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PHYSICS I
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PROPOSAL
for
AN EXPERIMENT ON K_2^o LEPTONIC AND 3 π DECAYS

by

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PROPOSAL FOR AN EXPERIMENT ON K_2^o LEPTONIC AND 3 π DECAYS.

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Summary.- An experiment is proposed to study the energy dependence of the form factors in the K_2^o leptonic decays and the behaviour of the matrix element in the 3π decay.

For this experiment we propose to use the same wire chambers spectrometer built by the CERN-Aachen group for the measure of the $K_1^o - K_2^o$ mass difference.

A very high statistics can be collected in a period of the order of one week exposure on the neutral beam b_{13}.

FORM FACTORS IN THE K° LEPTONIC DECAY

As it was pointed out in the recent Heidelberg Conference, the situation related to the form factors in K_{13} decay is rather confused. The discrepancy already existing between values of form factors obtained by polarization measurements and branching ratio was not solved, and the new experiments presented at the Conference have confirmed this strong inconsistency.

The matrix element corresponding to the leptonic decay of the K meson

$$K \to \pi \ell \nu$$

is:

$$M = \frac{1}{2}\left[f_+(q^2)(P_K + P_\pi)_\lambda + f_-(q^2)(P_K - P_\pi)_\lambda\right] \cdot \left[\bar{u}\gamma_\lambda (1+\gamma_5)\nu\right]$$

The hadron current is described by the form factors f_+, f_- functions of the momentum transferred to the pion. Experimental informations on f_+, f_- can be obtained by the study of the decay spectra of the π or μ's, of the $K_{\mu 3}/K_{e3}$ branching ratio and of the μ polarization. f_- is unmeasurable in K_{e3} decay because of the factor $(M_{lept}/M_K)^2$ in the terms containing f_-; therefore all the informations about this form factor are coming from $K_{\mu 3}$.

We propose a detailed investigation of the Dalitz plot in the decay modes

$$K_2^0 \to \pi^\pm e^\mp \nu$$

$$K_2^0 \to \pi^\pm \mu^\mp \nu$$

The distribution of the pion and lepton energies, averaging over spin directions, results

$$\rho(\xi, E_\pi, E_e) = \frac{f_+}{2\pi^3 M_K^2}\left[A(E_e, E_\pi) + B(E_e, E_\pi) + C(E_\pi)\,|\xi|^2\right]$$

where $\xi = f_-/f_+$ and A, B, C contain only kinematical variables.

At fixed π energy the ξ value can be determined by measuring the Dalitz plot density ρ as a function of the μ energy (along the vertical strip in fig. 1). On the other hand the q^2 dependence of the form factors requires measurements at different π energies (horizontal line, fig. 1).

The $\xi(o)$ dependence of the squared matrix element is dislpayed in fig. 2, where the $\xi(o)$ variation from - 0.5 to + 0.5 results in an appreciable change of slope of the constant ξ lines. For each E_π value, lines corresponding to different ξ are normalized to the same area.

Among topics related to the subject of this note we mention the following ones:

a) Time reversal invariance, implying a real value of ξ, equal decay rates and equal form factors for the decays $\bar{K}^o \rightarrow \pi^+ \mu^- \bar{\nu}$ and $K^o \rightarrow \pi^- \mu^+ \nu$

b) $\Delta I = 1/2$ for leptonic decays, implying equal form factors in the K^+ and K^o_2 leptonic decays and a ratio of the respective decay probabilities $\Gamma_+^{lept}/\Gamma_2^{lept} = 1/2$.

c) μ - e universality, implying equal form factors and equal coupling constants for the decays $K^o \rightarrow \pi e \nu$, $K^o \rightarrow \pi \mu \nu$, and a ξ dependence of the relative $K_{\mu 3}/K_{e3}$ rate of the type(1)

$$R(\xi) = 0.65 + 0.124\xi + 0.019\xi^2$$

if ξ is assumed constant.

The proposed experiment should obtain information on the form factors behaviour and a better understanding of the important questions listed above.

We intend to use the counters - wire chambers system built by Cern-Aachen group for the Δm and $\Delta \phi$ experiments. The system, at the present time, is under test at Cern and will be settled on the K^o beam b_{13} soon. The study of the K^o leptonic decay can be performed using the same apparatus, and the same beam, with different triggering conditions. On the other hand, some of us are interested to continue the study of the K leptonic decay, after the experiment on $K^+_{\mu 3}$ branching ratio and spectra in bubble chamber[4]. In our opinion it is useful to compare results from completely different techniques.

We would like to point out that, without any change of the experimental apparatus and during the same running time, a considerable number of $K_2^o \to \pi^+\pi^-\pi^o$ decays will be collected. (see Appendix)

Presents experimental situation.

The early attempts to fit the experimental data assuming constant form factors revealed important discrepancies. The data are now usually analyzed assuming a linear q^2 dependence:

$$f_\pm(q^2) = f_\pm(0)\left(1 + \lambda_\pm \frac{q^2}{m_\pi^2}\right)$$

Measurements of relative branching ratio $\Gamma(K_{\mu 3}) / \Gamma(K_{e3})$ and muon polarization are most suited to draw conclusions on our problem. Individual energy spectra are less sensitive and Dalitz plot distributions are not yet explored with sufficient statistics to overcome intrinsic difficulties due to not uniform efficiency.

A recent review[2] summarized in Table I, shows the inconsistency among the available experimental data, ξ has been taken as constant and calculated by taking an average over experimental data on branching ratio and μ polarization.

Table I

$\xi(0) = 0.65 \pm 0.2$	using $R = 0.73 \pm 0.03$	$(R = \Gamma(K^+_{\mu 3})/\Gamma(K^+_{e3}))$
$\xi(0) = -1.1 \pm 0.5$	using $P_L = 0.96 \pm 0.12$	(long.pol. of μ from $K_{\mu 3}$)
$\xi(0) = -1.2 \pm 0.4$	using $P_T = -0.32 \pm 0.08$	(transverse μ polariz. in μ, π plane)

One can remark, for instance[2], that $\xi(0) = -1.2$ corresponds to a value R = 0.5, that is more than 7 standard deviations from the measured value.

