CERN-PH-TH/2005-178

Can the cosmological constant undergo abrupt changes?

Alejandro Cabo[1,2], Alejandro Garcia-Chung[3] and Alejandro Rosabal[4]

[1] *Theory Division, CERN, Geneva, Switzerland*
[2] *Grupo de Física Teórica, Instituto de Cibernética, Matemática y Física, Calle E, No. 309, Vedado, La Habana, Cuba*
[3] *Departamento de Física, Facultad de Ciencias Naturales, Universidad de Oriente, Santiago de Cuba, Cuba and*
[4] *Departamento de Física, Universidad de Granma, Bayamo, Cuba.*

The existence of a simple spherically symmetric and static solution of the Einstein equations in the presence of a cosmological constant vanishing outside a definite value of the radial distance is investigated. A particular succession of field configurations, which are solutions of the Einstein equations in the presence of the considered cosmological term and auxiliary external sources, is constructed. Then, it is shown that the associated succession of external sources tend to zero in the sense of the generalized functions. The type of weak solution that is found becomes the deSitter homogeneous space-time for the interior region, and the Schwartzschild space in the outside zone.

PACS numbers: 04.20.Dw,04.70Bw

I. INTRODUCTION

The gravitational equations in the presence of a cosmological constant Λ have been the subject of continued attention in the literature about Classical and Quantum Gravity, since their introduction by Einstein as a means to define a static and homogeneous model for the Universe [1]. In particular the question about the physical relevance of these equations [2–5] have received a large amount of interest in recent times, thanks to the modern experimental evidence signalling the relevance of this quantity in describing the expansion rate of the Universe [6, 7].

It is an accepted fact that the satisfaction of the Einstein equations for spherically symmetric and static solutions forces the value of the cosmological constant Λ to be rigorously constant, that is, independent of the radial coordinate. The known solution in this situation is the deSitter space-time. Inside this space, the matter is pushed by the gravitational force away from the centre, much as it could be attracted to the origin, in the interior of a Schwartzschild black-hole [8].

In this work we discuss the possibility that the same original Einstein equations in the presence of a cosmological term, could show a form for it that would be rigorously non-vanishing and space-independent inside a certain sphere, but reducing to zero outside it. This field configuration will then define the deSitter solution as the internal space and the Schwartzschild solution as the external one.

The main circumstance suggesting the existence of this solution is the fact that, at the special radial distance in which the metric becomes singular for both the deSitter and Schwartzschild solutions, the non-linearity of the equations could allow for solutions in the sense of the generalized functions, showing a sudden change in Λ.

The plan of the work will be as follows. In Section 2, the Einstein equations for a diagonal energy-momentum tensor are written and the notation to be employed is defined. Section 3 continues by considering a rough motivation for the existence of the solution after solving the basic radial and temporal Einstein equations. A main point is that, assuming the exact equality between $-g_{00}$ and $1/g_{rr}$ the equations for these quantities become linear and are exactly satisfied in the sense of the generalized functions by the piecewise defined solution given by the Schwartzschild and the deSitter fields for the external and internal regions respectively. As is known, for this class of centrally symmetric problems, these two equations are sufficient to fully determine the only two unknown fields g_{00} and $1/g_{rr}$. Therefore, if the solution for these quantities would result in being to be non-vanishing and differentiable, the full set of equations could be automatically solved. However, the non-linearity of the resting Einstein equation makes it necessary to check that the full set of Einstein equations can be considered as solved in some generalized sense. For this purpose, in Section 4, a physically grounded solubility criterion is defined for the satisfaction of the Einstein equations. It rests in the natural assumption of considering as a weak solution of the non-linear equations, the limit of a particular succession of field configurations, for which each element solve the equation in the presence of external sources and for which, moreover, the corresponding succession of sources tends to zero in the sense of the generalized functions. The need for involving a particular succession in the definition comes from the assumed non-linearity of the equations. In the case where the equations are linear, the generalized functions, being the limit of the linear functionals associated to each field configuration in the succession, could be considered as the solution. However, the non-linearity makes it necessary to have a more precise definition by including the particular succession that allows the limit of the external sources to be vanishing.

Finally in Section 5 it is shown that the defined solubility criterion can be satisfied for the Einstein equations in the presence of a cosmological constant which reduces to zero outside the radial distance r_0 for which the deSitter temporal metric component g_{00} vanishes. Roughly speaking, we found a particular succession of field configurations that produce a vanishing limit for the externals sources associated to each of its elements, in the sense of the generalized functions.

It becomes clear that the resulting gravitational field configuration shows a kind of naked singularity, since for example, the radial derivative of the g_{00} metric components has a discontinuity at the boundary [9–11].

