Ref.TH.1687-CERN

DOUBLE SCATTERING AND FINAL STATE INTERACTION IN XD → YNN

G. Alberi
CERN - Geneva
and
Istituto di Fisica Teorica, Miramare, Trieste

M.A. Gregorio *) and Z.D. Thomé +)
International Centre for Theoretical Physics,
Miramare, Trieste

A B S T R A C T

A unified approach to double scattering, as well as the final state interaction of the two nucleons at small and large momentum transfers are given. The closure sum rule for the final state interaction at small momentum transfers is shown explicitly in a simple model for the deuteron wave function and nucleon interaction. An application for the process $K^+D \to K^0PP$ is given, trying to explain discrepancies present in recent experiments.

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*) Fellow of the Consejo Nacional de Investigaciones Científicas y Técnicas de Argentina, on leave of absence from Universidade Nacional de La Plata, Argentina.

+) On leave of absence from COPPE and Istituto di Fisica, Universidade Federal de Rio de Janeiro, Brasil.

Ref.TH.1687-CERN
14 June 1973

1. - INTRODUCTION

The interest in doing experiments of the type $XD \rightarrow YNN$ lies in the possibility of extracting information, from the data for this reaction, on the amplitude $X \rightarrow Y$ for a free neutron target. The extraction is performed using the single scattering impulse approximation [1] which reduces the initial three-body problem to two independent two-body problems. In the past, the data were not precise enough to show any discrepancy from this theory [2], but nowadays several experiments [3],[4] have high enough statistics to show a marked discrepancy for high momenta of the "spectator". The "spectator" is defined as the slower of the two nucleons and this discrepancy casts some doubt on its actual physical rôle : that is at least for high momenta, it seems to interact with the incident particle or with the recoiling nucleon. These two types of secondary interactions are distinguished in the following by the name of double scattering for the former and by final state interaction for the latter.

In this paper we are concerned with the calculation of both effects keeping the independence hypothesis contained in the impulse approximation, but allowing successive multiple collisions. The framework is the Watson multiple scattering theory in the impulse approximation [1] generated by Feynman graphs. This technique was exploited extensively by Shapiro's group [5] in nuclear reactions ; it was used by Ericson and Locher [6] to study the analytic properties of pion and nucleon-nucleus scattering amplitude, and it was successively used for scattering of elementary particles off deuteron [7],[8].

The formalism has the following features :

1) it is naturally relativistic invariant [9],[10] ;

2) it contains the Glauber theory as its limit for high energies and small angles, for a deuteron target [7], as well as for heavier nuclei as shown by Gribov in the forward direction [11] and recently generalized by Bertocchi [12] for small angles ;

3) as shown by Aitchison a long time ago [13], it shows remarkable correspondence to dispersion theory, in the treatment of final state interaction at low momentum transfer, and it is consistent, at least in a simple model, with the closure sum rule, as will be shown in this paper.

The plan of the paper is the following. In Section 2 we consider the evaluation of the double scattering, linearizing the internal propagator ; using a Gaussian parametrization of the deuteron wave function, in S and D waves, we are able to derive analytical expressions for it.

In Section 3, we consider final state interaction for high values of the momentum transfer, where the linearization procedure can be applied to the internal nucleon propagator, in a perfectly symmetrical way : even here the D state of the deuteron is taken into account.

In Section 4, we study the final state interaction at small momentum transfers, where the energies of the nucleons are non-relativistic, and the relative diagram can be calculated analytically. We prove the closure sum rule.

Section 5 is devoted to the application of the formalism to the process $K^+D \to K^o pp$.

In Appendix A, we give some details about the calculation of integrals entering the double scattering and the final state interaction at large momentum transfer. In Appendix B, we show in detail the cancellation which gives rise to the closure sum rule for the final state interaction at small momentum transfer.

2. - DOUBLE SCATTERING

Using traditional Feynman rules and an additional rule for the deuteron vertex [9], we can write down the diagrams of Fig. 1. The kinematics is self-understandable from the figure, provided the dashed line represents the incident particle (π , K meson) and possibly its excited states in the final state, and the continuous lines the nucleons.

Figure 1a gives :

$$\hat{T}_1 = (16\pi^3 M)^{1/2} \left[\psi(\vec{\epsilon} + \tfrac{\vec{\Delta}}{2}) \hat{T}_\alpha(t) + \psi(\vec{\epsilon} - \vec{\Delta}/2) \hat{T}_\beta(t) \right]^{*)} \qquad (1)$$

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*) We write this trivial transcription of the diagram 1a to make clear which normalization and which phase we choose. We emphasize that we do not consider the phase factor (-), which would come naturally from Feynman rules.

where M is the deuteron mass and α, β are the spin and isospin indices for the elementary scattering operator. The calculation of diagram 1b was already done in Ref. 14) and refined in Ref. 15) to include recoil effects. However, we repeat briefly the essential points, including the spin of the deuteron. Following the procedure for the elastic scattering [5] we take the lines $\vec{\eta}+\vec{\epsilon}$ and $\vec{k}_0-\vec{\eta}$ on the mass-shell ($\vec{k}_0=\vec{k}-\vec{\Delta}/2$) and we get

$$\hat{T}_2=\frac{i}{4}\sqrt{\frac{M}{\pi}}\int d^{(3)}\vec{\eta}\,\frac{\psi(\vec{\eta}+\vec{\epsilon})}{\omega(\vec{k}_0-\vec{\eta})\,E(\vec{\eta}+\vec{\epsilon})}\,\hat{T}_\alpha(t_1)\,\hat{T}_\beta(t_2)\,\delta(Q) \qquad (2)$$

where

$$Q=E^{+}+\omega^{+}-\omega(\vec{k}_0-\vec{\eta})-E(\vec{\epsilon}+\vec{\eta})$$

and

$$E^{\pm}=E(\vec{\Delta}/2\pm\vec{\epsilon})$$

and

$$\omega^{\pm}=\omega(\vec{k}_0\pm\vec{\Delta}/2)$$

E and ω represent the energies of the nucleon and of the hadron, where the notation is that of Ref. 14). The wave function in this case includes the spin-dependent part [16]

$$\psi(\vec{q})=\psi_0(|\vec{q}|)-\hat{O}(\vec{q})\,\psi_2(|\vec{q}|)$$

$$\hat{O}(\vec{q})=\frac{1}{\sqrt{2}}\left(3.\,\frac{(\vec{J}.\vec{q})^2}{q^2}-2\right) \qquad (3)$$

where $\vec{J}$ is the spin of the deuteron. Using a Gaussian parametrization of the wave function [16]

$$\psi_0(q)=\sum_i A_i\,e^{-\alpha_i q^2}$$

$$\psi_2(q)=\sum_i B_i\,e^{-\beta_i q^2}\,q^2 \qquad (4)$$

and a Gaussian form of the amplitudes, we can perform all integrals analytically, provided we linearize the propagator Q in the variable $\vec{\eta}$. The linearization can be done whenever $|\vec{\eta}|$ (and therefore $|\vec{\epsilon}|$ because $|\vec{\eta}+\vec{\epsilon}|$ is kept small by the deuteron wave function) is small with respect to k_o and we can neglect therefore second order corrections in the expansion of $\omega(\vec{k}_o-\vec{\eta})$. Under these conditions

$$Q \cong \frac{k_o}{\omega_o}\cdot(-Y_{\parallel}+\eta_{\parallel}) \tag{5}$$

and $Y_{\parallel} = \omega_o + m - E^+ - E^-$, $\omega_o = \omega(k_o)$ and m is the nucleon mass. After some standard integration we get

$$\hat{T}_2 = \frac{i}{4}\sqrt{\frac{M}{\pi}}\cdot\frac{\hat{T}_\alpha(0)\,\hat{T}_\beta(0)\,e^{b_\alpha t_1^o}\,e^{b_\beta t_2^o}}{\left(1-\frac{Y_{\parallel}\cdot k_o}{\omega_o^2}\right)\cdot k_o\cdot m}\cdot\left[I_S - I_D\right] \tag{6}$$

where

$$t_1^o = (\omega_o-\omega_+)^2 - 2Y_{\parallel}(\omega_o-\omega_+)\frac{k_o}{\omega_o} - Y_{\parallel}^2 - Y_{\parallel}\cdot\Delta_{\parallel} - \Delta^2/4$$

$$t_2^o = (\omega_- - \omega_o)^2 + 2Y_{\parallel}(\omega_- - \omega_o)\frac{k_o}{\omega_o} - Y_{\parallel}^2 + Y_{\parallel}\cdot\Delta_{\parallel} - \Delta^2/4 \tag{7}$$

and

$$I_S = \sum_i A_i\, J(\alpha_i, S, D)$$

$$I_D = \sum_i B_i\, J(\beta_i,S,D)\cdot\left\{\hat{O}(f_i) + \frac{1}{\sqrt{2}}\left[\frac{3}{2}\left(\vec{J}^2 - \frac{(\vec{J}\cdot\vec{k}_o)^2}{k_o^2}\right) - 2\right]\cdot\frac{1}{(\beta_i+S)}\right\} \tag{8}$$

where

$$J(\gamma,S,D) = \frac{\pi}{\gamma+S}\cdot e^{-\gamma(\epsilon_{\parallel}+Y_{\parallel})^2} * \exp\left\{-\gamma\,\vec{\epsilon}_\perp^{\,2} + \frac{(\gamma\vec{\epsilon}_\perp + D/2\cdot\vec{\Delta}_\perp)^2}{\gamma+S}\right\}$$

$$\vec{f}_i = \frac{S\,\vec{\epsilon}_\perp - \frac{D}{2}\vec{\Delta}_\perp}{\beta_i+S} + (Y_{\parallel}+\epsilon_{\parallel})\,\vec{k}_o/k_o$$

$$S = b_\alpha + b_\beta \qquad , \qquad D = b_\alpha - b_\beta$$

$$\vec{\epsilon}_\perp = \vec{\epsilon} - \frac{\vec{\epsilon}.\vec{k}_o}{k_o} \cdot \left(\frac{\vec{k}_o}{k_o}\right) , \qquad \vec{\Delta}_\perp = \vec{\Delta} - \frac{\vec{\Delta}.\vec{k}_o}{k_o} \cdot \left(\frac{\vec{k}_o}{k_o}\right)$$

are the components of the relative momentum of the two nucleons and of the tri-momentum transfer on the plane orthogonal to $\vec{k}_o$. As was shown in Ref. 14), this formula tends to the formula of Glauber and Franco [17] but as it is, it can be used at higher momentum transfers, containing the recoil effect.