On the other hand, with the assumption of linear dependence the best fit to experimental data is obtained with the choice of the values

$$|\lambda_+| \lesssim 0.05$$
$$\lambda_- \sim -0.3$$

The large values of λ_- is in disagreement with the current theoretical estimates[3] which provide no immediate explanation of so strong a dependence of $f_-(q^2)$ on q^2.

The quoted analysis[2] underlines the need for improved experimental determinations of the Dalitz plot distribution in $K_{\mu 3}$ decay.

The last experiments reported at the Heidelberg Internation. Conference on Elementary Particles (1967) do not contribute to clarify very much the situation. Taking into account the new results there presented (Table II)

Table II

$\frac{\Gamma(K^+_{\mu 3})}{\Gamma(K^+_{e3})} = 0.62 \pm 0.04$	(Aachen, Bari, CERN, Padova collaboration)
$\frac{\Gamma(K^0_{\mu 3})}{\Gamma(K^0_{e3})} = 0.76^{+0.14}_{-0.08}$	(P. Basile et al.)
$= 0.71 \pm 0.07$	(B. Aubert et al.)
$\lambda_+ = 0.045^{+0.017}_{-0.018}$	(Bellotti et al., from K^+_{13})
$\xi = -0.5 \pm 0.3$	(Aachen, Bari, CERN, Padova, Valencia, Madrid coll. from K^+_{13})
$\lambda_+ = 0.023 \pm 0.017$	(Basile et al., from K^0_{13})
$\xi = 1.0 \pm 0.7$	(Basile et al., from K^0_{13})
$\xi = 0.4 \pm 0.5$	(Orsay)

the following world averages are obtained:

$\lambda_+ = 0.023 \pm 0.008$ from K^+_{13}

$\lambda_+ = 0.013 \pm 0.009$ from K^o_{13}

$\xi = 0.3 \pm 0.4$ from K^+_{13} spectra and branching ratios

$\xi = -1.25 \pm 0.32$ from μ polarization in $K^+_{\mu 3}$

$\xi = 0.7 \pm 0.3$ from K^o_{13} spectra and branching ratios

$\xi = -1.15 \pm 0.35$ from μ polarization in K^o_{13}

from which appears the evident discrepancy between ξ values from polarization and from B.R.

It has to be noted that the available statistics for K° leptonic decay is one order of magnitude smaller than the corresponding one for K^+ decay(5).

Experimental setup

The apparatus is sketched in fig. 1 and is practically the same used by CERN-Aachen group from the $\Delta\phi$, Δm (phase and mass difference $K^o_1 - K^o_2$).

The K^o_2's from the neutral beam b_{13} decay in the volume defined by the scintillators $\bar{S}_1$, S_2. The charged decay products are momentum analysed by the spectrometer $W_1 MW_2$. $W_{1,2}$ are magnetic core wire-chambers, and M, a wide gap magnet (30×200 cm^2, $\int B\,dL = 370$ K G cm).

Behind the spectrometer are placed a threshold gas Čerenkov counter (Č) to detect the electrons and more than 10 iron interaction length (followed by a scintillator S_5) to separate muons from pions (μ detector). Two charged tracks are required to be present; one of them has to fire the electron or muon counter. π and leptons have to cross two differ

ent counters; for this purpose the counter $S_{3,4}$ and Cerenkov are subdivided in 4 sectors.
The main parameters of the Cerenkov counter and μ detector are summarized below:

maximum angular acceptance	$\pm 10°$
efficiency	$\geq 95\%$
wall thickness	$\sim .3$ mm Al
π feed-through	$\sim .5 \cdot 10^{-3}$
μ-detector(for $P_\mu \geq 1.5$ GeV/c) efficiency	$\sim$ 90%

The trigger requirement is given by $\bar{S}_1\, S_2\, S_3\, S_4$. The S_5 and Čerenkov signals are recorded for the separation of the different decay modes. The wire chambers coordinates and counters informations are transferred to magnetic tape after a preliminary data elaboration performed by a 1800 - IBM computer on-line.

Monte Carlo calculation

A Monte Carlo calculation was done in order to

1) evaluate the experimental errors
2) determine the sensitivity of the experiment

A sample of K° were allowed to decay randomly inside the decay volume and the charged decay products followed through the experimental apparatus. The dynamics of $K^\circ \to \ell \pi \nu$ decays is that predicted by a pure V - A interaction, with constant form factors.
The trajectories of the charged products through the magnetic field and their intersection with the wire-chambers and counters are computed.
We used different values of magnet current in order to have the den

sity of the Dalitz plot as uniform as possible.

Some indicative results are:

Average geometrical acceptance	$\sim$ 20%
K° momentum resolution	$\lesssim$ 10%
Decay point coordinates resolution(cm)	$\pm$ 0.2 in normal plane
	$\pm$ 2.0 along beam direction.

In figs. 4, 5, 6 is visible the effect on the Dalitz plot (fig. 4: events accepted by the spectrometer), of the muon detector p_μ cut (fig. 5) and of the experimental uncertainties in the reconstruction procedure (fig. 6). Fig. 7 represents the K° momentum spectrum of the accepted events (P.S. b_{13} neutral beam).

Neglecting the uncertainty in the magnetic field, the error on the charged particles momentum, due to wire chamber coordinates indetermination, is of the order of 1% half width (fig. 8).

There is a twofold ambiguity in the calculation of the laboratory energy of the K_2^o from the measured momenta of the charged particles. Consequently there is a related twofold ambiguity in the calculation of the energy of charged particles in the K_2^o cm. **system.** As usually done in similar experiments[(6),(7)] this difficulty may be overcome assigning to each solution a weight proportional to the jacobian of the transformation lab → CM systems, to the assumed Dalitz plot density and beam spectrum.

A test of the sensitivity of the experiment as a whole was made calculating by the standard likelyhood method the parameters $\xi(0), \lambda_+ \lambda_-$.

The "reconstructed" events were processed by a likelyhood program used in the X2 experiment[(5)]. Each Dalitz plot bin was corrected by the detection efficiency of the apparatus (spectrometer and μ detector).

