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THE PARTON MODEL, DUALITY AND DEEP INELASTIC SINGLE PARTICLE DISTRIBUTIONS

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A B S T R A C T

Within the quark parton model the two-component duality idea is generalized to single hadron inclusive distributions in deep inelastic scattering. The discussion is limited to the current fragmentation region. The number of independent distribution functions is reduced. Relations as well as inequalities among the structure functions are obtained. In particular, the excess of π^+ over π^- in the electroproduction off a proton comes out as a consequence and the differential cross-section for $e^+e^- \to \pi +$ + anything is fully determined by the electroproduction of pions on nucleons.

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1. INTRODUCTION

The parton model [1] has been applied to the deep inelastic interactions of leptons with hadrons. It has been extended later on [2] in a manner which enabled the treatment of single particle distributions in deep inelastic scattering and deep e^+e^- annihilation.

Further on, ideas borrowed from the duality scheme of hadron reactions have been applied to total deep inelastic structure functions both within the framework of the parton model [3] and without explicitly assuming this model [4),5]. In Refs. 3) and 5) the Harari-Freund [6] two-component duality idea and positivity constraints have been applied to the imaginary part of the forward current-nucleon amplitude. It has been realized in Ref. 5) that the internal symmetry properties of the structure functions can be reduced to those of a quark parton model with the quark parton diagrams interpreted as duality diagrams [7]. In such a model the quark distribution within a nucleon [8] is divided into two components: the neutral component ("sea" quarks) corresponding to the Pomeron background part of the amplitude, and the quantum number carrying component ("valence" quarks) which corresponds to the Regge resonance part. It is a typical duality element to assume separate positivity for these [5], a property which is not implied in a mere parton approach. This enables one to obtain tight bounds on the neutrino structure functions by using experimental data of the electroproduction ones.

The measurement of single hadron distributions in deep inelastic reactions can serve as a useful tool for the further understanding of the structure of the nucleon. Some general consequences following from symmetry principles have been derived by Lipkin and Paschos [9] for the target fragmentation region. The same region has been treated in detail by various authors using the light-cone approach [10]. The parton model has been applied to both this region and to the current fragmentation region [2),11),12]. Since single hadron distributions in electroproduction are now in the process of measurement, it is worthwhile to look for possible tests of duality involving such cross-sections.

In this paper we study the current fragmentation region within the framework of the quark-parton model as described by Feynman and Berman et al., [2], and in Ref. 12) since in this region the model has its strongest predictive power. Moreover, at present, it is more easily accessible from the experimental point of view. We shall make use of the valence and sea quark notion in order to show that in the quark-parton model the two-component duality idea [6] has a simple and straightforward generalization to the six-point function level of current hadron interactions. Moreover, the diagram content of resonant part of the six-point function is simplified to a large extent due to the dynamical picture of the parton model.

In Section 2 we describe the kinematics of the process under study, and the notation for structure functions in the parton model. In Section 3 we show how duality ideas lead naturally to the generalization of the two-component duality idea, and in Section 4 we discuss consequences of the approach. These include linear relations among the structure functions, and tighter bounds on experimental quantities. Parts of the results reported here have already been published elsewhere [13),14)]. Similar relations have been independently obtained by Kingsley [11)].

We conclude in Section 5 with a few remarks concerning the scheme, and its possible relationship to ideas encountered in hadronic physics.

2. NOTATION

We consider the process $J_V(q) + N(p) \to h + \text{anything}$ as shown in Fig. 1. The virtual current (electromagnetic or weak) carries four-momentum q, p is the momentum of the nucleon target, and the detected hadron carries four-momentum h. The current fragmentation region is defined by:

$$\begin{gathered} q^2 \longrightarrow -\infty \,,\; M\nu = p\cdot q \longrightarrow \infty \,,\; x = -q^2/2M\nu \text{ finite}\,, \\ h\cdot q \longrightarrow -\infty \,,\; z = \frac{h\cdot p}{M\nu} = \frac{2h\cdot q}{q^2} \text{ finite}\,,\; h_T \text{ finite}\,. \end{gathered} \tag{2.1}$$

Here h_T is the transverse momentum of the observed hadron.

It has been proposed by Feynman [2)] that this process occurs in two steps: in the first one, the photon is absorbed by a parton whose momentum is a fraction x of the nucleon momentum; in the second step, this parton fragments into hadrons which are current fragments, and the rest of the partons materializes in the form of hadrons which are target fragments. These two fragmentation processes are assumed to occur independently on the average (namely, not in individual events). As a result simple forms for the structure functions are obtained. For example, in electroproduction of hadrons one has:

$$L^{ep}_{1,h}(x,z,h_T^2) = \frac{1}{2}\sum_i Q_i^2\, d_p^i(x)\, D_i^h(z,h_T^2) \qquad . \tag{2.2}$$

$L^{ep}_{1,h}$ is the structure function for the inclusive electroproduction of a hadron h on a proton. It is the analogue of $F_1^{ep}(x)$ in the total deep inelastic cross-section. The index i runs over the types of quarks and anti-quarks. Q_i is the charge of a quark of type i, $d_p^i(x)$ is the momentum distribution of a parton type i within the nucleon *), and $D_i^h(z,h_T^2)$ is the function describing the fragmentation of this quark into h + anything. Here the hadron h carries a longitudinal momentum which is a fraction z of the quark (i) momentum, and its transverse momentum is h_T. The various D functions satisfy isospin and charge conjugation relations. Similar expressions can be written for neutrino and anti-neutrino structure functions [for details see Feynman [2] and Ref. 12)].

3. TWO-COMPONENT DUALITY IN THE PARTON MODEL

As mentioned in Section 2, the parton distributions $d^i(x)$ (we omit the subscript p, as from now on only nucleon targets will be dealt with) appear both in the total deep inelastic cross-sections and in the single hadron inclusive distributions. We can, therefore, use the results of Landshoff and Polkinghorne [3] and of Ref. 5) for the d functions. In the quark parton model for the total deep inelastic cross-sections the t channel is non-exotic to begin with. There, the duality constraints amount to:

a) the s channel exotic component is dual to a t channel SU(2) or SU(3) singlet (Pomeron);

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*) For these distributions we use the notation of Ref. 14) rather than that of Feynman [2] and Ref. 12) in order to emphasize the symmetric role played by the target and the produced hadron. This will become apparent later on.

b) the s channel resonant part is limited to non-exotic quantum numbers;

c) currents with the $\emptyset$ meson quantum numbers do not contribute to the resonant part of the amplitude.