The aspect of this formula tells us that, only in the region of small values of $|\vec{\epsilon}|$, the effect is important and it is suppressed for large values of $|\vec{\epsilon}|$.

We have not specified, to simplify the formalism, the possible intermediate isospin states of the incident hadron and the nucleon pair [17] ; however, they are taken in account in the final calculation.

3. - FINAL STATE INTERACTION AT HIGH MOMENTUM TRANSFER

When the momentum transfer Δ is large we expect the fast recoiling nucleon to interact with the spectator nucleon in a perfectly symmetric way as the fast hadron in the double scattering term. In a similar fashion, we put on the mass shell the spectator nucleon and the fast recoiling nucleon [*]. We can start again from formula (2), but it is convenient to change the variable and call the internal variable of the diagram, as shown on Fig. 2. We then obtain

$$\hat{T}_3 = \frac{i}{4}\sqrt{\frac{m}{\pi}} \int d^{(3)}\vec{\xi} \, \frac{\psi(\vec{\xi})}{E(\vec{\xi}-\vec{\Delta})\cdot E(\vec{\xi})} \cdot \left[T_\alpha(t)\, T_\nu(t_2) + T_\beta(t)\, T_\mu(t_2) \right] \delta(Q) \qquad (9)$$

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*) A similar treatment was given by Smith and Wilkin for the case of electron deuteron scattering [18].

where

$$Q = E_+ + E_- - E(\vec{\xi}) - E(\vec{\Delta} - \vec{\xi}) \qquad (10)$$

and

$$t_2 = (E_+ - E(\vec{\Delta} - \vec{\xi}))^2 - (\vec{\Delta}/2 + \vec{\epsilon} - \vec{\Delta} + \vec{\xi})^2 \qquad (11)$$

and t is the momentum transfer of the hadron. T_μ and T_ν denote nucleon-nucleon amplitudes, where μ, ν indicate the appropriate isospin combination *). If Δ is large enough, we can again linearize the propagator Q and get

$$Q \cong \frac{\Delta}{E(\Delta)} (\xi_{\parallel} - Z_{\parallel}) \qquad (12)$$

where

$$Z_{\parallel} = \frac{E(\Delta)}{\Delta} \cdot (E(\Delta) + m - E^+ - E^-) \qquad (13)$$

This time the reference axis is given by the direction of the momentum transfer, which is the forward direction for the nucleon-nucleon scattering. As a matter of fact, the transverse components of $\vec{\xi}$ are small, because of the damping factor in the deuteron vertex and by momentum conservation the same can be said of $\vec{\epsilon}_{\perp}$ while $\epsilon_{\parallel}$ is of the order of $\Delta/2$. From this observation, it follows that the momentum transfer in the second scattering is

$$\begin{aligned} t_2 &= -(\vec{\epsilon}_{\perp} + \vec{\xi}_{\perp})^2 - (\Delta/2 - \epsilon_{\parallel} - Z_{\parallel})^2 \cdot \left(1 - \frac{(\Delta/2 + \epsilon_{\parallel})^2}{E^2(\Delta/2 + \epsilon_{\parallel})}\right) \\ &= -(\vec{\epsilon}_{\perp} + \vec{\xi}_{\perp})^2 + t_{20} \end{aligned} \qquad (14)$$

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*) μ and ν are direct and charge-exchange scattering for neutron-proton in the final state and correspond both to pp scattering for charge-exchange processes but with opposite c.m. angle and opposite sign. In the high energy limit, however, the second term can be safely neglected.

The first part is the usual transverse part of the momentum transfer, while the second part is the recoil correction ; the integrals in (9) can be performed analytically, provided we assume for the nucleon-nucleon amplitude an exponential dependence on the momentum transfer and no spin dependence

$$\hat{T}_3 = \frac{i}{4}\sqrt{\frac{M}{\pi}}\;\frac{T_\alpha(t)\,T_\nu(0)\,e^{b_\nu t_{20}}(J_S^\nu - J_D^\nu) + T_\beta(t)\,T_\mu(0)\,(J_S^\mu - J_D^\mu)}{\left(1 - \frac{Z_{\parallel}\cdot\Delta}{E^2(\Delta)}\right)\Delta . m} \tag{15}$$

$$J_S^\nu = \sum_i A_i \frac{\pi}{b_\nu + \alpha_i} \exp\left(-\alpha_i Z_{\parallel}^2 - \frac{\alpha_i b_\nu}{b_\nu + \alpha_i}\epsilon_\perp^2\right)$$

$$J_D^\nu = \sum_i B_i \frac{\pi}{b_\nu + \beta_i} \exp\left(-\beta_i Z_{\parallel}^2 - \frac{\beta_i b_\nu}{b_\nu + \beta_i}\epsilon_\perp^2\right) \times \Big\{ \hat{O}(\vec{v}_i) + \tag{16}$$

$$+ \frac{1}{\sqrt{2}}\left[\frac{3}{2}\left(\vec{J}^2 - \frac{(\vec{J}\cdot\vec{\Delta})^2}{\Delta^2}\right) - 2\right]\frac{1}{b_\nu + \beta_i}\Big\}$$

where

$$\vec{v}_i = -\frac{b_\nu}{b_\nu + \beta_i}\vec{\epsilon}_\perp + Z_{\parallel}\frac{\vec{\Delta}}{\Delta}$$

The immediate consideration one can make, looking at the formula, is that the effect will be relevant only if the transverse part of the relative momentum is small and suppressed when $\epsilon_\perp$ is very large. This is reasonable, because we have assumed that at high momentum transfers, the nucleon-nucleon energy is high and the scattering is concentrated in the forward direction. Now the question arises which is the region of validity of this formula : the region is limited at high momentum transfers, for two reasons, first the approximation of linearizing the propagator is justified only if $<\xi^2>/<\Delta^2> \ll 1$, second the assumption that all nucleon-nucleon scattering in the forward direction is true only if $\sqrt{\Delta^2} \geq 1.5$ GeV/c, as is seen from displaying the slope of the nucleon-nucleon differential cross-section as function of incident momentum [19]. Therefore, we would suggest that the latter is the actual limit of application of this formula.

4. - FINAL STATE INTERACTION AT SMALL MOMENTUM TRANSFERS

When the momentum transfer is small, however, the situation changes completely : the momentum transfer is of the same order as the average Fermi momentum $<\Delta^2> \sim <\xi^2>$ and the diagram of Fig. 2 becomes non-relativistic. It is a fortunate circumstance that in this limit the diagram becomes integrable analytically : this circumstance will allow us to derive an interesting property and to make calculations at the end.

Although the calculation appears already in the literature [20),21)] we feel that for sake of completeness we have to repeat it, at least along its main lines. The starting point is still the diagram of Fig. 2, with the same convention for the variables. Using the same procedure as before, we put the line $\vec{\xi}$ on the mass shell : we do not proceed further because, at low energy, the cancellation with higher order terms is not definitely occurring as at high energy [22)]. A posteriori, we can say that this is the right choice, because in that way, we obtain a result exactly consistent with closure.

The formula for the triangular diagram is therefore

$$\hat{T}_4 = -\frac{1}{4}\sqrt{\frac{M}{\pi^3}}\int \psi(\xi)\,\frac{T_\alpha(t)\,T_\nu(t_2)+T_\beta(t)\,T_\mu(t_2)}{[E^+ + E^- - E(\vec{\xi})]^2 - (\vec{\Delta}-\vec{\xi})^2 - m^2 + i\varepsilon}\cdot\frac{d^{(3)}\vec{\xi}}{E(\vec{\xi})} \qquad (17)$$

If the momentum transfer is small $\Delta^2 \ll m^2$, we can take the non-relativistic limit for the propagator [20)]

$$\begin{aligned}[E^+ + E^- - E(\vec{\xi})]^2 - (\vec{\Delta}-\vec{\xi})^2 &\cong 2m\left(E^+ + E^- - E(\vec{\xi})\right) - (\vec{\Delta}-\vec{\xi})^2 \\ &\cong 2\left(\epsilon^2 - \Delta^2/4 - \xi^2 + 2\,\vec{\Delta}\cdot\vec{\xi}\right)\end{aligned}$$