For simplicity the lower K° momentum solution was choosen for each event in this preliminary analysis. The more refined 2 solu -

tions averaging procedure described above will improve the final precision in the form factors parameters determination.

The maximum likelyhood was found very close to the generated point, showing no systematic effect of the reconstruction procedure and of the experimental uncertainties.

In fig. 9 is shown the effect of statistics: the maximum of the curve b (higher statistics) reproduces with satisfactory agreement the ξ (o) value from which the events were originated in the Monte Carlo.

Background

The good resolution in the K° decay coordinate reconstruction will permit to reject with high efficiency triggers due to random coincidences of two charged tracks.

The neutron background was found to be negligible in the present CERN - Aachen experiment with optical chambers. Another possible source of background are the $K^0 \to \pi^+\pi^-\pi^0$ decay able to trigger the system via $\pi \to \mu$ decay in flight or via the feed-through the Cerenkov counter or via γ conversion. The reject of the events fitting the τ hypothesis will eliminate this background with a loss of the order of 1% of good events.

Events rate

The counting rate was evaluated assuming:

P.S. proton beam intensity	10^{10} protons/pulse
Solid angle defined by the beam collimator	$1.5\ 10^{-4}$ sterad.
K_2^0 beam intensity in the decay volume	$50\ 10^4\ K_2^0$ / pulse
decay volume length	3.5 meters

The resulting number of counts is about:

$$30 \quad K^{o}_{\mu 3} \; / \; \text{pulse}$$

$$100 \quad K^{o}_{e3} \; / \; \text{pulse}$$

sufficient to guarantee a very high statistics in a reasonable P.S. time, also taking into account a safety factor.

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-Appendix-

Pions spectra in the decay $K_2^o \rightarrow \pi^+\pi^-\pi^o$

The experimental apparatus described for the K_2^o form factors determination permits to collect in the mean time $K_L^o \rightarrow \pi^+\pi^-\pi^o$ decays.

The theoretical description of $K_{\pi 3}^o$ decay is not very advanced[8]. The reason is that little is known about a matrix element which involves only strongly interacting particles.

From a phenomenological point of view, an often used approach to this decay mode is the "linear matrix element" approximation:

$$M\,(X,Y) = a + b\,X + c\,Y$$

where

$$X = -\sqrt{3}\,(s_1 - s_2)/2\,m_K\,Q$$

$$Y = -\sqrt{3}\,(s_3 - s_o)/2\,m_K\,Q$$

$$Q = m_K - \Sigma\,m_i$$

$$s_i = (P_K - P_{\pi i})^2 \text{ with } i=1,2,3 \text{ for } \pi^+\pi^-\pi^o$$

If CP invariance is assumed, M does not contain odd powers of X:

$$M = a + c\,Y$$

Thus, assuming CP invariance and no strong π-π final state interaction, the spectrum is approximated (neglecting quadratic terms) by:

$$|M(x,y)|^2/\phi = 1 + 2\sigma_o\,(2T_3 - T_{max})\,\frac{m_K}{m_\pi^2}$$

where:

ϕ is the normalized invariant phase space

σ_o is the slope parameter

The main problems connected with the pions spectra in $K^o_{3\pi}$ decay are:

a) To reveal, if it exists, the quadratic term in the matrix element. This problem needs high statistics and high resolution. The existing data[8][9] are not accurate enough to draw useful conclusions about the presence of the quadratic term.

b) test of Δ I = 1/2 selection rule, which leads to the following prediction for the slope parameter of the odd pion spectrum in τ°, τ' and τ decays

$$\sigma_0(+-0) = \sigma_+(+00) = -2\sigma_-(++-)$$

and for the decay rates

$$R(+-0) = 2R(+00) = \frac{1}{2}R(++-) = \frac{2}{3}R(000)$$

By the comparison of the odd pion slope in τ° and τ' the presence of an I = 2 state may be investigated and informations otherwise not available may be obtained.

The existing data[10], though in agreement with Δ I = 1/2, do not exclude a Δ I = 3/2 adimixture of the order of 15%[8].

c) Test of CP invariance in K(+ - o) decay. The consequence of CP violation is an asymmetry between π^+, π^- corresponding to a value $\neq 0$ of the coefficient b in the matrix element M.

An experimental indication of different slope for π^+, π^- spectra was found[2] in bubble chamber, with inconsistency at 99% confidence level. A similar experiment[1] in spark chambers with comparable statistics does not confirm this result.

d) Test of the presence of a $\pi\pi$ final state interaction (single interaction or rescattering), to explain the spectra deviation from a pure phase space distribution. Such interactions in many models lead to a non linear matrix element.

In conclusion we can remark that, all the experimental results so far obtained are based on samples of the order of few thousands events[8]. With the proposed experimental apparatus the statistical accuracy does not represent any more a serious problem.

The expected event rate, in the same conditions assumed for the $K_{\mu 3}$, K_{e3} rate, is in fact

$$\sim 30 \quad K_2^o \rightarrow \pi^+ \pi^- \pi^o / \text{pulse}$$

which corresponds, in few PS days, to at least 2 orders of magnitude more than the existing data.

