Remarks on the low lying baryon isobars

R.E. Marshak *) and S. Okubo **)

CERN - Geneva

A B S T R A C T

The relative energies and widths of the low lying baryon isobars have been examined from the point of view of the global symmetry hypothesis. In particular, the energy shifts of the hyperon isobars Λ^* Σ^* due to the mass differences between the baryons has been calculated independent of the dynamics.

---

*) Guest Professor under the Ford Foundation and Guggenheim Fellow, on leave from the University of Rochester for the academic year 1960/61.

**) Supported by the National Science Foundation, U.S.A.

1217
TH/180
25 April 1961

1. Introduction

The lowest nucleon isobar N_1^* is identified with the well-known $I=3/2$, $J=3/2$ pion-nucleon resonance and lies at an excitation energy [1] $E(N_1^*)-m_N \simeq 2.12\mu$ (μ and m_N are the pion and nucleon masses respectively) with a total width $\Gamma(N_1^*) \simeq 0.90\mu$ *). The excitation energy is small compared to the nucleon rest mass and we say that N_1^* is a low lying nucleon isobar. The next nucleon isobar N_2^* already lies at a sufficiently high excitation energy $E(N_2^*) - m_N \simeq 3.85\mu$ that the term "low-lying" can hardly be applied to it. For the purpose of this discussion we shall only consider N_1^*. The physics underlying this limitation may perhaps be expressed by saying that the properties of N_1^* are determined chiefly by the strong p-wave pion-nucleon interaction whereas this need not be true of N_2^* and higher nucleon isobars (where the kaon-baryon interaction may play a role, etc.).

The other known baryons, Λ, Σ and Ξ may also exist in low lying isobaric levels and indeed there is now rather conclusive experimental evidence [2] for a low lying Λ isobar, Λ_1^*, having an excitation energy $E(\Lambda_1^*)-m_\Lambda = 1.92\mu$ and a total width $\Gamma(\Lambda_1^*) = 0.45\mu$. Further, if some form of global symmetry holds, low lying Σ and Ξ isobars (Σ_1^* and Ξ_1^*) should also exist. Whether there are additional "low-lying" hyperon isobars (i.e. having excitation energies small compared to the hyperon rest masses) is an open question and will depend on the complicated interplay of pion and kaon couplings. However, if a universally strong p-wave pion-baryon dominates the situation for the lowest lying hyperon isobars, it should be possible to establish some (necessarily approximate) relations among the energies and half-widths of N_1^*, Λ_1^* and Σ_1^* by treating the mass differences of the baryons as small perturbations. We note that the neglect of the kaon interaction for the $J=3/2$ isobar states may be reasonable since the p-wave kaon-nucleon interaction appears to be small in the low energy region. However, for the study of $J=1/2$ levels, it may be important.

---

*) This value has been estimated from the data given by S.J. Lindenbaum and L.C.L. Yuan, Phys.Rev. 111, 1380 (1958). The Chew-Low theory seems to predict a higher value $\Gamma \simeq 1.40\mu$, if we use their formula (see G.F. Chew, 7th Rochester Conference).

The essential results of this note are already contained in the papers of Amati, Vitale and Stanghellini [3] and of Lee and Yang [4]. However, we believe that our approach is physically more perspicuous than the methods given in refs. [3] and [4]. In section 2, we derive the formulae which connect the energies of the low lying baryon isobars N^*, Λ^*, Σ^* and Ξ^* (hereafter we drop the subscript 1) and compare our results with refs. [3] and [4]. In section 3, we briefly discuss the half widths of the low lying baryon isobars.

2. Energies of the low lying baryon isobars

We derive the displacements of the energies of the low lying baryon isobars (with respect to their global symmetric value) by treating the mass differences of the baryons as a small perturbation on the globally symmetric interaction. It is plausible that this method will become increasingly bad for the higher baryon isobars.

