ANGULAR DISTRIBUTION IN NUCLEON-NUCLEON "QUASI-ELASTIC DIFFRACTION" SCATTERING

II. EFFECTS OF THE NUCLEON SPIN

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A B S T R A C T

In this paper, the optical approximation for elastic and quasi-elastic diffraction scattering of nucleons by nucleons, developed in a previous paper (I), is extended to take into account effects of the nucleon and isobar spins. Expressions are derived for the angular distribution and polarization of the scattered nucleons, both for elastic and quasi-elastic scattering with $(3,3^{+})$ isobar excitation. Two special cases are considered: a spin-spin interaction between the nucleons, or nucleon and isobar, in the final state; and a spin-orbit interaction, but no spin-spin interaction; the angular distributions and resulting nucleon polarizations are derived in detail in these cases, and experimental possibilities are considered for distinguishing between them, and for deriving further information on the spin-spin interactions, especially as a result of polarization measurements. Finally, in a mathematical appendix, we consider various possible devices for taking into account, in a more realistic fashion, the nucleon "shape" in terms of appropriate approximations to the nucleon "form factor" in the optical approximation.

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Introduction

This paper is intended as a continuation and extension of the one previously written under the same title *). In the earlier paper, though motivated by observations which we interpreted as being due to the reactions **)

$$N+N \rightarrow N+N^* \tag{1}$$

we developed a "spin-independent" optical approximation for the angular distribution of the scattered nucleons. In particular, the spin of the projectile nucleon (but not of the target) was neglected; thus the considerations of that paper apply, more appropriately, to the reactions

$$\pi+N \rightarrow \pi+N^* \tag{2}$$

or to the corresponding reactions with kaon projectiles. In the following, the theory is developed taking full account of the spin of the projectile nucleon in reactions (1). Having done this, we consider the question of whether, and under what conditions, the scattered nucleon may be expected to be polarized, and derive expressions for its possible transverse polarization. In these computations, we confine our attention to the isobar of spin $(3/2)^+$, but the same considerations may be easily extended to any of the other nucleon isobars. Finally, some general considerations are presented for the improvement of the optical approximation, both for elastic and for "quasi-elastic" diffraction scattering, by taking into account the variation of the nucleon "opacity" with position (impact parameter) and the "diffuseness" of the nucleon boundary.

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*) CERN Report 1114/TH.178, 17 April, 1961, henceforth referred to as I.

**) N* represents an excited state of the target nucleon with definite spin, parity, isotopic spin, and energy (to within a natural width, Γ_o) ... i.e., a nucleon "isobar".

Effects of Nucleon Spin; General Expressions:

A. Elastic Scattering. Owing to the nucleon spins, there are two channels in which the nucleons can interact; the triplet, with total spin S=1, and the singlet, S=0. Furthermore, provided the nucleons are initially unpolarized, the reactions in the two channels are independent (incoherent) so that they may be treated separately insofar as their contributions to the cross-section and to the polarization are concerned. The computation of the differential scattering cross-sections for the two channels now proceeds along entirely conventional lines, such as outlined in I, except for the effect of the Pauli principle. Its effect is simple: scattering in the singlet channel proceeds only through states with even orbital angular momentum; triplet scattering involves only odd ℓ.

We may completely define the scattering in terms of six classes of scattering amplitudes, characterized by the channel spin (S), the total angular momentum (J), the incident orbital angular momentum (ℓ) and the outgoing orbital angular momentum (ℓ'). We write, following the usual optical approximation *)

$$ {}^{S}A^{J}_{\ell,\ell'} = a_j e^{i\alpha_j} f_j(\ell); \quad (j=1-6) \tag{3} $$

in which a_j and α_j are the average values of the magnitude and phase for a given class of scattering, and $f_j(\ell)$ is assumed to be slowly varying with ℓ **). In the reaction (1) under consideration, the amplitudes involved are

*) For elastic scattering $A_j = 1-\eta_j$ with $|\eta_j| \to 0$ for the usual case of diffraction arising out of strong absorption.

**) In I we have taken $f(\ell)=1$ for $\ell \leq kR$, $f(\ell)=0$ for $\ell > kR$. However, as will be discussed in a subsequent section, this represents an unnecessary and unphysical approximation. Still, we shall not carry things to their ultimate generality, in this paper, but, rather, make the further assumption that $f(\ell)$ is the same for all scattering processes with the same channel spin S.

$$ {}^{0}A^{\ell}_{\ell,\ell} = a_o e^{i\alpha_0} f_o(\ell) \tag{4a} $$

$$ {}^{1}A^{\ell}_{\ell,\ell} = a_1 e^{i\alpha_1} f_1(\ell) \tag{4b} $$

$$ {}^{1}A^{\ell\pm1}_{\ell,\ell} = a_{\pm} e^{i\alpha_{\pm}} f_1(\ell) \tag{4c} $$

$$ {}^{1}A^{\ell\pm1}_{\ell\pm2,\ell} = b_{\pm} e^{i\beta_{\pm}} f_1(\ell) \tag{4d} $$

and the differential scattering cross-section becomes

$$ 2k^2 \frac{d\sigma}{d\Omega} = |a_o|^2(\Sigma_{+}^{o})^2 + (\tfrac{1}{2}|A_1+\bar{A}-\bar{B}|^2 + |\bar{A}+\bar{B}|^2)(\Sigma_{-}^{o})^2 + $$

$$ + 2(|A'|^2 + |B'|^2)(\Sigma_{-}^{1})^2 \tag{5} $$

$$ + \tfrac{1}{2}|A_1-\bar{A}+\bar{B}|^2(\Sigma_{-}^{2})^2 $$

In Eq. (5), we have adopted the following abbreviations

$$ A_1 = a_1 e^{i\alpha_1} $$

$$ \bar{A} = \tfrac{1}{2}(a_{+}e^{i\alpha_{+}} + a_{-}e^{i\alpha_{-}}) $$

$$ \bar{B} = \tfrac{1}{2}(b_{+}e^{i\beta_{+}} + b_{-}e^{i\beta_{-}}) \tag{4e} $$

$$ A' = \tfrac{1}{2}(a_{+}e^{i\alpha_{+}} - a_{-}e^{i\alpha_{-}}) $$

$$ B' = \tfrac{1}{2}(b_{+}e^{i\beta_{+}} - b_{-}e^{i\beta_{-}}) $$

$$\sum\nolimits_{\{\pm\}}^{n} = \sum_{\ell \left\{\begin{smallmatrix}\text{even}\\ \text{odd}\end{smallmatrix}\right\}} f_{\left\{\begin{smallmatrix}0\\ 1\end{smallmatrix}\right\}}(\ell)\,\ell^{(1-n)} P_{\ell}^{(n)}(\cos\theta). \tag{6}$$

The evaluation of the summations (6) proceeds as in the Appendix of I, except for the halving in the number of terms resulting from the Pauli principle; however, as long as there are many terms involved $(kR \gg 1)$, this amounts only to a reduction of the summation by a factor 2. Thus, assuming the step function behaviour of $f(\ell)$, the summations become

$$\sum\nolimits_{\pm}^{n} \rightarrow \tfrac{1}{2}k^2R^2F_n(X) \tag{6a}$$

with the $F_n(X{=}kR\theta)$ as defined in I. The use of more reasonable expressions for $f(\ell)$ will be discussed in an appendix.

The transverse polarization of the scattered nucleon may also be evaluated in terms of the same parameters(4e). Defining the transverse axis as the direction given by $\underset{\sim}{k}'\times\underset{\sim}{k}$, we obtain, again by conventional means,

$$-2k^2 \frac{d\sigma}{d\Omega} P_t = \mathrm{Im}\left\{ \left[2(\bar{A}+\bar{B})(A'+B')^* + (A_1+\bar{A}-\bar{B})(A'-B')^*\right] \sum\nolimits_{-}^{0} \sum\nolimits_{-}^{0} + \right.$$

$$\left. + (A_1-\bar{A}+\bar{B})(A'-B')^* \sum\nolimits_{-}^{1} \sum\nolimits_{-}^{2} \right\} . \tag{7}$$

Thus, in the most general case, evaluation of the optical approximation for the elastic scattering, Eq. (5), and the polarization, Eq. (7), requires the specification of six complex amplitudes (twelve parameters) and six functions $f_j(\ell)$, which we have arbitrarily reduced to two $f_S(\ell)$. We shall discuss further simplifications in the next section. However, even without simplification, we may note from Eq. (7) that the appearance of a polarization in elastic scattering requires a spin-orbit interaction .. i.e., the non-vanishing of the combinations A' or B'.