The results are reviewed and commented in the conclusions.

II. EINSTEIN EQUATIONS

The squared line element for spherically symmetric systems will be written in the form

$$\begin{aligned} ds^2 &= g_{00}(r)dx^{0^2} + g_{rr}\, dr^2 + g_{\phi\phi}\, d\phi^2 + g_{\theta\theta}\, d\theta^2, \\ &= -v(r)dx^{0^2} + u(r)^{-1}dr^2 + r^2(\sin^2\theta\, d\phi^2 + d\theta^2), \end{aligned} \tag{1}$$

in which the functions u and v are defined in terms of the metric components as

$$\begin{aligned} v(r) &= -g_{00} = \exp(\nu(r)), \\ u(r) &= g_{rr} = \exp(-\lambda(r)). \end{aligned} \tag{2}$$

We will consider for the start, a set of equations slightly generalizing the usual Einstein equations with a cosmological term. For the case of a static and spherically symmetrical solution, when there are no external sources J_0^0,J_r^r,J_ϕ^ϕ and J_θ^θ acting on the system, the equations to be examined cab be written in the form

$$J_0^0 = -\Lambda_0(r) - \frac{u'(r)}{r} + \frac{1-u(r)}{r^2} = -\Lambda_0(r) + G_0^0(r) = 0, \tag{3}$$

$$J_r^r = -\Lambda_r(r) - \frac{u(r)}{v(r)}\frac{v'(r)}{r} + \frac{1-u(r)}{r^2} = -\Lambda_r(r) + G_r^r(r) = 0, \tag{4}$$

$$\begin{aligned} J_\phi^\phi &= J_\theta^\theta = -\Lambda(r) - \frac{u(r)}{2}\Big(\frac{v''(r)}{v(r)} - \frac{v'(r)^2}{2v(r)^2} + \\ &\quad \frac{v'(r)\ u'(r)}{2\ v(r)\ u(r)} + \frac{1}{r}\Big(\frac{u'(r)}{u(r)} + \frac{v'(r)}{v(r)}\Big)\Big), \\ &= -\Lambda(r) + G_\phi^\phi(r), \\ &= -\Lambda(r) + G_\theta^\theta(r) = 0, \end{aligned} \tag{5}$$

in which the Einstein tensor G_μ^ν is diagonal in the spherical coordinates, and its diagonal components are given as

$$\begin{aligned} G_0^0 &= \Lambda_0(r), \\ G_r^r &= \Lambda_r(r), \\ G_\phi^\phi &= G_\theta^\theta = \Lambda(r). \end{aligned}$$

These metric components are written in terms of the three functions Λ_0, Λ_r and Λ, which only depend on the radial coordinate. Since the vanishing of the covariant divergence of the Einstein tensor is an identity for any field configuration (see[12]) it follows that

$$G_{\mu\ ;\ \nu}^{\nu}(r) = \frac{1}{\sqrt{-g(r)}}\partial_\nu(\sqrt{-g(r)}G_\mu^\nu\ (r)) - \frac{1}{2}\partial_\mu(g_{\gamma\nu}(r))\ G^{\gamma\nu}(r) = 0,$$

As usual, g is the determinant of the metric tensor

$$g(r) = -\frac{v(r)\ r^4 \sin^2\theta}{u(r)}.$$

III. INDICATIONS FOR THE SOLUTION

Let us motivate the existence of a solution of the Einstein equation in the presence of a cosmological term that does not reduce itself to a fixed constant times the metric tensor for all the points of the space-time. For this purpose it can be first noticed that when all the Λ_0, Λ_r and Λ functions are selected as equal among them and to a given constant Λ, the set of equations (3)–(5) have the usual deSitter solution showing the regular behaviour at the origin

$$u(r) = 1 - \frac{\Lambda\ r^2}{3}, \tag{6}$$
$$= 1 - \frac{\phi_0^2\ r^2}{6},$$
$$v(r) = 1 - \frac{\Lambda\ r^2}{3} \tag{7}$$
$$= 1 - \frac{{\phi_0}^2\, r^2}{6},$$

in which the scalar field parameter ϕ_0 is related to Λ through

$$\Lambda = \frac{\phi_0^2}{2},$$

where ϕ_0^2 can be interpreted as the square of the mass times the square of the scalar field producing the same cosmological term as the Klein-Gordon Lagrangian, when the field is assumed to be a constant in all the space. The reason for introducing this parameter is the fact that the field configuration discussed here has a close relationship with a particular solution for the Einstein Klein Gordon system discussed in [13].