We write for the total Hamiltonian

$$H = H_o + H_1$$

where H_o = globally symmetric Hamiltonian (which need not be specified)

$$H_1 = \int d^3\underline{x} \left\{ (m_\Lambda - m_N)(\bar{\Lambda}\Lambda) + (m_\Sigma - m_N)(\bar{\Sigma}\cdot\Sigma) \right\} \tag{1}$$

We have omitted terms containing Ξ because obviously these terms do not affect this calculation. If we define Y and Z doublets as follows

$$Y^+ = \Sigma^+ , \qquad Z^0 = \frac{1}{\sqrt{2}}(\Lambda + \Sigma_c)$$
$$Y^0 = \frac{1}{\sqrt{2}}(\Lambda - \Sigma_c) , \quad Z^- = \Sigma^-$$

H_1 is transformed into

$$H_1 = \int d^3\underline{x} \; \{ [\tfrac{1}{4}(m_\Lambda + 3\, m_\Sigma) - m_N] (\bar{Y}\cdot Y + \bar{Z}\cdot Z) + \tfrac{1}{4}(m_\Sigma - m_\Lambda)(\bar{Y}\tau_3 Y - \bar{Z}\tau_3 Z) - \tfrac{1}{4}(m_\Sigma - m_\Lambda)[\bar{Y}(\tau_1 - i\tau_2) Z + \bar{Z}(\tau_1 + i\tau_2) Y] \} \qquad (1')$$

We start with the eigenfunctions $|Y^*\rangle$, $|Z^*\rangle$ of H_0, which have isospin $I=3/2$ corresponding to the $(3/2,3/2)$ nucleon isobar N^*. We then can calculate the new eigenvalues of the energy resulting from switching on H_1. This is done by using standard degenerate perturbation theory.

Let $|Y^*,M\rangle$, $|Z^*,M\rangle$ be the eigenfunctions of H_0 corresponding to Y^*, and Z^* with the third component of isospin $I_Z=M$. Hence M takes on the values $3/2$, $1/2$, $-1/2$, $-3/2$. However, in what follows, it is convenient to include $M=\pm 5/2$ by putting $|Y^*, M=\pm 5/2\rangle = |Z^*, M=\pm 5/2\rangle \equiv 0$.

Now, inspection of the perturbation H_1 shows that the eigenfunctions of (H_0+H_1) must have the following form

$$\Psi^*_M = \alpha_M |Y^*, M\rangle + \beta_M |Z^*, (M+1)\rangle \qquad (2)$$

where the constants α_M, β_M are determined by

$$\langle Y^*, M | H_1 | Y^*, M\rangle \alpha_M + \langle Y^*, M | H_1 | Z^*, M+1\rangle \beta_M = \Delta E^* \cdot \alpha_M$$
$$\langle Z^*, M+1 | H_1 | Y^*, M\rangle \alpha_M + \langle Z^*, M+1 | H_1 | Z^*, M+1\rangle \beta_M = \Delta E^* \cdot \beta_M \qquad (3)$$

with ΔE^* the energy shift.

If we now use global symmetry, and the isospin invariance of the theory, we may put

$$\langle Y^*, M | \int d^3\underline{x}\, (\bar{Y}Y + \bar{Z}Z) | Y^*, M\rangle = A$$

$$\langle Z^*, M+1 | \int d^3\underline{x}\, (\bar{Y}Y + \bar{Z}Z) | Z^*, M+1\rangle = A \tag{4a}$$

$$\langle Y^*, M | \frac{1}{2} \int d^3\underline{x}\, (\bar{Y}\tau_3 Y - \bar{Z}\tau_3 Z) | Y^*, M\rangle = M \cdot B$$

$$\langle Z^*, M+1 | \frac{1}{2} \int d^3\underline{x}\, (\bar{Y}\tau_3 Y - \bar{Z}\tau_3 Z) | Z^*, M+1\rangle = -(M+1) \cdot B \tag{4b}$$

$$\langle Y^*, M | \frac{1}{2} \int d^3\underline{x}\; \bar{Y}(\tau_1 - i\tau_2) Z | Z^*, M+1\rangle = [(\tfrac{3}{2}-M)(\tfrac{5}{2}+M)]^{\frac{1}{2}} \cdot B$$

$$\langle Z^*, M+1 | \frac{1}{2} \int d^3\underline{x}\; \bar{Z}(\tau_1 + i\tau_2) Y | Y^*, M\rangle = [(\tfrac{3}{2}-M)(\tfrac{5}{2}+M)]^{\frac{1}{2}} \cdot B \tag{4c}$$