The resulting solution for the parton distribution functions is [5]

$$
\begin{aligned}
d^{u} &= v^{u}(x) + s(x) \\
d^{d} &= v^{d}(x) + s(x) \\
d^{\bar{u}} &= d^{\bar{d}} \equiv s(x) \\
d^{s} &= d^{\bar{s}} \equiv s'(x)
\end{aligned}
\tag{3.1}
$$

$s(x)$ $[s'(x)]$ is the distribution function for quarks in the neutral sea, and $v(x)$ is the distribution in the valence component. In the SU(3) limit for the currents, $s(x) = s'(x)$. Notice, that one is not forced to assume a relation between v^u and v^d as in Kuti and Weisskopf [8] and Pantin [12]. Using these results one can obtain new relations among the structure functions in single hadron inclusive distributions. Of these we quote here the following [13]:

$$
\begin{aligned}
L^{ep}_{1,\pi^+} - L^{en}_{1,\pi^+} &= \frac{1}{2}\left(v^{u}(x) - v^{d}(x)\right)\left[\frac{4}{9} D^{\pi^+}_{u}(z, k_T^2) - \frac{1}{9} D^{\pi^+}_{d}(z, k_T^2)\right] \\
L^{ep}_{1,\pi^-} - L^{en}_{1,\pi^-} &= \frac{1}{2}\left(v^{u}(x) - v^{d}(x)\right)\left[\frac{4}{9} D^{\pi^+}_{d}(z, k_T^2) - \frac{1}{9} D^{\pi^+}_{u}(z, k_T^2)\right].
\end{aligned}
\tag{3.2}
$$

Using

$$
\int_0^1 \left(v^{u}(x) - v^{d}(x)\right) dx = 1
$$

and

$$
v^{u}(x) - v^{d}(x) = 6\left(F_1^{ep}(x) - F_1^{en}(x)\right)
\tag{3.3}
$$

one finds:

$$\frac{L^{ep}_{1,\pi^+} - L^{en}_{1,\pi^+}}{\int_0^1 [L^{ep}_{1,\pi^+} - L^{en}_{1,\pi^+}]\,dx} = v^u(x) - v^d(x) = 6\left(F_1^{ep}(x) - F_1^{en}(x)\right) , \qquad (3.4)$$

and a similar relation for the π^- distributions. Thus, the left-hand side becomes independent of the pion altogether. More important, it is easy to show that a <u>similar relation is true for any type of current fragment</u>, independent of its internal quantum numbers (π, K, N, ...). The x dependence is determined by the total inelastic structure functions. Equation (3.2) also implies that $(L^{ep}_{1,\pi^+} - L^{en}_{1,\pi^+})/(L^{ep}_{1,\pi^-} - L^{en}_{1,\pi^-})$ is independent of x. These results cannot be obtained without the use of the sea and valence structure of the quark distributions [restricted by Eq. (3.1)].

It is worth noting that in the quark parton model [2] one already obtains another type of factorization:

$$L^{ep}_{1,\pi^+} - L^{ep}_{1,\pi^-} = \frac{1}{2}\left(D_u^{\pi^+} - D_d^{\pi^+}\right)\left[\frac{4}{9}(d^u - d^{\bar{u}}) - \frac{1}{9}(d^d - d^{\bar{d}})\right] \qquad (3.5)$$

This implies that in the quantity

$$\frac{L^{ep}_{1,\pi^+} - L^{ep}_{\pi^-}}{\int [L^{ep}_{1,\pi^+} - L^{ep}_{1,\pi^-}]\,dz\,dk_T^2} = \frac{D_u^{\pi^+} - D_d^{\pi^+}}{\int [D_u^{\pi^+} - D_d^{\pi^+}]\,dz\,dk_T^2} \qquad (3.6)$$

the presence of the proton is not felt at all. Thus, <u>similar relations should hold for any target</u> (π, N, d, Σ, ...). It is the striking symmetry between equations (3.5), (3.6) and (3.2), (3.4) that we want to stress. However, the symmetry is not complete. In Eq. (3.4) only the valence component of the parton distribution within the nucleon plays a role. In Eq. (3.6), the full hadron content of the quark fragmentation function appears. The physical principles behind the description of the nucleon in terms of its constituents and behind the description of a quark fragmenting into hadrons are similar. Therefore, one is tempted to look for a scheme in which the D functions are decomposed into two terms [14] analogous to the valence and sea components of the d functions. This aim will turn out to coincide with our main goal which is the generalization of the two-component duality idea to the inclusive single hadron distributions in the deep inelastic lepton-nucleon interactions.

For this purpose, we rewrite the parton model expressions in a compact form which clearly exhibits its internal symmetry properties. As a reminder [5], let us first consider the total deep inelastic structure functions F_1, F_3 (in the quark parton model $F_2 = 2xF_1$). Denote

$$F_\pm = F_1 \mp \frac{1}{2} F_3 \tag{3.7}$$

As shown by Callan et al., [15], in the parton model, $F_\pm$ for the process $b + \beta \rightarrow a + \alpha$ can be represented as follows:

$$F^{ab}_{+,\alpha\beta} = \mathrm{Tr}\left[\lambda^a \lambda^b G_{+,\alpha\beta}\right] \tag{3.8a}$$

$$F^{ab}_{-,\alpha\beta} = \mathrm{Tr}\left[\lambda^b \lambda^a G_{-,\alpha\beta}\right] , \tag{3.8b}$$

where ab ($\alpha\beta$) are the SU(3) indices of the current (hadron). The matrices $G_{\pm,\alpha\beta}$, given by

$$G_{\pm,\alpha\beta} = \sum_{c=0}^{8} \frac{\lambda^c}{2} G^c_{\pm,\alpha\beta} , \tag{3.9}$$

correspond to the scattering of an antiquark (G_+) or a quark (G_-) off the hadron. Equations (3.8a) and (3.8b) can be described graphically by the two diagrams in Fig. 2. $G_{\pm,\alpha\alpha}$ are diagonal matrices in the quark space. If we identify their eigenvalues with the parton distribution functions:

$$G_+ = \frac{1}{2}\begin{pmatrix} d^u & & 0 \\ & d^d & \\ 0 & & d^s \end{pmatrix}, \quad G_- = \frac{1}{2}\begin{pmatrix} d^{\bar{u}} & & 0 \\ & d^{\bar{d}} & \\ 0 & & d^{\bar{s}} \end{pmatrix}, \tag{3.10}$$

we obtain the parton model expressions for the structure functions $F_{\pm}$.