Consider now the external region to the sphere having radius r_0 and assume that all three functions Λ_0, Λ_r and Λ in (6) vanish. Within this region, the Schwartzschild solution

$$u(r) = 1 - \frac{r_0}{r}, \tag{8}$$
$$v(r) = 1 - \frac{r_0}{r}, \tag{9}$$

satisfies equations (3) and (4), but for the case of a zero cosmological constant.

The above remarks suggest to check whether a composite configuration, coinciding with the above described solutions in the internal and the external regions, globally satisfies the Einstein equations (3) and (4) for a cosmological term being constant inside the mentioned sphere and vanishing out of it.

To examine this question, let us define the ansatz for $u = 1/g_{rr}$ and $v = -g_{00}$, for all values of the radial distance by:

$$u(r) = \left(1 - \frac{r_0}{r}\right)\ \theta(r - r_0) + \left(1 - \frac{\phi_0^2\, r^2}{6}\right)\ \theta(r_0 - r),$$
$$v(r) = u(r), \tag{10}$$

where the constraint of making u and v to vanish in the limits taken from both sides at $r = r_0$ has been imposed. This condition determines r_0 in terms of the cosmological constant through

$$\frac{\phi_0^2\, r_0^2}{6} = 1. \tag{11}$$

Henceforth, the derivative of u takes the explicit form

$$u'(r) = \left(\frac{r_0}{r^2}\right)\ \theta(r - r_0) + \left(1 - \frac{r_0}{r}\right)\ \delta(r - r_0) -$$
$$\frac{\phi_0^2\, r}{3}\theta(r_0 - r) - \left(1 - \frac{{\phi_0}^2\, r^2}{6}\right)\ \delta(r_0 - r),$$
$$= \left(\frac{r_0}{r^2}\right)\ \theta(r - r_0) - \frac{{\phi_0}^2\, r}{3}\ \theta(r_0 - r), \tag{12}$$

where the terms of Dirac's delta function cancel precisely owing to the selected condition (11).

After substituting u and u' in the Einstein equation (3), it follows that

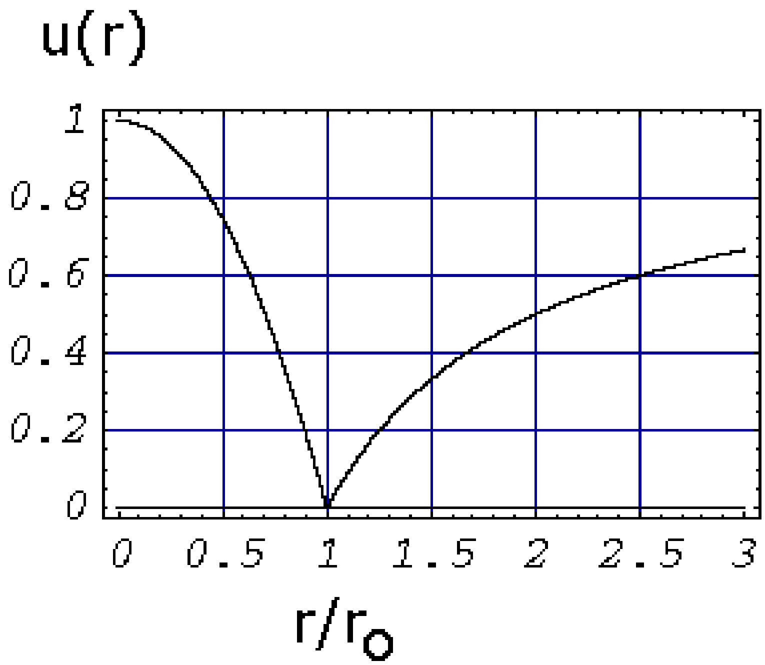

FIG. 1: The common radial dependence of the functions $u(r)$ and $v(r)$. Note the abrupt change in the slope that is produced by the sudden change in the cosmological constant.

$$\begin{aligned}\frac{u'(r)}{r}-\frac{1-u(r)}{r^2}+\Lambda(r) &= \left(-\frac{{\phi_0}^2}{3}\right)\theta(r_0-r)+\left(\frac{r_0}{r^3}\right)\theta(r-r_0)+\\ &\quad \frac{1}{r^2}\left(1-\frac{{\phi_0}^2 r^2}{6}\right)\theta(r_0-r)+\frac{1}{r^2}\left(1-\frac{r_0}{r}\right)\theta(r-r_0)+\\ &\quad (\frac{{\phi_0}^2}{2})\theta(r_0-r)-\frac{1}{r^2},\\ &\equiv 0.\end{aligned}$$

Therefore, an exact solution of the couple of Einstein equations (3) and (4), in the sense of the distribution functions can be written, in the simple form:

$$\begin{aligned}u(r) &= v(r)=\left(1-\frac{r_0}{r}\right)\theta(r-r_0)+\left(1-\frac{r^2}{r_0^2}\right)\theta(r_0-r),\\ r_0 &= \frac{\sqrt{6}}{|\phi_0|}.\end{aligned} \tag{13}$$

Fig.1 illustrates the radial dependence of the $u(r)=v(r)$ functions. It becomes clear that there is, for example, a finite change in the slope of the g_{00} component of the metric. Thus a singularity is a associated to the boundary.

