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IS α' ALSO THE $e^{\pm}$ ANNIHILATION SCALE ?

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ABSTRACT

A model for the annihilation of electron-positron pairs into hadrons for $q^2 = s > 4\ \mathrm{GeV}^2$ and up to the highest measured energies is developed. The main ingredients are pointlike couplings between three resonances and photons and the high level of opening thresholds that follow from an effective Lagrangian formalism. The model gives an almost constant annihilation cross-section, soft pions everywhere and at all energies, high multiplicities and a large absolute cross-section. The details of all exclusive channels are also predicted. Other consequences of this point of view concern the discovery of hitherto ignored couplings in low energy photon initiated reactions. We argue that the energy regime explored by SPEAR is not the asymptopia and we discuss possible further constancy of the cross-section in a range of s.

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It has been widely believed that $e^{\pm}$ annihilation into hadrons would scale early, as happens in deep inelastic scattering [1]. A point-like constituent model predicts a falling cross-section of the form $e^{\pm} \to$ hadrons $\sim$ const/s, where s is the virtual mass. It has become clear that Nature has chosen otherwise [2],[3].

Here we discuss a model that may explain much of what is seen, though it might not be the only mechanism at work.

Although there is a bias in favour of assuming that bilinear and trilinear photon couplings are the whole story, it is natural - as we shall show - to expect quadrilinear pointlike vertices between a photon and three hadrons. A posteriori [4], the data suggest very strongly that, in the absence of fast pions and with high multiplicities, most of them are decay debris of resonances. Hence we propose a model that couples resonances in a pointlike scheme and whose structure is fully determined by a Lagrangian to which a covariant derivative procedure is applied.

The second element that makes the calculation possible and appealing is that the number of thresholds is large, about 10/100 MeV. Hence, each particular channel can only survive very little after it is opened. Its contribution is damped by a one-parameter absorption procedure which at the same time takes care of the form factor. In this case, competing channels and form factors are intimately related. Once the absorption parameter is given, the whole problem is defined and it is just a question of performing the calculations. They are straightforward but complicated.

Other consequences, like the presence of these terms in other processes and new couplings are discussed at the end.

To motivate the discussion, we start by a well-known process in electro- and photoproduction : $\gamma N \to \pi^- \Delta^{++}$ [5]. This channel is a large piece (about 1/3) of the total photon cross-section at $E_\gamma = 700$ MeV. Writing a strong interaction Lagrangian for the hadrons and performing a covariant derivative trick, one obtains contact terms that, together with the pole terms of the external states is called the electric Born term model [6)-8)]. For photoproduction, its success is beyond expectations and, as seen in Fig. 1, it describes, for a large range of energies, all angular distributions, the size of the cross-section, and decay correlations. This effective Lagrangian is the best phenomenological model for intermediate energy we know of. The absorption, applied when the channel competes with new channels opening, works very well.

When the photon becomes massive, the multiplication of Feynman terms by a form factor works well [9], this may be thought of as the absorption in the current channel. Indeed, the procedure has been generalized to many (all the known ones) two-body and quasi-two-body reactions with similar success, though the data are unfortunately not as good [10].

Numerically, it is always the case in all standard gauges that the contact term is large near threshold and the pole terms characterize the small decreasing term of the amplitude, mainly because it contributes to the less absorbed peripheral waves. The explicit term that describes the process is given below.

Our starting point is to write a Lagrangian which reproduces in an economical way the well-known low-lying mesonic and baryonic states and their couplings. For this purpose we write an $SU(6)_W$ Lagrangian [11] for the lowest-lying baryons (56) and mesons (35). Furthermore, higher excitations with $\ell = 1$ and higher appear. This strong interaction Lagrangian will have the form

$$\mathcal{L}_0 = \mathcal{L}_{0\,free} + \mathcal{L}_{0\,int}$$

where the interaction terms are of the form $35\times35\times35$ and $\overline{56}\times56\times35$, and so on. There is one coupling constant per triplet of representations which is determined from some decay process involving only strong interactions. As an example, we write explicitly a typical mesonic term [11]

