtained from I his magnitude of $D_+(\omega)$ is substantially larger than ours, and hence in less good agreement with the experimental data.

To illustrate the contributions to $D_+(\omega)$ and to $\Delta D_+(\omega)$ from different parts of Eq. 1, we divide the right-hand side of Eq. 1 into six different pieces c_i ($i = 1,\ldots,6$), defined as follows :

$c_1 = \Lambda + \Sigma^0 + Y_1^*$ pole term contributions

c_2 = the low energy K^-p contributions from $\omega_{\Lambda\pi} = 240$ MeV to $\tilde{\omega}_- = 574.2$ MeV including the $D_-(m_K)$ term. (The uncertainty in c_2 includes all the errors from the low energy K^-p solutions.)

c_3 = the K^-p contributions from $\tilde{\omega}_-$ to 20 GeV

c_4 = the low energy K^+p contributions from m_K to $\tilde{\omega}_+ = 914$ MeV including the $D_+(m_K)$ term

c_5 = the K^+p contributions from $\tilde{\omega}_+$ to 20 GeV

c_6 = the K^+ and K^- cross-section integrals in the asymptotic region, $\omega > 20$ GeV.

The values of c_i and Δc_i at several representative momenta are listed in Table 1. We assume that the final error for $D_+(\omega)$ is given by a statistical combination of the errors of the six individual parts. This assumption is not rigorously correct since the errors in the hyperon pole terms are complicated functions of the experimental cross-sections used in I and again in Eq. (1). However this method of calculating the errors should give a reasonable order of magnitude estimate of the uncertainties.

$\underline{K^-p}$:

The calculated values of $\alpha_-(\omega) = D_-(\omega)/A_-(\omega)$ are displayed in Fig. 2, along with the available experimental values [13]. $D_-(\omega)$ is more sensitive to the S wave K^-p parameters so that there is a non-negligible difference between the results obtained using the Kim [9] and Sakitt [10] solutions, especially at low momenta. Again only the Kim solution is plotted. The Sakitt solution II prediction is rather close to the lower limit error curve of the Kim solution. The errors in the Sakitt case are about 30% larger than for Kim. The structure in $D_-(\omega)$ versus ω reflects the resonant structure in the K^-p total cross-sections at $Y_0^*(1520)$, $Y_0^*(1815)$, and $Y_0^*(2100)$ [5]. The agreement between the dispersion theory prediction and experiment is not very good at $k_- = 1.97$ GeV/c, Cook et al. [13], which is also the experimental point with the smallest apparent error. Confirmation of such a discrepancy would certainly require some substantial modifications of our analysis. The small values for α_- predicted in the vicinity of $Y_0^*(1520$ MeV), $k_- = 400$ MeV/c, are consistent with solution III of Watson et al. [13] but not with their solution I, which was their favoured one. This latter predicts values for $\alpha_- \sim +0.4$ in the vicinity

of the resonance, which is not compatible with the dispersion relation predictions. Recently Kittel and Otter [14] have given arguments based on an effective range treatment of $I = 0$, $\bar{K}n$ interactions that also favour Watson et al.'s solution III over solution I. The essential point is that the S wave parts of solution III of Watson et al. are much closer to the low energy solutions of Kim and Sakitt et al. than are the S wave parts of solution I.

Again, to illustrate the contributions to $D_-(\omega)$ and to $\Delta D_-(\omega)$ from different parts of Eq. (1), we divide the r.h.s. of Eq. (1) into the same six pieces $c_1,\ldots,c_6$ as defined previously for K^+ with the necessary sign changes incorporated in the definition. The values of c_i and Δc_i at several representation momenta are listed in Table II.