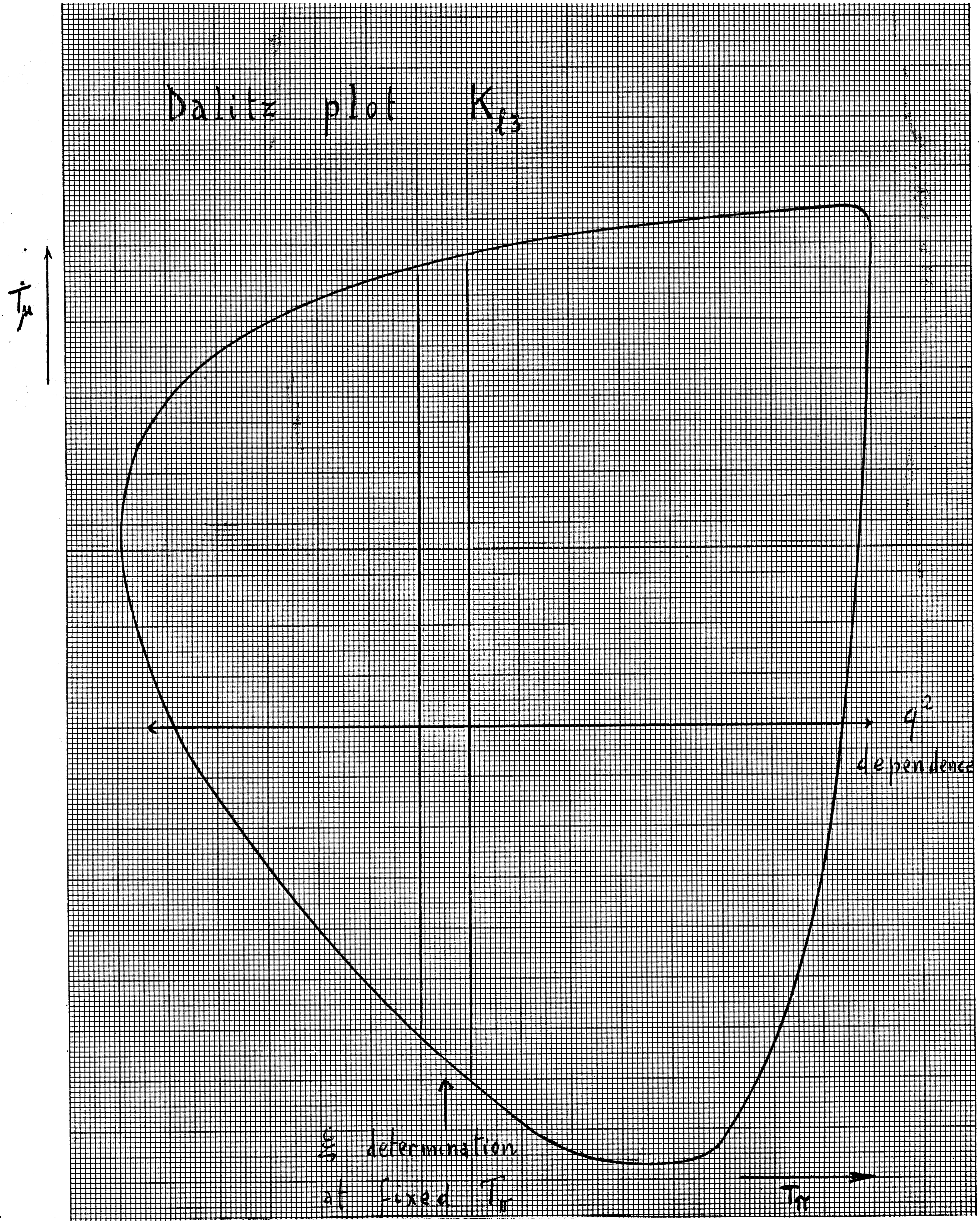


Fig. 1 –

- K13 Dalitz plot
- ξ determination along vertical strips (at constant T_π)
- q^2 dependence of ξ along horizontal lines.