$$\langle Y^*, M | \frac{1}{2} \int d^3\underline{x}\; \bar{Z}(\tau_1 + i\tau_2) Y | Z^*, M+1\rangle = 0$$

$$\langle Z^*, M+1 | \frac{1}{2} \int d^3\underline{x}\; \bar{Y}(\tau_1 - i\tau_2) Z | Y^*, M\rangle = 0 \tag{4d}$$

where A and B are constants independent of M. If, furthermore, we define

$$a = \frac{1}{4}(m_\Lambda + 3m_\Sigma - 4m_N) \cdot A$$

$$b = \frac{1}{2}(m_\Sigma - m_\Lambda) \cdot B \tag{5}$$

then Eq. (3) becomes

$$(a + bM - \Delta E^*)\,\alpha_M - [(\tfrac{3}{2}-M)(\tfrac{5}{2}+M)]^{\frac{1}{2}}\, b \cdot \beta_M = 0$$

$$-[(\tfrac{3}{2}-M)(\tfrac{5}{2}+M)]^{\frac{1}{2}}\, b\, \alpha_M + (a - b(M+1) - \Delta E^*)\,\beta_M = 0 \tag{6}$$

The two solutions of Eq. (6) are given by

$$\Delta E_2^* = a + \frac{3}{2} b\,, \quad \alpha_M^{(2)} = \frac{1}{2}\left(\frac{5}{2}+M\right)^{\frac{1}{2}}, \quad \beta_M^{(2)} = -\frac{1}{2}\left(\frac{3}{2}-M\right)^{\frac{1}{2}} \tag{7a}$$

$$\Delta E_1^* = a - \frac{5}{2} b\,, \quad \alpha_M^{(1)} = \frac{1}{2}\left[\frac{3}{2}-M\right]^{\frac{1}{2}}, \quad \beta_M^{(1)} = +\frac{1}{2}\left[\frac{5}{2}+M\right]^{\frac{1}{2}} \tag{7b}$$

and, hence, the corresponding wave functions by

$$\Psi_2^*(M) = \frac{1}{2}\left[\frac{5}{2}+M\right]^{\frac{1}{2}} |Y^*, M\rangle - \frac{1}{2}\left[\frac{3}{2}-M\right]^{\frac{1}{2}} |Z^*, M+1\rangle \tag{8a}$$

$$\Psi_1^*(M) = \frac{1}{2}\left[\frac{3}{2}-M\right]^{\frac{1}{2}} |Y^*, M\rangle + \frac{1}{2}\left[\frac{5}{2}+M\right]^{\frac{1}{2}} |Z^*, M+1\rangle \tag{8b}$$

From Eqs. (8a) and (8b), it follows that Ψ_2^* has five non-zero components corresponding to $M=3/2,\ 1/2,\ -1/2,\ -3/2,\ -5/2$ and Ψ_1^* has three non-zero components corresponding to $M=1/2,\ -1/2,\ -3/2$. It is not difficult to check that if one uses the usual isospin assignments instead of the Y and Z doublet formalism, then $\Psi_2^*(M)$ behaves as $y^{(2)}_{(M+1/2)}$, and $\Psi_1^*(M)$ as $y^{(1)}_{(M+1/2)}$, apart from a common numerical factor. Hence, $\Psi_2^*(M)$ corresponds to $I=2$ with $I_Z=M+1/2$ and $\Psi_1^*(M)$ to $I=1$ with $I_Z=M+1/2$ in the usual isospin notation. We identify states represented by the Ψ_2^* and Ψ_1^* wave functions with the Σ^* and Λ^* isobars respectively, namely

$$\begin{aligned} |\Sigma^*, I_z\rangle &= \Psi_2^*\left(I_z - \tfrac{1}{2}\right) \\ |\Lambda^*, I_z\rangle &= \Psi_1^*\left(I_z - \tfrac{1}{2}\right) \end{aligned} \tag{9}$$

We believe that this notation is more consistent than that given in refs. 3) and 4) where Y^* and Z^* are taken as the baryon isobars. The quantities ΔE^* given by Eqs. (7a) and (7b) then represent $E(\Sigma^*) - E(N^*)$ and $E(\Lambda^*) - E(N^*)$ respectively.