B. (3,3$^+$) Isobar Excitation. For reaction (1) with $N^* \neq N$ the two initial spin states (S=0 and 1) are still independent and the Pauli principle still requires even and odd ℓ-values to be associated, respectively, with the two initial spin channels. However, both possible outgoing spin channels, with

$$S' = S^* \pm \tfrac{1}{2} \tag{8}$$

are open to both incident channels (although not through the same intermediate states, and corresponding ℓ'-values, owing to the necessity for parity conservation), which results in a considerable multiplication of the classes of amplitudes involved in the reaction, as compared to elastic scattering. Thus, generalizing Eq. (3)

$$^{S,S'}A^{J}_{\ell,\ell'} = a_j e^{i\alpha_j} f_j(\ell) \tag{3'}$$

we now require 16 amplitude types for the complete specification of the reaction (1) with $S^* = 3/2^+$, viz *)

*) Again, only for purposes of simplification, we reduce the sixteen functions $f_j(\ell)$ to two, $f_S(\ell)$ with S=0 and 1.

$$ {}^{0,1}A^{\ell}_{\ell,\ell} = a_{o1} e^{i\alpha_o} \; f_o(\ell) \tag{9a} $$

$$ {}^{0,2}A^{\ell}_{\ell,\ell} = b_{o2} e^{i\beta_o} \; f_o(\ell) \tag{9b} $$

$$ {}^{0,2}A^{\ell\pm2}_{\ell\pm2,\ell} = b^{\pm}_{o2} e^{i\beta_o^{\pm}} \; f_o(\ell) \tag{9c} $$

$$ {}^{1,1}A^{\ell}_{\ell,\ell} = a_{11} e^{i\alpha_1} \; f_1(\ell) \tag{9d} $$

$$ {}^{1,1}A^{\ell\pm1}_{\ell,\ell} = a^{\pm}_{11} e^{i\alpha_1^{\pm}} \; f_1(\ell) \tag{9e} $$

$$ {}^{1,1}A^{\ell\pm1}_{\ell\pm2,\ell} = b^{\pm}_{11} e^{i\beta_1^{\pm}} \; f_1(\ell) \tag{9f} $$

$$ {}^{1,2}A^{\ell}_{\ell,\ell} = c_{12} e^{i\gamma_1} \; f_1(\ell) \tag{9g} $$

$$ {}^{1,2}A^{\ell\pm1}_{\ell,\ell} = c^{\pm}_{12} e^{i\gamma_1^{\pm}} \; f_1(\ell) \tag{9h} $$

$$ {}^{1,2}A^{\ell\pm2}_{\ell\pm2,\ell} = d^{\pm}_{12} e^{i\delta_1^{\pm}} \; f_1(\ell) \tag{9i} $$

$$ {}^{1,2}A^{\ell\pm1}_{\ell\pm2,\ell} = g^{\pm}_{12} e^{i\nu_1^{\pm}} \; f_1(\ell) \tag{9j} $$

The differential cross-section is

$$8k^2 \frac{d\sigma}{d\Omega} = \left|\sqrt{\tfrac{1}{2}}B_{02}+\sqrt{3}\,\bar{B}_{02}\right|^2(\Sigma_+^0)^2$$

$$+\left(\left|A_{11}+\bar{A}_{11}-\bar{B}_{11}\right|^2+2\left|\bar{A}_{11}+\bar{B}_{11}\right|^2+\left|C'_{12}-\sqrt{2}D'_{12}-G'_{12}\right|^2\right)(\Sigma_-^0)^2$$

$$+\left(2\left|A_{01}\right|^2+4\left|B'_{02}\right|^2\right)(\Sigma_+^1)^2$$

$$+\left(\left|\frac{\sqrt{3}}{2}C_{12}+\bar{C}_{12}-\sqrt{\tfrac{1}{2}}\,\bar{D}_{12}-\bar{G}_{12}\right|^2+\left|\sqrt{\tfrac{1}{2}}C_{12}+\sqrt{3}\,\bar{D}_{12}\right|^2+\right. \tag{10}$$

$$\left.+2\left|\bar{C}_{12}+\bar{G}_{12}\right|^2+4\left|A'_{11}\right|^2+4\left|B'_{11}\right|^2\right)(\Sigma_-^1)^2$$

$$+\left|\sqrt{3/2}\,B_{02}-\bar{B}_{02}\right|^2(\Sigma_+^2)^2$$

$$+\left(\left|A_{11}-\bar{A}_{11}+\bar{B}_{11}\right|^2+\left|C'_{12}+\sqrt{2}\,D'_{12}-G'_{12}\right|^2+2\left|C'_{12}+G'_{12}\right|^2\right)(\Sigma_-^2)^2$$

$$+\left|\sqrt{3/2}\,C_{12}-\bar{C}_{12}-\sqrt{\tfrac{1}{2}}\,\bar{D}_{12}+\bar{G}_{12}\right|^2(\Sigma_-^3)^2$$

Here, again, the amplitudes are abbreviated

$$A_{01} = a_{01}e^{i\alpha_0}$$

$$\bar{B}_{02} = \tfrac{1}{2}(b_{02}^+e^{i\beta_0^+} + b_{02}^-e^{i\beta_0^-}) \tag{9k}$$

$$B'_{02} = \tfrac{1}{2}(b_{02}^+e^{i\beta_0^+} - b_{02}^-e^{i\beta_0^-})$$

etc.

and the $\Sigma_\pm^n$ are defined by Eq. (6).

The expression for the transverse polarization of the (quasi-elastically) scattered nucleon is complicated and long, containing almost all possible combinations of the 16 amplitudes. We write it, symbolically, as

$$-8k^2 \frac{d\sigma}{d\Omega} P_t = \mathrm{Im} \left\{ \sum_{i \neq j, n} \sigma^n_{ij} Q_i Q^*_j \Sigma^n_{\pm} \Sigma^{n+1}_{\pm} \right\} \quad (11)$$

in which the Q_i are the amplitudes defined by Eqs. (9h) (i, j go from 1 to 16), n goes from 0 to 2, and the σ^n_{ij} are numbers, given in Table I.

Clearly, without some drastically simplifying assumptions concerning the possible inter-relationships among the amplitudes, we cannot make any quantitative statements concerning either the angular distribution or the polarization of the scattered nucleons. However, even at this stage, we may note that there exists, in the case of quasi-elastic scattering with isobar excitation, another possibility, which can lead to polarization of the scattered nucleons, in addition to the conventional "spin-orbit" interaction. Thus, as may be seen in Table I, even if all of the "primed" amplitudes vanish (which would be the case in the absence of a spin-orbit interaction in the conventional sense ... i.e., see Eqs. 9h) there still remain terms in the polarization equation (11), of the form $\mathrm{Im}\left\{A_{01}B^*_{02},\ A_{11}C^*_{12}, \text{ etc.}\right\}$; the existence of such terms requires a difference in the amplitudes, for a given value of the incident channel spin S, corresponding to the two possible values of the outgoing channel spin S'. Hence, the presence of such terms implies a kind of "spin-spin" interaction, between the nucleon and the isobar, in the final state. In the following section we consider the possible effects of such a spin-spin interaction, as well as of a spin-orbit interaction of the conventional type.

TABLE I

Coefficients *) σ^n_{ij} in the expression for the transverse polarization of nucleons (Eq. 11) in the reaction N+N → N+N*(3,3$^+$).

Q_i	Q_j	n=0	1	2
A_{o1}	B_{o2}	1/4	- 3/4	
	$\bar{B}_{o2}$	$(3/8)^{\frac{1}{2}}$	$(3/8)^{\frac{1}{2}}$	
B_{o2}	B'_{o2}	$(3/8)^{\frac{1}{2}}$	$(3/8)^{\frac{1}{2}}$	
$\bar{B}_{o2}$	B'_{o2}	3/2	- 1/2	
A_{11}	C_{12}	- 2	1/2	- 3/2
	$\bar{C}_{12}$	$-(3)^{\frac{1}{2}}$		$(3)^{\frac{1}{2}}$
	$\bar{D}_{12}$		$(3/2)^{\frac{1}{2}}$	$(3/2)^{\frac{1}{2}}$
	$\bar{G}_{12}$	$(3)^{\frac{1}{2}}$		$-(3)^{\frac{1}{2}}$
	A'_{11}	- 1	- 1	
	B'_{11}	1	1	
$\bar{A}_{11}$	C_{12}	- 2	- 1/2	3/2
	$\bar{C}_{12}$	$-3(3)^{\frac{1}{2}}$		$-(3)^{\frac{1}{2}}$
	$\bar{D}_{12}$		$-(3/2)^{\frac{1}{2}}$	$-(3/2)^{\frac{1}{2}}$
	$\bar{G}_{12}$	$-(3)^{\frac{1}{2}}$		$(3)^{\frac{1}{2}}$
	A'_{11}	- 3	1	
	B'_{11}	- 1	- 1	

*) Terms not included have $\sigma^n_{ij}=0$. Note that, in Eq. (11), the Σ_+ go with the Q_o and the Σ_- with the Q_1.

cont.