Concerning the resting equation (5), it is not clear that it can be satisfied. Its non-linear nature is the main source of the difficulty, since the candidate solutions (13) for u and v and their derivatives are singular quantities, and their products at the point r_0 are not well defined. In spite of this, the above discussion leads to the expectation that a weak solution could exists, showing the considered sudden change in the cosmological term.

IV. SOLUBILITY CRITERION

The results of the previous section suggest the existence of solutions of the Einstein equations in the presence of a cosmological constant which, suddenly reduces to zero outside a sphere of radius r_0.

In this section we will argue that this system of equations can be solved in a concrete sense to be defined below.

Definition

Consider the linear functionals in the space D' of test functions f (see [14]) and the generalized functions u_g and v_g defined in D' by a given succession S of fields $(u_n(r), v_n(r)), n = 1, 2, 3, ...\infty$ through

$$\begin{aligned} u_g &= \lim_{n\to\infty} \int_0^\infty dr\ f(r) u_n(r), \\ v_g &= \lim_{n\to\infty} \int_0^\infty dr\ f(r) v_n(r). \end{aligned}$$

We will say that the set of equations (3)-(5) are solved in a weak sense by (u_g,v_g), if the three successions of linear functionals associated to the external sources J_0^0, J_r^r and $J_\phi^\phi = J_\theta^\theta$ and determined by each field configuration $(u_n(r), v_n(r))$ through

$$\begin{aligned} F_0^0[u_n, v_n] &= \int_0^\infty dr\ f(r) J_0^0(u_n, v_n), \qquad (14)\\ F_r^r[u_n, v_n] &= \int_0^\infty dr\ f(r) J_r^r[u_n, v_n], \\ F_\phi^\phi[u_n, v_n] &= F_\theta^\theta[u_n, v_n], \\ &= \int_0^\infty dr\ f(r) J_\phi^\phi(u_n, v_n), \ \ \textit{for all } f \in D', \end{aligned}$$

all vanish in the limit $n \to \infty$.

It should be stressed that any field configuration can be considered as a solution of the Einstein equations associated to its corresponding external sources. These sources are nothing other than the result of evaluating the given fields in the equations. Therefore the above definition declares as a solution of the equations in the absence of sources, a generalized function defined by a particular succession of field configurations, whenever it turns out that the sources for the configurations forming the succession converge to zero in the sense of the generalized functions. Such a definition implies, in particular, the physically desirable property that the limit $n \to \infty$ of the variation of the action defining the equations, after being evaluated in the fields (u_n, v_n) tends to vanish.

In the next section we will show that the given solubility criterion for the equations (3)-(5) is satisfied by a specially chosen succession of functions $(u_n(r), v_n(r))$, $n = 1, 2, 3, ..., \infty$.

For the purposes of the next section, we will consider a useful representation of any spherically symmetric field configuration of the Einstein equations. It can be found for example in Ref. [12], and will be employed here as specialized for the static situation under consideration. The first two equations (3) and (4) allow us to obtain, for the functions λ and ν in (2):

$$\begin{aligned} \lambda(r) &= -\log\left(1 - \frac{1}{2}\int_0^r r^2 G_0^0(r)\ dr\right), \qquad (15)\\ \nu(r) &= \int_0^r dr\left(\frac{\exp(\lambda(r)) - 1}{r} - r\exp(\lambda(r)) G_r^r(r)\right), \qquad (16) \end{aligned}$$

expressing these functions in terms of the two components of the Einstein tensor $G_r^r(r)$ and $G_0^0(r)$. Then, the field configurations (u, v) are also fully defined as functions of $G_1^1(r)$ and $G_0^0(r)$ by (2) in the form

$$\begin{aligned} v(r) &= g_{00}(r) = \exp(\nu(r)), \qquad (17)\\ u(r) &= -g_{rr}(r) = \exp(-\lambda(r)). \qquad (18) \end{aligned}$$

Moreover, the vanishing of the covariant divergence of the Einstein tensor as an identity, allows the expression of the components G_ϕ^ϕ a G_θ^θ as functions of $G_1^1(r)$ and $G_0^0(r)$ through [12]

$$G_\phi^\phi = G_\theta^\theta = \frac{r}{2}\partial_r G_r^r\ (r) + G_r^r(r) + \frac{r}{4}\partial_r \nu(r)(G_r^r(r) - G_0^0(r)). \qquad (19)$$