Up to this point, the results contained in Eqs. (7a) and (7b) are correct for the relativistic theory. However, the quantities A and B (entering into Eqs. (7a) and (7b) through the definitions (4a) and (4b)) are very difficult to compute in general and it is necessary to make certain approximations. (We shall attempt to justify these approximations below by repeating our perturbation calculation for the ground states, i. e. rest masses of the Λ and Σ hyperons). If we suppose that baryon pairs do not make an appreciable contribution to the positions of the low lying nucleon isobars (the static theory is the limiting case of this approximation), then we may replace the expression $(\bar{Y}Y+\bar{Z}Z)$ by (Y^+Y+Z^+Z) in Eq. (4a) and we obtain

$$A \simeq \langle Y^*, M | \int d^3\underline{x}\, (Y^+Y + Z^+Z) | Y^*, M \rangle = 1 \tag{10}$$

Eq. (10) follows, since $\int d^3\underline{x}(Y^+Y+Z^+Z)$ is the baryon number operator.

In order to evaluate B, we not only neglect the contribution of the baryon pairs but we also neglect the pion contribution to the third component of the total isospin. More explicitly, the neglect of the baryon pair contribution permits us to rewrite Eq. (4b) as follows

$$\begin{aligned} M \cdot B &\simeq \frac{1}{2} \langle Y^*, M | \int d^3\underline{x}\, (Y^+\tau_3 Y - Z^+\tau_3 Z) | Y^*, M \rangle \\ &\simeq \frac{1}{2} \langle Y^*, M | \int d^3\underline{x}\, (Y^+\tau_3 Y + Z^+\tau_3 Z) | Y^*, M \rangle \end{aligned} \tag{11}$$

The second step in Eq. (11) is already permissible if we neglect the Z baryon loops (a condition less stringent than the complete neglect of baryon pair contributions) since then

$$\langle Y^*, M | \int d^3x\, Z^+\tau_3 Z \,| Y^*, M \rangle \doteq 0 \tag{12}$$

Furthermore, the third component of the total isospin operator is given by

$$I_z = \frac{1}{2}\int d^3\underline{x}\,(Y^+\tau_3 Y + Z^+\tau_3 Z) + i\int d^3\underline{x}\,(\phi^+\cdot\pi^+ - \pi\cdot\phi) \tag{13}$$

where ϕ is the pion field operator and π is the canonically conjugate operator of ϕ. If we now neglect the pion contribution to (13), we obtain from Eq. (11) that

$$B \simeq 1 \tag{14}$$

Thus, neglecting the contribution of baryon pairs and the pion contribution to the third component of the total isospin, Eqs. (7a) and (7b) reduce to

$$\Delta E^*(I=2) \equiv E(\Sigma^*) - E(N^*) \simeq (m_\Lambda - m_N) + \frac{3}{2}(m_\Sigma - m_\Lambda) \tag{15a}$$

$$\Delta E^*(I=1) \equiv E(\Lambda^*) - E(N^*) \simeq (m_\Lambda - m_N) - \frac{1}{2}(m_\Sigma - m_\Lambda) \tag{15b}$$

which are precisely the formulae given in refs. [3] and [4]. It is easy to show further that in our approximation

$$E(\Xi^*) - E(N^*) \simeq m_\Xi - m_N \tag{15c}$$

The advantage of our derivation has been its straightforwardness and the clear identification of the approximations which must be made to lead to the simple result (15). In the derivation of ref. [3] the features of the Chew-Low approximation responsible for the final result are not too transparent (particularly with regard to the pion contribution to the third component of the isospin) while in ref. [4], [Eq. (22)], an assumption must be made which is not easily justified.