Now, to go any further, we are bound to assume that the amplitudes do not depend on internal variables : this is certainly justified for the hadron-nucleon amplitude and it is justified even for the angular dependence of the nucleon-nucleon amplitude, at least at low energy. As far as the off-shell dependence of the nucleon-nucleon amplitude is concerned, we cannot tell too much, if we limit ourselves to the experimental evidence, and therefore we can assume, as a working hypothesis, that it is constant. Once this dependence is eliminated, we are left with an integral, which can be done analytically provided the form of the wave function is chosen to be a pole term in $\vec{\xi}$ or, in a more realistic case, as a sum of poles [23]. We will take the one pole case to work out the result for the amplitude, but that result is immediately generalized to more realistic wave functions. We get, therefore, by first integrating on the angle between $\vec{\xi}$ and $\vec{\Delta}$ and after, doing an integration in the complex plane :

$$\hat{T}_4 = -\sqrt{\frac{M}{\pi}}\sqrt{N}\,\frac{i}{4\Delta . E(i\chi)}\,(T_\alpha T_\nu + T_\beta T_\mu)\log\left(\frac{\epsilon - \frac{\Delta}{2} + i\chi}{\epsilon + \frac{\Delta}{2} + i\chi}\right) \quad (18)$$

if the wave function of the deuteron is defined (only S wave) as

$$\psi(\xi) = \frac{\sqrt{N}}{\xi^2 + \chi^2} \quad (19)$$

We will now show that when we have a spin triplet state for the nucleon-nucleon system in the final state and the corresponding scattering amplitude is approximated by a pole term, we get a result which is graphically expressed in Fig. 3. That is exactly the result of closure [17]. We will see also that in the case of a spin singlet state for the final nucleons, we do not get any effect of the final state interaction. If we consider only the S wave, there is no way to distinguish between T_ν and T_μ and we get for the total amplitude

$$\hat{T}_1 + \hat{T}_4 = (16\pi^3 M)^{1/2}\Big[T_\alpha \psi(\vec{\epsilon} + \vec{\Delta}/2) + T_\beta . \psi(\vec{\epsilon} - \vec{\Delta}/2) + \frac{i}{16\pi^2}\cdot\frac{\sqrt{N}}{\Delta . \xi_0} \times$$

$$\times (T_\alpha + T_\beta). T_\nu \log\frac{\epsilon + \Delta/2 + i\chi}{\epsilon - \Delta/2 + i\chi}\Big] \quad (20)$$

where $\xi_0 = E(-i\chi)$.

Transforming the amplitudes T in the centre-of-mass, one gets [10]

$$\hat{T}_1 + \hat{T}_4 = (16\pi^3 M)^{1/2} . 8\pi\sqrt{S_\alpha}\, f_\alpha . \Big[\psi(\vec{\epsilon} + \vec{\Delta}/2) + C . \psi(\vec{\epsilon} - \vec{\Delta}/2) +$$

$$+ \frac{i\sqrt{S_\nu}}{2\Delta\xi_0}(1 + C) f_\nu . \sqrt{N} . \log\frac{\epsilon + \Delta/2 + i\chi}{\epsilon - \Delta/2 + i\chi}\Big] \quad (21)$$

where f are the centre-of-mass amplitudes and $f_\beta = Cf_\alpha$, C being a complex function of the momentum transfer ; $\sqrt{s_\alpha}$, $\sqrt{s_\nu}$ are the centre-of-mass energies of the elementary process and of the nucleon-nucleon scattering, respectively.

Let us now integrate on the phase space of the broken deuteron, neglecting the fact that the angle of the fast particle is related to $\vec{\epsilon}$ through energy and momentum conservation. f_α can be taken out of the integral and we are left with the integration of the square brackets

$$\int [\quad]^2 d^{(3)}\vec{\epsilon} = \int |\psi(\vec{\epsilon}+\vec{\Delta}/2) + C.\psi(\vec{\epsilon}-\vec{\Delta}/2)|^2 d^{(3)}\vec{\epsilon} +$$

$$+ \frac{S.N}{\Delta^2 . 4\xi_0} |1+C|^2 . \int |f_\nu . \log \frac{\epsilon+\Delta/2+i\chi}{\epsilon-\Delta/2+i\chi}|^2 d^{(3)}\vec{\epsilon} - \tag{22}$$

$$- \frac{\sqrt{N}\sqrt{S}}{2\Delta\xi_0} . 2 \int \mathrm{Re}\left\{ i(1+C)^* f_\nu^* . \log \frac{\epsilon+\Delta/2+i\chi}{\epsilon-\Delta/2+i\chi} . \left[\psi(\vec{\epsilon}+\vec{\Delta}/2) + C.\psi(\vec{\epsilon}-\vec{\Delta}/2)\right]\right\} d^{(3)}\vec{\epsilon}$$

If we now assume that f_ν is just a pole term

$$f_\nu = \frac{i}{\epsilon - i\chi} \tag{23}$$

we can make all the integrations in (22) analytically and by making the approximation that $\sqrt{s} \sim 2\,\xi_0$, we can cancel one part of the third integral with the second one *) and we are left with

$$\int [\quad]^2 d^{(3)}\vec{\epsilon} = \int |\psi(\vec{\epsilon}+\vec{\Delta}/2) + C.\psi(\vec{\epsilon}-\vec{\Delta}/2)|^2 d^{(3)}\vec{\epsilon} -$$

$$- |1+C|^2 \frac{\pi N}{\Delta^2} \left(\int_{-\infty}^{\infty} \frac{1}{\epsilon - i\chi} \log^2 \frac{\epsilon+\Delta/2+i\chi}{\epsilon-\Delta/2+i\chi}\, \epsilon\, d\epsilon + \int_{-\infty}^{\infty} \frac{1}{\epsilon+i\chi} \log^2 \frac{\epsilon+\Delta/2-i\chi}{\epsilon-\Delta/2-i\chi}\, \epsilon\, d\epsilon \right) \tag{24}$$

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*) More details in Appendix B.

The latter integrals can be solved again by the complex plane method and we get

$$-|1+C|^2 \cdot \frac{4\pi^2 N}{\Delta^2} \cdot \chi \log^2 \frac{\chi - i\Delta/4}{\chi + i\Delta/4}$$

which can be simply related to the deuteron form factor $S(\Delta/2)$ in the one pole approximation and we get the final formula

$$\int [\quad]^2 d^{(3)}\varepsilon = 1 + |C|^2 + 2\,\mathrm{Re}[C^*]\, S(\Delta) - |1+C|^2 S^2(\Delta/2) \qquad (25)$$

This is the result of closure [17], which is remarkably reproduced in this simple model. In this model we have done the implicit assumption that the hadron-nucleon amplitude is pure non-spin flip and charge preserving. If the elementary amplitude is pure charge-exchange non-spin flip, the result is still (25), with $C = -1$. In this case we do not have an effect of the final state interaction in the S wave, because of the spatial antisymmetry of the final state. If we consider spin singlet states for the nucleon pair (with $I = 1$), which can be obtained using a pure spin-flip amplitude (charge preserving or charge exchange), the scattering length for the singlet nucleon-nucleon interaction is negative and the integrals in (24) are identically zero. The picture is therefore perfectly consistent with the closure sum rule.

This consistency is encouraging and we can use this model for actually calculating the final state interaction in deuteron dissociation induced by hadrons. However, one has to consider with better care the spin structure of the nucleon-nucleon amplitude, together with the spin structure of the elementary amplitude. The spin structure of the amplitude is changed in a Lorentz transformation and therefore one should in principle use the invariant forms for the hadron-nucleon and the nucleon-nucleon amplitude [24),25)]. However, since the energy is low, we can neglect such a Lorentz transformation of the amplitudes ; furthermore if the hadron-nucleon is diagonal in the space of the total spin of the nucleon pair, we can factorize some combinations of amplitudes in the differential cross-section. To do this we calculate the matrix elements for the spin triplet and spin singlet of the nucleon pair in the final state.

$$\langle 1m'' | \hat{T} | 1m \rangle = \langle m'' | f + i\sigma_{\alpha z} g | m' \rangle \cdot K \cdot \Big\{ \big[\psi(\vec{\epsilon} + \vec{\Delta}/2) + C\psi(\vec{\epsilon} - \vec{\Delta}/2) \big] \delta_{m'm} + \frac{i\sqrt{S_V}\cdot\sqrt{N}}{2\Delta\xi_0} \cdot \langle m' | f_v + c f_\mu | m \rangle \log \frac{\epsilon + \Delta/2 + iX}{\epsilon - \Delta/2 + iX} \Big\}$$

$$\langle 00 | \hat{T} | 1m \rangle = i g K \Big\{ \psi(\vec{\epsilon} + \vec{\Delta}/2) - c\psi(\vec{\epsilon} - \vec{\Delta}/2) + i \frac{\sqrt{S_V}\sqrt{N}}{2\Delta\xi_0} \langle s | f_v - c f_\mu | s \rangle \log \frac{\epsilon + \Delta/2 + iX}{\epsilon - \Delta/2 + iX} \Big\} \delta_{m0} \qquad (26)$$

where $K = (16\pi^3 M)^{\frac{1}{2}} 8\pi\sqrt{s_\alpha}$ and $C = +1$ for the isospin singlet state of the nucleon pair and -1 for the isospin triplet. Using these matrix elements, we can trivially calculate the differential cross-section for the charge exchange process and for the charge preserving one [10]. In this calculation a good guess for the centre-of-mass angle for the nucleon-nucleon scattering is $\cos\theta^* = (\vec{\epsilon}\cdot\vec{\Delta}/\epsilon\Delta)$: indeed, $\vec{\Delta}$ is the momentum of the centre-of-mass and $|\vec{\epsilon}|$ is the momentum of each nucleon in their centre-of-mass. For the actual calculation, one has to abandon the one-pole wave function and use the Hulthén or the Gartenhaus-Moravcsik [23] wave function.

5. - APPLICATION TO THE PROCESS $K^+D \to K^0pp$

We will now work out numerically an example of application and we will choose as incident particle the K meson : the reason is that the K nucleon interaction is weak compared with the nucleon-nucleon interaction, so that double scattering and final state interaction are distinguishable. Moreover we choose the charge exchange, because the K^0 is detectable in the bubble chamber and by detecting the recoiling proton, the complete kinematical reconstruction can be done even if the spectator proton is not visible and therefore we have the whole "spectator distribution" [3),4)], while the other experiment, with the neutron recoiling (like $K^+D \to K^+np$), does not allow it [26].