$$\mathcal{L}_{VVV} = 3g\Big\{\Big[m_\rho \,\mathrm{Tr}\Big(\Big[\frac{F_{\mu\nu}}{m}\Big]V_\mu V_\nu\Big) + \frac{m_\rho^2}{2}\,\mathrm{Tr}\Big[\Big(\frac{F_{\mu\nu}}{m}\Big)\Big(\frac{V_\mu}{m}\Big)V_\nu\Big] +$$
$$+ \mathrm{Tr}\Big[\Big(\frac{F_{\mu\nu}}{m}\Big)V_\mu\Big(\frac{V_\nu}{m}\Big) - \mathrm{Tr}\Big[F_{\mu\nu}\Big(\frac{V_\mu}{m}\Big)\Big(\frac{V_\nu}{m}\Big)\Big] - \frac{2}{3}m_\rho \mathrm{Tr}\Big[\Big(\partial_\rho\frac{V_\nu}{m}\Big)\Big(\partial_\nu\frac{V_\rho}{m}\Big)\Big(\partial_\mu\frac{V_\mu}{m}\Big)\Big]\Big]$$

where $\left(\frac{V}{m}\right)^\alpha_\beta = \frac{V^\beta_\alpha}{m_{T\,\beta}^{\ \alpha}}$ and $m_T = \begin{bmatrix} m_\rho & m_\rho & \frac{m_{K^*}^2}{m_\rho} \\ m_\rho & m_\rho & m_{K^*}^2/m_\rho \\ \frac{m_{K^*}^2}{m_\rho} & \frac{m_{K^*}^2}{m_\rho} & \frac{m_\phi^2}{m_\rho} \end{bmatrix}$ (1)

and a typical baryonic term :

$$\mathcal{L}_{\Delta N\pi} = g\,\Delta_\mu\, p\, \partial_\mu \phi_\pi \qquad (2)$$

where Δ_μ is the Rarita-Schwinger wave function for $\Delta(1236)$ and p is the proton spinor. This, of course, is the term discussed previously.

It should be clear that we are not using the $SU(6)$ Lagrangian to count states and to relate economically all couplings. Unitarity problems never appear since these terms are used as effective Lagrangians near threshold.

We now introduce electromagnetism by a covariant derivative technique. We need not worry about renormalizability as we just explained, but it is indeed amusing to notice that some terms are renormalizable according to modern theories[12]. In this model, as opposed to the resonance photoproduction region, the competing channels and the form factor can be parametrized by a single absorption parameter. To be able to argue in general, we first discuss a specific channel in detail. A typical interaction term reads :

$$\mathcal{L}_{\rho\pi\pi\gamma} = e\, g_{\rho\pi\pi}\left[2 A_\mu \rho^0_\mu \pi^+\pi^- - A_\mu \rho^-_\mu \pi^0\pi^+ - A_\mu \rho^+_\mu \pi^-\pi^0\right] \qquad (3)$$

The cross-section becomes :

$$\sigma = \frac{3}{s^2}\,\alpha^2\,\frac{g^2_{\rho\pi\pi}}{4\pi}\;\frac{1}{\pi^2}\int dR_3\;\delta^T_{\mu\nu}\;\mathrm{Pol}_{\mu\nu}(\rho) * \text{absorption} \qquad (4)$$

where R_3 is the three-particle invariant phase space with dimensions in GeV^2 and

$$\frac{s}{2}\,\delta^T_{\mu\nu} = \frac{1}{4}\,\mathrm{Tr}\left\{ \not{p}_{e^-}\gamma_\mu \not{p}_{e^+}\gamma_\nu \right\} + O(m_e^2) \qquad (5)$$

where the superscript T indicates transversality with respect to the beams. $\mathrm{Pol}_{\mu\nu}(\rho)$ is the ρ polarization tensor. Near threshold, the expression reduces to

$$\sigma = \frac{3\pi}{s^2}\,\alpha^2\,\frac{g^2_{\rho\pi\pi}}{4\pi}\sqrt{\frac{m_\pi^2 m_\rho}{(m_\rho+2m_\pi)^3}}\left[\sqrt{s}-(m_\rho+2m_\pi)\right]^2 \qquad (6)$$

