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STUDY
OF INTRA-BEAM SCATTERING EFFECT
AT CESR AND VEPP-4M STORAGE RINGS
AT 1.8 GeV ENERGY

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Abstract

Intra-beam scattering (IBS) phenomena can cause significant particle loss rate from the beam as well as the beam transverse emmitances and energy spread grow. In the present paper we briefly describe modified method for calculation of beam particles loss rate. The method takes into account two-dimensional character of collisions of particles inside beam. Results of calculation are compared with experimental data from two electron-positron colliders: CESR (Cornell University, USA) and VEPP-4M (BINP, Novosibirsk, Russia).

1 Introduction

Intra-beam scattering (IBS) processes[1] may cause a significant particle loss rate resulted in an unacceptable short beam lifetime. The beam emittances can be affected by IBS too. Because these processes become stronger at low energy, it is of great concern in relation to plan of CESR to operate at 1.8 GeV energy.

In the present paper we are following the well-known theory of IBS described in [1]. However, there are two specifics. First, we take into account two-dimensional character of collisions of particles inside beam, i.e., in addition to the transverse momentum distribution in horizontal plane we consider the distribution in vertical plane too. This allows us to avoid assumption about beam flatness. Two dimensional approach has been considered earlier on in [2, 3, 4]. The second specific is the way we determine energy aperture. In IBS theory, the energy aperture is defined as the maximum energy deviation of the beam particle caused by the IBS event at which the particle motion is still stable and the particle does not interact with vacuum pipe walls. Note that, if occurred in location with dispersion, IBS excites betatron oscillations in addition to synchrotron motion. The energy aperture will be equal to the RF buckets height, if the latter measured in beam energy units is smaller than a half of the ratio between aperture available for horizontal motion and the dispersion (see the exact expression in text below). In this case, which is applicable to VEPP-4M collider, energy aperture can be accurately calculated from the well-measured or well-defined parameters. At CESR we have an opposite situation. Both the high RF voltage needed for bunch shortening and the large closed orbit distortion required for the multi-bunch collision operation make the RF bucket higher and the space available for horizontal motion smaller. In result, the RF bucket is larger than the space available for horizontal motion. In this case the energy aperture will depend on betatron motion and may be affected by many factors such as nonlinearities of the leading magnetic field, proximity of betatron

[1]For electron storage rings it is known as Toushek effect

tunes to resonance conditions, coupling between betatron motion in vertical and horizontal planes and etc. Because it is impossible to take into account all the above effects in analytical calculation we use tracking.

In Section 2 we consider the method and in Section 3 we present details and result of tracking used for energy aperture simulation. Description of experimental data obtained on CESR and VEPP-4 electron-positron colliders and comparison with calculation result are given in Section 4 and 5. In Section 6 we give a brief summary.

2 Method for calculation of Touschek effects

The method which is briefly described below has been developed in [4]. In fact, basing on theory given in [1] it expands one-dimensional (flat beam) approximation used in [1] into theory which takes into account two-dimensional character of two-particle Coulomb interaction inside a bunch.

To describe two-dimensional character of motion let's introduce the interaction parameter: $k = \sigma_{X'}/\sigma_{Y'}$, where $\sigma_{X'}$ and $\sigma_{Y'}$ are respectively the spreads of trajectory angles in the horizontal plane (X) and vertical one (Y). The contribution to the transverse velocity from the dispersion function is neglected as well as in [1]. In the so-called "round" beam $k \to 1$ and in the flat one $k \to \infty$. The modified function of distribution as to momentum (p) in the center-of-mass system (CMS) has the following form [4]:

$$f(k,p)dp = \frac{2kp}{\sigma_p^2} \cdot S(w,k)dp, \quad (p > 0)\,,$$

$$S(w,k) = \exp\left[-\frac{w}{2}(1+k^2)\right] I_0\left[\frac{w}{2}(1-k^2)\right].$$

Here $p = m\nu/2$, the momentum in CMS (similarly to [1], we use the non-relativistic description of motion in CMS); m is the rest electron mass; ν is the relative velocity of colliding particles inside a beam ($\nu^2 = \nu_X^2 + \nu_Y^2$); $w = p^2/\sigma_p^2$; $\sigma_p = mc\gamma\sqrt{\sigma_{X'}^2 + \sigma_{Y'}^2}$, the transverse momentum spread in a beam; c is the speed of light; γ is the Lorentz factor; $I_0(x)$ is the modified Bessel function.