The "spectator" distribution is important, because on that distribution it is easy to check the validity of the single scattering approximation, just plotting against experimental data the probability distribution of the deuteron wave function (Moravcsik-Gartenhaus [23]) in Fig. 4)

$$\mathbb{P}(p_s) = 4\pi \left[\Psi_0^2(p_s) + \Psi_2^2(p_s) \right] p_s^2 \tag{27}$$

If we do that, including the D wave of the deuteron, which is not usually included in the comparison, done in the literature, we get a good agreement for $p_s \leq 250$ MeV/c and a rising discrepancy at higher momenta (see Fig. 4). The attitude generally taken is to throw away events with high spectator momenta, and limit the analysis to low momenta. This procedure can cause normalization problems, as shown in Ref. 3) for the case of K^* production, where the total production rate on a proton extracted from deuterium data is 20% lower than the same measured directly in hydrogen.

In order to solve the problem, one should try to calculate the effects of secondary interaction at low momenta and extrapolate it for large momenta, filling the gap between theory and experimental data.

Before starting it, it is necessary to understand the kinematics of the spectator distribution. For this purpose we divide the interval of centre-of-mass angles of the elementary process, in this case the charge-exchange $K^+N \to PP$, into several slices, and we calculated - using a program with three-body kinematics [10]) - the average value of the three-momentum transfer as a function of the momentum of the spectator

$$\langle \Delta^2 \rangle = \frac{\int \Delta^2 \sum_{i,f} |\langle f | \hat{T}_1 | i \rangle|^2 \, dV}{\int \sum_{i,f} |\langle f | \hat{T}_1 | i \rangle|^2 \, dV} \tag{28}$$

Figures 5a,b show the result for 0.98 and 1.51 GeV/c incident momentum of the K meson. The lines in the plot show the behaviour of the average value of Δ for bins of 0.15 in $\cos\theta_{c.m.}$ of the $K^+N \to PP$ process from -0.95 to 0.95 : for small angles (at the bottom of the plot) Δ is small for small values of p_s and it is increasing with p_s, but remaining in the non-relativistic regime. For large angles (at the top of the plot) Δ is large and we can eikonalize the theory along the direction of $\vec{\Delta}$. Therefore for small angles, we can apply the non-relativistic approach and, at large angles, the eikonal model. As far as the double scattering is concerned, only the bottom part on the left of the diagram seems to be safe, for the application of the Glauber theory, but one could certainly isolate better the region of small ϵ , considering the spectrum of the invariant mass of the two protons, possibly in the low momentum transfer region for the K meson, where k_o is large.

Figures 6a and 7a show the result of the calculation at small momentum transfers of the K meson at 0.98 and 1.51 GeV/c of incident momentum. The line with crosses shows the result of the calculation, using for the final state interaction the non-relativistic theory with the Hulthén wave function [23] and the McGregor proton-proton amplitudes [27].

The agreement with experiment is extremely encouraging, but the model is not giving the suppression of the peak, which would be required by the closure sum rule. In this paper, however, the emphasis is put on the interpretation of the tail of the spectrum. The problem of closure is being investigated and will be published in a forthcoming paper [28]. Some preliminary results give a very good consistency with closure of the same model in the case of the neutron-proton system, with the Hulthén wave function and effective range expansion for the nucleon-nucleon amplitude.

In the same Figs. 6a and 7a, the effect of the double scattering is shown to be very small in the region of high momentum of the spectator ; however, it causes a suppression in the region of small spectator momenta ($p_s \leq 250$ MeV/c), which amounts to an integral value of 4.8% for 0.98 GeV/c and 3.1% for 1.51 GeV/c. Here, and in the following calculations, a Gaussian parametrization [16] of the Gartenhaus-Moravcsik wave function was used.

Figures 6b and 7b show the result for the model of final state interaction "à la Glauber" at high momentum transfers. The crosses show the result with the D wave for the deuteron and the dotted line without the D wave. The result is consistent with the one at low momentum transfers.

While in the single scattering and in the non-relativistic model for the final state interaction we have used the amplitudes obtained by a complete phase shift analysis [29] for the double scattering and for the final state interaction at large momentum transfers, the kaon-nucleon amplitudes are parametrized as a pure non-spin flip Gaussian

$$f = A(i+\alpha)\, e^{bt} \qquad (29)$$

and the values of the parameters are extracted from the differential cross-sections [26),30)] and from a dispersion relation calculation [31)].

The nucleon-nucleon amplitude is parametrized in the same way, with A determined from total cross-section data [32)], α determined from dispersion relations [33)] and b parametrized in the following way

$$b = \frac{\sigma_T \, q^2_{c.m}}{20} \qquad (30)$$

following Ref. 35).

6. - CONCLUSIONS

We have been facing in this paper the problem of unifying the formalism of the final state interaction for large and small momentum transfers. The results we are getting are surprisingly filling the gap between single scattering theory and experiment at high momenta of the spectator for the case of $K^+D \rightarrow K^0PP$; therefore we interpret the high momentum tail as the final state interaction of the protons in this case. The intuitive reason is that high momenta correspond to small distances and, if the interaction with the incident particle occurs for small distances between the two nucleons, we expect double collision effects to be important, and in this case the nucleon-nucleon interaction is much stronger than the kaon-nucleon interaction and is likely to dominate. Other processes like $\pi D \rightarrow \pi$'s NN will require obviously other explanations for the high momentum tail and the annihilation channel for incident antiprotons is a different problem, but basically a secondary interaction is required to explain it.

We want now to conclude, commenting on a recent interpretation [34)] of the high momentum tail as an evidence of high momentum components in the deuteron wave function. This interpretation is not inconsistent with what we are saying in this paper, because when two nucleons are emitted, their form factor is affected by the final state interaction, and therefore one can generalize the concept of the wave function, including final state interaction effects.

ACKNOWLEDGEMENTS

We are deeply indebted to L. Bertocchi, T.E.O. Ericson, R.J. Glauber, R.E. Peierls and C. Wilkin for very interesting discussions ; we are grateful to E. Castelli and P. Poropat for providing us with the raw data of the spectator distribution.

Two of us (M.G. and Z.T.) wish to thank A. Salam and P. Budini for hospitality at the International Centre for Theoretical Physics in Trieste.

APPENDIX A

To perform the integration of Eq. (2) we can take as the z axis the direction of k_o, and after having made the integration, we can rotate again the z axis to coincide with the quantization axis of the nucleon spin, the amplitude being rotationally invariant.

The integral for the S wave is easily made

$$\int d^{(3)}\vec{\eta}\; \delta(\eta_{\parallel}-\Upsilon_{\parallel})\, e^{-\gamma(\vec{\eta}+\vec{\epsilon})^2}\, e^{-D\vec{\eta}_\perp\cdot\vec{\Delta}_\perp}\, e^{-S\vec{\eta}_\perp^2} =$$

$$= \frac{\pi}{\gamma+S}\, e^{-\gamma(\Upsilon_{\parallel}+\epsilon_{\parallel})^2}\, e^{-\gamma\epsilon_\perp^2 + \frac{(\gamma\vec{\epsilon}_\perp^2 + \frac{D}{2}\vec{\Delta}_\perp)^2}{\gamma+S}} \qquad (A.1)$$

while that for the D wave requires first of all a variable transformation $\vec{\eta}_\perp' = \vec{\eta}_\perp + \vec{Q}_o$ where $\vec{Q}_o = (\gamma\vec{\epsilon}_\perp + \frac{D}{2}\vec{\Delta}_\perp)/(s+\gamma)$ and $\vec{Q}_o' = \vec{Q}_o\sqrt{s+\gamma}$

$$\int d^{(3)}\vec{\eta}\; \delta(\eta_{\parallel}-\Upsilon_{\parallel})\, e^{-\gamma(\vec{\epsilon}+\vec{\eta})^2}\, e^{-D\vec{\eta}_\perp\cdot\vec{\Delta}_\perp}\, e^{-S\vec{\eta}_\perp^2} \times \left\{3[\vec{J}\cdot(\vec{\eta}+\vec{\epsilon})]^2 - (\vec{\eta}+\vec{\epsilon})^2\right\} =$$

$$= e^{-\gamma(\epsilon_{\parallel}+\Upsilon_{\parallel})^2}\, e^{-\gamma\epsilon_\perp^2 + Q_o'^2} \times \int d\vec{\eta}_\perp'\, e^{-(\gamma+S)\eta_\perp'^2} \times \Big\{3\Big[J_z^2(\Upsilon_{\parallel}+\epsilon_{\parallel})^2 + (\vec{J}_\perp\cdot(\vec{\eta}_\perp'+\vec{\epsilon}_\perp-\vec{Q}_o))^2 +$$

$$+ J_z\cdot(\Upsilon_{\parallel}+\epsilon_{\parallel})\,\vec{J}_\perp\cdot(\vec{\eta}_\perp'+\vec{\epsilon}_\perp-\vec{Q}_o) + \vec{J}_\perp\cdot(\vec{\eta}_\perp'+\vec{\epsilon}_\perp-\vec{Q}_o)\cdot J_z(\Upsilon_{\parallel}+\epsilon_{\parallel})\Big] -$$

$$- (\vec{\eta}_\perp'+\vec{\epsilon}_\perp-\vec{Q}_o)^2 - (\Upsilon_{\parallel}+\epsilon_{\parallel})^2\Big\} = \frac{\pi}{\gamma+S}\, e^{-\gamma(\epsilon_{\parallel}+\Upsilon_{\parallel})^2}\, e^{-\gamma\epsilon_\perp^2 + Q_o^2} \times$$

$$\times\left[\hat{O}(\vec{f}) + \frac{1}{\sqrt{2}}\left(\frac{3}{2}\left(\vec{J}^2 - \left(\frac{\vec{J}\cdot\vec{k}_o}{k_o}\right)^2\right) - 2\right)\frac{1}{\gamma+S}\right] \qquad (A.2)$$

where

$$\vec{f} = \vec{\epsilon}_\perp - \vec{Q}_o + (\Upsilon_{\parallel}+\epsilon_{\parallel})\frac{\vec{k}_o}{k_o}$$

This proves Eq. (8). An analogous task is to prove Eq. (16).