where the non-relativistic form of the phase space has been used. Notice that the dimensionality of the three-body phase space is such as to guarantee the cancellation of one power of s as desired. The numerical factors include a multiplicity factor for charge and spin, the latter being 2. This channel opens at ~1 GeV and is isolated. Hence, it is probably competing with the tail of the vector meson terms and the other presumed resonant states (ρ'). Nevertheless, we apply to it the same absorption of the form $e^{-(E-E_{th})^2/\Theta^2}$ where Θ is the absorption parameter fixed at one energy. The result is shown in Fig. 2. The channel gives about 1.3×10^{-33} cm^2 near threshold and grows unless absorbed. It is reassuring that this is the magnitude observed for the channel $2\pi^+2\pi^-$ and that it seems that its dominant part is $\rho^0\pi^+\pi^-$ and not the uncorrelated four-pion production [13]. It is important to emphasize that obtaining the appropriate size of the cross-section is not a minor achievement. Indeed, these local couplings are a poor man's parton model in which the resonances are the partons.

Since this channel can be measured well, it is interesting to notice that one also predicts $(\sigma \to 2\pi^+2\pi^-)/(\sigma \to 2\pi^0\pi^+\pi^-) = 4$.

Since we mentioned before the large number of channels that open, it is hopeless to attempt to compute each of them explicitly. Nevertheless, and to be able to substantiate the arguments given below, we calculated a large number of channels that differ in mass relations, spin, couplings, momentum dependence and charge structure [14]. Some of them are depicted in Fig. 2. These channels always contribute the same amount within a factor of two. There are many fluctuating factors, but these are random and tend to compensate. Hence, it is possible to define an average cross-section by the formula

$$\sigma = K \cdot C \ \frac{1}{s^2} [\text{Polarization sums}] * \text{Density of states}$$

The last factor is slowly varying, as can be seen in Fig. 3, and it is well approximated by a continuous function, as can be checked from the Table [15]. K contains threshold factors that are momentum and mass dependent, and also includes the absorption factor. C contains all numerical factors and couplings. At 2.5 GeV, the cross-section is about 25 nanobarns. Notice that K and C are s independent. <u>The sole s dependence is left in the spin sum and the flux propagator factor</u>. Essentially only two spin sums are independent because only one partial wave is present. The resonances themselves are on linear trajectories and hence the numerator goes as $\alpha'^2 s^2$ and the whole expression is approximately constant ! We have verified explicitly channels at 4 and 5 GeV, and the effect obtains [14]. Clearly, we do not expect an exact linear law; indeed, we expect fluctuations. In this mesonic resonance annihilation region α' is the relevant scale, once more as in resonant hadron physics.

Let us now discuss other specific predictions of the model. The first interesting point is the change in the over-all properties expected at about 2.5 GeV, in which this mechanism is presumably dominant. This giant threshold marks the replacement of one and two resonance final states into three resonance final states.

The angular distributions are for the one-pion inclusive system rather flat, though forward and backward extra terms are present. The two-body correlations are also flat since pions are spilled from three resonances in all directions. Clearly, for a specific channel, all exact distributions can be given. The inclusive scaling distribution can be computed and we show a typical channel. Figure 4 shows the rapid growth and exponential decay of the distribution of pions that can be parametrized as e^{-6p_π} for large p_π. Physically, this is due to the fact that at all energies one is producing resonances at rest. These resonances cascade and as their mass grows it is well known that the number of pions also grows. A small increase in average energy seems possible, but we have not calculated these fine effects. The few fast pions require special treatment. These particles come probably from channels with one pion and two resonances in the final state. We will present predictions concerning them in another paper. The multiplicity can be calculated from the list of states. Since there are three resonances in the final state, the multiplicity is high. At $s \sim 2.5$ GeV, the multiplicity is ~ 5, while it rises very fast to reach about 8 at 5 GeV. As opposed to most "smooth" models, the multiplicity in this theory is very narrowly distributed. One does not expect 8 or less than 4 particles in the final state, except for corrections like the η decay.

The abundance of particles is also fully determined. Though the Lagrangian conserves SU(3), since the resonances decay into pions, there is - as a consequence - a net increase of the pion-to-kaon ratio. Without corrections for decays one expects about 6% of kaons and a slight decrease of their proportion as energy increases, essentially because higher resonances decay always into one kaon, but more and more pions. The baryon pairs are highly suppressed because the channel density is small. It is amusing to remakr that the Lagrangian of Eq. (2) gives a large contribution in this region as well. One expects about 0.5% baryon pairs. Because of the positive G parity of the Lagrangian, purely pionic states should have an even number of pions before decay corrections. The ratio of charged to neutral pions is also given at all energies and we will give details elsewhere.

