NOT FOR PUBLICATION

ON ELECTRON-DEUTERON INELASTIC SCATTERING WITH COINCIDENCES

by

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I. KINEMATICAL CONSIDERATIONS

1) We consider the reaction

$$e+D \rightarrow e+n+p$$

in the one-photon approximation with the momenta as defined in Fig. 1 in the lab. system.

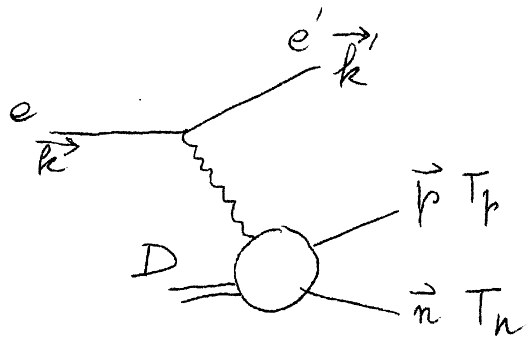

Fig. 1

The momentum transfer for the electron is defined by

$$\begin{aligned} \vec{q} &= \vec{k}-\vec{k}' \\ q_o &= k-k' \\ q^2 &= \vec{q}^2-q_o^2 \\ q^2 &= 4kk' \sin^2 \frac{\Theta}{2} \end{aligned} \qquad (1)$$

where Θ is the scattering angle of the electron.

$$T_n+T_p = q_o-B = T_o \qquad (2)$$

where B is the deuteron binding energy T_n and T_p are the kinetic energies of the outgoing nucleons.

2) The quasi-elastic peak in the electron spectrum corresponds to a choice of q^2 and q_o such that one of the outgoing nucleons may have zero kinetic energy. For a given momentum transfer q^2 this is only possible if q_o has a well-defined value given by

$$q_o = \frac{q^2+BM_D}{2(M-B)} \tag{3}$$

(M = nucleon mass, $M_D = 2M-B$ = deuteron mass).

Using Eq. (1) we see that for a given incident energy the choice of q^2 alone determines both the energy and the scattering angle of the outgoing electron. In this particular situation T_o defined in relation (2) is the maximum possible kinetic energy of one of the nucleons

$$T_o = \frac{q^2+B^2}{2M-B} \quad . \tag{4}$$

3) Let us consider the case where, in addition to the angle and energy of the electron, one of the nucleons is detected in the quasi-elastic situation. The reaction is entirely specified by three parameters:

- electron parameter q^2 (which furnishes both θ and k').
- azimuthal angle between the planes (ee') and (np) which plays no role in <u>unpolarized</u> cross-sections.
- angle between the momentum of one nucleon (for instance, the proton) and the momentum transfer $\vec{q} = \vec{k}-\vec{k}'$.

Let us call this angle α : for given q^2 and α the kinetic energies of the emerging nucleons are determined.

If we choose to consider the angle between the proton and q we get

$$\cos\alpha = \sqrt{\frac{T_p}{T_o}\,\frac{T_o+2M}{T_p+2M}} \,. \qquad (5)$$

Actually for $|q^2| < 5\ \text{fermi}^{-2}$

$$T_p < T_o \leqslant 106\ \text{MeV}$$

and to a very good approximation

$$\cos^2\alpha \cong \frac{T_p}{T_o} \qquad (5')$$

then $\sin^2\alpha \simeq \frac{T_n}{T_o}$, which means that the angle between the directions of the two nucleons is $\sim 90^o$.

As an illustration let us consider the case

$$q^2 = 5\ \text{fermi}^{-2}$$

Then $T_o = 106$ MeV. Then the protons (neutrons) of energy $>$ 96 MeV are emitted in a cone of axis $\vec{q}$ and semi-angle 17^o they correspond to neutrons (protons) of energy $<$ 10 MeV emitted outside a cone of axis $\vec{q}$ and semi-angle 73^o. It then appears clearly that by detecting fast neutrons in a narrow cone around direction $\vec{q}$ one detects events where the kinetic energy of the proton is very small. Alternative methods to detect such events may consist in

i) direct detection of low energy protons in a ring counter

ii) anticoincidence against protons emitted in a cone of large angle (?).