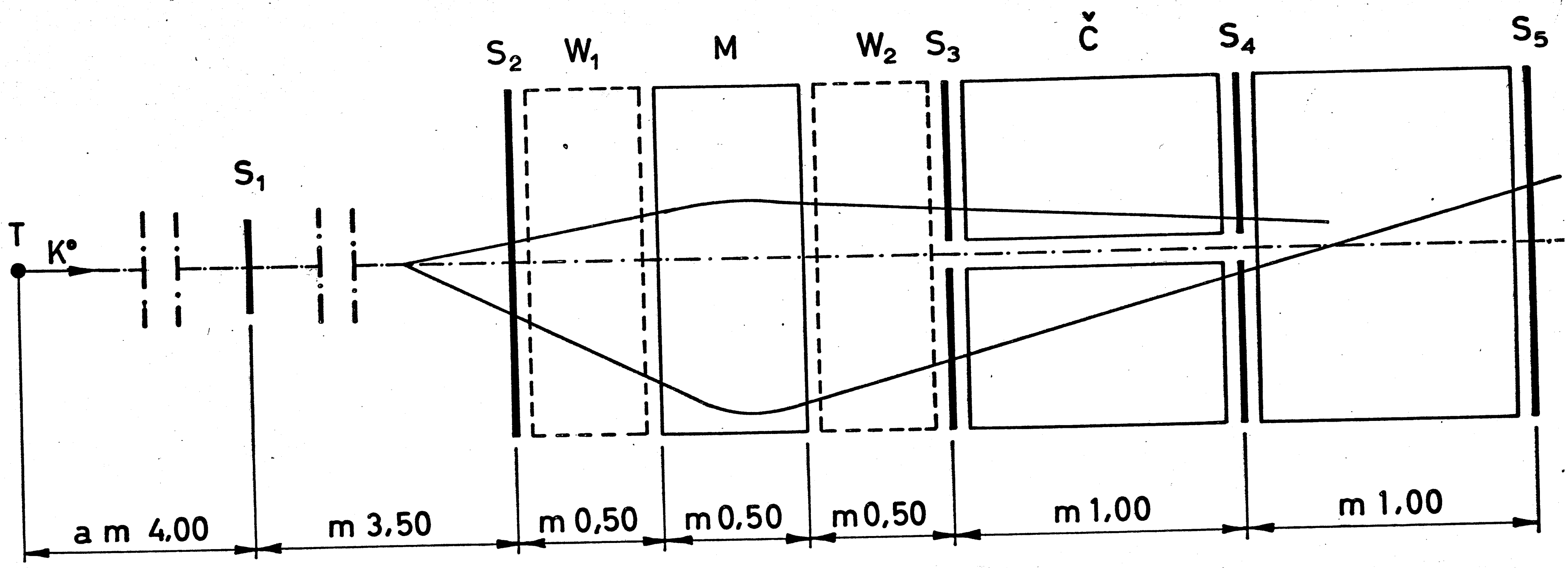


Fig. 3. - Experimental setup

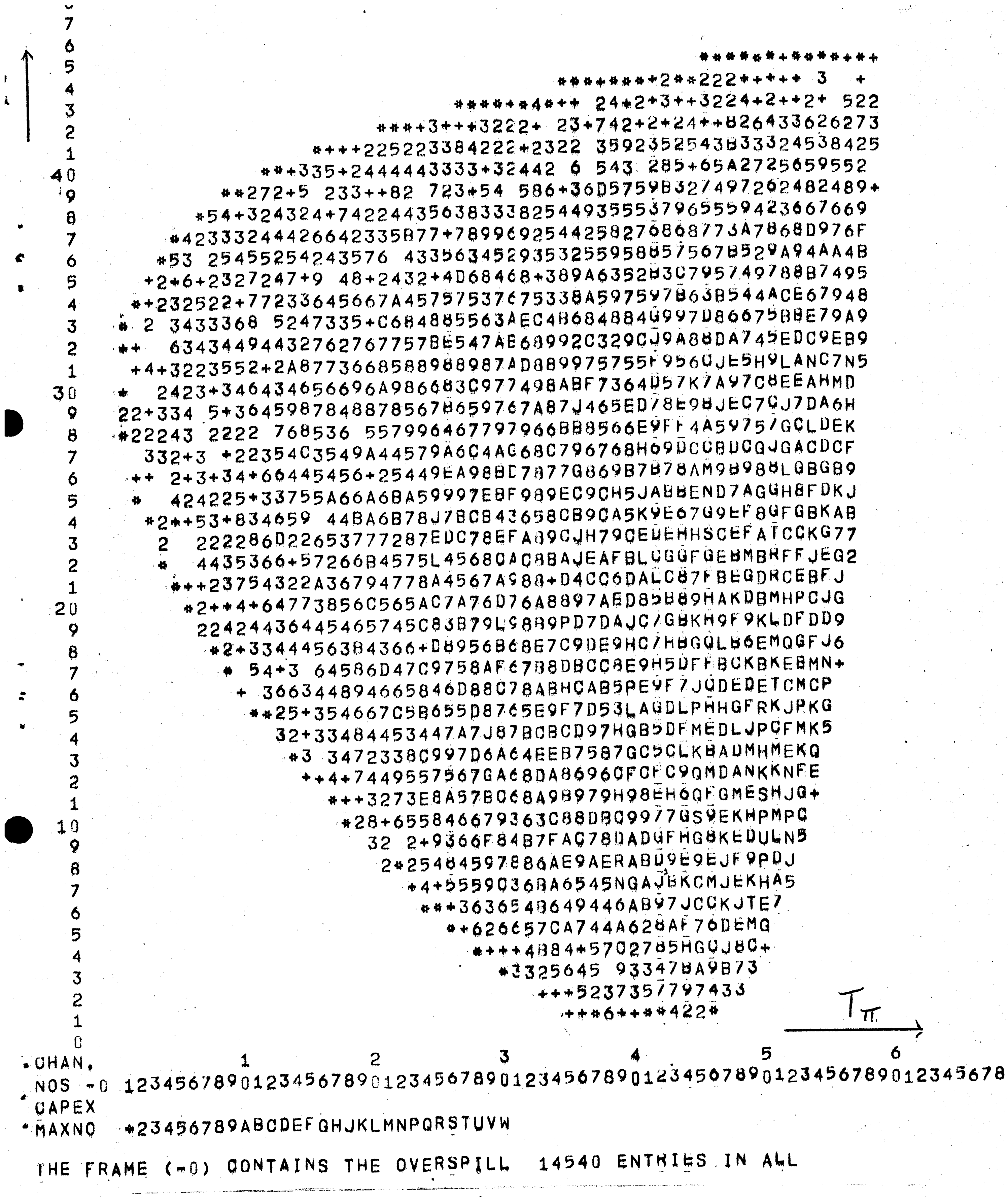
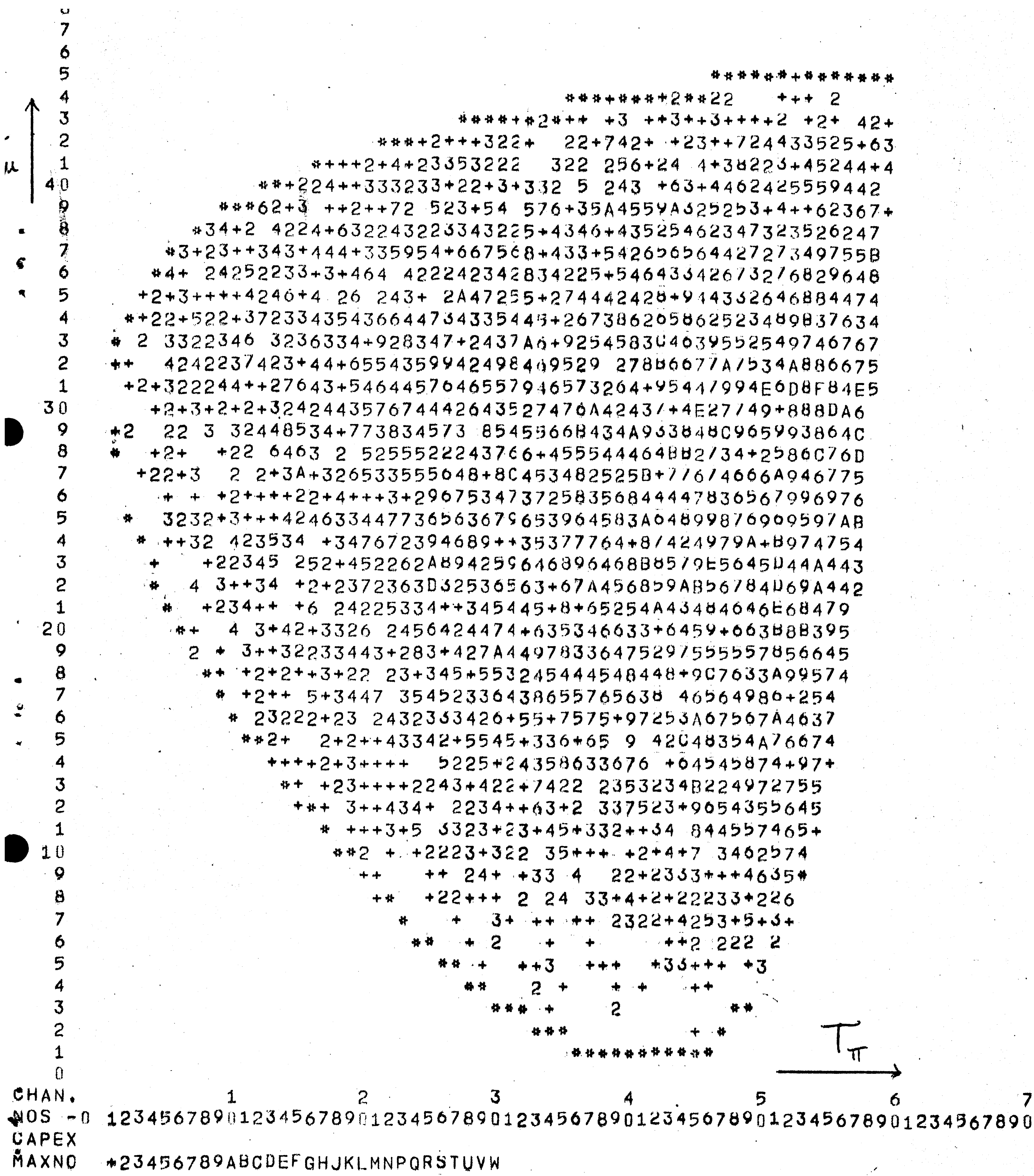