For simplicity, we mention here the SU(3) decomposition of the vertex functions $G^c_{\pm}$ for an octet of target hadrons. At the end, results will be stated both in the SU(2) and in the SU(3) limits. In general,

$$G^c_{\pm,\alpha\beta} = i f_{\alpha\beta c} G^f_{\pm} + d_{\alpha\beta c}(1-\delta_{co}) G^d_{\pm} + \sqrt{\tfrac{2}{3}}\,\delta_{\alpha\beta}\delta_{co} G^o_{\pm} . \tag{3.11}$$

For meson targets

$$G^f_+ = -G^f_- \equiv G^f , \quad G^d_{\pm} \equiv G^d , \quad G^o_{\pm} \equiv G^o . \tag{3.12}$$

It has been shown [5] that under the conventional two-component duality assumptions $G_{\pm}$ take the form:

$$G^c_{\pm,\alpha\beta} = (\mp i f_{\alpha\beta c} + d_{\alpha\beta c})\hat{G} + \sqrt{\tfrac{2}{3}}\,\delta_{\alpha\beta}\delta_{co} G^P \quad \text{(for mesons)}, \tag{3.13a}$$

$$\begin{aligned} G^c_{\pm,\alpha\beta} &= i f_{\alpha\beta c}\hat{G}^f + d_{\alpha\beta c}(1-\delta_{co})\hat{G}^d + \sqrt{\tfrac{2}{3}}\,\delta_{\alpha\beta}\delta_{co}(\hat{G}^o + G^P) \\ 2\hat{G}^o + \hat{G}^d + 3\hat{G}^f &= 0 \qquad \text{(for baryons)}. \end{aligned} \tag{3.13b}$$

Here $\hat{G}$ denotes the resonant component, and G^P the Pomeron component. This finally leads to the following decomposition:

$$F^{ab}_{\pm,\alpha\beta} = \hat{F}^{ab}_{\pm,\alpha\beta} + F^{P}\,\delta_{\alpha\beta}\,\delta_{ab} \quad . \tag{3.14}$$

Let us now return to the semi-inclusive production of hadrons. In the quark parton model [2], after absorbing the photon, the quark independently emits hadrons. Thus, in the upper part of Fig. 2, we have to add a quark hadron "blob". The corresponding graphs are shown in Fig. 3. There, τ , ρ are the SU(3) indices of the observed hadron. The formal expression for the structure functions is:

$$(L_{1\pm})^{ab}_{\alpha\beta,\tau\rho} = \mathrm{Tr}\left[\lambda^{a} D_{\pm,\rho\tau}\,\lambda^{b}\, G_{\pm,\alpha\beta}\right] \quad . \tag{3.15}$$

$G_{\pm}$ is the same matrix as occurs in the total cross-sections and by Eq. (3.10) is related to the parton distribution functions. The matrix $D_{\pm}$ denotes the quark (anti-quark) fragmentation function, and similarly to Eq. (3.9) can be decomposed:

$$D_{\pm,\rho\tau} = \sum_{\sigma=0}^{8} \frac{\lambda^{\sigma}}{2} D^{\sigma}_{\pm,\rho\tau} \quad . \tag{3.16}$$

In terms of Feynman's notation [2] (as introduced in Section 2) one has,

$$D_{+,\rho\rho} = \frac{1}{2}\begin{pmatrix} D^{\rho}_{u} & & 0 \\ & D^{\rho}_{d} & \\ 0 & & D^{\rho}_{s} \end{pmatrix} \quad , \quad D_{-,\rho\rho} = \frac{1}{2}\begin{pmatrix} D^{\rho}_{\bar{u}} & & 0 \\ & D^{\rho}_{\bar{d}} & \\ 0 & & D^{\rho}_{\bar{s}} \end{pmatrix} , \tag{3.17}$$

where ρ denotes different particles in a given multiplet.

For an octet of hadrons, the SU(3) decomposition of $D_{\pm}$ is:

$$\begin{aligned} D^{\sigma}_{\pm,\rho\tau} &= i f_{\rho\tau\sigma} D^{f}_{\pm} + d_{\rho\tau\sigma}(1-\delta_{\sigma 0}) D^{d}_{\pm} + \sqrt{\tfrac{2}{3}}\,\delta_{\rho\tau}\,\delta_{\sigma 0} D^{0}_{\pm} \quad , \\ \text{for mesons:}\quad & D^{f}_{+} = -D^{f}_{-} \equiv D^{f} \;,\; D^{d}_{\pm} \equiv D^{d} \;,\; D^{0}_{\pm} \equiv D^{0} \quad . \end{aligned} \tag{3.18}$$

Our next step is to introduce duality constraints. We first follow Müeller [16] in describing the cross-section for the process $J(a) + t(\alpha) \to h(\tau)$ + anything by the discontinuity in the missing mass variable of the amplitude of the process

$$J(a) + t(\alpha) + \bar{h}(\tau) \longrightarrow J(a) + t(\alpha) + \bar{h}(\tau) \quad . \tag{3.19}$$

In the dual resonance picture there are eighteen distinct graphs contributing to the discontinuity of the amplitude. These graphs correspond to the various permutations of adjacent lines on each side separately [17]. For simplicity, let us consider the case in which both the target and the observed hadron are mesons. The end results will be stated for baryons as well.