APPENDIX B

We show here in more detail the cancellation which leads to Eq. (24) and therefore to the closure result (25). Let us simplify the notation, defining

$$A^{\pm} = \frac{\epsilon + \Delta/2 \pm i\chi}{\epsilon - \Delta/2 \pm i\chi} \tag{B.1}$$

Factorizing the elementary amplitude in the last two terms of (22), we get, integrating on the angle

$$|1+c|^2 \left(\frac{SN}{4\Delta^2 \xi_o^2} 4\pi \int_0^\infty \frac{1}{\epsilon^2+\chi^2} \log A^+ \log A^- \epsilon^2 d\epsilon \; - \right.$$
$$- \frac{N\sqrt{s}}{2\Delta\xi_o} \frac{2\pi}{\Delta} \int_0^\infty \left[\frac{1}{\epsilon - i\chi} \log A^+ + \frac{1}{\epsilon + i\chi} \log A^- \right] \times$$
$$\left. \times \left[\log A^+ + \log A^- \right] \epsilon \, d\epsilon \right) \tag{B.2}$$

The last term is expanded giving

$$\frac{N\sqrt{s}}{2\Delta\xi_o} \cdot \frac{2\pi}{\Delta} \cdot \left(\int_0^\infty \left(\frac{1}{\epsilon - i\chi} \log^2 A^+ + \frac{1}{\epsilon + i\chi} \log^2 A^- + \right. \right.$$
$$\left. \left. + \frac{2\epsilon}{\epsilon^2+\chi^2} \log A^+ \log A^- \right) \epsilon \, d\epsilon \right) \tag{B.3}$$

which proves the cancellation between the last term and the first term of (B.2) provided $S/\xi_o^2 = 2(\sqrt{s}/\xi_o)$, which is the kinematical condition for the validity of the closure [36]. This condition is satisfied because the relative energy of the two nucleons is low and therefore $\sqrt{s} \simeq 2m$ and $\xi_o \sim m$.

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FIGURE CAPTIONS

Figure 1 Diagrams representing the single and double scattering. The dotted lines represent the incident hadron and possibly its excited states in the final state.

Figure 2 Diagram representing the final state interaction between the two nucleons.

Figure 3 Closure sum rule expressed in diagrammatic language.

Figure 4 Spectrum of the slower proton for the reaction $K^+D \to K^oPP$ at 0.98 GeV/c. The crosses are the result of the calculation using the single scattering approximation and proper three-body kinematics, and the open circles are the plot of the probability density of the deuteron normalized at $p_s = 0.01$ GeV/c to the other curve. The effect of the phase space is to lower the curve and it results in a suppression of 2% in the area of the spectrum for $p_s \leq 0.25$ GeV/c.

Figure 5 Plot of the quantity $\sqrt{<\Delta^2>}$ as function of the momentum of the spectator p_s for 12 intervals of 0.15 in $\cos \theta_{c.m.}$ of the process $K^+N \to K^oP$ from -0.95 to 0.95 leaving the interval -0.05 ÷ 0.05, to separate clearly the forward from the backward hemisphere. The lower part of the plot corresponds to small angles and the higher one to large angles.

a) Incident momentum 0.98 GeV/c.

b) Incident momentum 1.51 GeV/c.

Figure 6 Plot of $d\sigma/dp_s$, calculated theoretically, using the same prescription as for the experimental analysis ; the dash-dotted line corresponds to the single scattering approximation, the line with crosses corresponds to the inclusion of the final state interaction between the two protons. The incident momentum is 0.98 GeV/c.

a) The calculation is done for $0.8 \leq \cos \theta_{c.m.} \leq 0.95$ and the curve is renormalized to fit the first part of the spectrum. The final state interaction is calculated using the non-relativistic model. The dashed curve shows the contribution of the double scattering.

b) Here $\cos\theta_{c.m.}$ ranges between -0.95 and -0.8 and the final state interaction is calculated using the eikonal model ; the dotted curve is the result without the D wave of the deuteron, neither in the single scattering term, nor in the final state interaction.

Figure 7 The same, for 1.51 GeV/c of incident momentum.

a) $0.9 \leq \cos\theta_{c.m.} \leq 0.95$.

b) $-0.95 \leq \cos\theta_{c.m.} \leq 0$.