In assessing the validity of the approximations which led to Eqs. (15a) and (15b) we would add the following remarks. It seems rather plausible that the relative positions of the low lying baryon isobars will only be slightly affected by the baryon pair (or loop) contributions. However, at first sight, it would appear that the neglect of the pion contribution to the third component of total isospin [cf. Eq. (13)] would be a poor approximation even for the low lying baryon isobars. We justify this approximation essentially "ad hoc" by computing the "ground states" of the Λ and Σ hyperons by means of the identical perturbation procedure which we have employed for computing the first excited states (i.e. Λ^* and Σ^*). We find, in analogy to Eqs. (8a) and (8b) :

$$\Lambda\Psi_1(M) = [\tfrac{3}{2}+M]^{\frac{1}{2}}\,|Y, M\rangle - [\tfrac{1}{2}-M]^{\frac{1}{2}}\,|Z, M+1\rangle \tag{16a}$$

$$\Lambda\Psi_0(M) = [\tfrac{1}{2}-M]^{\frac{1}{2}}\,|Y, M\rangle + [\tfrac{3}{2}-M]^{\frac{1}{2}}\,|Z, M+1\rangle \tag{16b}$$

where $\Lambda\Psi_1(M)$ has three non-zero components $M=1/2, -1/2, -3/2$ and $\Lambda\Psi_0(M)$ one non-zero component $M=-1/2$. The analogue of Eq. (9) becomes

$$\begin{aligned} |\Sigma, I_z\rangle &= \Lambda\Psi_1(I_z-\tfrac{1}{2}) \\ |\Lambda, I_z\rangle &= \Lambda\Psi_0(I_z-\tfrac{1}{2}) \end{aligned} \tag{17}$$

and, finally, Eqs. (7a) and (7b) are converted into

$$\Delta E(I=1) \equiv E(\Sigma) - E(N) = a + \tfrac{1}{2}\,b \tag{18a}$$

$$\Delta E(I=0) = E(\Lambda) - E(N) = a - \tfrac{3}{2}\cdot b \tag{18b}$$

where a and b are defined by Eq. (5), as before, and A and B are, in turn, defined by Eq. (4a) and (4b) with Y and Z replacing Y^* and Z^* respectively. The approximations of neglecting the baryon pair (or loop) contributions and the pion contribution to the third component of the isospin permit us to set again $A \simeq 1$, $B \simeq 1$. With these values of A and B, Eqs. (18a) and (18b) reduce to

$$E(\Sigma) - E(N) \simeq m_\Sigma - m_N \tag{19a}$$

$$E(\Lambda) - E(N) \simeq m_\Lambda - m_N \tag{19b}$$

which are exactly correct. The same approximations lead to

$$E(\Xi) - E(N) \simeq m_\Xi - m_N \tag{19c}$$

It should be pointed out that B can be calculated in the static limit. Miyazawa [5] obtains the value $B \simeq 0.37$ whereas Fubini and Thirring [6] get $B \simeq 0.28$. This is small compared to 1 so that Λ and Σ masses come out quite incorrect, since in the same static limit $A \simeq 1$.

The fact that Eqs. (19a) and (19b) are exact encourages us to believe that the same values $A \simeq 1$ and $B \simeq 1$ leading to Eqs. (15a) and (15b) may not be too bad.

The above comments are based on the usual view that the mass differences among the baryons are due to the kaon-baryon couplings Thus as far as the pure kaon interactions are concerned, m_Λ, m_Σ and m_Ξ in our original Hamiltonian can be regarded as the renormalised masses instead of the bare ones.

However, we are still not taking account of the renormalisation effects of the combined pion and kaon interactions, so that the baryon masses are still not the observed renormalised masses, unless these combined renormalisation effects are negligible. One way out of this difficulty is to suppose that the quantities A and B, which may be different from unity (thus representing the combined pion-kaon renormalisation effects) are the same respectively for the Y, Z doublets and the low lying Y^*, Z^* doublets. It would follow then that the formulae Eqs. (19a), (19b), (19c) and Eqs. (15a), (15b) are still true provided that we replace the masses m_Λ, m_Σ and m_N by their corresponding renormalised values. Roughly speaking, this assumption corresponds to the idea that the pion clouds in the low lying Y^* and Z^* doublets do not affect so much the properties of the cores represented by the Y and Z doublets.