The pure resonant part in the structure function, denoted by $(\hat{L}_{1\pm})^{ab}_{\alpha\beta,\rho\tau}$, can be written in the duality scheme as

$$(\hat{L}_{1+})^{ab}_{\alpha\beta,\tau\rho} = \hat{L}_I \,\mathrm{Tr}\left[\lambda^a\lambda^\tau\lambda^\rho\lambda^b\lambda^\beta\lambda^\alpha\right] + \sum_{P\neq I}\hat{L}_P \,\mathrm{Tr}\left[P(\lambda^a\lambda^\tau\lambda^\rho\lambda^b\lambda^\beta\lambda^\alpha)\right] . \tag{3.20}$$

Here P runs over all the allowed permutations [17]. We now compare the dual resonance description (3.20) with the parton model description (3.15) which, after substitution of the dual form for $G_\pm$ [5], [Eq. (3.13)], can be written as:

$$\begin{aligned}(\hat{L}_{1+})^{ab}_{\alpha\beta,\tau\rho} = \frac{1}{2}\hat{G}\Big\{ &(D^f + D^d)\,\mathrm{Tr}\left[\lambda^a\lambda^\tau\lambda^\rho\lambda^b\lambda^\beta\lambda^\alpha\right] \\ &+ (-D^f + D^d)\,\mathrm{Tr}\left[\lambda^a\lambda^\rho\lambda^\tau\lambda^b\lambda^\beta\lambda^\alpha\right] \\ &+ \frac{2}{3}(D^o - D^d)\,\mathrm{Tr}\left[\lambda^a\delta_{\tau\rho}\lambda^b\lambda^\beta\lambda^\alpha\right]\Big\} \quad .\end{aligned} \tag{3.21}$$

Comparison of Eqs. (3.20) and (3.21) implies that only the first term survives. Thus,

$$L_P = 0 \quad , \quad P \neq I$$

$$D^f = D^d = D^o \equiv \hat{D} \tag{3.22}$$

Finally, duality restricts the resonant part of the quark-meson fragmentation function $D^{\sigma}_{\pm,\rho\tau}$ to

$$\hat{D}^{\sigma}_{\pm,\rho\tau} = (\mp i f_{\rho\tau\sigma} + d_{\rho\tau\sigma})\hat{D} \quad . \tag{3.23}$$

In the notation of Section 2, for the fragmentation into pions this reads:

$$\hat{D}^{\pi^+}_u = 2\hat{D} \equiv V^{\pi} , \quad \hat{D}^{\pi^+}_d = \hat{D}^{\pi^+}_s = 0 \tag{3.24}$$

Notice, that the intermediate step of Eq. (3.21) is not necessary. By direct comparison of Eqs. (3.15) and (3.20) one can simultaneously obtain the duality restrictions on $\hat{G}$ and $\hat{D}$.

We now have a dual description of the pure resonant component. However, in order to construct a successful phenomenological model, one must include Pomeron terms. At the four-point function level [5] duality diagrams were identified with quark parton graphs. As a result, the resonant component only involved valence quarks, and the Pomeron part-sea quarks only. We have used the same identification principle for the purely resonant part of the six-point function. In this spirit the inclusion of Pomeron terms in the latter is obvious: replace the valence quark lines in either the target hadron or the observed hadron by "sea" quark lines. It is important to note that this is the only possible way allowed by the production mechanism of the quark parton model (see Figs. 3 and 4). This amounts to the assumption that not only the target hadron but also the observed hadron could be looked upon as made up of valence and sea quark partons. Since the sea is an SU(3) singlet, its contribution is

$$G^{c}_{\pm,\alpha\beta} = \sqrt{\tfrac{2}{3}}\, G^{P} \delta_{\alpha\beta} \delta_{c0}$$
$$D^{\sigma}_{\pm,\rho\tau} = \sqrt{\tfrac{2}{3}}\, D^{P} \delta_{\rho\tau} \delta_{\sigma 0} \tag{3.25}$$

To summarize Eqs. (3.23) and (3.25), we see that the D_i^h functions can be decomposed into a sum of two terms [14]:

$$D_i^h = V_i^h + S^h \tag{3.26}$$

In the quark parton model V_i^h, the resonance (valence) component is that part of D_i^h in which the fragmenting quark can be found in the hadron h (observed in the final state) as a valence quark. S^h, the background (sea) component is that part of D_i^h in which the quark i ends up anywhere else in the final state.

Strictly speaking, the derivation for the resonance part presented above only applies to six-point functions where all hadrons are mesons. However, in the case where either the target or the observed hadron (but not both) is a baryon, one can still apply the duality constraints at the level of the six-point function. This can be done only within the framework of the quark parton model where the two baryon lines are adjacent (see Fig. 4a). In this way, one can also decompose the D_i^h (where now h is a baryon) into a sum of a valence component and a sea component. Due to the well-known baryon-anti-baryon problem in duality, we cannot impose duality constraints at the six-point function level when both the target and the detected hadron are baryons. However, we can restrict both the d_t^i and D_i^h in such a process by considering two separate cases: one in which only the target is a baryon, and another in which only the observed hadron is a baryon. Combining now the two component duality decompositions of d_t^i for a baryon target with that of D_i^h for an observed baryon we can apply our results to the current fragmentation region of current + baryon → baryon + anything.

In the case where at least one of the hadrons is a meson, the duality constraints imposed on each hadron vertex, coupled with the factorization property of the parton model [Eq. (2.2)] cover the full duality content of the six-point function. When the two hadrons are baryons this cannot be said, and the baryon-anti-baryon problem remains.

The correspondence between the production mechanism in the quark parton model and the duality description is graphically represented in Fig. 4.

The results mentioned above can be obtained as well in a different manner. For the d_t^i one uses the two-component duality description of the total deep inelastic structure functions. To constrain the D_i^h one can assume that they appear in the single hadron inclusive distributions in deep e^+e^- annihilation [2),12)]. This follows from assuming the production mechanism shown in Fig. 5. There, the virtual photon is converted into a quark-anti-quark pair, and each of these now fragments into hadrons. The single particle inclusive cross-section obtains the form [2),12)]:

$$\frac{d\sigma}{dz}(e^+e^- \longrightarrow h + X) = \frac{8\pi\alpha^2}{3Q^2}\tilde{F}^h(z) \quad , \tag{3.27}$$

where

$$\tilde{F}^h(z) = \frac{1}{2}\sum_i Q_i^2 \int D_i^h(z, h_T^2)\, dh_T^2 \quad . \tag{3.28}$$

Q^2 is the virtual photon mass, and z is the fraction of the parton momentum carried by the observed hadron. Here z is given by

$$z = \frac{2h\cdot Q}{Q^2} \tag{3.29}$$

In a manner analogous to the procedure in the total deep inelastic case [5)], one can apply the two-component duality ideas here, and rederive Eq. (3.26) [13)].