As discussed in Ref. [12], the meaning of the written expressions is that it becomes possible to determine the metric by arbitrarily fixing the two components $G_1^1(r)$ and $G_0^0(r)$ of the Einstein tensor, and afterwards simply select the resting components G_ϕ^ϕ and G_θ^θ as defined by (19) in order to complete the satisfaction of the Einstein equations. Clearly, in these equations the energy-momentum tensor is assumed to be equal to the Einstein tensor, fixed in the explained way.

Let us now consider that all three functions Λ are equal to a common one, $\Lambda(r)$, for all the radial axis except within a small open neighbourhood B of the boundary point r_0. Then, the eq. (19) implies, for these regions:

$$\partial_r \Lambda(r) = 0.$$

Thus, the function $\Lambda(r)$ should be strictly constant in each of the two zones. However, the possibility is not yet discarded that the two values associated to each of the two disjoint regions in which r_0 divides the radial axis could be different. Let us look into this. In the case that they could effectively differ, the term $\partial_r G_r^r(r) = \partial_r \Lambda(r)$ in eq. (19) will contribute with a Dirac delta like singularity. Thus, whenever the factor $\partial_r \nu(r)$ is a regular one in the neighbourhood of r_0, there will be no possibility other than the coincidence of the two values of the function Λ. However, since the factor $\partial_r \nu(r)$ can show singularities at some special points, the opportunity is yet open for cancelling the Dirac delta term. This is only possible by breaking the equality between Λ_r and Λ_0, which allows the mentioned singular dependence of the factor $\partial_r \nu(r)$ to play a role near special points. In this way the Einstein equations could be obeyed in the sense of the generalized functions.

In the next section the satisfaction of the Einstein equations (3)-(5) will be discussed, in accordance with the given solubility criterion, when all the Λ functions have a Heaviside step function behavior.

V. THE SOLUTION

Let us search in this section a solution of the equations (3)-(5) for the specific form of the cosmological term suggested by the discussion in Section 2, that is

$$\begin{aligned} \Lambda_r(r) &= \Lambda_r(r) = \Lambda(r), \\ &= \Lambda\ \theta(r_0 - r). \end{aligned} \tag{20}$$

The first step in showing the satisfaction of the solubility criterion by a succession of functions (u_n, v_n) tending to (10) in the limit $n \to \infty$ will be to define a corresponding succession of the values for the radial and temporal components of the Einstein tensor. These components will be selected to approach the step-like cosmological function (20) in the limit $n \to \infty$ as:

$$G_r^r(r|n) = \Lambda\ \theta_n^{(1)}(r_0, r), \tag{21}$$

$$G_0^0(r|n) = \Lambda\ \theta_n^{(0)}(r_0 - r), \tag{22}$$

$$\sigma_n(r) = \theta_n^{(1)}(r_0 - r) - \theta_n^{(0)}(r_0 - r), \tag{23}$$

where the succession of functions $\theta_n^{(1)}$ and $\theta_n^{(0)}$ for all values of n are both chosen to define the step functions appearing in (20) in the limit $n \to \infty$. The precise expression for $\theta_n^{(0)}$ will be

$$\theta_n^{(0)}(r_0 - r) = \theta(r_0 - \epsilon'(n) - r),$$

where $\theta(x)$ is the Heaviside step function and $\epsilon'(n)$ is a small quantity with respect to r_0, which is taken as vanishing in the limit $n \to \infty$. The regularization for $\theta_n^{(1)}(r_0, r)$ is chosen as given by

$$\begin{aligned} \theta_n^{(1)}(r_0, r) &= \theta(r_0 - \epsilon'(n) - r) + \sigma_n(r), \\ \sigma_n(r) &= g_n(r)\theta(r - r_0 + \epsilon'(n))\theta(r_0 - \epsilon(n) - r), \\ \epsilon'(n) &> \epsilon(n), \end{aligned} \tag{24}$$

where $\epsilon(n)$ is another segment, also tending to zero in the limit $n \to \infty$, but smaller as a real number than $\epsilon'(n)$. The up to now arbitrary (but assumed to be bounded) function $g_n(r)$) will be fixed in what follows.

Therefore, for this n-dependent selection of the two arbitrary components of the Einstein tensor, G_0^0 and G_r^r, the quantities u_n and v_n determined through using (2) and (15), are solutions of the Einstein equations whenever the angular components are calculated from (19). Thus, in order to show that the succession (u_n, v_n) satisfies the solubility criterion for the space-dependent cosmological constant (20), it is only needed to prove that the generalized functions associated to their external sources (14) have vanishing limits $n \to \infty$. Let us show this property below.