It is interesting to note th t this statement is equivalent to the assumption that $\alpha' = \alpha$, $\beta' = \beta$, $\gamma' = \gamma$ in the formulation of Lee and Yang [cf. Eqs. (19) and (20) of their paper [4]]. Indeed, we can easily write down the following empirical formulae for the "ground states" and "first excited states" of the baryons [7]:

$$E = a + b\, I(I+1) + c\, S + d\, S^2, \quad (N, \Lambda, \Sigma, \Xi) \tag{20a}$$

$$E = a^* + b\, I(I+1) + c\, S + d\, S^2, \quad (N^*, \Lambda^*, \Sigma^*, \Xi^*) \tag{20b}$$

with the same b, c, d. In Eqs. (20a) and (20b), I = isospin, S = strangeness number and

$$
\begin{aligned}
a &= m_N - \frac{3}{8}(m_\Sigma - m_\Lambda) \quad , \quad a^* = E(N^*) - \frac{15}{8}(m_\Sigma - m_\Lambda) \\
b &= \frac{1}{2}(m_\Sigma - m_\Lambda) \\
c &= \frac{1}{2}(m_\Xi - m_N) - 2(m_\Lambda - m_N) - \frac{3}{4}(m_\Sigma - m_\Lambda) \\
d &= \frac{1}{2}(m_\Xi - m_N) - (m_\Lambda - m_N) - \frac{3}{8}(m_\Sigma - m_\Lambda)
\end{aligned}
\tag{21}
$$

Eqs. (20a) and (20b) can be translated into Eqs. (19) and (20) of ref. [4] provided we choose $\alpha' = \alpha$, $\beta' = \beta$, $\gamma' = \gamma$.

Finally, we remark that once we know the wave functions for the low lying baryon isobars [as given for example by Eqs. (8) and (9)], we can readily calculate the weight factors for the various partial transitions, namely

Type	Weight
$(N^*)^{++} \to P + \pi^+$	1
$(\Lambda^*)^+ \to \Lambda + \pi^+$	2/3
$(\Lambda^*)^+ \to \Sigma^0 + \pi^+$	1/6
$(\Lambda^*)^+ \to \Sigma^+ + \pi^0$	1/6
$(\Sigma^*)^{++} \to \Sigma^+ + \pi^+$	1
$(\Xi^*)^+ \to \Xi^0 + \pi^+$	1

These weight factors are independent of the values of A and B (i.e. the approximations of neglecting baryon pairs and the pion contribution to the third component of the total isospin) and depend only on the structure of H_1 [cf. Eq. (1)]. It is interesting that they are identical with the weight factors obtained by Lee and Yang [4] on the basis of group theoretic considerations and, while not at all surprising, perhaps the physical basis of their results is now more clear.

3. Widths of low lying baryon isobars

In this section, we discuss briefly the half-widths of the "first excited states" of the baryons. The intention is to spell out in an intuitive fashion the essential parameters which contribute to this width and thereby to make some crude estimates of the relative widths of the low lying baryon isobars. For this purpose, we have recourse to a simple formula for the width of a metastable level of a baryon isobar B^* derived on the basis of classical strong coupling theory [8], namely :

$$\Gamma_{B^*} \simeq \frac{C_1\, a_B \cdot p_{B^*}^3}{[E(B^*) - m_B]} \qquad (22)$$

where a_B is the baryon "source size". p_{B^*} is the momentum of the pion in the rest system of B^*, $[E(B^*)-m_B]$ is the excitation energy of B^*, and C_1 is a constant depending on the quantum numbers of B^* (i.e. I,J, etc.) which must be assigned in accordance with the "correspondence principle". Eq. (22) follows in classical strong coupling theory for a strong p-wave pion-baryon interaction, provided that $a_B \mu \ll 1$, i.e. the "source size" is small compared to the pion Compton wavelength. Eq. (22) can also be derived quantum mechanically in the strong coupling application [9] and the only difference is a slight change in the constant C_1 (which cancels out when the relative widths are compared). Indeed, Eq. (22) maintains its essential form in the Chew-Low theory [10] if one identifies the "source size" a_B with the inverse of the cut-off energy ω_{max} of Chew and Low [cf. their equations (52) - (53)]; the "natural" choice of the baryon Compton wave length for a_B also leads to good quantitative accord with the Chew-Low theory. This simple physical picture yields the crude estimate that

$$a_B \sim \frac{1}{\omega_{max}(B)} \sim \frac{1}{m_B}$$

The strong coupling theory also makes a prediction concerning the excitation energy of the baryon isobar, namely ;