4. SUMMARY OF RESULTS

The duality constraints as explained in Section 3 imply the following decomposition of D_i^h. For pion emission, if the sea is an SU(2) singlet [from now on called "SU(2) duality"; in this context see a remark in Section 5]:

$$\left.\begin{aligned} D_u^{\pi^+} &= V^\pi + S^\pi \\ D_d^{\pi^+} &= S^\pi \\ D_s^{\pi} &= S'^\pi \end{aligned}\right\} \quad \text{SU(2) duality}, \tag{4.1}$$

The other D_i^π functions are related to those of Eq. (4.1) by isospin and charge conjugation invariance. For SU(3) duality [the sea, an SU(3) singlet]:

$$S^\pi = S'^\pi \tag{4.2}$$

For the production of K mesons:

$$\left.\begin{aligned} D_u^{K^+} &= V^K + S^K \\ D_{\bar{s}}^{K^+} &= V'^K + S'^K \\ D_{\bar{u}}^{K^+} = D_d^{K^+} &= D_{\bar{d}}^{K^+} = S^K \\ D_s^{K^+} &= S'^K \end{aligned}\right\} \quad \text{SU(2) duality}, \tag{4.3}$$

and for SU(3) duality:

$$S'^K = S^K \quad , \quad V'^K = V^K \quad . \tag{4.4}$$

For the emission of a proton one obtains:

$$\left.\begin{aligned} D_u^p &= V_u^p + S^p \\ D_d^p &= V_d^p + S^p \\ D_{\bar{u}}^p &= D_{\bar{d}}^p = S^p \\ D_s^p &= D_{\bar{s}}^p = S'^p \end{aligned}\right\} \quad \text{SU(2) duality} \tag{4.5}$$

SU(3) duality implies:

$$S'^p = S^p \tag{4.6}$$

The complete analogy between (4.5) (for a produced proton) and (3.1) (for a target proton) is obvious.

a. Sum Rules and Inequalities

By using the SU(2) duality results of Eq. (3.1) alone one obtains, in the zero approximation for the Cabibbo angle, the following sum rules:

$$\begin{aligned} 5\left(L_{1,\pi^+}^{\nu n} - L_{1,\pi^+}^{\nu p}\right) &= 6\left[4\left(L_{1,\pi^+}^{ep} - L_{1,\pi^+}^{en}\right) + L_{1,\pi^-}^{ep} - L_{1,\pi^-}^{en}\right] \\ 5\left(L_{1,\pi^-}^{\nu n} - L_{1,\pi^-}^{\nu p}\right) &= 6\left[4\left(L_{1,\pi^-}^{ep} - L_{1,\pi^-}^{en}\right) + L_{1,\pi^+}^{ep} - L_{1,\pi^+}^{en}\right] \end{aligned} \tag{4.7}$$

The difference of these two sum rules does not depend on duality constraints and is valid in the quark parton model [2),12)].

Using SU(2) duality constraints for the d^i [Eq. (3.1)], and SU(3) duality for the D_i^π [Eqs. (4.1), (4.2)] and the relations:

$$\int_0^1 v^u \, dx = 2$$
$$\int_0^1 v^d \, dx = 1 \qquad (4.8)$$

we can now relate the e^+e^- structure functions [Eq. (3.28)] to the electroproduction structure functions [Eq. (2.2)]. We obtain the following sum rule:

$$\frac{d\sigma}{dz}(e^+e^- \longrightarrow \pi^{\pm,0} + X) = \frac{4\pi\alpha^2}{3} \cdot \frac{2}{7} \int dx \, dk_T^2 \cdot$$
$$\left[13\, L^{ep}_{1,\pi^+} + 15\, L^{ep}_{1,\pi^-} - 14\, L^{en}_{1,\pi^+} - 14\, L^{en}_{1,\pi^-} \right] \quad . \qquad (4.9)$$

Within the framework of the quark parton model one can obtain inequalities among the semi-inclusive distribution functions analogous to inequalities obtained by Nachtmann [19], since all the functions d^i and D_i^h are positive quantities. The basic inequalities are:

$$2d^u - d^d \geq 0$$
$$2d^{\bar{d}} - d^{\bar{u}} \geq 0 \qquad (4.10)$$

and

$$2D_u^p - D_d^p \geq 0$$
$$2D_{\bar{d}}^p - D_{\bar{u}}^p \geq 0 \qquad (4.11)$$
$$3D_u^{\pi^+} - D_d^{\pi^+} \geq 0 \ , \ \text{etc.}$$

These follow from isospin invariance and positivity constraints.

The bounds obtained using the general quark parton model are not so stringent, e.g., they do not imply an excess of π^+ over π^- in electroproduction [20),21)]. These bounds are given in the first row of the Table for the structure function L_1.

However, if we apply duality constraints with separate positivity for the Pomeron and resonance components, Eqs. (4.10) and (4.11) give:

$$2v^u - v^d \geq 0 \quad , \quad s, s' \geq 0 \tag{4.12}$$

and

$$\begin{gathered} 2V_u^p - V_d^p \geq 0 \quad , \quad S^p, S'^p \geq 0 \\ V^\pi \geq 0 \quad , \quad S^\pi, S'^\pi \geq 0 \quad , \quad \text{etc.} \end{gathered} \tag{4.13}$$

From there we obtain the inequalities shown on the second row of the Table. In the third row we present inequalities for structure functions integrated over x.