In conclusion, we find that the dispersion relation predictions, for $_+(\ )$, using the results of I, are in reasonable agreement with experiment. Less data for $\alpha_-(\omega)$ is available, but there may be a discrepancy at $k_- = 1.97$ GeV/c. Clearly much more data on $D_\pm(\omega)$ would be of value for such comparisons, which can then also be used to make a more accurate determination of the KYN coupling constants. In the absence of such data, we think that the values of $\alpha_\pm(\omega)$ given in Figures 1 and 2 can be used to help decide between alternative $(K^\pm, p)$ phase shift solutions in the momentum region below ~ 2 GeV/c, as illustrated by our discussion of the 400 MeV/c K^-, p analysis.

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Table I : Contributions to $D_+(\omega)$ from different parts of the dispersion relation given by Eq. (1) for K^+p.
[See text for detailed explanation of c_i $(i = 1,\ldots,6)$]

k (GeV/c)	c_1 (Pole terms)	c_2 (K^- low energy)	c_3 (K^- higher energy)	c_4 (K^+ low energy)	c_5 (K^+ higher energy)	c_6 *) (Regge contrib)	D_+ (GeV^{-1}) (Total)
0.520	0.457 ±0.087	-0.802 ±0.027	+0.905 ±0.010	-3.493 ±0.252	0.665 ±0.005	0.032 ±0.001	-2.24 ±0.27
0.970	1.100 ±0.21	-2.067 ±0.074	2.620 ±0.03	-5.707 ±0.600	1.957 ±0.043	0.112 ±0.003	-1.99 ±0.64
1.970	2.56 ±0.50	-5.13 ±0.19	7.71 ±0.07	-8.30 ±0.76	-1.00 ±0.06	0.46 ±0.02	-3.71 ±0.92
3.500	4.78 ±0.93	-9.89 ±0.35	17.01 ±0.15	-13.03 ±1.16	-5.21 ±0.22	1.46 ±0.08	-4.88 ±1.56
9.800	13.87 ±2.80	-29.51 ±1.10	61.64 ±0.48	-33.00 ±2.88	-34.19 ±0.65	12.25 ±0.68	-8.92 ±4.20

*) The errors in c_6 reflect only the errors in the Regge pole model of Foggart and Phillips and do not include uncertainties related to choosing different asymptotic models.

Table II : Contributions to $D_-(\omega)$ from different parts of the dispersion relation given by Eq. (1) for K^-p.
[See text for detailed explanation of c_i $(i = 1,\ldots,6)$]

k (GeV/c)	c_1 (Pole terms)	c_2 (K^- low energy)	c_3 (K^-higher energy)	c_4 (K^+ low energy)	c_5 (K^+higher energy)	c_6 *) (Regge contrib.)	D_- (GeV^{-1}) (Total)
0.415	- 0.61	-0.30	- 0.45	0.468	0.157	0.020	-0.71
	± 0.13	±0.14	± 0.28	±0.039	±0.001	±0.0004	±0.34
0.950	- 1.48	3.41	- 2.25	1.80	0.711	0.107	-2.30
	± 0.30	±0.17	± 0.23	±0.15	±0.003	±0.003	±0.42
1.975	- 3.02	6.97	-11.96	4.84	2.401	0.466	-0.30
	± 0.60	±0.26	± 0.16	±0.41	±0.009	±0.020	±0.78
3.460	- 5.18	11.67	-22.58	9.48	5.64	1.44	+0.27
	± 0.73	±0.40	± 0.37	±0.81	±0.02	±0.06	±1.22
5.000	- 7.43	16.49	-36.45	14.34	9.55	3.06	-0.44
	± 0.15	±0.55	± 0.53	±1.23	±0.04	±0.15	±2.10
9.000	-13.18	28.96	-76.99	27.05	21.06	10.56	-2.55
	± 2.65	±0.93	± 0.69	±2.33	±0.08	±0.49	±3.74

*) The errors in c_6 reflect only the errors in the Regge pole model of Foggart and Phillips and do not include uncertainties related to choosing different asymptotic models.

REFERENCES

1) M. Lusignoli, M. Restignoli, G.A. Snow and G. Violini - Phys.Letters 21, 229 (1966), hereafter called I.

