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Comments on Backreaction and Cosmic Acceleration

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Abstract

In this brief WEB note we comment on recent papers related to our paper *On Acceleration Without Dark Energy* [1].

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In Ref. [1] we elaborated on the proposal that the observed acceleration of the Universe might be the result of the backreaction of sub-horizon cosmological perturbations, rather than the effect of a negative-pressure dark-energy fluid or a modification of general relativity. Through studying the effective Friedmann equations describing an inhomogeneous Universe after smoothing (see the work of Buchert [2]), we suggested that acceleration in our Hubble volume might be possible even if local fluid elements do not individually undergo accelerated expansion.

To describe the time evolution of a region of the Universe as large as our local Hubble volume one has to construct the effective dynamics from which observable average properties can be inferred. This is intimately connected with the general problem of how the (possibly nonlinear) dynamics of cosmological perturbations on small scales affects the large-scale "background" geometry, and with the process of averaging over a given domain $\mathcal{D}$ of volume $V_{\mathcal{D}}$.

The scale factor averaged over a domain $\mathcal{D}$ is defined by $a_{\mathcal{D}} \equiv (V_{\mathcal{D}})^{1/3}$. The very simple fact that the averaging of the time derivative of a locally defined quantity differs from the time derivative of the averaged quantity implies that acceleration is possible in principle for the dynamics described by the average scale factor $a_{\mathcal{D}}$.

Indeed, from the effective equations of motion, it is easy to show that acceleration may be achieved if $Q_{\mathcal{D}} > 4\pi G\langle\rho\rangle_{\mathcal{D}}$, where $Q_{\mathcal{D}}$ is the kinematical backreaction term encoding the effect of nonlinearities [1, 2] and ρ the local energy density.

Nambu and Tanimoto [3], propose an explicit example of an inhomogeneous Universe that leads to accelerated expansion after taking spatial averaging. The model contains both a region with positive spatial curvature and a region with negative spatial curvature. It was found that after the region with positive spatial curvature begins to re-collapse, the deceleration parameter of the spatially averaged Universe becomes negative and the averaged Universe starts an accelerated expansion phase. While this is not represented as a realistic cosmological model, it illustrates several important concepts.

Generalizing the model of Ref. [3], to understand how backreactions can lead to acceleration, one can think of the domain $\mathcal{D}$ as a collection of smaller regions which are individually homogeneous and isotropic and expanding with a scale factor a_i and rate H_i. The corresponding average acceleration in this case is given by

$$\begin{aligned}\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} &= \left\langle \frac{\ddot{a}(x)}{a} \right\rangle + 2\left[\left\langle \left(\frac{\dot{a}(x)}{a(x)} \right)^2 \right\rangle - \left(\left\langle \frac{\dot{a}(x)}{a(x)} \right\rangle \right)^2 \right] \\ &= \sum_i \frac{\ddot{a}_i}{a_i} \frac{V_i}{a_{\mathcal{D}}^3} + 2\sum_i \left(\frac{\dot{a}_i}{a_i} \right)^2 \frac{V_i}{a_{\mathcal{D}}^3} - 2\left(\sum_i \left(\frac{\dot{a}_i}{a_i} \right) \frac{V_i}{a_{\mathcal{D}}^3} \right)^2 \end{aligned} \tag{1}$$

where V_i is the volume of region i. A necessary (but not sufficient) condition for getting acceleration is that some of the regions evolve with a different Hubble rate than others. In Ref. [4] this strategy was criticized on the basis that one can envisage situations in which, upon averaging, the Universe accelerates, despite the fact that the observers of each individual separate region experience deceleration ($\ddot{a}_i < 0$).

Indeed, Ishibashi and Wald [4] consider the case in which there are two of such separate regions for which $a_1 = a_2$ and $\dot{a}_1 = -\dot{a}_2$ (how these conditions may be preserved by the dynamics, *e.g.*, in the model of Ref. [3] is not clear to us). They claim that this "graphically illustrates" that the requirement $\ddot{a}_{\mathcal{D}} > 0$ is "far from sufficient to account for the physically observed effects of acceleration in our universe." But our basic point is *exactly* that acceleration of the mean scale factor can occur even though individual elements decelerate. The fact that $a_{\mathcal{D}}$ is related to observables is strongly suggested by the work of Tomita [5], who calculated observables such as the luminosity distance and the angular-diameter distance in an inhomogeneous models and finds apparent acceleration.