Q_i	Q_j	n=0	1	2
$\bar{B}_{11}$	C_{12}	2	1/2	$-3/2$
	$\bar{C}_{12}$	$-(3)^{\frac{1}{2}}$		$(3)^{\frac{1}{2}}$
	$\bar{D}_{12}$		$(3/2)^{\frac{1}{2}}$	$(3/2)^{\frac{1}{2}}$
	$\bar{G}_{12}$	$-3(3)^{\frac{1}{2}}$		$-(3)^{\frac{1}{2}}$
	A'_{11}	-1	-1	
	B'_{11}	-3	1	
C_{12}	C'_{12}		$(3/4)^{\frac{1}{2}}$	$(3/4)^{\frac{1}{2}}$
	D'_{12}		$(3/2)^{\frac{1}{2}}$	$(3/2)^{\frac{1}{2}}$
	G'_{12}		$-(3/4)^{\frac{1}{2}}$	$-(3/4)^{\frac{1}{2}}$
$\bar{C}_{12}$	C'_{12}	-1	2	-1
	D'_{12}	$(2)^{\frac{1}{2}}$		$-(2)^{\frac{1}{2}}$
	G'_{12}	1	2	1
$\bar{D}_{12}$	C'_{12}	$(8)^{\frac{1}{2}}$	$(9/2)^{\frac{1}{2}}$	$-(1/2)^{\frac{1}{2}}$
	D'_{12}	-4	3	-1
	G'_{12}	$-(8)^{\frac{1}{2}}$	$-(9/2)^{\frac{1}{2}}$	$(1/2)^{\frac{1}{2}}$
$\bar{G}_{12}$	C'_{12}	1	2	1
	D'_{12}	$-(2)^{\frac{1}{2}}$		$(2)^{\frac{1}{2}}$
	G'_{12}	-1	2	-1
A'_{11}	C'_{12}	$(3)^{\frac{1}{2}}$	$-3(3)^{\frac{1}{2}}$	
	D'_{12}	$-(6)^{\frac{1}{2}}$	$-(6)^{\frac{1}{2}}$	
	G'_{12}	$-(3)^{\frac{1}{2}}$	$-(3)^{\frac{1}{2}}$	
B'_{11}	C_{12}	$-(3)^{\frac{1}{2}}$	$-(3)^{\frac{1}{2}}$	
	D'_{12}	$(6)^{\frac{1}{2}}$	$(6)^{\frac{1}{2}}$	
	G'_{12}	$(3)^{\frac{1}{2}}$	$-3(3)^{\frac{1}{2}}$	

Effects of Nucleon Spin; Specific Models:

A. Spin-Spin Interaction: In order to obtain some feeling for the form of the angular distribution and polarization, both in elastic and quasi-elastic scattering, which might result from a spin-spin interaction of the type mentioned above, we consider the appropriate expressions under the assumption that the amplitudes, Eqs. (4) and (9), are entirely independent of the particular combination of ℓ and ℓ' involved in the scattering; we assume, however, that they do depend on the channel spins, S and S'. Thus, for elastic scattering, we take

$$A_1 = \bar{A} = \bar{B} \tag{12a}$$

$$A' = B' = 0 \tag{12b}$$

while for the quasi-elastic scattering, we assume

$$B_{o2} = \bar{B}_{o2} \tag{12c}$$

$$A_{11} = \bar{A}_{11} = \bar{B}_{11} \tag{12d}$$

$$C_{12} = \bar{C}_{12} = \bar{D}_{12} = \bar{G}_{12} \tag{12e}$$

$$B'_{o2} = A'_{11} = B'_{11} = C'_{12} = D'_{12} = G'_{12} = 0 \tag{12f}$$

For the elastic scattering, we obtain

$$4k^2 \frac{d\sigma}{d\Omega} = 2|A_o|^2(\Sigma_+^o)^2 + 9|A_1|^2(\Sigma_-^o)^2 + |A_1|^2(\Sigma_-^2)^2 \tag{13a}$$

$$P_t = 0 \tag{13b}$$

And, for quasi-elastic scattering leading to excitation of the $(3,3^+)$ isobar *)

$$8k^2 \frac{d\sigma}{d\Omega} = 6|B_{02}|^2 (\Sigma_+^0)^2 + 9|A_{11}|^2(\Sigma_-^0)^2$$

$$+ 2|A_{01}|^2(\Sigma_+^1)^2 + 14|C_{12}|^2(\Sigma_-^1)^2$$

$$+ 0.05|B_{02}|^2(\Sigma_+^2)^2 + |A_{11}|^2(\Sigma_-^2)^2 \qquad (14a)$$

$$+ 0.025|C_{12}|^2(\Sigma_-^3)^2$$

$$8k^2 \frac{d\sigma}{d\Omega} P_t = (-0.86\, \Sigma_+^0 \Sigma_+^1 + 0.138\, \Sigma_+^1 \Sigma_+^2)\mathrm{Im}A_{01}B^*_{02}$$

$$+ (15.86\, \Sigma_-^0 \Sigma_-^1 - 1.725\, \Sigma_-^1 \Sigma_-^2 + 0.275\, \Sigma_-^2 \Sigma_-^3)\mathrm{Im}A_{11}C^*_{12} \qquad (14b)$$

Eqs. (13) and (14), while still not transparently simple, have now been reduced to manageable proportions. Since we are primarily concerned, at this point, with the effects of a spin-spin interaction in the final state, we shall simplify the expressions still further by the additional (completely arbitrary) assumption that the amplitudes depend only on the outgoing channel spin S' and not on S. In this case, we may write, for the elastic scattering amplitudes

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*) The numerical coefficents while not exact, are accurate to $\lesssim 1\%$

$$A_o = \chi A_1 e^{i\delta} \tag{12a'}$$

and for the quasi-elastic

$$A_{o1} = A_{11} \tag{12c'}$$

$$B_{o2} = C_{12} \tag{12d'}$$

$$A_{o1} = y B_{o2} e^{i\gamma} \tag{12e'}$$

Finally, assuming $f_o(\ell) = f_1(\ell)$, and adopting the notation *) of Eq..(6a) for the $\sum_{\pm}^{n} = \frac{1}{2}k^2 R^2 F_n$,

$$\frac{16}{|A_1|^2 k^2 R^4} \frac{d\sigma}{d\Omega}(el) = (9+2\chi^2)F_o^2 + F_2^2 \tag{13a'}$$

$$\frac{16}{|B_{o2}|^2 k^2 R^4} \frac{d\sigma}{d\Omega}(q\text{-}el) = (3+\frac{9}{2}y^2)F_o^2 + (7+y^2)F_1^2 + \frac{1}{2}(0.05+y^2)F_2^2 + 0.0125F_3^2 \tag{14a'}$$

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*) Note that F_o, as defined by Eq. (6a) is larger than the F_o defined in the Appendix of I by a factor 2. Thus, for the "step-function" approximation for $f(\ell)$, our $F_o = (J_1(X)/X)$.

$$\frac{16}{|B_{o2}|^2 k^2 R^2} \frac{d\sigma}{d\Omega}(\text{q-el}) \frac{P_t}{y \sin\gamma} = 7.50\, F_o F_1 - 0.79 F_1 F_2 + 0.14 F_2 F_3. \qquad (14b')$$

For purposes of illustration, we show, in Figure 1, plots of the right-hand sides of Eqs. (13a') and (14a') and, in Fig. 2, a plot of $P_t/\sin\gamma$, Eq. (14b'), for the special case of $\chi = y = 1$ and assuming $f(\ell)=1$ for $\ell \leq kR$, $f(\ell)=0$ for $\ell > kR$, using the expressions developed in the Appendix of I. The abscissa is $X = kR\Theta$, the "universal" variable appropriate to the optical approximation (see I). We note, from Fig. 2, that, provided $\gamma \to \pm\pi/2$, the polarization of the quasi-elastically scattered nucleon can be quite large.