2) See I for the references to the experimental data used, besides those listed below.

3) N. Zovko - Z.für Physik 192, 346 (1966).

4) R.H. Dalitz and S.F. Tuan - Ann.Phys. N.Y., 10, 307 (1960).

5) R.L. Cool, G. Giacomelli, T.F. Kycia, B.A. Leontic, K.K. Li, A. Lundby and J. Teiger - Phys.Rev.Letters 16, 1228 (1966) (K^-) ; and Phys.Rev.Letters 17, 102 (1966) (K^+).

6) See Refs. 2), 3) of I.

7) F. Buccella, M. Lusignoli and G. Violini - Phys.Letters 21, 572 (1966).

8) a) T.F. Stubbs, H. Bradner, W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Slater, D.M. Stork and H.K. Ticho - Phys.Rev.Letters 7, 188 (1966).

b) V. Cook, D. Keefe, L.T. Kerth, P.G. Murphy, W.A. Wenzel and T.F. Zipf - Phys.Rev. 129, 2743 (1963).

c) W. Chinowsky, G. Goldhaber, S. Goldhaber, T. O'Halloran and B. Schwarzschild - Phys.Rev. 139, B1411 (1965).

d) J. De Baisieux, F. Grard, J. Heughebaert, L. Pape, R. Windmolders, R. George, Y. Goldschmidt-Clermont, V.P. Henri, D.W.G. Leith, G.R. Lynch, F. Muller, J.M. Perreau, G. Otter and P. Sällstrom - Nuovo Cimento 43, A142 (1966).

e) W. De Baere, J. De Baisieux, P. Dufour, F. Grard, J. Heughebaert, L. Pape, P. Peeters, F. Verbeure, R. Windmolders, R. George, Y. Goldschmidt-Clermont, V.P. Henri, B. Jongejans, D.W.G. Leith, A. Moisseev, F. Muller, J.M. Perreau and V. Yarba - Nuovo Cimento (to be published).

f) K.J. Foley, S.J. Lindenbaum, W.A. Love, S. Ozaki, J.J. Russell and L.C.L. Yuan - Phys.Rev.Letters 11, 503 (1963).

9) J.K. Kim - Phys.Rev.Letters 14, 29 (1965).

10) M. Sakitt, T.B. Day, R.G. Glasser, N. Seeman, J. Friedman, W.E. Humphrey and R.R. Roos - Phys.Rev. 139, 572 (1965).

11) W. Kittel, G. Otter and I. Wacek - Phys.Letters 21, 349 (1966).

12) N.M. Queen - University of Birmingham Preprint (1966).

13) a) M.B. Watson, M. Ferro-Luzzi and R.D. Tripp - Phys.Rev. 131, 2248 (1963).

b) A. Fridman, O. Benary, A. Michalon, B. Schiby, R. Strub and G. Zech - Phys.Rev. 145, 1136 (1966).

c) V. Cook, B. Cork, T.F. Hoang, D. Keefe, L.T. Kerth, W.A. Wenzel and T.F. Zipf - Phys.Rev. 123, 320 (1961).

d) M.N. Focacci, S. Focardi, G. Giacomelli, P. Serra, M.P. Zerbetto and L. Monari - Phys.Letters 19, 441 (1965).

e) J. Gordon - Phys.Letters 21, 117 (1966).

14) W. Kittel and G. Otter - Phys.Letters 22, 115 (1966).

FIGURE CAPTIONS

Figure 1 K^+P dispersion relation predictions of $\alpha_+ = D_+/A_+$ as a function of the K^+ laboratory momentum. The references for the experimental points are given in Ref. [8].

Figure 2 K^-P dispersion relation predictions of $\alpha_- = D_-/A_-$ as a function of the K^- laboratory momentum. The references for the experimental points are given in Ref. [13].