The linear functionals in D' being equivalent to the auxiliary external sources for which each of the pairs (u_n, v_n) turns to be a solution of Einstein equations (with cosmological term defined by (20)) can be explicitly written as

$$\begin{aligned}
F_0^0[u_n,v_n] &= \int_0^\infty dr\ f(r)\ J_0^0(u_n,v_n),\\
&= \int_0^\infty dr\ f(r)\left(-\Lambda(r)+G_0^0(r|n)\right),\\
&= \int_0^\infty dr\ f(r)\left(-\Lambda(r)-\frac{u_n'(r)}{r}+\frac{1-u_n(r)}{r^2}\right), \qquad (25)\\
F_r^r[u_n,v_n] &= \int_0^\infty dr\ f(r)J_r^r(u_n,v_n),\\
&= \int_0^\infty dr\ f(r)\left(-\Lambda(r)+G_r^r(r|n)\right),\\
&= \int_0^\infty dr\ f(r)\left(-\Lambda(r)-\frac{u_n(r)}{v_n(r)}\frac{v_n'(r)}{r}+\frac{1-u_n(r)}{r^2}\right), \qquad (26)\\
F_\phi^\phi[u_n,v_n] &= \int_0^\infty dr\ f(r)J_\phi^\phi(u_n,v_n)=\int_0^\infty dr\ f(r)J_\theta^\theta(u_n,v_n),\\
&= \int_0^\infty dr\ f(r)(-\Lambda(r)+G_\phi^\phi(r|n)),\\
&= \int_0^\infty dr\ f(r)\left(-\Lambda(r)-\frac{u_n(r)}{2}\left(\frac{v_n''(r)}{v_n(r)}-\frac{v_n'(r)^2}{2v_n(r)^2}+\frac{v_n'(r)\ u_n'(r)}{2\ v_n(r)\ u_n(r)}+\frac{1}{r}\left(\frac{u_n'(r)}{u_n(r)}+\frac{v_n'(r)}{v_n(r)}\right)\right)\right), \qquad (27)\\
\Lambda(r) &= \Lambda\ \theta(ro-r). \qquad (28)
\end{aligned}$$

Our purpose in what follows will be to show that these functionals tend to zero by properly selecting the regularization, that is, to argue that there exists a succession of field configurations, approaching the fields (13) in the sense of the generalized functions, for which the Einstein tensor also tends to the step-like cosmological term (20) in the same sense.

For eq.(25) and (26), thanks to the same definitions of the succession of Einstein tensors (21),(22), it follows that

$$\begin{aligned}
\lim_{n\to\infty} F_0^0[u_n,v_n] &= \lim_{n\to\infty}\int_0^\infty dr\ f(r)\left(-\Lambda(r)+G_0^0(r|n)\right),\\
&= \lim_{n\to\infty}\int_0^\infty dr\ f(r)\left(-\Lambda(r)+\Lambda\ \theta(r_0-\epsilon'(n)-r)\right),\\
&= -\Lambda\lim_{n\to\infty}\int_{r_0-\epsilon'(n)}^{r_0} dr\ f(r),\\
&= 0, \qquad (29)\\
\lim_{n\to\infty} F_r^r[u_n,v_n] &= \lim_{n\to\infty}\int_0^\infty dr\ f(r)\left(-\Lambda(r)+G_r^r(r|n)\right),\\
&= \lim_{n\to\infty}\int_0^\infty dr\ f(r)\left(-\Lambda(r)+\Lambda\ \theta(r_0-\epsilon'(n)-r)+\sigma_n(r)\right),\\
&= -\Lambda\lim_{n\to\infty}\int_{r_0-\epsilon'(n)}^{r_0} dr\ f(r)+\lim_{n\to\infty}\int_{r_0-\epsilon'(n)}^{r_0-\epsilon(n)} dr\ f(r)\ \sigma_n(r),\\
&= 0, \qquad (30)
\end{aligned}$$

where the last equality follows thanks to the fact that $\sigma_n(r)$ is assumed to be a bounded function for all n. Therefore, the first two functionals associated to the external sources vanish in the limit $n \to \infty$.