As we pointed out at the beginning, there are probably other competing mechanisms in this region. We point a few to underline the difficulties of this intricate region.

1) Direct channel resonances. It is conceivable that even at 3 GeV or more there are some vector particles being formed both of positive and negative G parity.

2) The free Lagrangian also generates terms [16]. The density of states is very low, but not negligible.

3) Kroll–Ruderman–like terms. If one considers the reaction : photon of mass s plus pion (soft) going through a resonance of the same mass s and decaying into another resonance and a soft pion, one avoids denominators. This requires a G parity degeneracy of the spectrum in the absence of zeros in external momenta. We do not believe it is a likely possibility.

It is indeed difficult to find ways to produce large cross-sections with couplings other than pointlike. Of course, if the strong effective Lagrangian has direct couplings with more than three hadrons, then other possibilities are open. However, this seems unlikely, with the only exception perhaps of a four–pion coupling, as in the sigma model.

The final question one would like to discuss is asymptopia. In this model, given the strong interaction resonance spectrum, one can generate terms that will keep the cross–section constant. Since resonances are known with masses up to 3 GeV, it is conceivable that the cross-section will go constant or even grow, up to $s = 80\,\mathrm{GeV}^2$. This depends, of course, on whether couplings are still large and the density of states remains at the observed values.

It is also important to look for consequences of this approach elsewhere. To begin with, we know that in the resonance region it works. In fact, this was our starting point. Terms like the one we discussed contribute to the radiative ρ decay. This term, which may be renormalizable, gives a width

$$\Gamma_{\rho\to\pi\pi\gamma} = \frac{1}{2m_\rho} 4 e^2 g^2_{\rho\pi\pi} \frac{1}{\pi^3} \int dR_3 = .3\,\mathrm{MeV} \qquad (7)$$

It is amusing that this contribution makes this decay larger than the $\rho\to\pi\gamma$. Its presence would be a nice test of the general ideas and would show once more that there are new terms in photon initiated reactions. Notice that the spectrum of gammas will be quite different as compared to bremsstrahlung. These terms may also be relevant to explain the large inelastic Compton cross-section [17] and can also contribute to two–photon processes. For deep inelastic scattering, these ideas seem of little use. There, the photon vertex shows a large mismatch of masses because the photon is spacelike and there is no hope that the first order effective Lagrangian can help. In some specific channels, however, these terms may show up.

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TABLE : Thresholds in MeV of the opening channels

π	π	ρ	1050
π	K	K^*	1527
π	π	f	1550
π	ρ	ω	1694
K	K	ρ	1760
K	K	ω	1774
π	π	f_1	1794
π	K^*	K^*	1924
π	ρ	φ	1929
η	K	K^*	1936
π	π	G	1960
π	η	A_2	1999
K	K	φ	2009
π	A_1	ρ	2010
π	K	K^{**}	2055
η	ρ	ρ	2089
⋮	⋮	⋮	⋮

REFERENCES AND FOOTNOTES

1) J.D. Bjorken, Cornell Conference on Photon Interactions at High Energy (1971).

2) A. Litke et al., Phys.Rev.Letters 30, 1189 (1973).

3) B. Richter, Invited talk at the Irvine Conference, January 1974.

4) Most of the results of this paper were established some time ago.

5) For a review of these processes, see :
A. Lucke and P. Söding, Springer Tracts of Modern Phys., Vol.59 (1971).

6) R. Cutkosky and F. Zachariasen, Phys.Rev. 103, 1108 (1956).

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10) P.V. Collins, H. Kowalski, H. Romer and H.R. Rubinstein, Phys.Letters 44B, 183 (1973).

11) B. Sakita and K. Wali, Phys.Rev. 139, B1355 (1965).

12) M. Veltman, invited talk at the Bonn Conference on Photon Interactions, (1973), and references therein.

13) F. Ceradini et al., Phys.Letters 43B, 341 (1973).

14) We used the program for the algebraic reduction of Feynman diagrams : Schoonship by M. Veltman, and performed the integrations numerically with FOWL. Thus we have checked about 15 channels explicitly. A few are depicted in Fig. 2.