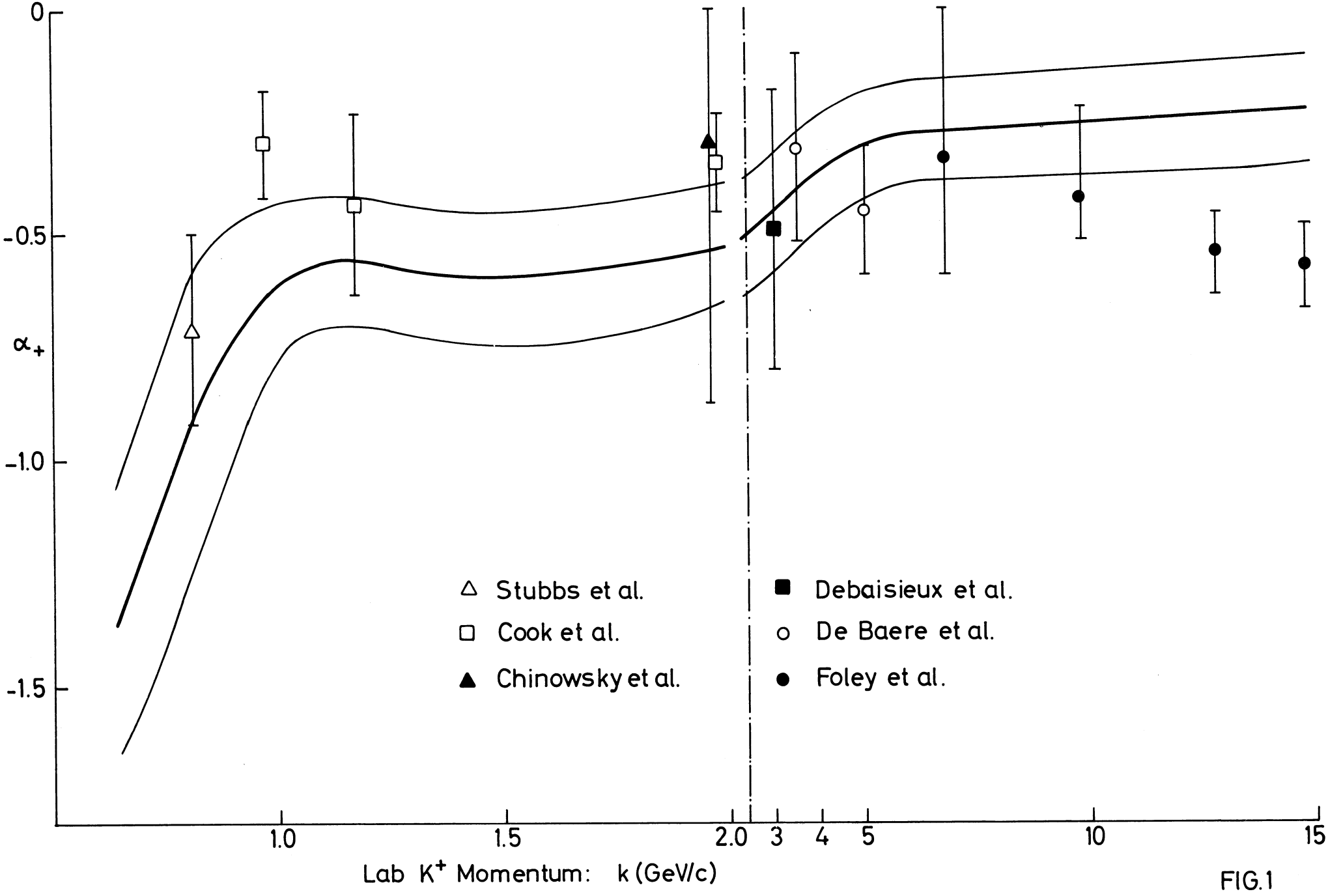

FIG.1