For the last functional it is possible to write first

$$
\begin{aligned}
\lim_{n\to\infty} F_\phi^\phi[u_n,v_n] &= \lim_{n\to\infty}\int_0^\infty dr\ f(r)(-\Lambda(r)+G_\phi^\phi(r|n)),\\
&= \lim_{n\to\infty}\int_0^\infty dr\ f(r)(-\Lambda(r)+\frac{r}{2}\partial_r G_r^r\ (r|n)+G_r^r(r|n)+\frac{r}{4}\partial_r\nu_n(r)(G_r^r(r|n)-G_0^0(r|n))\ ),\\
&= \lim_{n\to\infty}\int_0^\infty dr\ f(r)(\frac{r}{2}\partial_r G_r^r(r|n)+\frac{r}{4}\partial_r\nu_n(r)(G_r^r(r|n)-G_0^0(r|n))\ ,\\
&= \lim_{n\to\infty}\int_0^\infty dr\ f(r)\left(\frac{r}{2}\partial_r(\Lambda\ \theta(r_0-\epsilon'(n)-r)+\Lambda\ \sigma_n(r))+\frac{r}{4}\partial_r\nu_n(r)(G_r^r(r|n)-G_0^0(r|n))\right),\\
&= -\frac{1}{2}\Lambda\ f(r_0)+\lim_{n\to\infty}\int_0^\infty dr\ f(r)\frac{r}{4}\partial_r\nu_n(r)(G_r^r(r|n)-G_0^0(r|n)).
\end{aligned} \tag{31}
$$

But, from the general relations (15)-(18) it follows that

$$
\lambda_n(r) = -\log(1-\frac{1}{2}\int_0^r r^2G_0^0(r|n)\ dr), \tag{32}
$$

$$
\nu_n(r) = \int_0^r dr\left(\frac{\exp(\lambda_n(r))-1}{r}-r\exp(\lambda(r))\,G_1^1(r|n)\right), \tag{33}
$$

which, after taking the derivative of $\nu_n(r)$, gives

$$
\begin{aligned}
\partial_r\nu_n(r) &= \frac{\exp(\lambda_n(r))-1}{r}-r\exp(\lambda_n(r))\,G_1^1(r|n),\\
&= -\frac{1}{r}+\frac{(\frac{1}{r}-\Lambda r\theta_n^{(0)}(r_0-r))}{1-\int_0^r drr^2\theta_n^{(1)}(r_0-r)}.
\end{aligned}
$$

Further, the denominator in the last expression can be explicitly evaluated as

$$
\begin{aligned}
\exp(-\lambda_n(r)) &= 1-\int_0^r drr^2\theta_n^{(1)}(r_0-r),\\
&= \theta(r_0-\epsilon'(n)-r)\left(1-\frac{\Lambda r^2}{3}\right)+\theta(r-r_0+\epsilon'(n))\left(1-\frac{\Lambda}{r}\frac{(r_0-\epsilon'(n))^3}{3}\right).
\end{aligned} \tag{34}
$$

After inserting (24) and (34) in (31) the functional F_ϕ^ϕ can be transformed in the following way:

$$
\begin{aligned}
\lim_{n\to\infty} F_\phi^\phi[u_n,v_n] &= -\frac{1}{2}\Lambda\ r_0\ f(r_0)+\lim_{n\to\infty}\int_0^\infty dr\ f(r)\frac{r}{4}\partial_r\nu_n(r)(G_r^r(r|n)-G_0^0(r|n)),\\
&= -\frac{1}{2}\Lambda\ r_0\ f(r_0)-\lim_{n\to\infty}\int_0^\infty dr\ f(r)\frac{\Lambda}{4}\sigma_n(r)+\\
&\quad \frac{\Lambda}{4}\lim_{n\to\infty}\int_{r-r_0+\epsilon'(n))}^{r_0-\epsilon(n)-r} dr\ f(r)\frac{\sigma_n(r)(1-\Lambda r^2\sigma_n(r))}{(1-\frac{\Lambda}{r}\frac{(r_0-\epsilon'(n))^3}{3})},\\
&= -\frac{1}{2}\Lambda\ r_0\ f(r_0)+\frac{\Lambda}{4}\lim_{n\to\infty}\int_{r-r_0+\epsilon'(n))}^{r_0-\epsilon(n)-r} dr\ f(r)\frac{\sigma_n(r)(1-\Lambda r^2\sigma_n(r))}{(1-\frac{\Lambda}{r}\frac{(r_0-\epsilon'(n))^3}{3})}.
\end{aligned}
$$