15) We have computed the density of states by feeding all well-known resonances from the Tables and we have used their physical masses. The density above 4 GeV of our list may receive contributions of hitherto unknown states.

16) These terms were originally calculated by :
N. Cabibbo and R. Gatto, Phys.Rev. 124, 1577 (1961).

17) This process shows also an anomalous large cross-section. See Bloom's talk at Bonn, 1973.

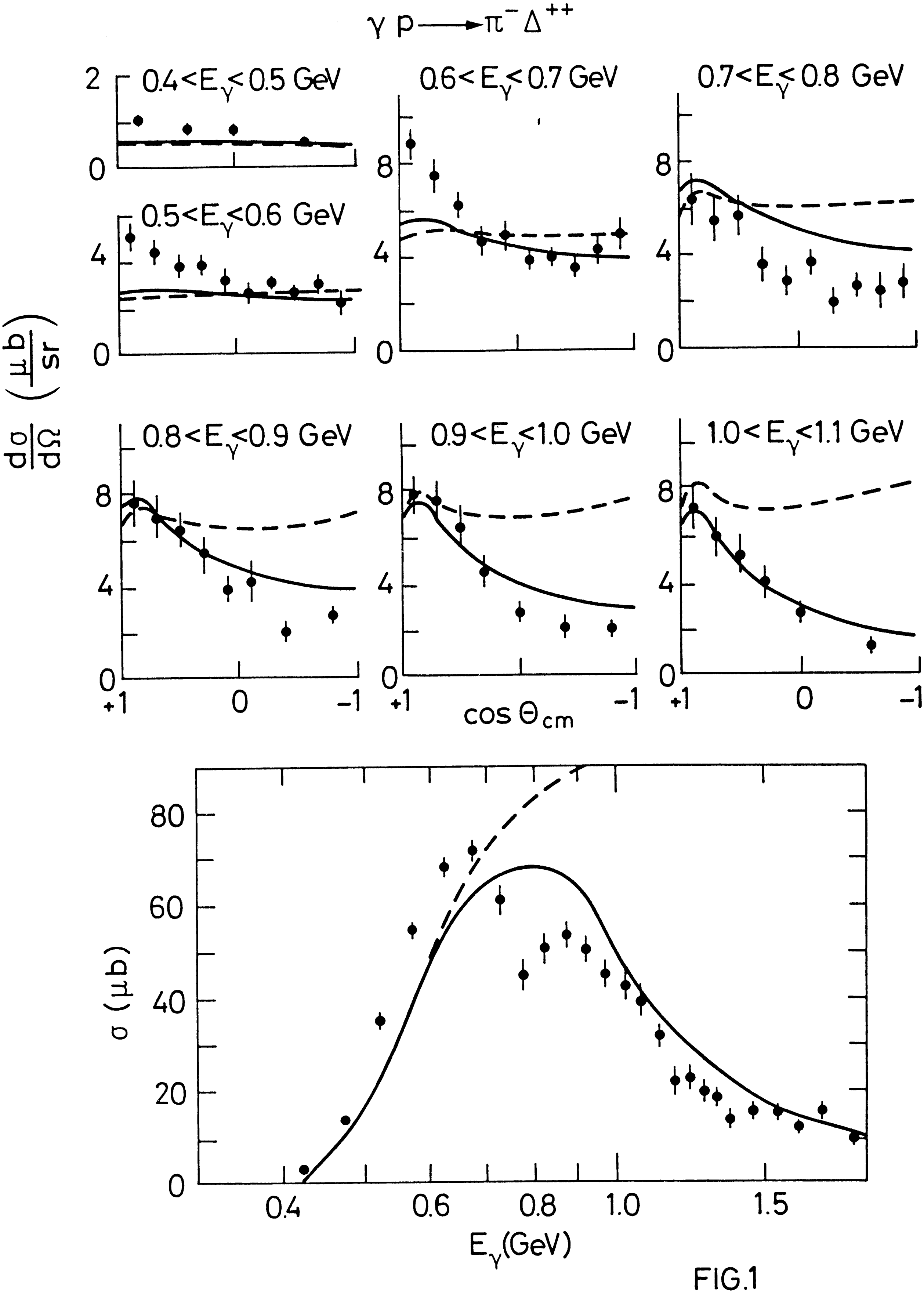
FIGURE CAPTIONS

Figure 1 : Over-all energy dependence and angular distributions in the stated reaction taken from Ref. 5). Helicity density matrices are shown in that reference.