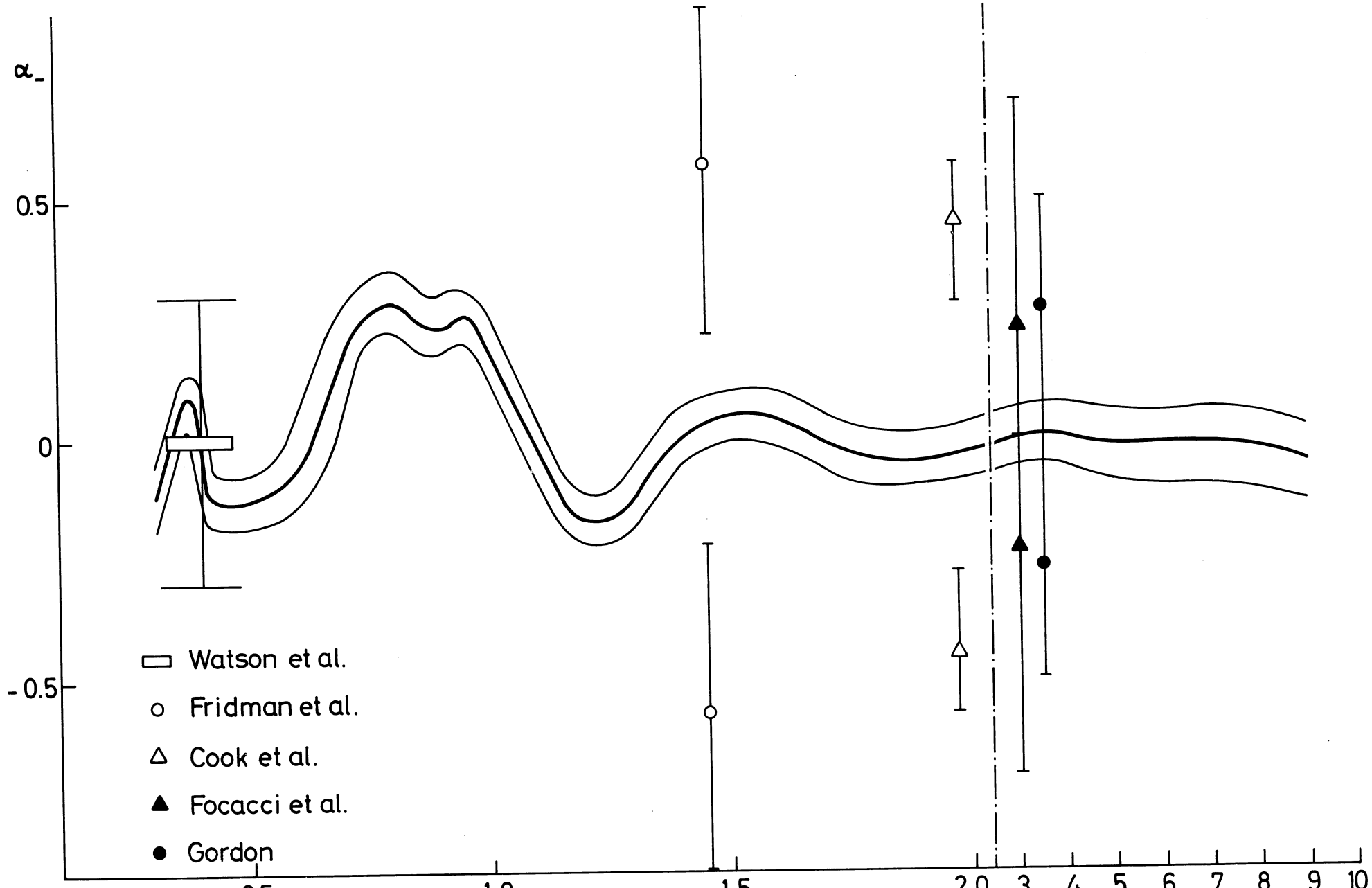

FIG. 2