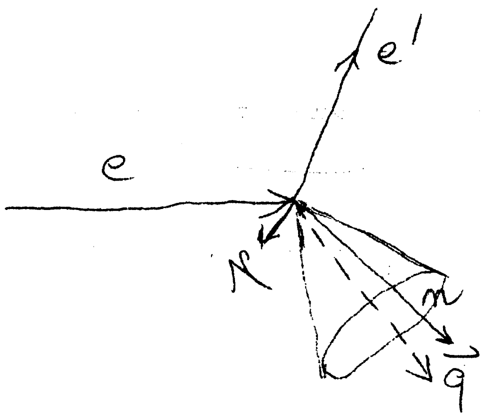

Fig. 2

II. ESTIMATE OF THE ACCURACY OF COINCIDENCE EXPERIMENTS

We wish to give a rough estimate of the error in neutron measurements and in particular to show the advantage one finds in performing coincidence experiments.

Since we wish only to give a first indication we shall here disregard all complications due to spins. A calculation in the realistic case has no essential difficulty but need just some algebra. We shall be glad to look into this problem if you will, indeed, plan to perform a coincidence experiment. In any case the spinless case can give an especially good idea of the problem of the neutron charge.

The amplitude for $(\gamma)+D \rightarrow p+n$ has the form

$$A = \frac{\alpha_p}{T_p + \frac{B}{2}} + \frac{\alpha_n}{T_n + \frac{B}{2}} + \frac{\alpha_o}{T_o} \qquad (6)$$

where T_p, T_n are the proton and neutron kinetic energies, B the binding energy of deuteron. α_p, α_n are the amplitudes for (ep) and (en) scattering. Finally, α_o represents the sum of all effects in which the D does not simply behave as a sum of free proton and free neutron. An accurate estimate of α_o is not yet available, however, we can say that α_o has the order of magnitude of $|\alpha_p| + |\alpha_n|$. Eq. (6) can be represented in the following graph

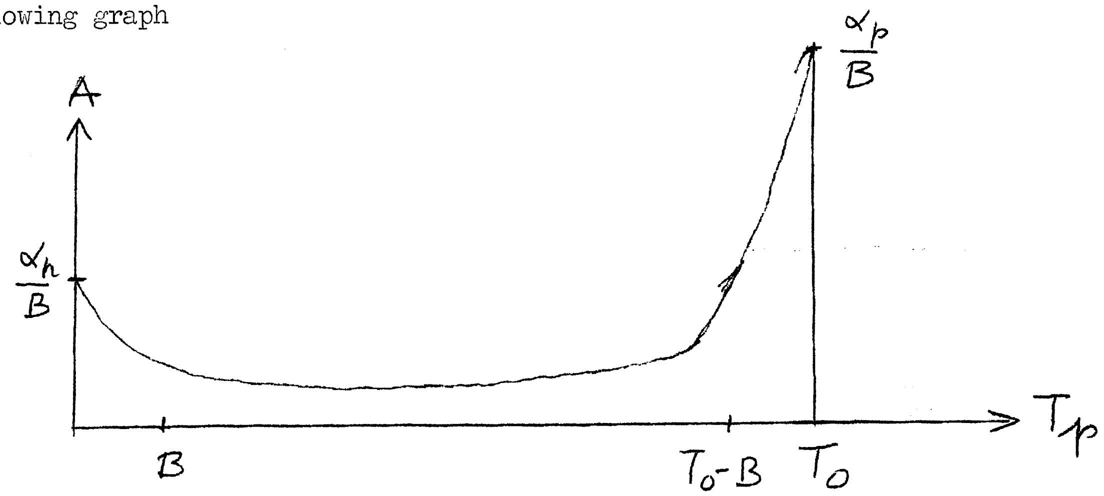

Fig. 3

We see that the proton and the neutron terms are dominant only in small regions around $T_n \simeq 0$ and $T_p \simeq 0$; in the remaining region the proton and neutron terms are of the same order of magnitude of the unwanted term in α_o . In a coincidence experiment we wish to select only the interesting region, i.e., if one wants to see the neutron effect one has to look for very slow protons and very quick neutrons.