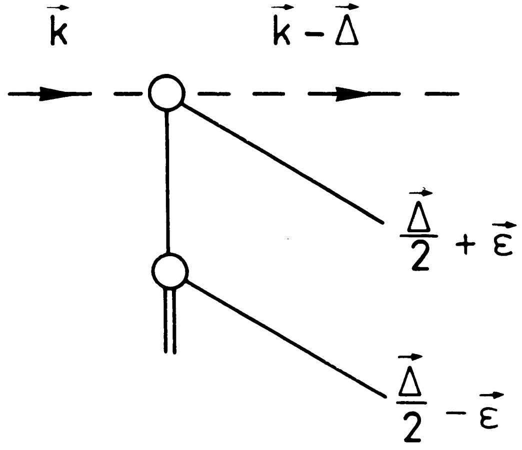


FIG.1a

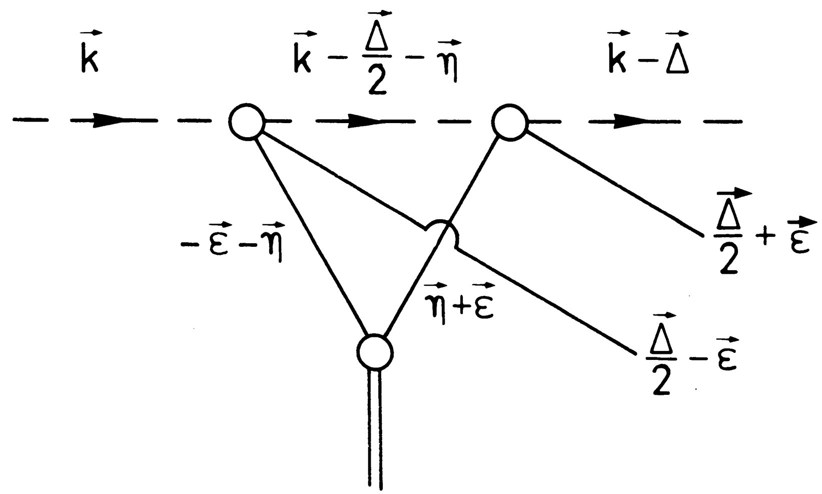


FIG.1b

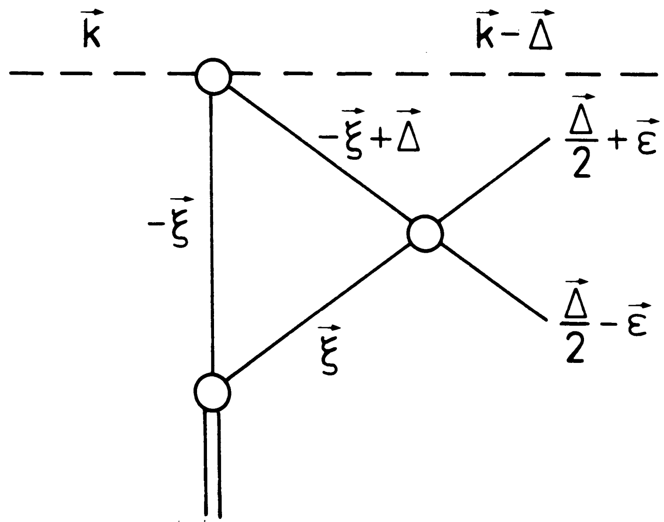


FIG.2

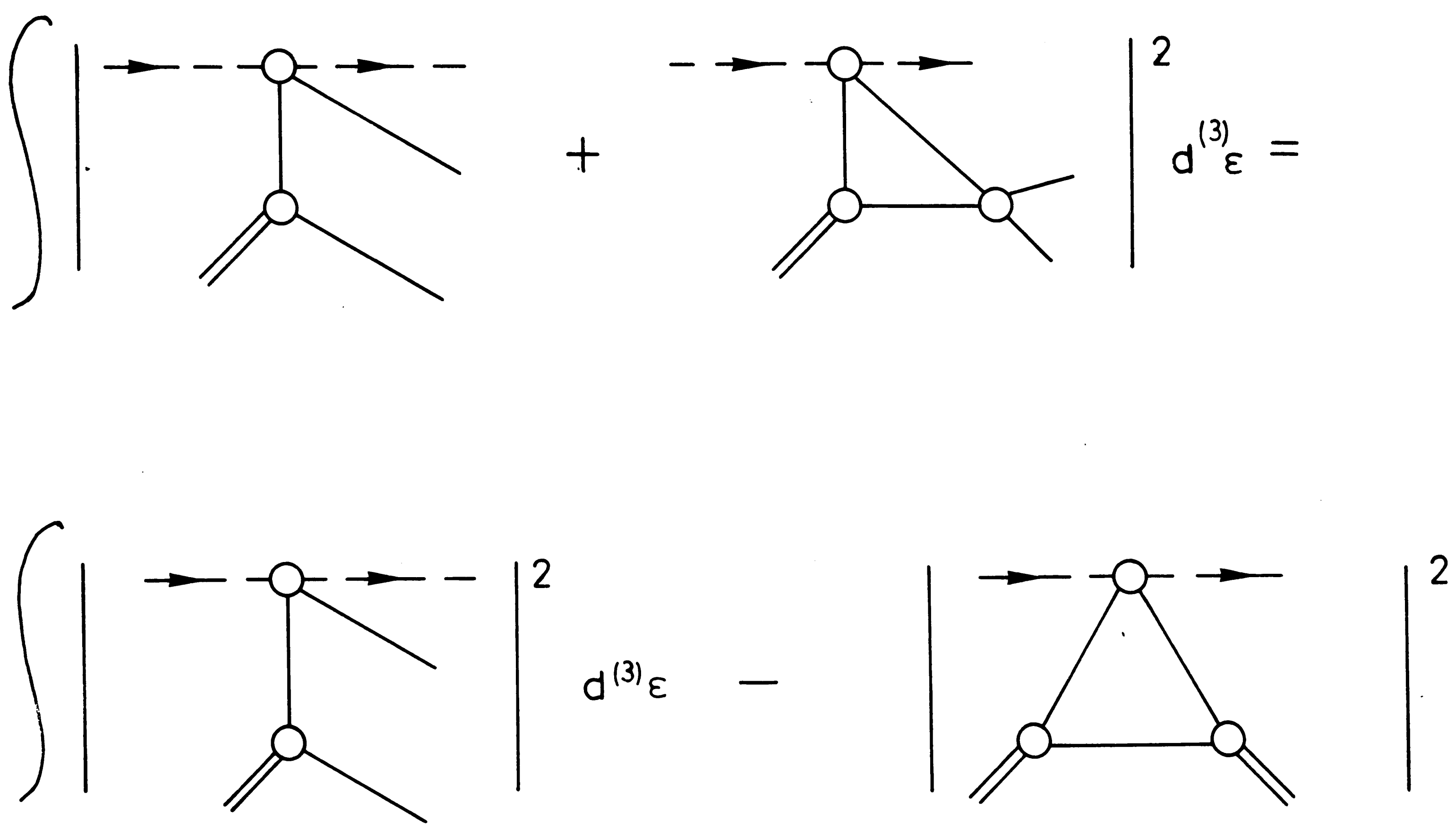