In order to proceed, we complete the specification of the forms of u_n and v_n by fixing the functions σ_n as given by

$$
\sigma_n(r)=\frac{1}{6}\ \theta(r_0-\epsilon(n)-r)\ \theta(r-r_0+\epsilon'(n));
$$

after defining the new integration variable z and parameter Δ according to

$$
\begin{aligned}
z &= \frac{1}{\epsilon'(n)}(r-r_0),\\
\epsilon(n) &= -\Delta\ \epsilon'(n),
\end{aligned}
$$

this allows us to write

$$
\begin{aligned}
\lim_{n\rightarrow\infty} F^{\phi}_{\phi}[u_n,v_n] &= -\frac{1}{2}\Lambda r_0 f(r_0)+ \\
&\frac{\Lambda}{24}\lim_{n\rightarrow\infty}\int_0^{\infty} dz\ f(r_0+\epsilon'(n)z)\frac{\frac{\Lambda}{3}(\frac{1}{2}r_0^2-\epsilon'(n)r_0 z-\frac{1}{2}\epsilon'(n)^2z^2)(r_0+\epsilon'(n)\ z)\theta(1+z)\theta(z-\Delta)}{\frac{\Lambda}{3}(z\ r_0^2+3r_0^2-\epsilon'(n)r_0-\epsilon'(n)^2)}, \\
&= -\frac{1}{2}\Lambda r_0 f(r_0)+f(r_0)\frac{\Lambda\ r_0}{48}\lim_{n\rightarrow\infty}\int_{-1}^{\Delta} dz\frac{1}{z+3}, \\
&= f(r_0)\frac{\Lambda r_0}{2}(-1+\frac{1}{24}\log\frac{(\Delta+3)}{2}).
\end{aligned}
$$

In this way, after selecting Δ defined by

$$\Delta = 2\exp(24)-3,$$

it follows that the succession of regularized fields $S=\{(u_n,v_n)\}$ has a corresponding succession of associated externals sources, which vanish in the limit $n\rightarrow\infty$. Therefore the generalized function (u_g,v_g) defined with the precision of being the limit of the linear functionals associated to the specific configurations in S, satisfies the Einstein equations in the weak sense defined here. It can noticed that for linear systems this definition is less restrictive and the class of successions of field configurations allowed for expressing (u_g,v_g) as their limit is very much more wider.

VI. CONCLUSIONS

A criterion for a generalized function to be a solution of the non-linear Einstein equations is proposed. Then, a particular solution satisfying the criterion for the Einstein equations in the presence of a cosmological term which suddenly vanishes outside a given radial distance, is found. The considered space-time shows a homogeneous deSitter Universe being at an internal region and the Schwartzschild space for the external one. The characterization of the appearing singularity at the boundary will be considered in future works.

Acknowledgements

The helpful conversations and remarks of R. F. Parada, F. Ongay, O.P. Fernández, R. Acharya, L. Jiayu and L. Dung Trang during the preparation of the work are gratefully acknowledged. The careful revision of Ms. Suzy Vascotto at the TH Division of CERN Secretariat is also very much appreciated.

[1] A. Einstein, Sitzungsber. Preuss. Akad. Wiss., p. 142 (1917), (English translation in: *The principle of Relativity,* p. *177* (Dover, New York, 1952).
[2] I. Dymnikova, *Variable Cosmological Constant: Geometry and Physcis,* talk given at the: International Seminar on Mathematical Physics and Applied Mathematics, St. Petersburg, Russia, 16 June 2000, e-Print Archive: gr-qc/0010016.
[3] S. Weinberg, *The Cosmological Constant problems*, (Texas U.), talk given at the 4th International Symposium on Sources and Detection of Dark Matter in the Universe (DM 2000), Marina del Rey, California, 23-25 Feb. 2000, e-Print Archive: astro-ph/0005265.
[4] M. S. Turner, *Dark Energy and the New Cosmology,* FERMILAB-01-463-A, Aug 2001. 11pp. Contribution to the SNAP Yellow Book (Snowmass 2001), e-Print Archive: astro-ph/0108103.
[5] P. J. E. Peebles, Rev. Mod. Phys. 75, 559 (2003).
[6] A. Riess et. al., Astron. J. 116, 1009 (1998).
[7] S. Perlmutter et al, Astrophys. J 517, 565 (1999).
[8] S. Weinberg, *Gravitation and Cosmology* (John Wiley & Sons, Inc. 1972).
[9] R. Giambo et al, Gen. Rel. Grav. 36, 1279 (2004).
[10] P. S. Joshi, N. Dadhich and R. Marteens, Phys. Rev. D 65, 101501-1 (2002).

[11] T. Harada, Pamana-J., Phys. 53, 1 (1999).
[12] J.L. Synge, *Relativity: the General Theory* (North-Holland, 1966)
[13] A. Cabo and E. Ayon-Beato, Int. J. Mod. Phys. A14, 2013 (1999), e-Print Archive: gr-qc/9704073.
[14] V.S. Vladimirov, *Generalized Functions in Mathematical Physics* (Mir Publishers, Moscow, 1979).