B. <u>Spin-Orbit Interaction.</u> In low energy nucleon-nucleon scattering ($E_N \approx$ 100 MeV), appreciable nucleon polarizations are known to arise from a spin-orbit interaction between nucleons. Although we have no evidence that the same kind of a spin-orbit interaction is operative at the energies with which we are concerned ($E_N \sim$ 1-25 GeV), it is nevertheless of interest to investigate, from our phenomenological point f view, the effects of a spin-orbit interaction on the angular distribution and nucleon polarization for both the elastic and quasi-elastic diffraction scattering. To do this, we shall assume that the amplitudes can all be approximated by expressions of the form

$$A_j = A + B \underset{\sim}{S} \cdot \underset{\sim}{\ell} / S\ell + C \underset{\sim}{S'} \cdot \underset{\sim}{\ell'} / S'\ell' \qquad (15)$$

with $\ell, \ell' >> 1$ (i.e., we treat $\underset{\sim}{\ell}$ and $\underset{\sim}{\ell'}$ as classical vectors). Furthermore, although, in the general case, A, B and C will depend on the channel spins (S and S'), we shall take them to be independent of

S and S', since we have already considered, in the previous section, the effects of a spin-spin interaction. We also take $f_1(\ell)=f_o(\ell)=f(\ell)$.

In the classical limit ($\ell, \ell' >> S, S' \sim 1$) we have

$$S\cdot\ell/S\ell \cong m/S \tag{15a}$$

$$S'\cdot\ell'/S'\ell' \cong m'/S' \tag{15b}$$

where

$$J = \ell+m = \ell'+m' \tag{15c}$$

Hence, for the amplitudes used to describe the elastic scattering, Eqs. (4)

$$A_o = A_1 = A \tag{16a}$$

$$A_{\pm} = A\pm B\pm C \tag{16b}$$

$$B_{\pm} = A\mp B\pm C \tag{16c}$$

giving

$$\bar{A} = \bar{B} = A \tag{16d}$$

$$A' = B+C \tag{16e}$$

$$B' = -B+C \tag{16f}$$

Substituting, the elastic scattering cross-section, Eq. (5) becomes

$$2k^2 \frac{d\sigma}{d\Omega} = \frac{11}{2}|A|^2(\Sigma_{\pm}^{o})^2+4(|B|^2+|C|^2)(\Sigma_{-}^{1})^2+\tfrac{1}{2}|A|^2(\Sigma_{-}^{2})^2 \tag{17}$$

and the polarization Eq. (7)

$$-2k^2 \frac{d\sigma}{d\Omega} P_t = 2\mathrm{Im}\left\{A(B+4C)^* \Sigma_-^0 \Sigma_-^1 + AB^* \Sigma_-^1 \Sigma_-^2\right\} \tag{18}$$

For the purposes of computation, in the absence of further information, we put

$$B = C = \chi A e^{i\delta_{s.o.}} \tag{16g}$$

whence, using Eq. (6a)

$$\frac{16}{|A|^2 k^2 R^4} \frac{d\sigma}{d\Omega}(el) = 11F_o^2 + 16\chi^2 F_1^2 + F_2^2 \tag{17'}$$

$$\frac{16}{|A|^2 k^2 R^4} \frac{d\sigma}{d\Omega} \frac{P_t(el)}{\chi \sin\delta} = 20F_oF_1 + 4F_1F_2 \tag{18'}$$

These are plotted, respectively, in Figs. 3 and 4, for $\chi = 1$.

Using the same assumptions for the spin-orbit-interaction amplitudes, Eqs. (15), the quasi-elastic amplitudes, Eqs. (9), become, in the appropriate combinations

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$$A_{o1} = B_{o2} = \overline{B_{o2}} = A_{11} = \overline{A_{11}} = \overline{B_{11}} = C_{12} = \overline{C_{12}} = \overline{D_{12}} = \overline{G_{12}} = A$$

$$\begin{aligned} B'_{o2} &= D'_{12} = C \\ A'_{11} &= B+C \\ B'_{11} &= -B+C \\ C'_{12} &= B+\tfrac{1}{2}C \\ G'_{12} &= -B+\tfrac{1}{2}C \end{aligned} \qquad (19)$$

Substituting into Eqs. (10) and (11) (Table I),

$$\begin{aligned} 8k^2 \frac{d\sigma}{d\Omega}(\text{q-el}) = & \left(15\,|A|^2 + 4\left|B - \sqrt{\tfrac{1}{2}}\,C\right|^2\right)(\Sigma^{o}_{\pm})^2 \\ & + (16|A|^2 + 8|B|^2 + 12|C|^2)(\Sigma^{1}_{\pm})^2 \\ & + \left(1.05|A|^2 + 4\left|B + \sqrt{\tfrac{1}{2}}\,C\right|^2 + 2|C|^2\right)(\Sigma^{2}_{\pm})^2 \\ & + 0.025|A|^2\,(\Sigma^{3}_{-})^2 \end{aligned} \qquad (20)$$

$$\begin{aligned} 8k^2 \frac{d\sigma}{d\Omega} P_t(\text{q-el}) = \mathrm{Im}\Big\{ & (-3.66AB^*+9.89AC^*+4.90BC^*)\,\Sigma^{o}_{\pm}\,\Sigma^{1}_{\pm} \\ & -(3.97AB^*+8.34AC^*-4.90BC^*)\,\Sigma^{1}_{\pm}\,\Sigma^{2}_{\pm} \\ & -(0.318AB^*+0.225AC^*)\,\Sigma^{2}_{-}\,\Sigma^{3}_{-} \Big\} \end{aligned} \qquad (21)$$

Finally, again for the purposes of illustrative computation only, we consider the special case

$$B = C = yAe^{i\delta_{s.o.}} \tag{19'}$$

Then

$$\frac{16}{|A|^2k^2R^4}\frac{d\sigma}{d\Omega}(q\text{-el}) = (7.48+0.172y^2)F_o^2 + (8.0+10y^2)F_1^2 + (0.525+6.83y^2)F_2^2 + 0.0125F_3^2 \tag{20'}$$

$$\frac{16}{|A|^2k^2R^4}\frac{d\sigma}{d\Omega}\frac{P_t(q\text{-el})}{y\sin\delta} = -3.12F_oF_1+6.16F_1F_2+0.271F_2F_3 \tag{21'}$$

These have also been plotted, in Figs. (3) and (4) respectively, again for the special case y=1.

On the assumption of a spin-orbit interaction, both the elastically and the quasi-elastically scattered nucleons can exhibit polarization. But, under the special set of assumptions adopted in this section, the effects appear to be rather larger for the case of elastic scattering than for the quasi-elastic $(3,3^+)$ isobar excitation.

Discussion

Although the description of the elastic and quasi-elastic nucleon scattering, as developed in the foregoing, is perfectly general -- being based on a "phase-shift" analysis in terms of a set of phenomenologically determinable amplitudes -- we have, in order to obtain some insight into the possible interpretation of the observable features of the scattering processes, been forced, step by step, to adopt a series of simplifying assumptions concerning the behaviour of the amplitudes. The most important assumption is the one normally adopted in the optical (diffraction) approximation; it is expressed by Eq. (3) -- that the individual amplitudes, which depend on the spins and the orbital angular momenta involved in the scattering process, can be replaced by their average values (depending on the spins) multiplied by slowly varying functions of the orbital angular momenta, $f^J_{S,S'}(\ell)$. This assumption enables the evaluation of the summations involved in terms of tabulated analytical functions, as has been discussed, for a special case, in I and will be further discussed in the Appendix to this paper.

In our illustrative examples, both here and in I, we have adopted a very special form for the $f(\ell)$, viz., a step function. This is equivalent to the assumption of uniform absorption of the incident wave (and emission of the scattered wave) over a sphere with a sharp boundary at its radius, R. But these assumptions, especially that of the sharp boundary, are precisely what lead to the clearly pronounced maxima and minima in the cross-sections and polarizations plotted in Figs. 1-4.

Presumably, a more reasonable mathematical approximation to the $f(\ell)$ will remove most (but not all) of the secondary maxima in the cross-section. The qualification is required because the various F_n, which determine the shape of the angular distribution, have their first maxima at increasing values of X; in the step-function approximation, that of F_0

occurs at $X=0$, F_1 at $X \cong 2.6$, F_2 at $X \cong 3.9$, F_3 at $X \cong 4.2$. More reasonable assumptions on $f(\ell)$ will maintain, more or less, the shape of these first maxima even if they lead to a disappearance of secondary maxima and minima.