Fig. 4 -

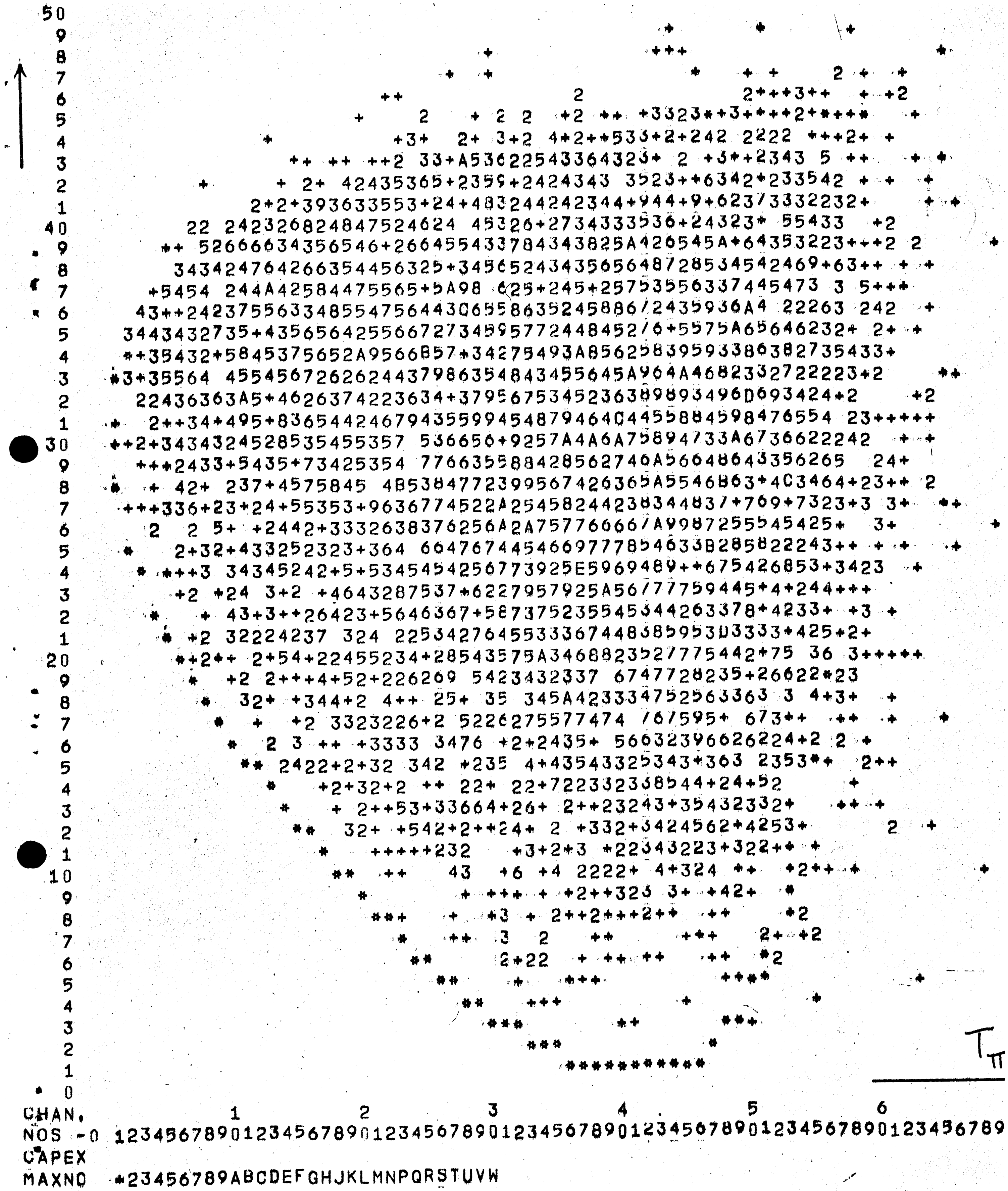
- Dalitz plot of $K^0_{\mu 3}$ events accepted by the spectrometer (T_μ vs T_π). Events produced according vector matrix elements with $\xi(0) = 0$, $\lambda_+ = 0.02$, $\lambda'' = 0.037$



THE FRAME (-0) CONTAINS THE OVERSPILL 6804 ENTRIES IN ALL

Fig.5 _

- Dalitz plot of $K^0_{\mu 3}$ events. Only events with $P_\mu > 1.5$ GeV/c are retained (muon detector threshold). The effect of this cut is visible in the low μ energy region.