$$E(B^*) - m_B = C_2 \cdot \frac{a_B}{f_B^2} \tag{23}$$

where C_2 is a constant like C_1 and f_B^2 is the renormalised coupling constant for the p-wave pion-baryon interaction. The Chew-Low theory again yields a similar equation to (23) if a_B is replaced [11] by $\omega^{-1}_{max}(B)$. Combining Eqs. (22) and (23), we obtain

$$\Gamma_{B^*} \simeq C \cdot f_B^2 \cdot p_{B^*}^3 \tag{24}$$

From Eq. (24), it is evident that global symmetry (f_B^2 = constant for all baryons) implies that

$$\Gamma_{B^*} \sim p_{B^*}^3 \tag{25}$$

which is essentially the p-wave phase space factor used by Lee and Yang [4] [cf. their Eq. (18)]. By the same token, it is understandable from Eq. (23) (since $a_B \sim 1/m_B$) that the heavier baryons will have somewhat smaller excitation energies for their low lying isobars. When this is combined with the subtractive term in Eq. (15b), it is easy to explain a reduction by a factor 2 in the half-width of Λ^* compared to that of N^*. On the other hand, the additive term in Eq. (15a) [even when one measures $E(\Sigma^*)$ with respect to m_Σ] leads to the expectation that the half-width of Σ^* will be as large, or even larger, than that of N^*. Similar qualitative arguments predict that the half-width of Ξ^* will again be comparable to (perhaps smaller than) that of N^*. These statements are roughly in accord with the numbers given in Table III of ref. [4].

In conclusion, it should be emphasized that the purpose of this note was to obtain a more intuitive understanding of the relative energies and widths of the low lying baryon isobar within the framework of global symmetry. We find, in agreement with refs. [3) and 4)], that the observations concerning the excitation energy and half-widths of Λ^* can readily be understood on this basis. If it turns out that the observed low lying Λ isobar does not have the quantum numbers [12)] $J=3/2$ the globally symmetric starting point for our calculations [as well as those of refs. [3) and 4)]] would then become doubtful because opposite parity of Λ and Σ would be likely [2),12)].

REFERENCES

1) The energies of all baryon isobars will be measured with respect to the rest masses of the corresponding baryons.

2) M. Alston et al., Phys.Rev. Letters 5, 520 (1960) and
H.J. Martin et al., Phys. Rev. Letters 6, 283 (1961);
in the second paper, the energy of Λ^* agrees with that of the first paper but the width is lower by at least a factor of 3.

3) D. Amati, A. Stanghellini and B. Vitale, Nuovo Cimento 13, 1143 (1959) and Phys.Rev. Letters 5, 524 (1960);
Ph. Meyer, J. Prentki and Y. Yamaguchi, Nuovo Cimento 14, 794 (1961).

4) T.D. Lee and C.N. Yang, preprint (Institute for Advanced Study, Princeton) to be published in Physical Review.

5) H. Miyazawa, Phys.Rev. 101, 1564 (1956).

6) S. Fubini and W. Thirring, Phys.Rev. 105, 1382 (1957).

7) It can be shown, on the basis of the symmetrical Sakata model (see e.g., Y. Yamaguchi, Prog.Theor.Phys.Suppl. 11, 1, 37 (1959), and M. Ikeda, S. Ogawa and Y. Ohnuki, Prog.Theor.Phys. 22, 1073 (1960)) that the mass formula for particles in the same representation must have the form of Eq. (20a); however, the symmetrical Sakata model further demands that $d=-b/4$, a condition which is not fulfilled by our coefficients. Of course, one can always argue that the particles represented by such a formula do not belong to the same representation or that the symmetrical Sakata model is only true at high energies. Details concerning these subjects will be published separately by one of the authors (S.O.).

8) R.E. Marshak, "Meson Physics", Dover publications, New York, p.359;
cf. also W. Pauli, "Meson Theory of Nuclear Forces", Interscience Publishers, New York.

9) Y. Fujimoto and H. Miyazawa, Prog.Theor.Phys. 5, 1052 (1950).

10) G.F. Chew and F.E. Low, Phys Rev. 101, 1570 (1956).

11) There seems to be a misprint of a factor 3 in the expression for the excitation energy $E(B^*)-m_B$ of the first paper of reference [3].

12) R.H. Dalitz, Phys.Rev. Letters 6, 239 (1961).