It is important to note that one of the effects of the nucleon spin is already to smooth out the angular distributions, as compared to the predictions of the approximation neglecting spins (see I), even with the step-function assumption on the $f(\ell)$. This is seen in Fig. 1 and, even more drastically, in Fig. 3 and is of course due to the fact that the existence of nucleon spin permits the angular momentum charges required for isobar excitation (or possible in elastic scattering) to take place through a variety of charges in the orbital angular momentum; the subscript n in the F_n reflects, essentially, the Δm_ℓ required in the reaction. The superposition of the angular distributionscorresponding to the various possible Δm_ℓ has the effect of "washing out" the sharp maxima and minima corresponding to any one value of F_n.

All this has the consequence that, as a result of the nucleon spin, it becomes more difficult to distinguish between those features of the scattering angular distribution which reflect the nucleon "shape" (or form factor) and those which reflect the spin dependence of the nucleon scattering amplitudes (i.e., spin-orbit vs. spin-spin interaction, etc.). It would appear, from such considerations, that a study of the elastic and quasi-elastic scattering of pions on nucleons will provide a more effective tool for the investigation of such nucleon shape effects.

However, nucleon-nucleon scattering does make available an additional tool -- namely, the study of the polarization of the scattered nucleons. The two extreme examples considered in this paper, the results of which are shown in Figs. 2 and 4, indicate quite clearly that appreciable polarizations are possible and that their magnitude and angular dependence will depend strongly on the spin dependence of the scattering amplitudes.

Our assumption of a sharp nucleon boundary gives rise to rather violent fluctuations in the polarization, especially for large values of $X = k R\Theta$; such fluctuations will not be present in the case of a diffuse nucleon boundary *). On the one hand, this makes the polarization at relatively large X a sensitive tool for the study of the nucleon "shape" and, in particular, for the resolution of the question of whether the nucleon boundary is sharp enough to lead to F_n's which go through zero (and therefore have secondary minima and maxima). On the other hand, it makes these predictions much less likely to correspond to the physical situation at large values of X without some pre-knowledge of the form factor at large momentum transfer, and therefore makes the polarization at large X a much less reliable tool for the study of the spin dependence of the scattering amplitudes.

However, at small values of X (say, $\lesssim 4$) the amplitudes are likely to be much less sensitive to the shape of the nucleon boundary; rather, they will depend on the choice of a single parameter, like the value of $\langle r^2 \rangle$ for the nucleon. In this region of relatively smaller momentum transfers, while the shape of the angular distribution will tell us little about the spin-dependence of the amplitudes, the magnitude of the polarization, and especially its dependence on X, may tell us a great deal. Thus, for example, a large polarization would indicate a strong spin-spin interaction in the final state (see Fig. 7), and its magnitude would provide a measure of the phase difference between the nucleon-isobar scattering amplitudes in the two possible spin channels, a kind of information which would be difficult to obtain by any other means.

On the other hand, small polarization would mean no appreciable phase difference (or $\delta \rightarrow \pi$) in the two spin channels. But such a small polarization, if observable, could arise from a spin-orbit interaction;

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*) Incidentally, the fluctuations are not as "violent" as they appear in the figures. Thus, in the CERN experiments at Θ=56mrad, and assuming $R \simeq 1$ fermi, the value of X varies between ~ 3 and ~ 7 in going from k=10 to 25 GeV/c. Thus, it would require only very moderate angular resolution to observe the variations in Figs. 1-4.

In this case, however, in contrast to a spin-spin origin, it would change sign in the region of $X \sim 2$ (see Fig. 4), since its smallness would then arise from the cancellation of two opposing terms (see Eq. 21'). Thus, in the region of small momentum transfer, the polarization, if observable, is a sensitive tool for the study of the spin dependence of the scattering amplitudes.

A final word is in order concerning the applicability of the "form factor" concept to elastic and quasi-elastic scattering at the high energies under consideration $(kR \gg 1)$. The use of a form factor is equivalent to the statement *) that the dependence of the scattering amplitude on the initial momentum k and on the invariant 4-momentum transfer q may be factored into the form

$$A(k,q) = kF(q) \tag{22}$$

In the optical approximation, as developed in I and in the following Appendix, this possibility arises from the assumption that the energy dependence of the slowly varying factor $f(\ell)$ derives completely from its parametric dependence on the parameter kR, $f(\ell) = f(\ell/kR) = f(\ell\theta/kR\theta)$, as well as from the "small-angle" approximation which permits the replacement **) $P_\ell(\cos\theta) \cong J_0(\ell\theta)$. In general, however,

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*) H. Lehmann, report at the CERN Conference on Theoretical Aspects of Very High Energy Phenomena, June 5-9, 1961

**) Note that $k\theta \cong \frac{1}{2} k\sin\theta/2 \equiv q$ in the small-angle approximation for elastic scattering. For quasi-elastic diffraction scattering this is still a good approximation, provided the momentum change $\Delta p_2 \ll k$.

there is no guarantee that this simple factorization is permissible, and we should write

$$A(k,q) = kF(q,k) \quad . \tag{22'}$$

Under these conditions, it would no longer be possible to use the form factor concept, and to compare the scattering for different incident energies on the basis of a "universal" optical relationship of the form

$$\frac{1}{k^2}\frac{d\sigma}{d\Omega} = F^2(q) \tag{23}$$

It remains an open question to determine experimentally the reliability of the description of the scattering by Eq. (23), especially for values of $qR \gg 1$.

I would like to express my appreciation to the group consisting of Messrs. Cocconi, Diddens, Lillethum, Manning, Taylor, Walker and Wetherell for their cooperative attitude concerning the divulgence and discussion of their experimental results, even in their preliminary stages; to CERN and all of its staff for the warm hospitality extended to me during my stay; and to the John Simon Guggenheim Foundation for generous financial support.

APPENDIX

Mathematics of the Optical Approximation

As noted in the text, the assumption of slow variation of the scattering amplitudes with ℓ, according to Eq. (3), leads to the expression of the angular dependences in terms of the summations

$$\Sigma^n \equiv \sum_{\ell} f(\ell)\, \ell^{(1-n)} P_{\ell}^{(n)}(\cos\theta) \tag{A1}$$

The optical approximation consists in the assumption of a characteristic (maximum) angular momentum $L = kR \gg 1$, such that

$$f(\ell) = f(\ell/L) \tag{A2}$$

and in the application of the small-angle approximation

$$P_{\ell}^{(0)}(\cos\theta) \cong J_o(\ell\theta) \tag{A3}$$

In this approximation, the summation may be replaced by an integral

$$\Sigma^n = L^2 F_n(X{=}L\theta) \tag{A4}$$

with

$$F_n(X) = \frac{1}{X^2}\int_o^{\infty} f(\chi/X)\,\chi\, J_n(\chi)\,d\chi \quad . \tag{A5}$$

The problem is thus reduced to the evaluation of the "form factors", $F_n(X)$, under appropriate assumptions concerning the form of $f(\chi/X)$.

1) The step-function approximation. The simplest form of f, and the one discussed in some detail in I, is the approximation

$$f(\chi/X) = 1 \quad \text{for} \quad \chi \leq X \tag{A6a}$$

$$f(\chi/X) = 0 \quad \text{for} \quad \chi > X \tag{A6b}$$

In the Appendix to I, we have given expressions for the F_n, for $n < 3$, evaluated by using the recursion relations for the Bessel functions

$$\chi J_n(\chi) = 2(n-1)J_{n-1} - \chi J_{n-2} \tag{A7}$$

and integrals of the form

$$\int_0^X \chi J_o(\chi)\, d\chi = XJ_1(X) \tag{A8}$$

etc. *) We have also given asymptotic expressions for the $J_n(X \ll 1)$ in I.

*) In this paper we have also required $F_3(X)$, which, for the step-function approximation, is given by

$$F_3 = \frac{1}{X^2} \int_0^\infty \left(\frac{8}{x}J_1 - \chi J_1 - 4J_o\right)dx \tag{A9a}$$

$$F_3(X \ll 1) \rightarrow X^3/240 \tag{A9b}$$

Other, more reasonable, forms of $f(x/X)$ lead to integrals which can sometimes be evaluated in terms of tabulated functions; more often, the integrals require numerical evaluation. The interested reader is referred to Watson's monumental work on the Bessel function *) for details on the evaluation of the integrals. In the following we give a number of illustrative examples of possible form factors.