The experimentally observed excess of π^+ over π^- in electroproduction off protons [20),21)], as well as the excess of K^+ over K^- [20)] are now explained by imposing duality on D_i^π and D_i^K. Similarly, an excess of π^+ over π^- is predicted in neutrino reactions (see the Table).

b. Threshold Behaviour

In the duality scheme, the dominance of the resonant contribution to the total deep inelastic structure function at $x \to 1$, which is the approach to threshold, implies that $s(x)$ and $s'(x)$ vanish faster than either $v^u(x)$ or $v^d(x)$. If the experimental ratio $F_2^{en}(x)/F_2^{ep}(x)$ actually approaches its boundary value, $1/4$, as $x \to 1$, one finds in addition that

$$\frac{v^d(x)}{v^u(x)} \longrightarrow 0 \ , \ \frac{s(x)}{v^u(x)} \longrightarrow 0 \ , \ \frac{s'(x)}{v^u(x)} \longrightarrow 0 \quad \text{as } x \to 1 \ . \tag{4.14}$$

As $z \to 1$, which is the approach to threshold, one similarly expects:

$$\frac{S^\pi}{V^\pi} \ , \ \frac{S'^\pi}{V^\pi} \longrightarrow 0$$

which translates into (in this connection, see a remark by Feynman [2]):

$$\frac{D_d^{\pi^+}}{D_u^{\pi^+}}\ ,\ \frac{D_s^{\pi}}{D_u^{\pi^+}} \longrightarrow 0 \qquad \text{as } z \longrightarrow 1 \tag{4.15}$$

(4.15) implies that in the limit $z \to 1$:

$$\frac{L^{en}_{1,\pi^-} - L^{ep}_{1,\pi^-}}{L^{ep}_{1,\pi^+} - L^{en}_{1,\pi^+}} \longrightarrow \frac{1}{4} \quad (\text{for all } x)\ . \tag{4.16}$$

In the limit $x \to 1$ and $z \to 1$, Eqs. (4.14) and (4.15) imply

$$\frac{L^{ep}_{1,\pi^-}}{L^{ep}_{1,\pi^+}} \longrightarrow 0\ ,\ \frac{L^{en}_{1,\pi^+}}{L^{ep}_{1,\pi^-}} \longrightarrow 0\ ,\ \frac{L^{ed}_{1,\pi^-}}{L^{ed}_{1,\pi^+}} \longrightarrow \frac{1}{4}\ , \tag{4.17}$$

where d stands for the sum $p + n$.

5. REMARKS AND CONCLUSIONS

In this paper we have discussed the current fragmentation region of the single hadron inclusive distributions in the deep inelastic lepton-nucleon interactions. We have shown that by identifying quark parton graphs with duality diagrams the purely resonant component obtains a simple form. It is given by one duality diagram only, shown in Fig. 4a. In addition, within the quark parton model, the generalization of the two-component duality idea is simple and straightforward.

By imposing duality constraints the number of independent structure functions describing the process under study is reduced to a large extent. We obtain sum rules and a special factorization property. Separate positivity for the resonance and background components borrowed from strong interaction phenomena, namely, that the cross-section for non-exotic reactions is larger than for the exotic ones ($\sigma_{K^-p} > \sigma_{K^+p}$, $\sigma_{\bar{p}p} > \sigma_{pp}$), enables us to obtain stringent bounds on various reactions. In particular, the observed excess of π^+ over π^- and K^+ over K^- in electroproduction [19),20)] comes out as a result of our approach.

We would like to conclude with a few remarks.

1) The factorizable form of Eq. (2.2) is an important ingredient in the quark parton model, which we have strongly used when imposing duality constraints. However, even if a weaker form of factorization were valid, e.g., only after integration over the transverse momentum h_T, all the results presented here would remain the same for the structure functions, now integrated over h_T. Such a weaker factorization property of the quark parton model can indeed be derived in a field theoretical treatment [22].

2) In Sections 3 and 4 we have used the concepts "SU(2) duality" and "SU(3) duality" implying that the sea of a hadron is either an SU(2) or an SU(3) singlet. This corresponds to the object exchanged in the t channel being an SU(2) or an SU(3) singlet. However, we have not used an SU(3) classification of the hadron states. For instance, even if the seas of a pion and a kaon are SU(3) singlets, they can be different. In the total deep inelastic case SU(2) or SU(3) duality amounts only to the assumption that the currents couple in an SU(2) or an SU(3) symmetric way, correspondingly. An approach of a similar spirit has been employed in Refs. 9), 23).

3) In the total deep inelastic case it is sometimes assumed that the Regge limit can be achieved from the scaling limit of the structure functions by allowing x to go to zero (or $\omega = 1/x \to \infty$). It is perhaps of interest to make a similar conjecture in the semi-inclusive case, especially in view of the recent interest in exoticity criteria in hadron physics [24].

Since we have introduced the two-component duality idea into the description of the semi-inclusive cross-section, we are able to decompose it to its various contributions (Fig. 4). Thus, we find that a necessary and sufficient condition for the absence of all resonant terms in the cross-section is that ab and $a\bar{c}$ be exotic (where a is the current, b is the target, and c is the observed hadron). Thus, our duality scheme, imposed on the quark parton model, dictates a definite exoticity criterion.

In the single Regge limit of the current fragmentation region, $x \to 0$ ($\omega = 1/x \to \infty$) and z fixed and different from 1, the condition that the expansion of the structure function has no $\omega^{-\frac{1}{2}}$ term is: ab exotic. This results from the fact that the whole ω dependence is contained in the target vertex $(d^i(x))$.

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Many helpful discussions were held with Bruno Renner. We dedicate this work to his memory.