FIG. 3

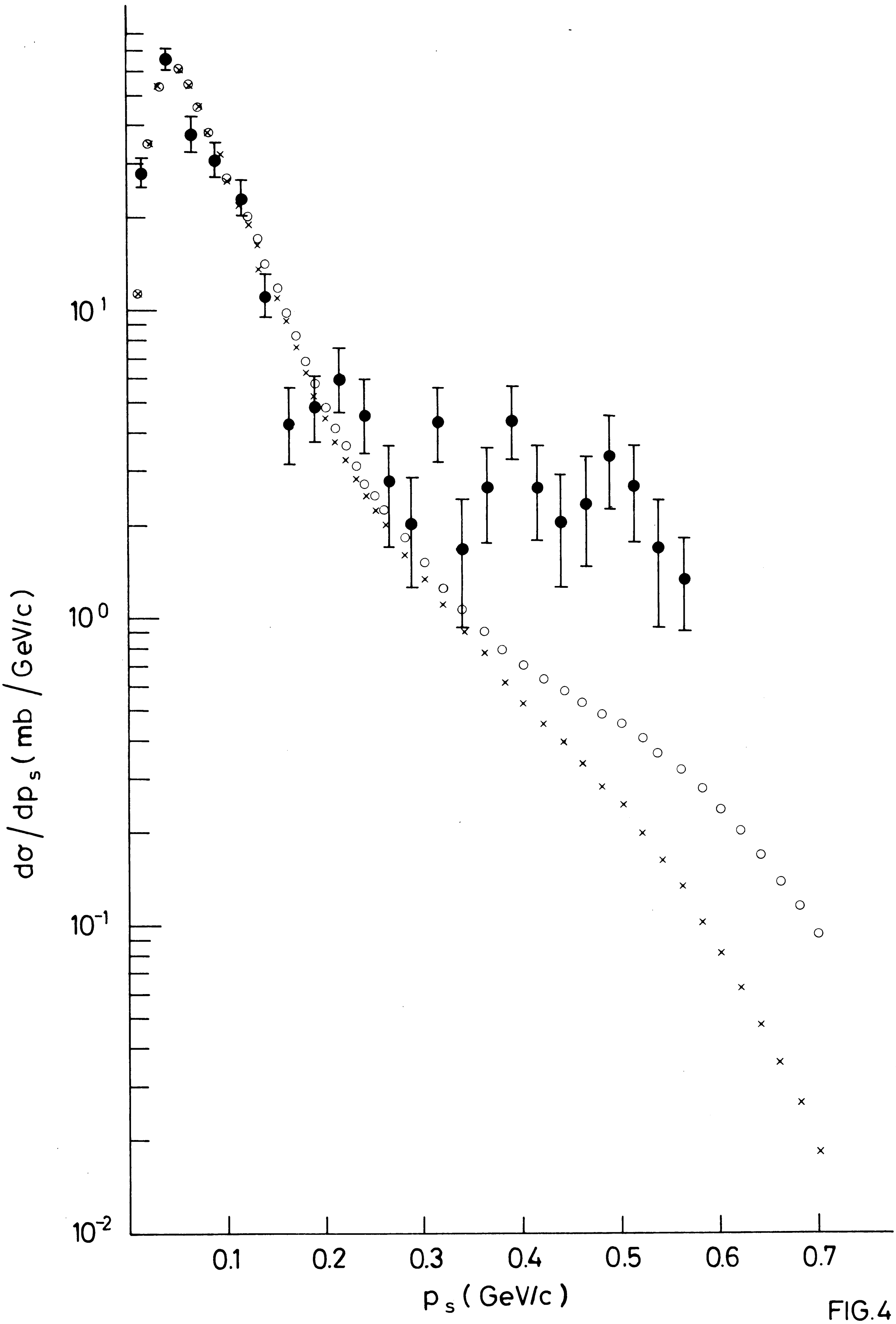


FIG.4

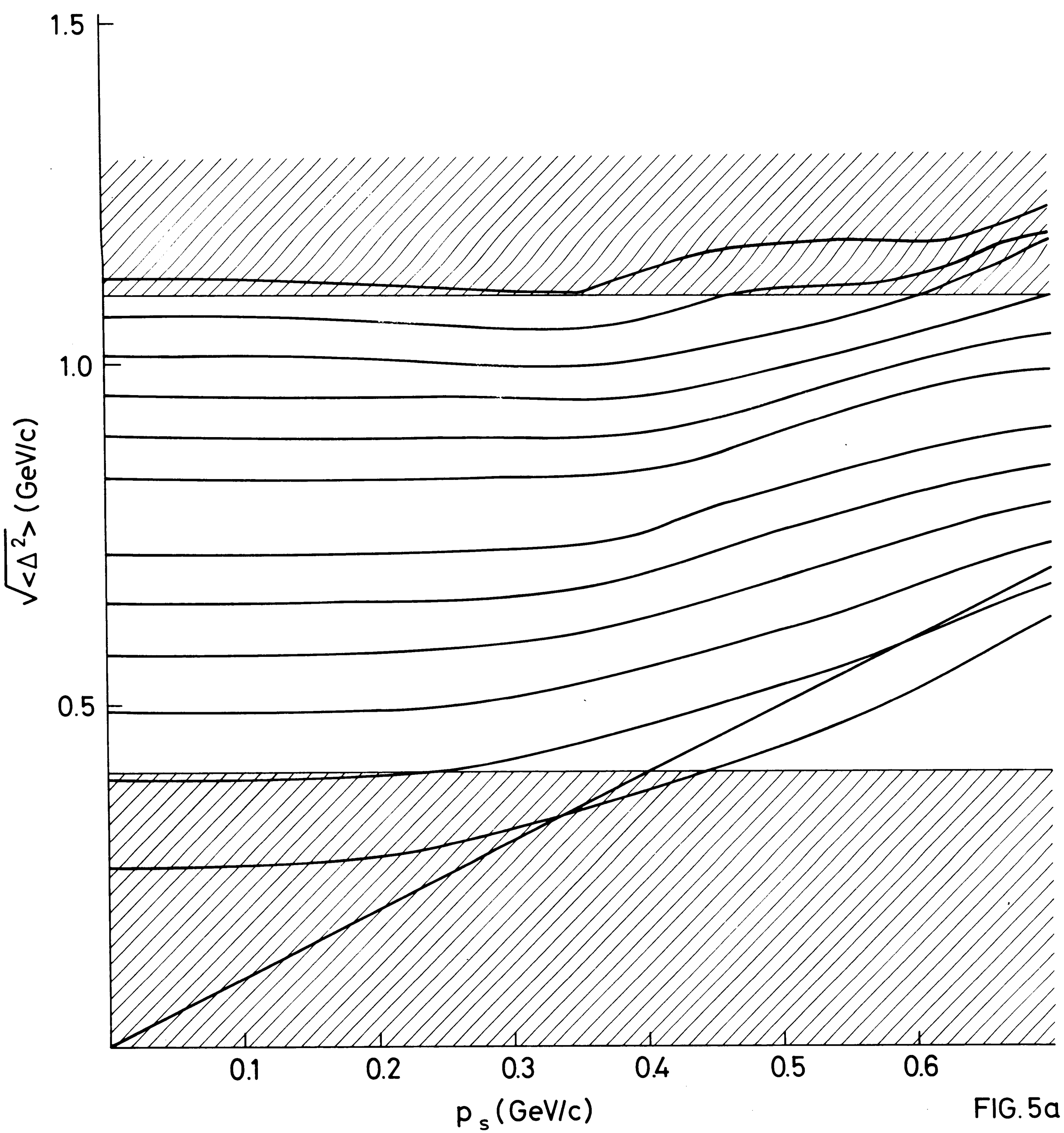


FIG. 5a

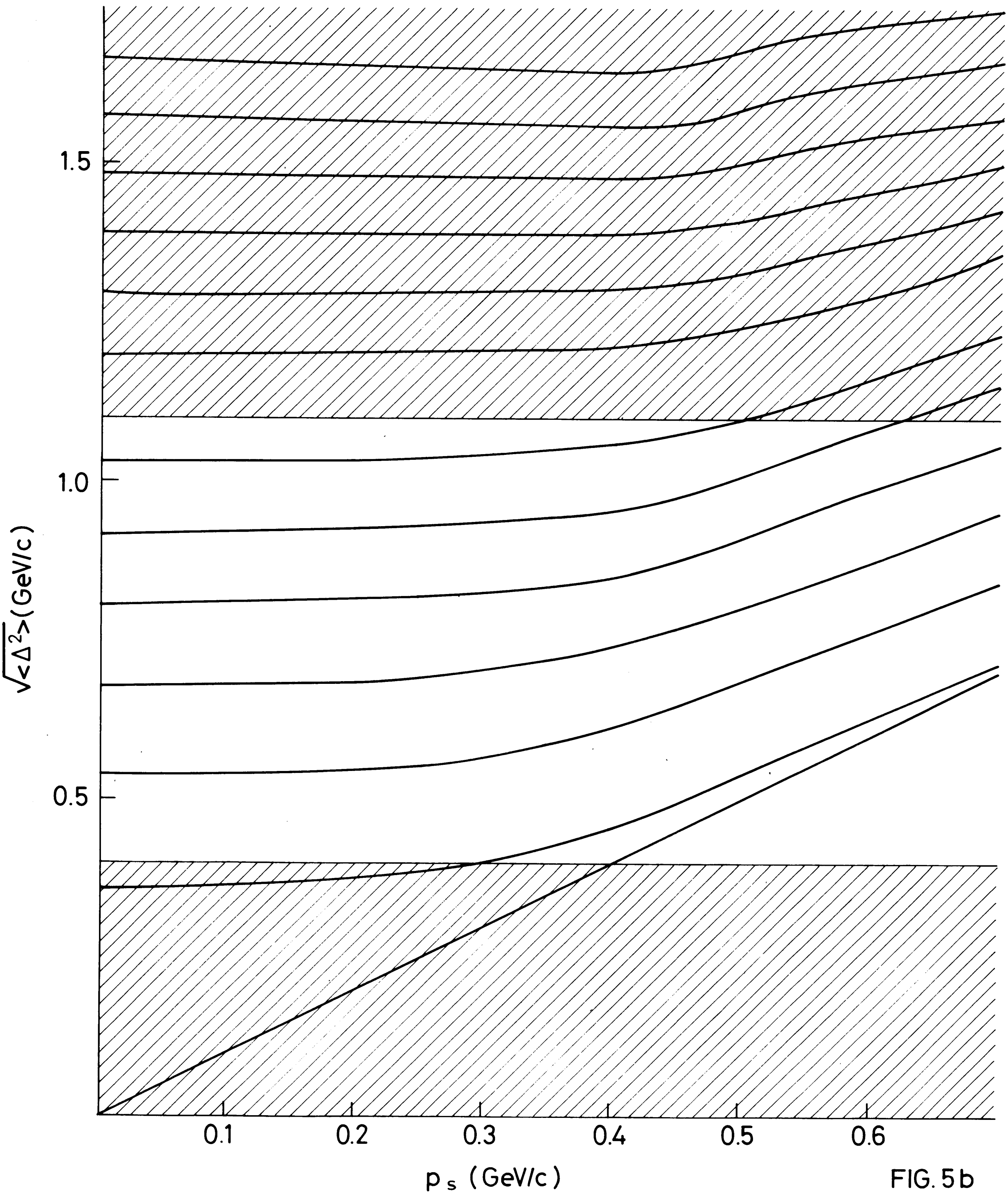


FIG. 5b