Let us now estimate the errors in the cross-section. Let us start with a "no coincidence" experiment

$$\sigma_D \cong \frac{2}{B}\left[\alpha_n^2 + \alpha_p^2 + \frac{B}{2T_o}\left[(4\alpha_n\alpha_p + 2\alpha_n\alpha_o + 2\alpha_p\alpha_o)\ln\frac{2T}{B} + \alpha_o^2\right]\right] \qquad (7)$$

Equation (7) shows that in a no coincidence experiment the error is of the order B/T_o, i.e., $\sim 5\%$. However, the fact that α_n^2 is much smaller than α_p^2 makes the error in the determination of α_n very large. Let us now consider coincidence experiments picking up protons in the interval E_1 to E_2 (where E_1 must be as small as possible). Then we have

$$\sigma_D(E_1E_2) = \frac{2}{B}\left[\alpha_n^2 F(E_1E_2) + \frac{B}{2T_o}(2\alpha_n\alpha_p + 2\alpha_n\alpha_o)\ln\left(\frac{E_2+\frac{B}{2}}{E_1+\frac{B}{2}}\right) + \frac{B}{2T_o}\varkappa\left[\alpha_p^2 + 2\alpha_p\alpha_o + \alpha_o^2\right]\right] \qquad (8)$$

where $\varkappa = \dfrac{E_1-E_2}{T_o}$ and

$$F(E_1E_2) \simeq \left(\frac{1}{E_1+\frac{B}{2}} - \frac{1}{E_2+\frac{B}{2}}\right)\frac{B}{2}$$

is of the order of $\frac{1}{2}$ if E_1 is chosen of the order of $\frac{B}{2}$.

In order to design an experiment to detect the neutron we have to maximize the coefficient of α_n^2 and minimize all other terms. So an ideal experiment would be with $E_1 = 0$ and $E_2 \sim 3 - 4$ B. In this case the coefficient of α_n^2 would be ~ 1, the terms containing $\alpha_p^2, \alpha_p\alpha_o, \alpha_o^2$ will be completely negligible and the coefficient of the interference terms with α_n will be reduced of a factor 2 - 3 as compared to the no coincidence case. We note that here the only cause of error is the interference with α_n. Since α_n is much smaller than α_p this will give a great advantage to coincidence experiments.

III. POSSIBLE EXPERIMENTS

1) From the preceding analysis we see that an experiment, in which one wants to get information about neutron electromagnetic structure, must be planned in such a way that the neutron pole plays a dominant role in the scattering process. It is obviously necessary to exclude small kinetic energies of the neutron ($T_n \gg B$); this will suppress the square term coming from the proton pole but the interference terms might still be rather large, though smaller than in a "classical" experiment. It is therefore better to retain only small kinetic energies of the proton ($T_p < 2\text{-}3\ B$, $T_n \simeq T_o$). Then, formula (8) shows that the interference is considerably reduced. It is important, however, to have an experimental arrangement which does not reject the events with very small T_p; if, for instance, one excludes the events with $0 < T_p < 1$ MeV the cross-section will be lowered by a factor of 2.

We have seen that the laboratory angles of emission of the nucleons and their energies are related to each other by conservation laws. The situation we are considering here corresponds to neutrons emitted in a narrow cone of axis $\vec{q}$ with a semi-angle α of the order of $\sqrt{B/T_o}$, and protons approximately distributed isotropically in a plane perpendicular to $\vec{q}$ (Fig. 2). We realize that, experimentally, detection of very low energy protons might be very difficult. Alternative ways of performing the experiments might be:

i) detection of fast neutrons in the direction $\vec{q}$

ii) anticoincidence against protons emitted is a large angle cone with axis $\vec{q}$. If the angle of this cone cannot be made large enough to satisfy the condition $T_p < 2\text{-}3\ B$, the interference terms will have some importance, but, nevertheless, this experiment is better than a coincidence experiment with a threshold of detection of protons $T_p > 1$ MeV.

As a check of the procedure one can study electron-proton scattering by detecting fast protons in the direction $\vec{q}$. A comparison of the results with direct experiments in hydrogen can give some estimate of the importance of the interference terms.

2) If the above conditions are fulfilled in one way or the other, one might ask what angles of scattering of the electron should be chosen for a given q^2 ? We think that the basic problem is to determine as well as possible the two parameters $A(q^2)$ and $B(q^2)$ which enter in the differential cross-section

$$\frac{d\sigma}{d\Omega}(e\text{-}n) = \frac{d\sigma}{d\Omega}_{\text{Mott}} \left[A(q^2) + B(q^2) \tan^2 \theta/2 \right] .$$

This means that one has to perform measurements for small and large angles plus a few (but not too many) intermediate angles. The small angle scattering will be dominated by the electric neutron form factor (roughly speaking) and the large angle scattering can give a direct measurement of the magnetic part which might be the easiest one to detect.