2) The Gaussian approximation. An obvious way of describing a diffuse nucleon boundary is to assume a Gaussian form for $f(\ell)$

$$f(x/X) = e^{-x^2/\xi^2 X^2} \tag{A9}$$

with $\xi \sim 1$ as an adjustable parameter, which can be chosen to provide a best fit to the observed angular distribution. In this case, the form factors can be evaluated in terms of the hypergeometric functions **)

$$F_n(X) = \frac{1}{2}\xi^2 \frac{\Gamma(\frac{n}{2}+1)}{\Gamma(n+1)} \; {}_1F_1\left(\frac{n}{2}+1;\; n+1;\; -\frac{\xi^2 X^2}{4}\right) \tag{A10}$$

In particular, for $n=0$ we have

$$F_o(X) = \frac{1}{2}\xi^2 e^{-\xi^2 X^2/4} \tag{A10a}$$

*) G.N. Watson, A Treatise on the Theory of Bessel Functions (Cambridge University Press, 1944), Chapt. XIII.

**) E.T. Whittaker and G.N. Watson, A course of Mathematical Analysis (Cambridge University Press, 1927), Chapt. XIV.

a result which immediately bears out the assertion, made in the foregoing, that the assumption of a diffuse nucleon boundary may lead to the disappearance of the secondary maxima and minima, characteristic of the form factors for a sharp nucleon boundary.

3) <u>The power-law approximation.</u> Another possible description of a diffuse boundary -- rather more diffuse than the Gaussian -- is through a power law form

$$f(\chi/X) = \frac{(\xi^2 X^2)^m}{(\chi^2 + \xi^2 X^2)^m} \qquad \text{(A11)}$$

The form factor

$$F_n(\xi X) = \xi^2 (\xi^2 X^2)^{m-1} \int_0^\infty \frac{\chi \, J_n(x)dx}{(\chi^2 + \xi^2 X^2)^m} \qquad \text{(A12)}$$

cannot be simply evaluated except in the case $n=0$

$$F_o(\xi X) = \xi^2 \frac{(\xi X/2)^{m-1}}{\Gamma(m)} K_{1-m}(\xi X) \quad . \qquad \text{(A12a)}$$

An alternative version of the power-law approximation is

$$f(\chi/X) = \frac{2\xi^2 X^2}{(\xi^4 + 4\xi^4 X^4)^{\frac{1}{2}}} \qquad \text{(A11')}$$

for which

$$F_o(\xi X) = 2\xi^2 K_o(\xi X) J_o(\xi X) \quad . \qquad \text{(A12a')}$$

4) The Yukawa approximation. The approximations discussed above correspond to a picture of a nucleonic absorption which is essentially constant for relatively small impact parameters ($\chi \ll X$) and then becomes diffuse in the region of the boundary ($\chi \sim X$) and beyond. Another, rather extreme, possibility is that of a weakly absorbing sphere which, however, becomes rather suddenly and strongly absorbing at small impact parameters. As a crude approximation to this situation *), we take

$$f(\chi/X) = e^{-\chi/\xi X}/(\chi/\xi X). \tag{A13}$$

and obtain

$$F_n(X) = \frac{\xi^2}{(1+\xi^2X^2)^{\frac{1}{2}}}\left\{\frac{(1+\xi^2X^2)^{\frac{1}{2}}-1}{\xi X}\right\}^n \tag{A14}$$

5) The exponential approximation. A less extreme form of the above ("absorbing core") approximation is to take

$$f(\chi/X) = e^{-\chi/\xi X}. \tag{A15}$$

In this case, we may obtain a general expression for the form factor in terms of the hypergeometric functions of the second kind

$$F_n(X) = \xi^2\left(\frac{n+1}{2^n}\right)(\xi X)^n\ {}_2F_1\left(\frac{n+2}{2}, \frac{n+3}{2}; n+1; -\xi^2X^2\right). \tag{A16}$$

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*) This form of $f(\ell)$ should not be confused with the one which would result from absorption by the meson cloud of the nucleon, whose density distribution is given by the Yukawa function (squared). In this case, the absorption at a given impact parameter is proportional to the total mass of (pionic) material transversed by a particle passing at that impact parameter. The resulting, rather complicated, absorption coefficient, and its effects on the variation of the elastic scattering amplitude with ℓ (and, correspondingly on the scattering angular distribution) has been considered by the Dubna group **).

**) D. Blokhintsev, report at the CERN Conference on Theoretical Aspects of Very High Energy Phenomena, June 5-9, 1961.

However, without resort to the ${}_2F_1$ we may evaluate all of the F_n by taking advantage of the recursion relation, Eq. (A8) and the following integrals

$$\int_0^\infty e^{-a\chi} J_n(b\chi)\,dx = (a^2+b^2)^{-\frac{1}{2}} \left\{ \frac{(a^2+b^2)^{\frac{1}{2}}-a}{b} \right\}^n \tag{A17a}$$

$$\int_0^\infty e^{-a\chi} J_n(b\chi)\,\chi^n dx = \frac{(2b)^n \Gamma(n+\frac{1}{2})}{(a^2+b^2)^{n+\frac{1}{2}} \sqrt{\pi}} \tag{A17b}$$

$$\int_0^\infty e^{-a\chi} J_n(b\chi)\,\chi^{n+1} dx = \frac{2a(2b)^n \Gamma(n+3/2)}{(a^2+b^2)^{n+3/2} \sqrt{\pi}} \tag{A17c}$$

with these, we obtain

$$F_o(X) = \xi^2(1+\xi^2X^2)^{-3/2} \tag{A16a}$$

$$F_1(X) = \xi^2(1+\xi^2X^2)^{-3/2}(\xi X) = \xi X F_o(X) \tag{A16b}$$

$$F_2(X) = F_o(X)\left\{ \frac{2(1+\xi^2X^2)}{\xi^2X^2} \left[(1+\xi^2X^2)^{\frac{1}{2}}-1 \right] -1 \right\} \tag{A16c}$$

$$F_3(X) = F_1(X)\left\{ \frac{4(1+\xi^2X^2)}{\xi^4X^4} \left[(1+\xi^2X^2)-2(1+\xi^2X^2)^{\frac{1}{2}}+1 \right] -1 \right\} \tag{A16d}$$

6) <u>The "peripheral" approximation.</u> Although the exponential approximation is, in itself, of not too great interest, we have gone into some detail because it provides the necessary basis for another useful approach, which we call the "peripheral" approximation. This would apply if the amplitudes become large only at large impact parameters -- i.e. for $\ell \sim kR$. Such a situation would prevail, for example, if the quasi-elastic excitation would result, primarily, from a "one-meson exchange" process; another possible application would be to the elastic scattering, if it were believed to be primarily due to "two-meson exchanges" *) (in which case the appropriate value of R would be $R \simeq \frac{1}{2}\mu^{-1} \simeq 0.7f$).

One way of approximating such a peripheral interaction would be to take

$$f(x/X) = e^{-x/\xi X} - e^{-x/\eta X} \tag{A18}$$

with

$$0 < \xi - \eta << \xi \quad . \tag{A18a}$$

With this approximation, $f(0) = 0$, and the maximum value of $f(x/X)$ occurs at $x/\xi X \simeq 1$. The form factors are now simply derived from those of the previous section

$$F_n(X) = F_n(\xi X) - F_n(\eta X) \ . \tag{A16'}$$

With the appropriate choice of the parameters ξ and $\xi - \eta$ it would be possible to approximate the effects of interactions whose maximum strengths occur at various impact parameters.

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*) e.g. S. Fubini, report at the CERN Conference on Theoretical Aspects of Very High Energy Phenomena, June 5-9, 1961.

FIGURE CAPTIONS

Figure 1

Comparison of the angular distributions, for elastic and quasi-elastic ($3,3^+$ isobar excitation) nucleon-nucleon scattering on the assumption of a spin-spin interaction in the final state. The curves assume no spin-orbit interaction, and certain other simplifying assumptions concerning the behaviour of the scattering amplitudes, as explained in the text. The ordinate scale is arbitrary, different for the two curves. The abscissa is $X = kR\Theta$.

Figure 2

Angular distribution (or energy dependence) of the polarization of the quasi-elastically scattered nucleons, on the assumption of a spin-spin, but no spin-orbit, interaction. The assumptions concerning the amplitudes are the same as for the cross-sections in Fig. 1.