Model \ Ratios of L's	$\frac{ep \to \pi^-}{ep \to \pi^+}$	$\frac{en \to \pi^-}{ep \to \pi^+}$	$\frac{ed \to \pi^-}{ed \to \pi^+}$	$\frac{en \to \pi^+}{ep \to \pi^+}$	$\frac{en \to \pi^-}{ep \to \pi^-}$	$\frac{(en \to \pi^+) + (en \to \pi^-)}{(ep \to \pi^+) + (ep \to \pi^-)}$	$\frac{\nu p(n) \to \pi^-}{\nu p(n) \to \pi^+}$	$\frac{\nu d \to \pi^+}{ed \to \pi^+}$	$\frac{\nu d \to \pi^-}{ed \to \pi^-}$	$\frac{(\nu d \to \pi^+) + (\nu d \to \pi^-)}{(ed \to \pi^+) + (ed \to \pi^-)}$
Quark Parton Model	$0 \leqq r \leqq 8$	$\frac{1}{4} \leq r \leq 4$	$\frac{1}{4} \leq r \leq 4$	$0 \leq r < 12$	$\frac{1}{12} \leq r < \infty$	$\frac{1}{4} \leq r \leq 4$	$0 \leq r \leq 3$	$0 \leq r \leq 18$	$0 \leq r \leq \frac{54}{7}$	$0 \leq r \leq \frac{18}{5}$
QPM constrained by duality	$0 \leq r \leq 1$	$\frac{1}{4} \leq r \leq \frac{3}{2}$	$\frac{1}{4} \leq r \leq 1$	$0 \leq r \leq 2$	$\frac{1}{4} \leq r < \infty$	$\frac{1}{4} \leq r \leq \frac{3}{2}$	$0 \leq r \leq 1$	$3^* \leq r \leq \frac{36}{5}$	$0 \leq r \leq \frac{18}{5}$	$3^* \leq r \leq \frac{18}{5}$
QPM + duality for L integrated over x	$\frac{1}{8} \leq r \leq 1$	$\frac{1}{4} \leq r \leq 1$	$\frac{1}{4} \leq r \leq 1$	$\frac{1}{2} \leq r \leq 1$	$\frac{2}{3} \leq r \leq 2$	$\frac{2}{3} \leq r \leq 1$	$0 \leq r \leq 1$	$3^* \leq r \leq \frac{36}{5}$	$0 \leq r \leq \frac{18}{5}$	$3^* \leq r \leq \frac{18}{5}$

Model \ Ratios of L's or $\tilde{F}$'s	$\frac{ep \to n}{ep \to p}$	$\frac{ed \to n}{ed \to p}$	$\frac{en \to p}{ep \to p}$	$\frac{ep \to K^-}{ep \to K^+}$	$\frac{ed \to K^-}{ed \to K^+}$	$\frac{e^+e^- \to K^0}{e^+e^- \to K^+}$	$\frac{\nu d \to p}{ed \to p}$	$\frac{(\nu d \to p) + (\nu d \to n)}{(ed \to p) + (ed \to n)}$
Quark Parton Model	$0 \leq r \leq 8$	$\frac{1}{4} \leq r \leq 4$	$0 \leq r \leq 8$	$0 \leq r \leq \infty$	$0 \leq r \leq \infty$	$\frac{1}{4} \leq r \leq 4$	$0 \leq r \leq 18$	$0 \leq r \leq \frac{18}{5}$
QPM constrained by duality	$0 \leq r \leq 2$	$\frac{1}{4} \leq r \leq \frac{3}{2}$	$0 \leq r \leq 2$	$0 \leq r \leq 1$	$0 \leq r \leq 1$	$\frac{1}{4} \leq r \leq 1$	$3^* \leq r \leq \frac{18}{5}$	$3^* \leq r \leq \frac{18}{5}$

Table : Bounds on semi-inclusive electroproduction of pions, nucleons and kaons in current fragmentation region and on e^+e^- - annihilation processes. Letter d stands for the sum of proton and neutron e.g. $(ed \to \pi^-) \equiv (ep \to \pi^-) + (en \to \pi^-)$, etc. By duality in this table we imply SU_2 duality on both d_i and D_i^h. However a number with star indicates that it has been derived from SU_3 duality.

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FIGURE CAPTIONS

Figure 1 :

The process $\ell + N \rightarrow \ell' + h + \text{anything}$.

Figure 2 :

Diagrammatic representation of the total inclusive deep inelastic cross-section within the quark parton model.

Figure 3 :

Diagrammatic representation of the single particle inclusive cross-section in the deep inelastic region within the quark parton model.

Figure 4 :

Correspondence between the dynamical description in the quark parton model (left-hand side) and the duality content (right-hand side).

a) Resonance-resonance (vV) term.
b) Resonance-Pomeron (vS) term.
c) Pomeron-resonance (sV) term.
d) Pomeron-Pomeron (sS) term.

Figure 5 :

Hadron production in e^+e^- annihilation. The photon is converted into a quark anti-quark pair and these fragment into hadrons.