Figure 2 : Contributions to the cross-sections with $\Theta = 800$ MeV (see text). The dotted lines are the contributions without absorption.

Figure 3 : Density of thresholds as a function of energy.

Figure 4 : Pion spectrum for the stated channel. The smooth curve is passed through the points coming from a numerical integration.



γ p ⟶ π⁻ Δ⁺⁺
0.4<Eγ<0.5 GeV
0.5<Eγ<0.6 GeV
0.6<Eγ<0.7 GeV
0.7<Eγ<0.8 GeV
0.8<Eγ<0.9 GeV
0.9<Eγ<1.0 GeV
1.0<Eγ<1.1 GeV
dσ/dΩ (μb/sr)
cos Θcm
+1
0
−1
σ (μb)
Eγ(GeV)
0.4
0.6
0.8
1.0
1.5
20
40
60
80

FIG.1

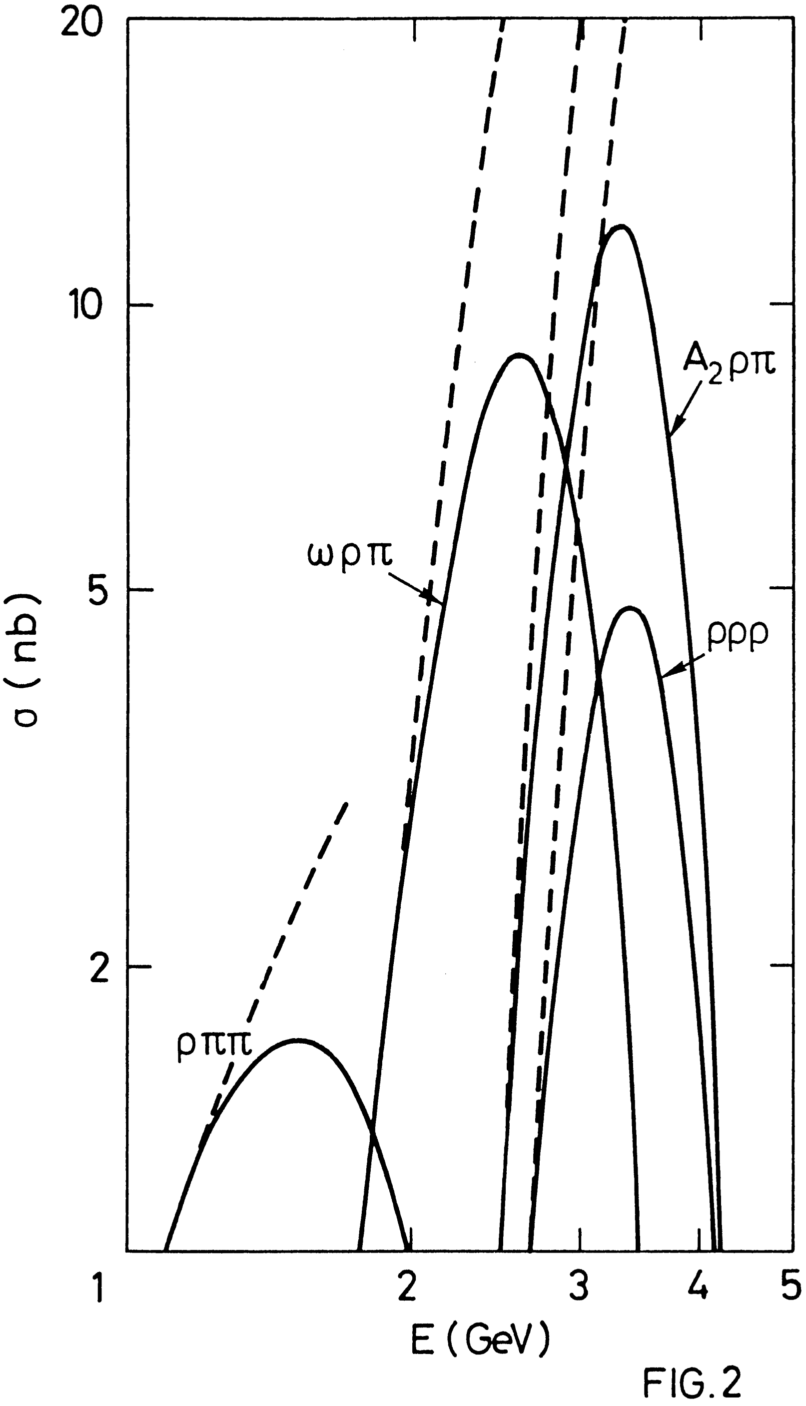


FIG.2

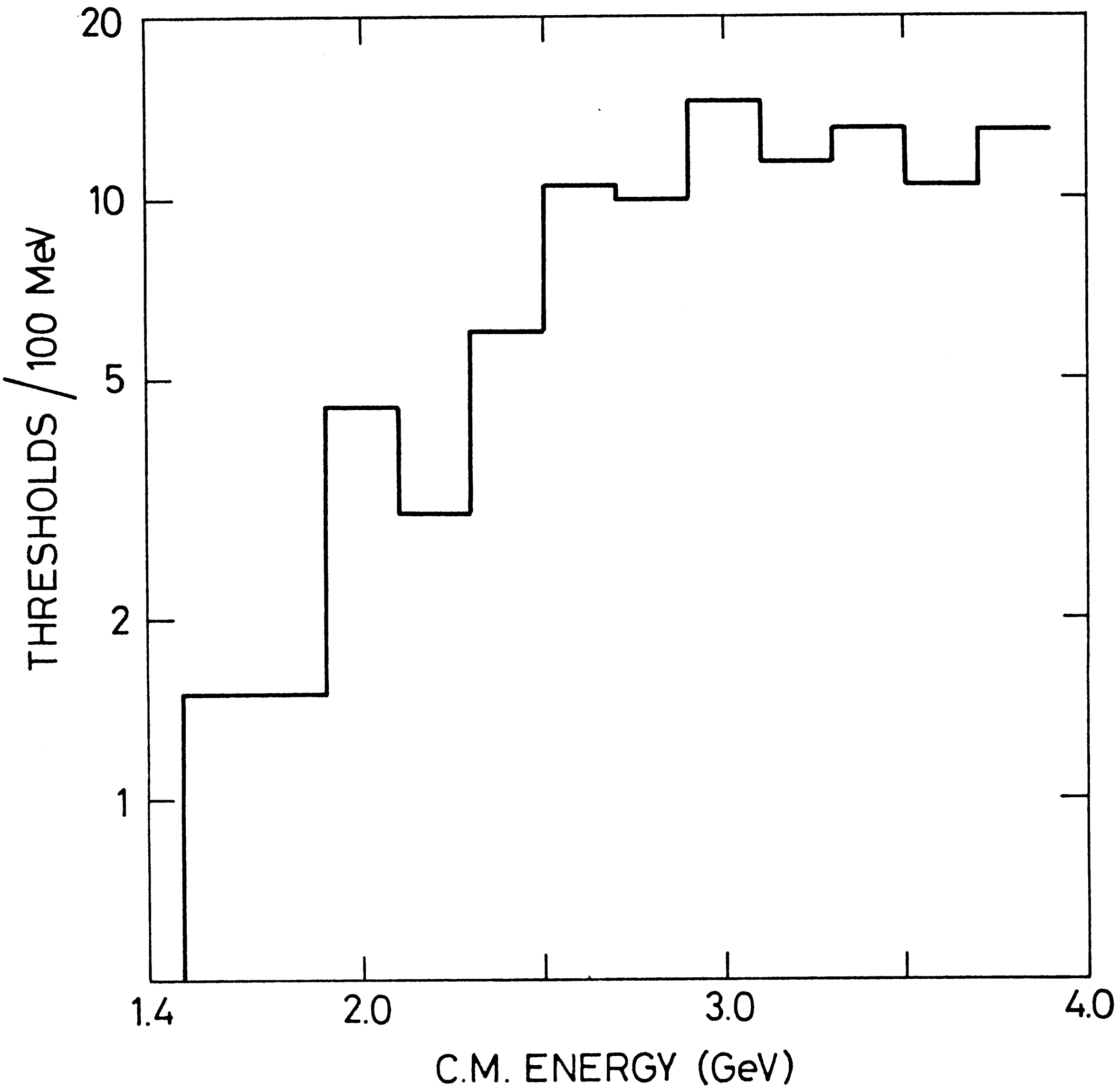
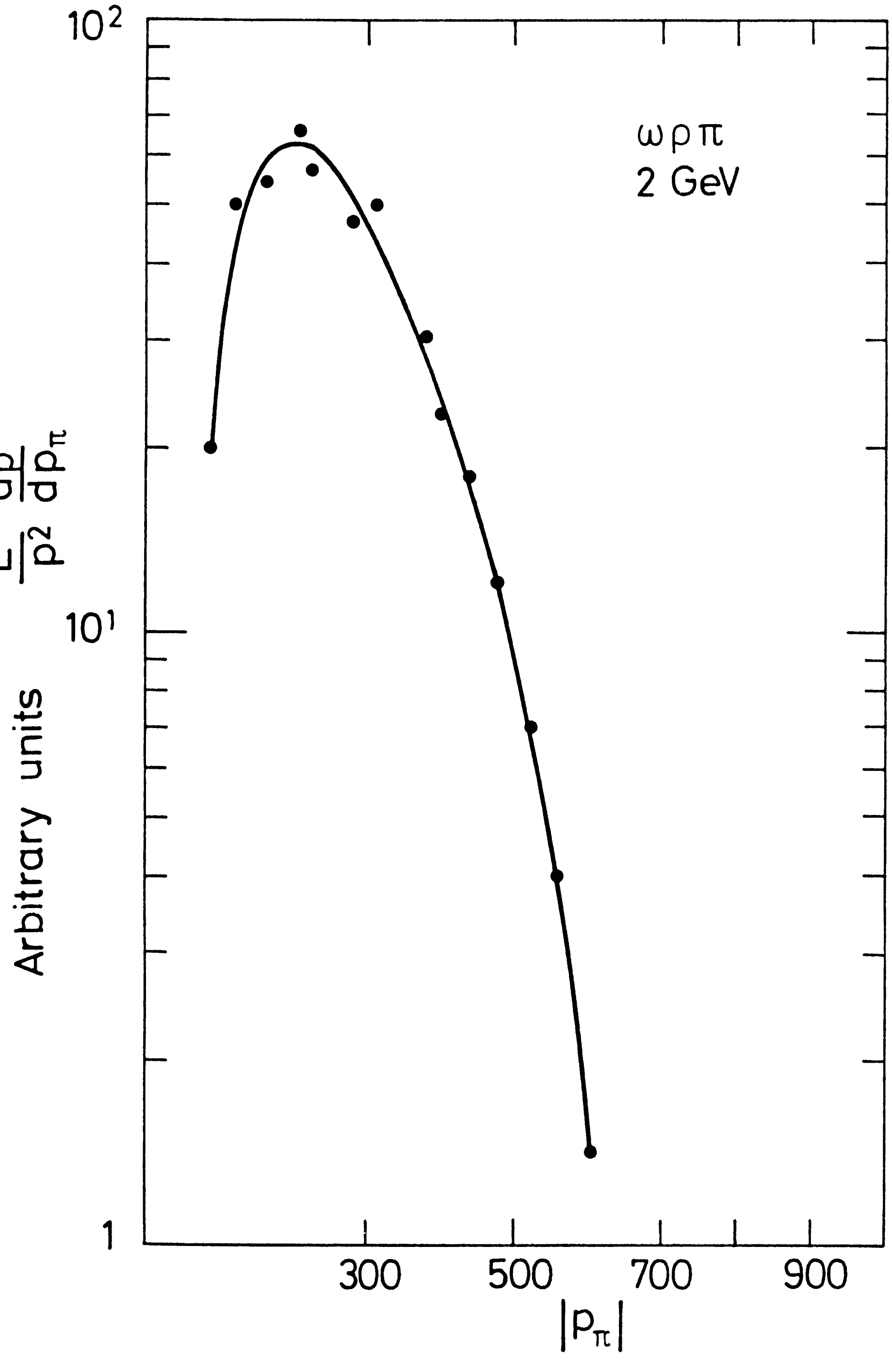


FIG. 3



10^2
10^1
1
ω ρ π
2 GeV
Arbitrary units $\frac{E}{p^2}\frac{d\rho}{dp_\pi}$
300
500
700
900
$|p_\pi|$

FIG. 4