. THE FRAME (-0) CONTAINS THE OVERSPILL 6804 ENTRIES IN ALL

Fig. 6 –

– Final sample of reconstructed events, experimental errors taken into account.

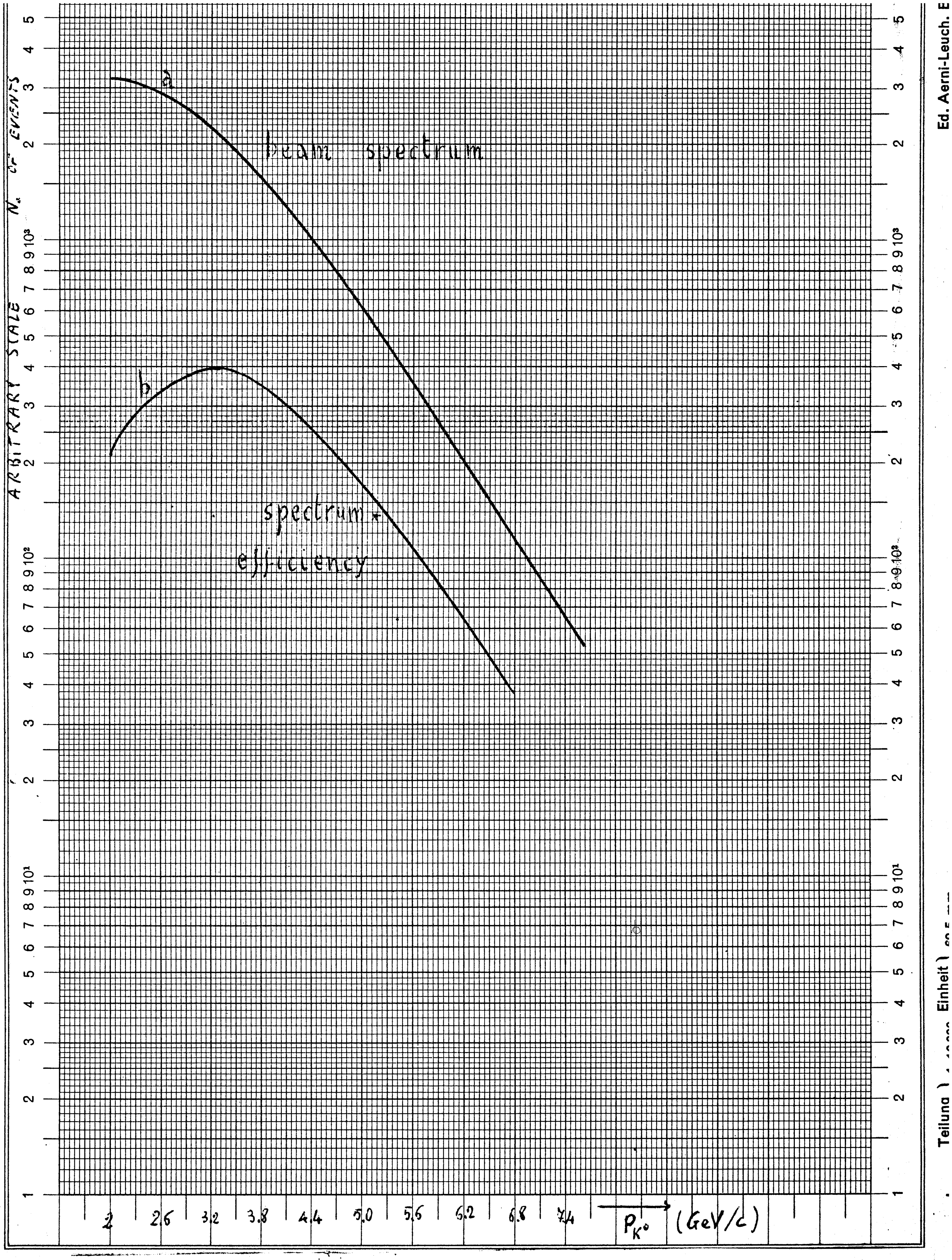


Fig. 7 –

– K^0 momentum distribution of the accepted events (curve b). Curve a is the neutral incident beam spectrum.