Figure 3

Angular distributions on the assumption of a spin-orbit, but no spin-spin, interaction. See the text for the other assumptions concerning the scattering amplitudes.

Figure 4

Nucleon polarization vs $X = kR\Theta$ for elastic and quasi-elastic scattering on the assumption of a spin-orbit, but no spin-spin, interaction.

ERRATA

to

ANGULAR DISTRIBUTION IN NUCLEON-NUCLEON "QUASI-ELASTIC DIFFRACTION" SCATTERING

II. Effects of the Nucleon Spin

B.T. Feld

CERN, Geneva

1700/TH. 193

p. 4	Eq. (7), end of the first line: $\sum_{-}^{0} \sum_{-}^{0}$ should be $\sum_{-}^{0} \sum_{-}^{1}$.
p. 8	Line after Eq. (11) and also 10th line of the next paragraph: the reference should be to Eqs. (9k), not (9h).
Table 1	2nd page, column 2, third from the bottom, should be C'_{12}.
p. 19	7th line from the bottom, should read (see Fig. 2), not 7.
p. 19	Reference, 2nd line, θ=56mrad.
Fig. 1	On the ordinate scale, the lowest number should be 10^{-3}, not 10^{-4}.

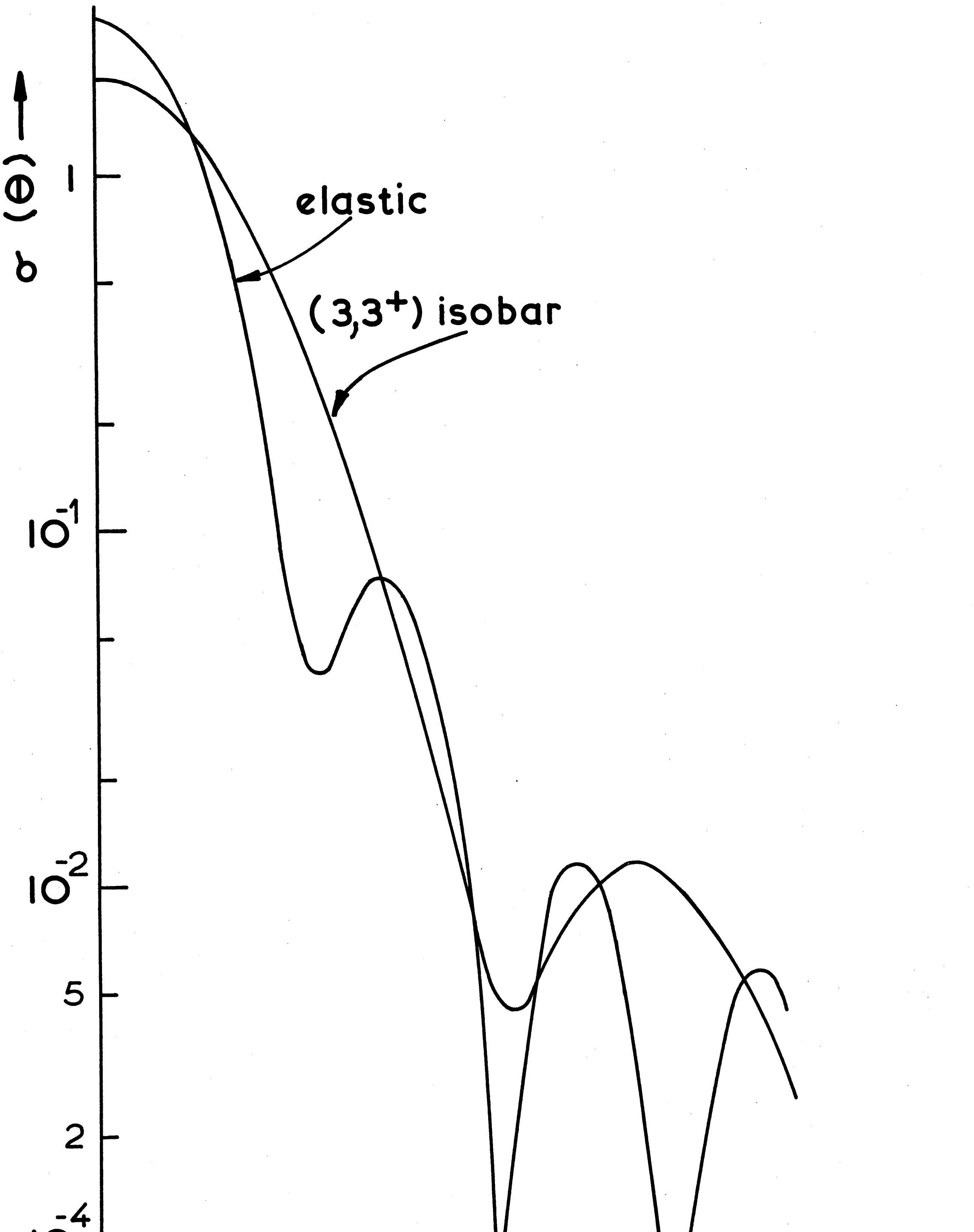

FIG. 1 N+N ⟶ N+N* spin-spin interaction only

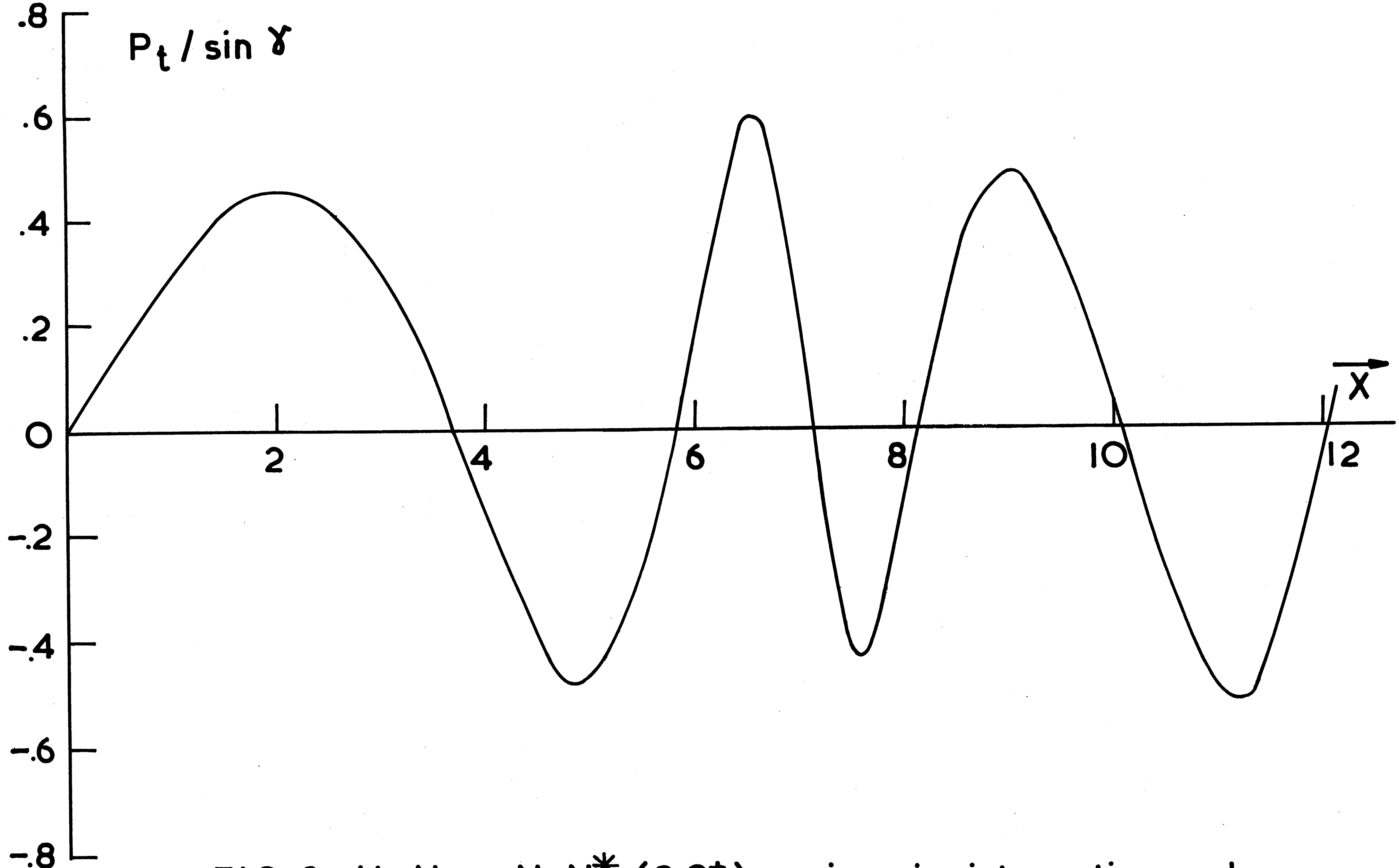

FIG.2 N+N ⟶ N+N* (3,3⁺) spin-spin interaction only

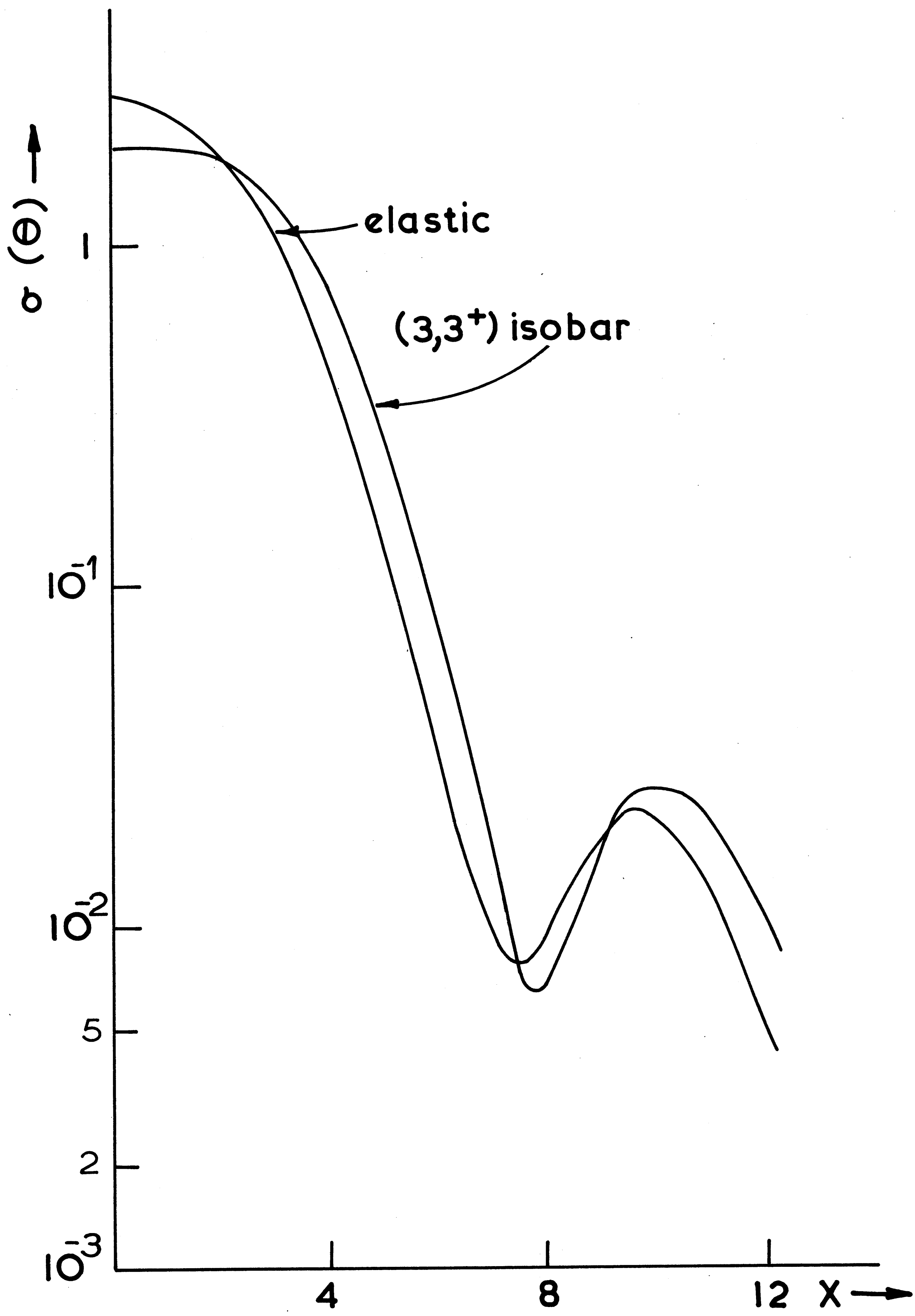

FIG. 3 N+N → N+N* spin-orbit interaction only

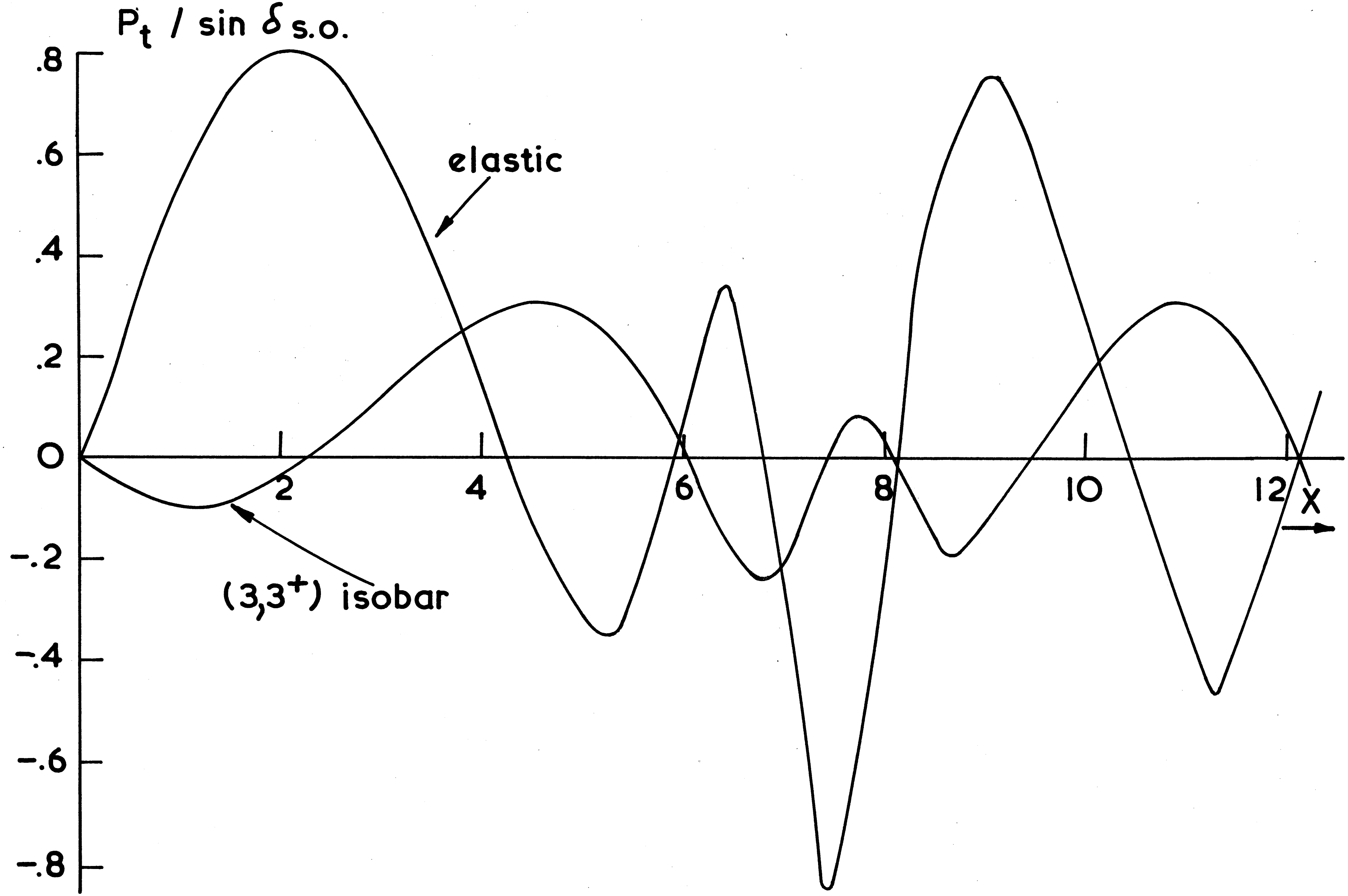

FIG. 4 N+N → N*+N spin-orbit interaction only