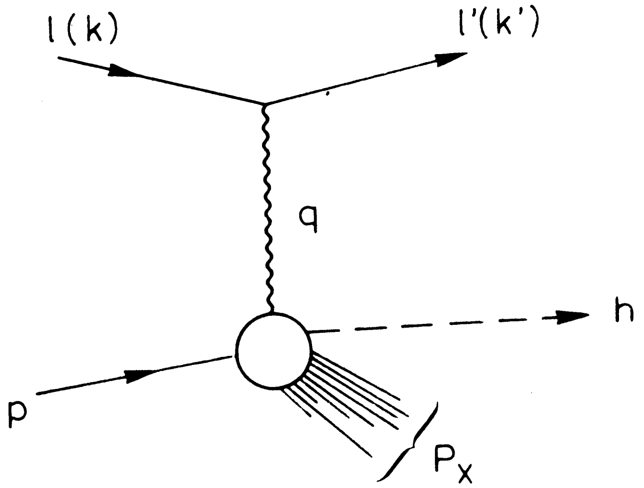

FIG.1

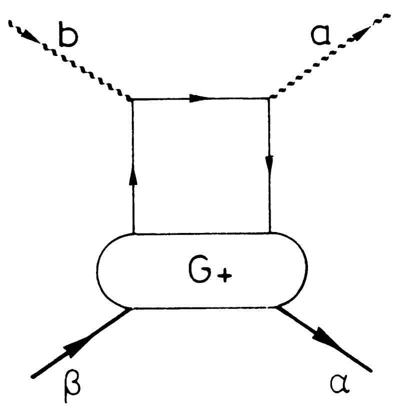

FIG. 2

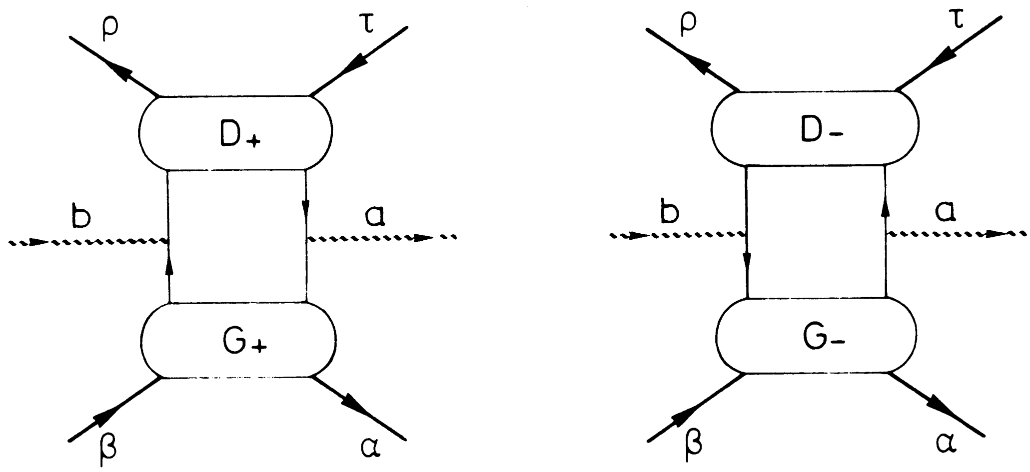

FIG. 3

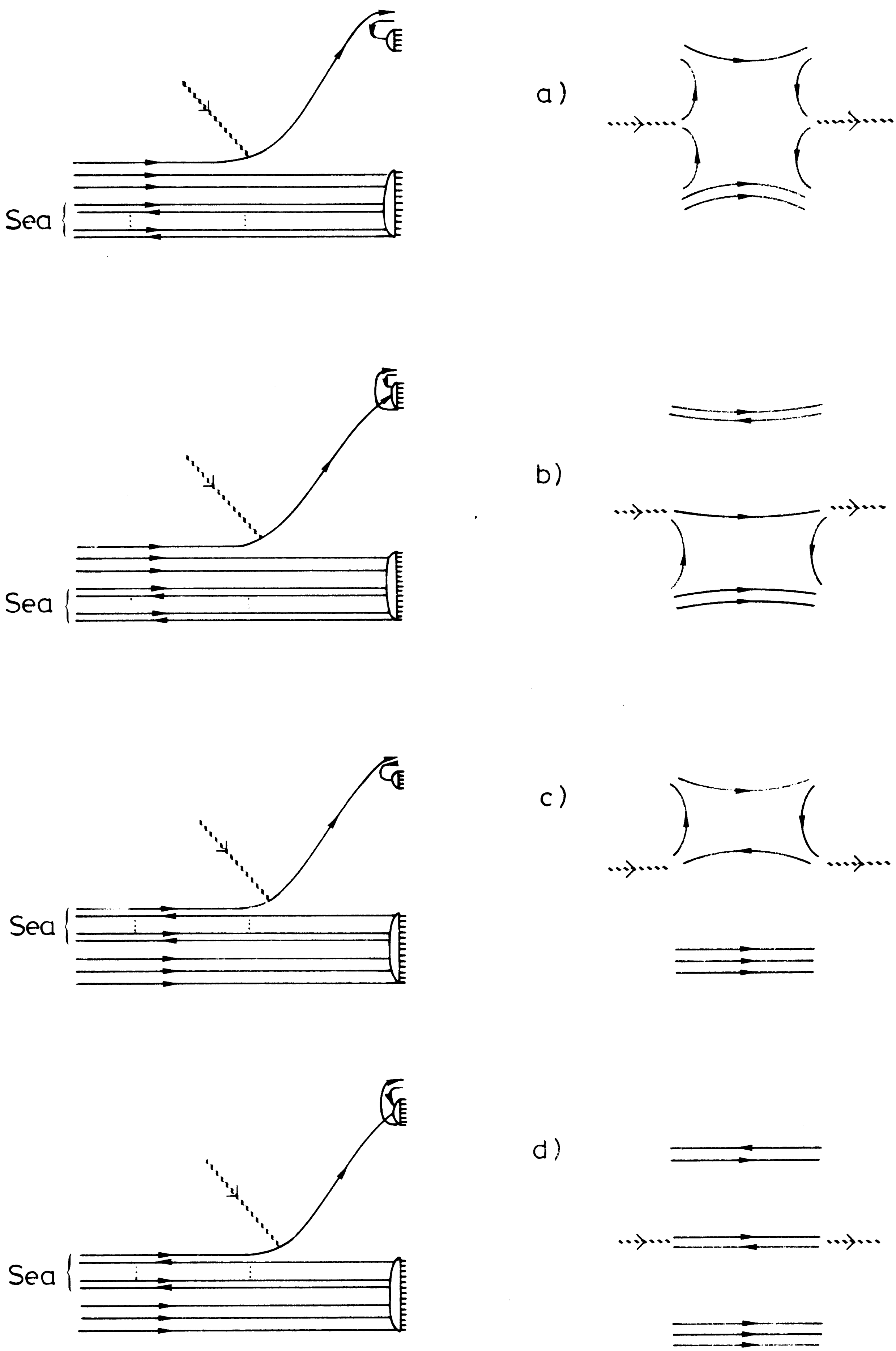

FIG. 4

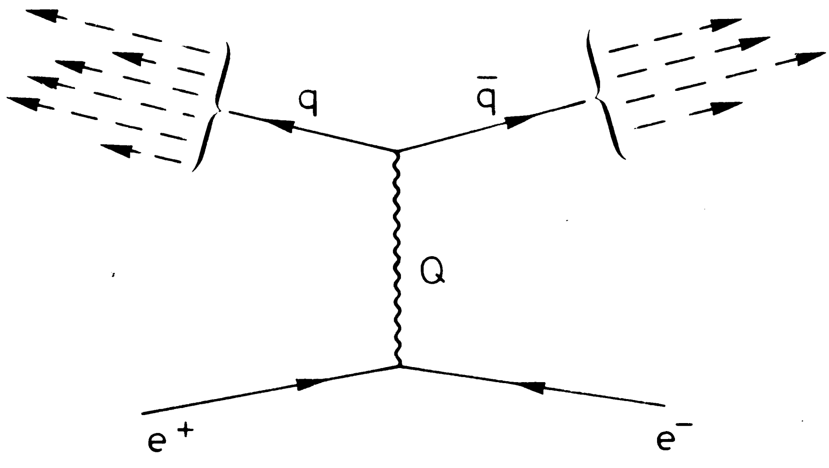

FIG. 5