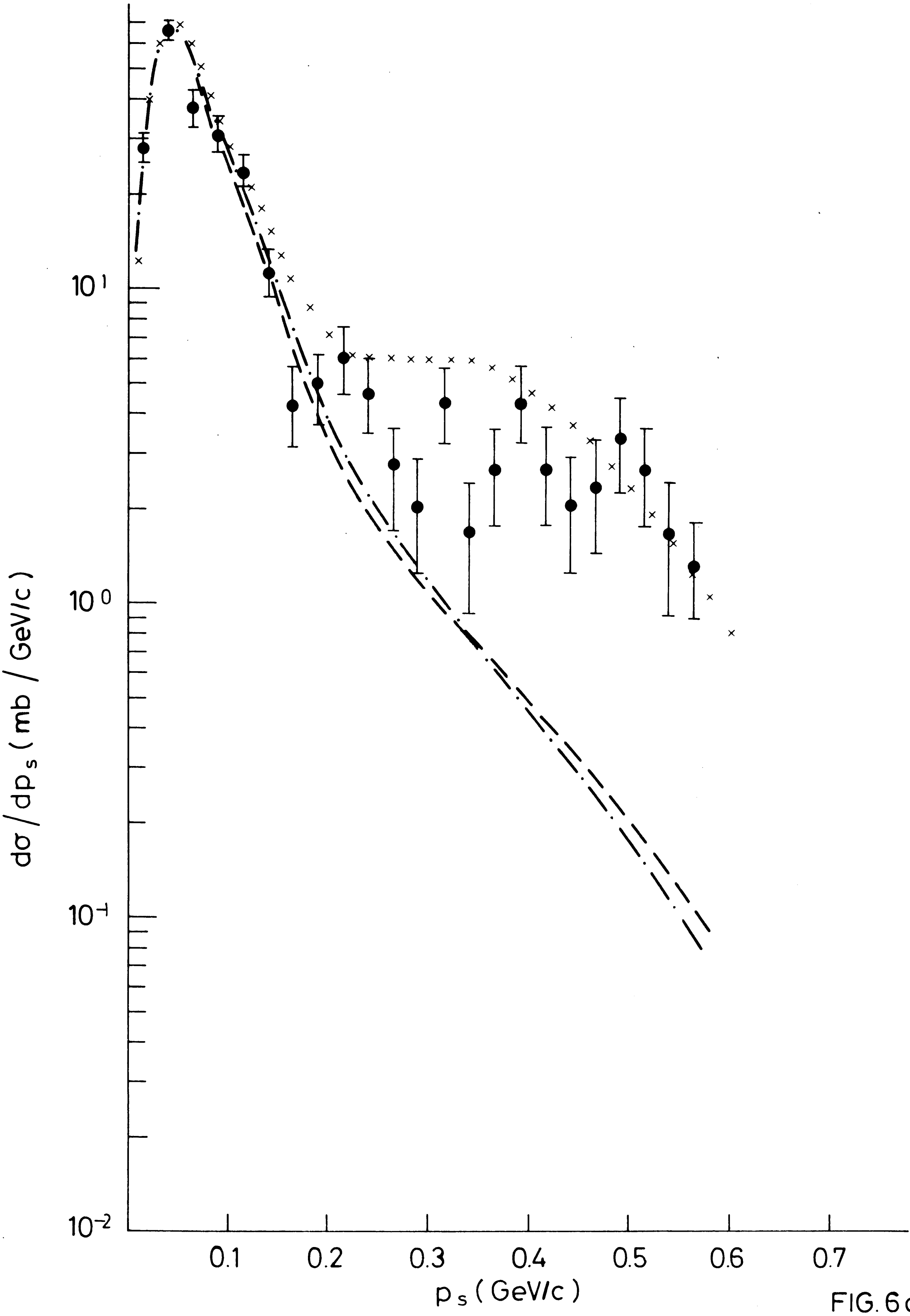


FIG. 6 a

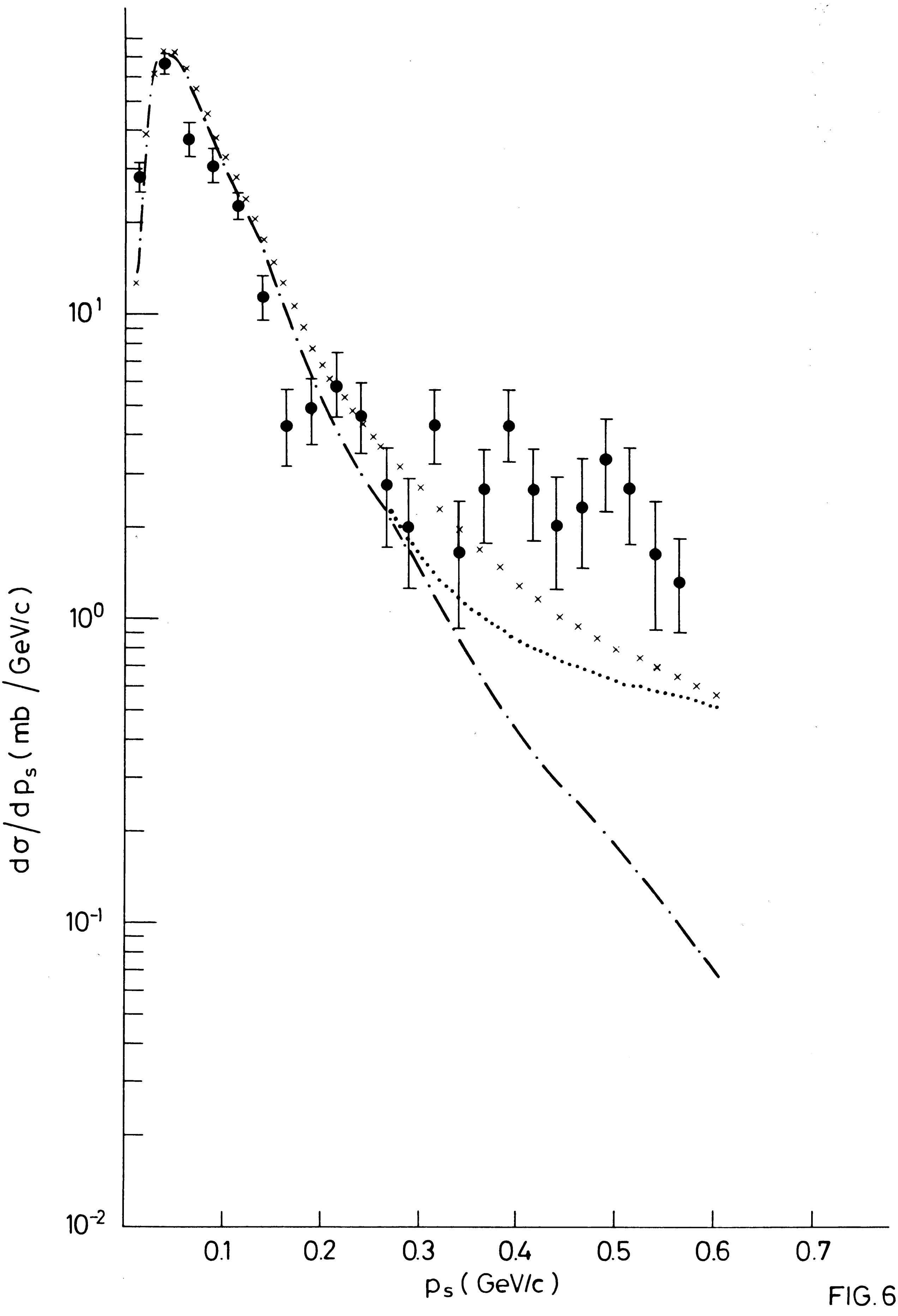


FIG. 6 b

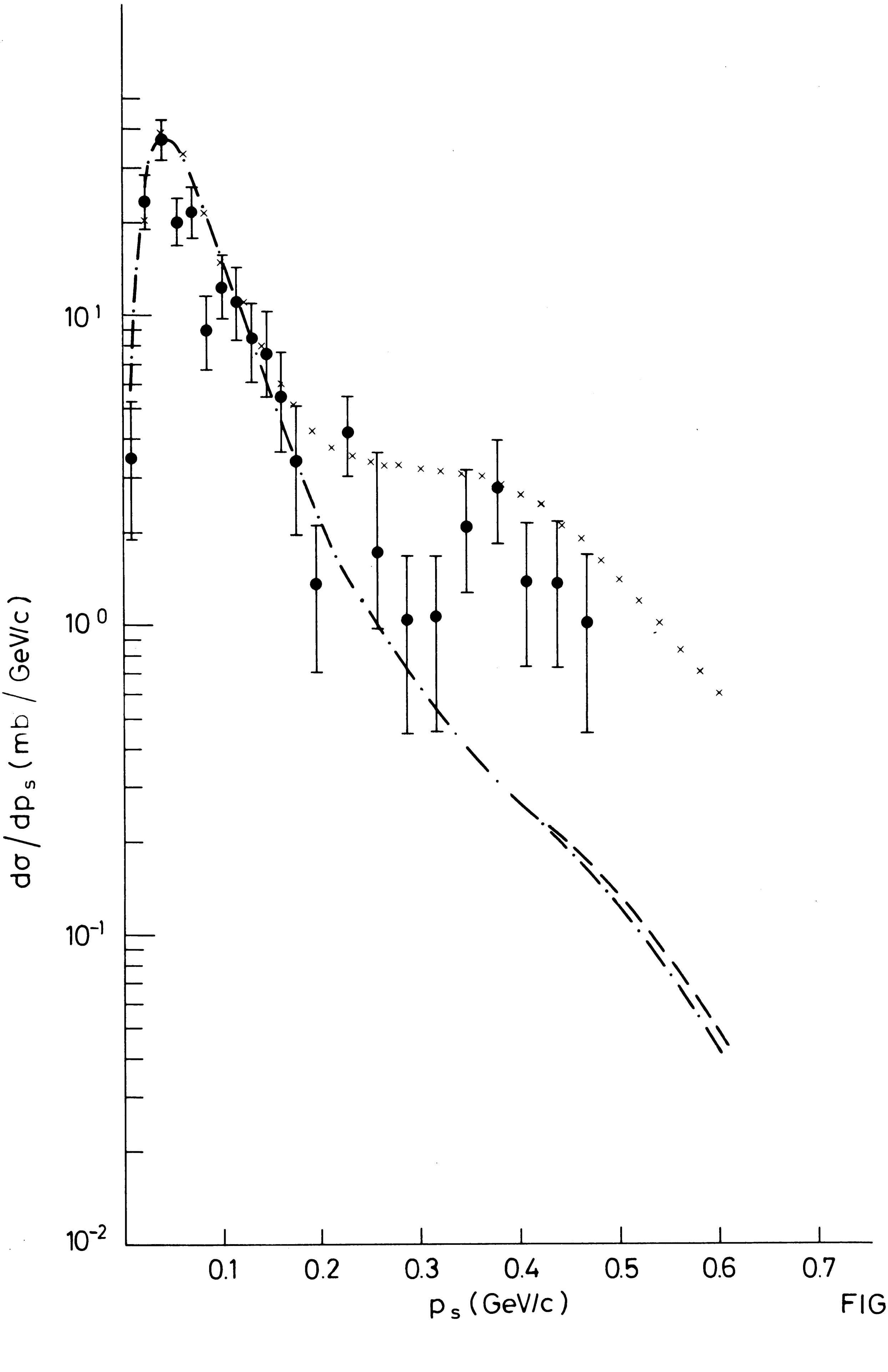
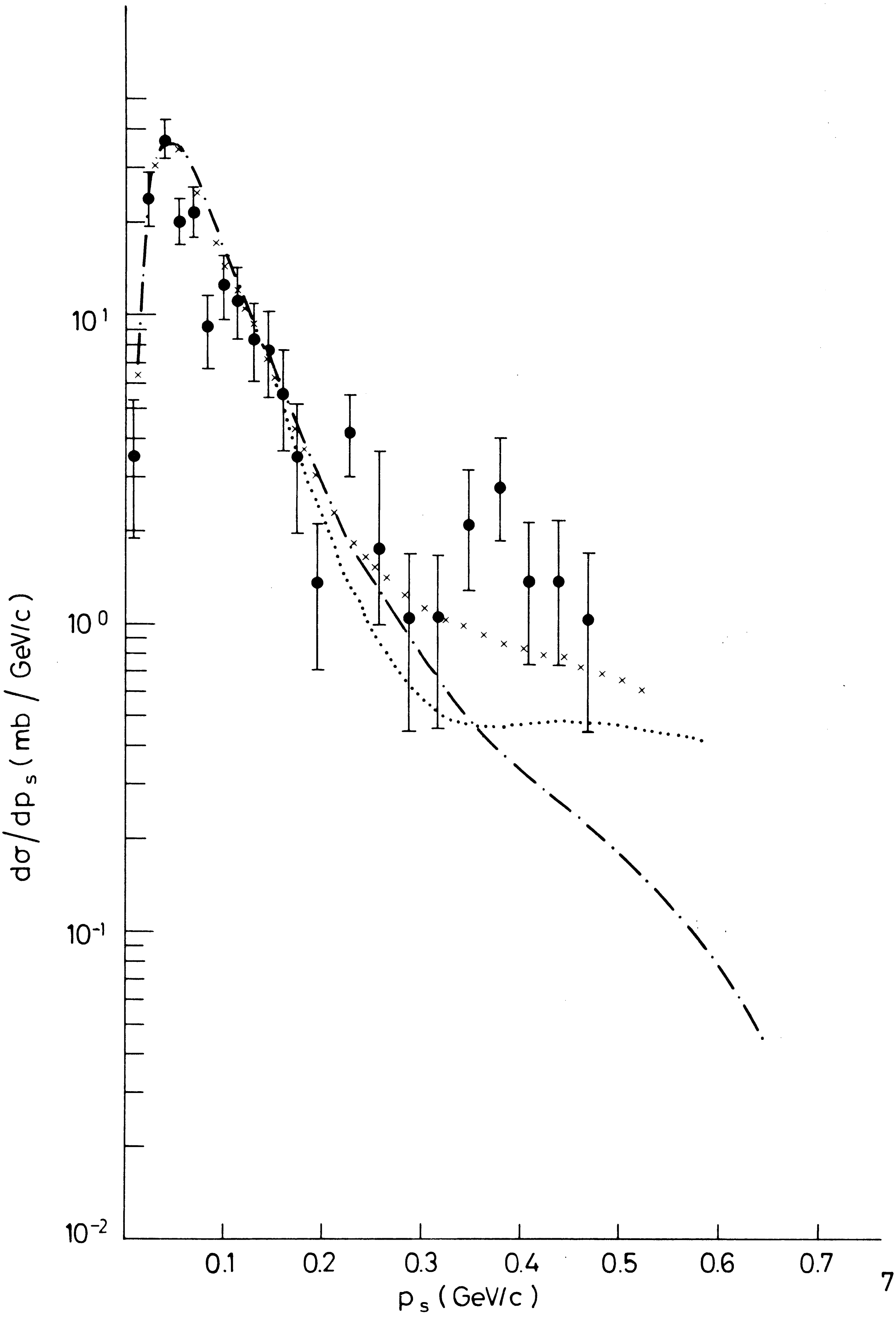


FIG.7a



7 b