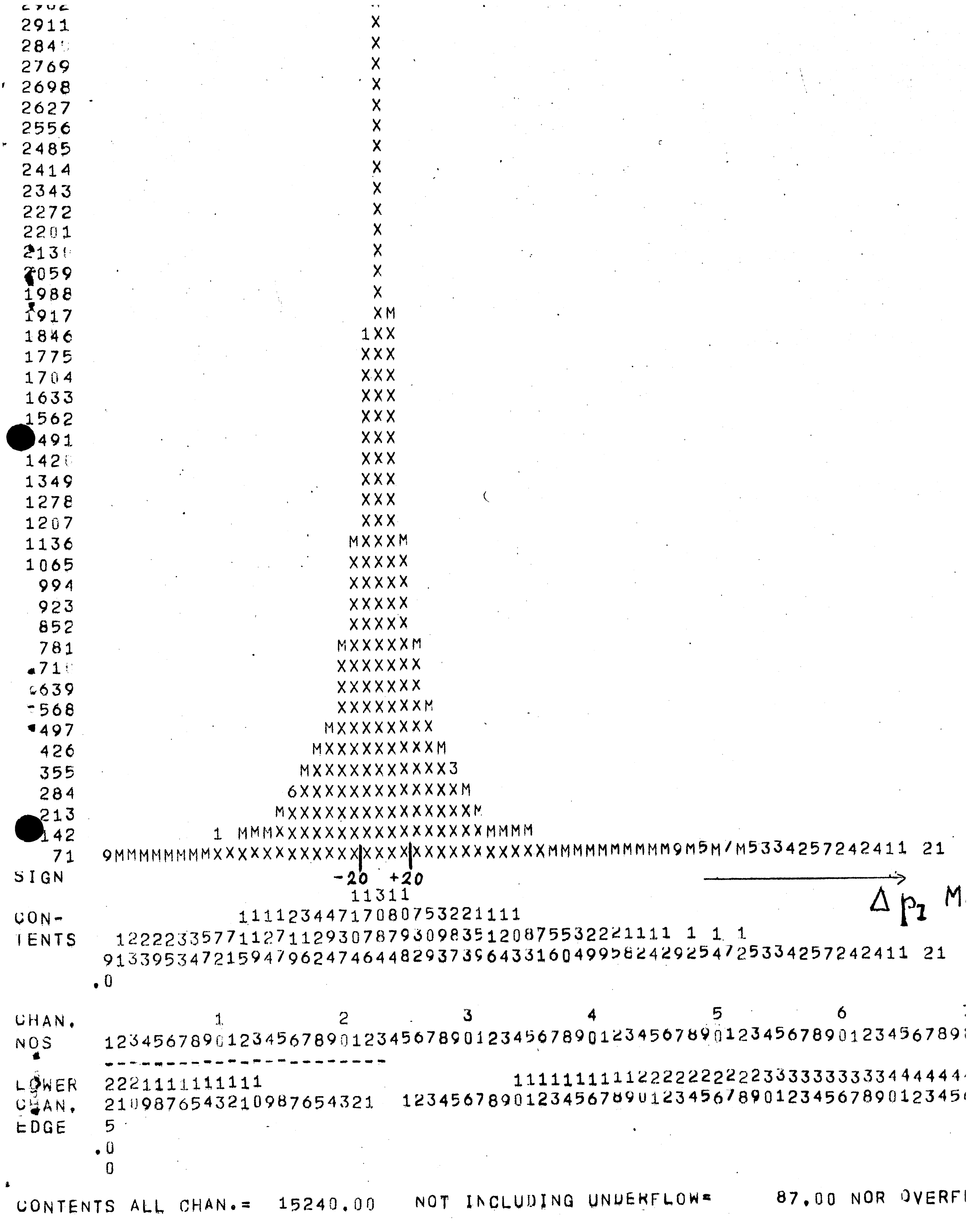


Fig. 8 –

– Absolute error in lepton momentum determination

$(\Delta p_1 = p_1^{real} - p_1^{reconstructed})$.

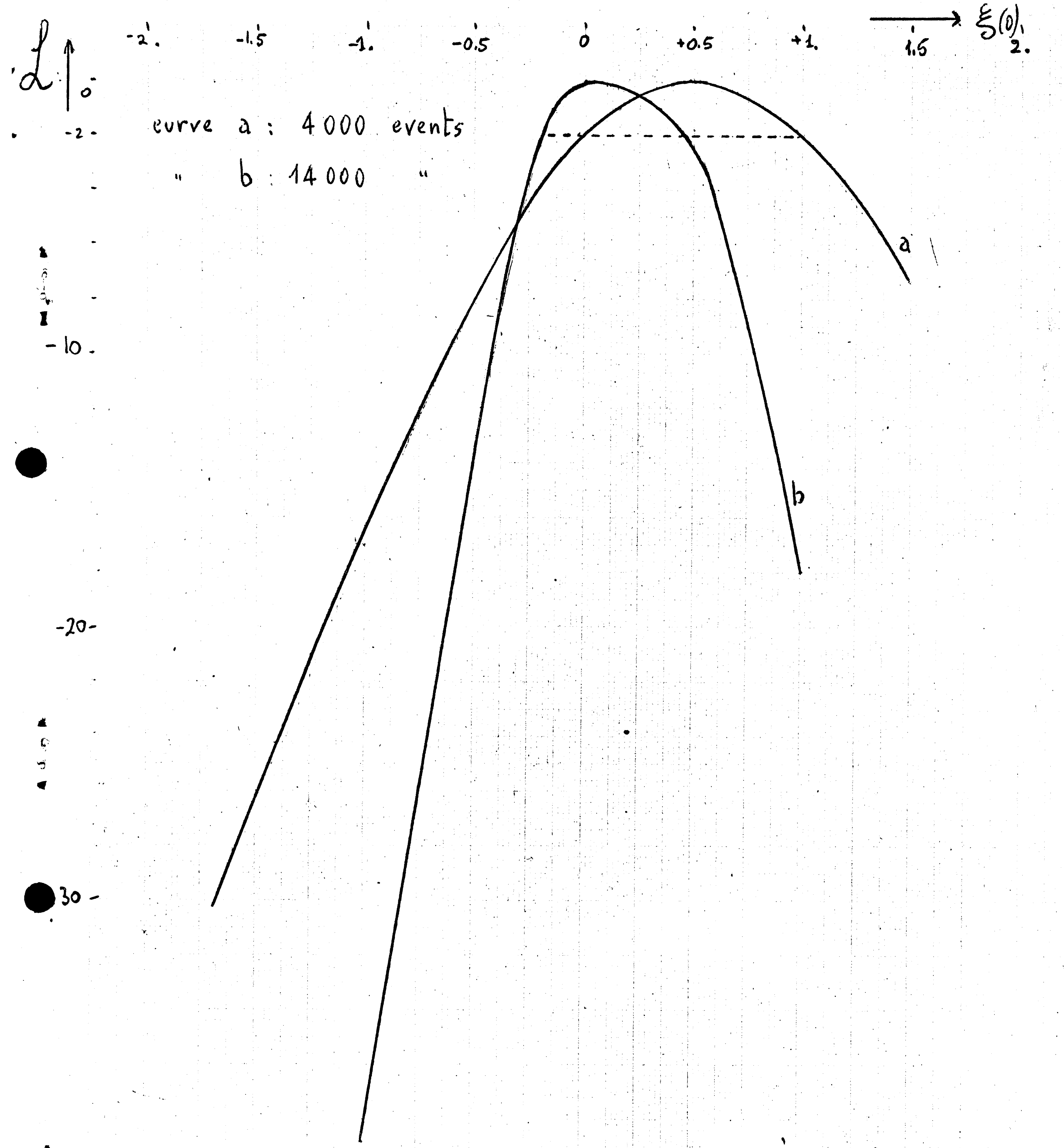


Fig. 9 _

- Likelyhood analysis on reconstructed events with experimental errors folded in.