The authors of Ref. [4] also argue that the averaging procedure is affected by ambiguity both in regard to the choice of time slicing and the choice of the domain $\mathcal{D}$. However, the statement about the time slicing is equivalent to say that whether acceleration is experienced or not depends upon the observer. This is true even for an unperturbed Friedmann-Robertson-Walker (FRW) spacetime in which only those observers comoving with the perfect fluid source would say that the Universe is homogeneous and isotropic. We do not see any ambiguity in choosing the observer comoving with the matter flow. The dependence of the average parameters on the choice of the domain $\mathcal{D}$ is an unavoidable consequence of the standard procedure of fitting an FRW model to a real perturbed Universe. That said, the real issue is the appropriate scale over which inhomogeneities need to be smoothed out, given the specific dataset one wishes to fit. For instance, Tomita has investigated the possibility that we live in a locally underdense region of size about $(200 - 300)$ Mpc and studied the magnitude-redshift relation in a cosmological model with such a local void [5]. The accelerating behavior of high redshift supernovae can be explained in this model, because the local void plays a role similar to the positive cosmological constant. Acceleration is experienced by the observer living inside the void as her/his region expands faster than the outer region despite the fact that both are decelerating. In the average language, this situation precisely results in the simultaneous presence of largely under-dense and over-dense regions giving rise to a large kinematical backreaction [1]. Notice also that the dependence over the volume $V_{\mathcal{D}}$ disappears and one can safely replace the spatial average over $\mathcal{D}$ with the ensemble average as soon as the volume is large enough for the Ergodic Theorem to hold.

Another criticism raised in Ref. [4] is that if the Universe is accurately described by a Newtonian perturbed FRW metric, then no backreaction may give rise to acceleration. We fully agree on the fact that Newtonian approximation yields an accurate description of our Universe on all relevant scales (as long as the considered wavelength is much larger than the Schwarzschild radius of collapsing bodies) as we explicitly emphasized in Section IIIC of Ref. [1]. The real issue is which quantity should be computed for a proper evaluation of the impact of the backreaction. Acceleration requires the kinematical backreaction term to be of the same order of the average curvature of comoving hypersurfaces. The latter is well known to vanish at the Newtonian level, but it nevertheless enters the dynamics of the Universe. One needs therefore a genuine relativistic description. In the Newtonian case, it is immediate to verify that $Q_{\mathcal{D}}$ is *exactly* (*i.e.,* at any order in perturbation theory) given by the volume integral of a total-derivative term in Eulerian coordinates, so that by the Gauss theorem it can be transformed into a pure boundary term. It is precisely for this reason that any consideration of the backreaction based on the Newtonian approximation is not helpful: it will invariably lead to a tiny effect, and to the absence of any acceleration. What is important for us here is that $Q_{\mathcal{D}}$ clearly displays sizeable non-Newtonian terms even in the weak-field gauge. The key point is that the backreaction has to be calculated adopting the proper time of observers comoving with the matter flow. It is precisely for this reason that sizeable post-Newtonian backreaction terms, of the type $\sim H^2\langle\delta^2(v/c)^2\rangle$ (where $\langle\cdot\rangle$ stands for the ensemble average), appear in the effective Friedmann equations describing an inhomogeneous Universe after smoothing. Notice that this occurs *in spite of the fact* that in the Poisson gauge the metric itself is very well approximated by the weak-field form, *i.e.* Eq. (77) of Ref. [1] with $\phi_P = \psi_P \equiv \Phi_N/c^2 \ll 1$. (Here the subscript N stands for Newtonian. Our form is, of course, identical to Eq. (1) of Ref. [4], where $\Psi \equiv \Phi_N/c^2$). We take the opportunity to stress once again that in the weak-field approach the number of gradients is finite and the complexity of the problem resides in the non-perturbative evaluation of the evolved potentials in terms of the initial seeds. The situation is reversed when approaching the problem in the synchronous and comoving gauge, where the expressions for backreaction terms have to be expanded through an infinite series of gradients of the initial seed itself.

A possible objection to the use of the synchronous and comoving gauge in addressing the backreaction problem is the occurrence of shell-crossing singularities (caustics) in the evolution of collisionless fluids, which might prevent the analysis to be carried over into the fully non-linear regime. We would like to point out that the instability we find in Ref. [1] in the gradient expansion is unrelated to shell-crossing singularities. This can be immediately appreciated by noting that: i) shell-crossing instabilities imply the emergence of *divergent* gradients terms, while our instability shows up through an infinite number of *finite* gradient terms; ii) shell crossing is well known to lead to an infinite Newtonian term, while our effect involves a tiny Newtonian term. It should also be stressed that the occurrence of caustics does not represent a serious limitation of our approach; indeed, the very fact that caustics only carry a small amount of mass implies that they can be easily smeared over a finite region out in such a way that their presence does not affect the mean expansion rate of the Universe.

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[1] E. W. Kolb, S. Matarrese and A. Riotto, arXiv:astro-ph/0506534.

[2] T. Buchert, arXiv:gr-qc/0509124, and references therein.

[3] Y. Nambu and M. Tanimoto, arXiv:gr-qc/0507057.

[4] A. Ishibashi and R. M. Wald, arXiv:gr-qc/0509108.

[5] K. Tomita, Mon. Not. Roy. Astron. Soc. **326**, 287 (2001).