At $k \to \infty$ the distribution function approaches the form corresponding to the one-dimensional collision case [4]:

$$f(p)dp = \frac{2}{\sqrt{\pi}\sigma_p} \exp\left(-\frac{p^2}{\sigma_p^2}\right) dp, \quad (p > 0).$$

At $k \to 1$ the distribution becomes the two-dimensional Maxwell one:

$$f(p) \propto p \cdot \exp\left(-p^2/\sigma_p\right).$$

The use of the modified distribution function changes the forms of universal characteristic functions which describe the diffusion rate and the losses rate in the theory of IBS [4].

The determinative process in the integrated Touschek effect is the multiple scattering provided the latter significantly contributes to the energy diffusion in compare with the synchrotron radiation (SR). Losses of particles (beam lifetime) due to single scattering depend on the steady beam dimensions determined by the total (SR + IBS) diffusion rate, radiative damping and betatron coupling. For the storage rings like CESR and VEPP-4M, it would suffice to describe the betatron coupling in the " weak coupling" terms when the beam cross section tilt is mostly negligible and the basic parameter is the ratio between the vertical ($\mathcal{E}_Y$) and horizontal emittances ($\mathcal{E}_X$): $æ = \mathcal{E}_Y/\mathcal{E}_X$.

Let denote

$$u = (\sigma_\gamma/\gamma)^2 = u_Q + u_T$$

– the squared relative energy dispersion;

$$v = \mathcal{E}_X = v_Q + v_T$$

– the radial phase volume;

$$k = \sqrt{(1+\alpha_X^2)\beta_Y/(æ(1+\alpha_Y^2)\beta_X)};$$

$$\mathcal{H} = [\eta_X^2 + (\beta_X\eta'_X + \alpha_X\eta_X)^2]/\beta_X$$

– the function describing the excitation of radial betatron oscillation due to an instant change in a particle energy; $\sigma_Z = R\alpha\sqrt{u}/Q_S$ – the longitudinal beam size, Q_S – the synchrotron tune, α – the momentum compaction, R – the machine radius; β_Y, α_Y, β_X, α_X, η_X, η'_X are the amplitude and dispersion functions. Here the indexes Q and T mark the contribution respectively of synchrotron radiation (quantum diffusion) and Touschek effect. Total diffusion coefficients in respect of the particle energy and the radial emittance are given through the following sums:

$$D_u = D_u^Q + D_u^T,$$

$$D_v = D_v^Q + D_v^T,$$

where D_u^Q and D_v^Q are determined, for example, in [1]. The Touschek diffusion coefficients may be written as

$$D_u^T = \frac{N r_0^2 c Q_S}{16\pi\gamma^3 R\alpha\sqrt{uv}} \left\langle \frac{\beta_X B(k,\chi_m)}{(\beta_X v + \eta_X^2 u)\sqrt{æ\beta_Y(1+\alpha_X^2)}} \right\rangle ,$$

$$D_v^T = \frac{N r_0^2 c Q_S}{16\pi\gamma^3 R\alpha\sqrt{uv}} \left\langle \frac{\beta_X B(k,\chi_m)\mathcal{H}}{(\beta_X v + \eta_X^2 u)\sqrt{æ\beta_Y(1+\alpha_X^2)}} \right\rangle .$$

Here angle brackets mean the averaging over the machine azimuth (ϑ); N is the number of particles in a bunch. The quantity $B(k,\chi_m)$ is a modified diffusion rate function [4] which in contrast to the one-dimensional collision theory [1] depends on the coupling parameter k:

$$B(k,\chi_m) = \sqrt{\pi} k \int\limits_{\chi_m}^{\infty} \sqrt{\frac{1}{\chi}} \cdot \ln\left(\frac{\chi}{\chi_m}\right) \cdot S(\chi,k) d\chi \, ,$$

r_0 is the classical electron radius; $\chi_m = (p_m/\sigma_p)^2$; $p_m = p_0\sqrt{r_0/b_{\max}}$, the classical lower limit of momentum transfer; $b_{\max}$ is the maximal scale of the impact parameter in CMS. We determine the latter as usually through the average particle density in the co-moving system:

$$b_{\max} = \left(\frac{\gamma V}{N}\right)^{1/3} ,$$

$V = 8\pi^{3/2}\sigma_X\sigma_Y\sigma_Z$ is the bunch volume in the laboratory system. The steady values of u and v are determined from the system of equations

$$u = u_Q + \frac{\tau_E}{2} D_u^T ,$$

$$v = v_Q + \frac{\tau_X}{2} D_v^T , \qquad (1)$$

τ_E and τ_X are respectively the damping times for synchrotron and radial betatron oscillations.

The loss rate (the inverse beam lifetime) due to Touschek processes may be found from the corrected formula:

$$\frac{1}{\tau} = 2\sqrt{\pi} r_0^2 m^3 c^4 N \left\langle \frac{C(k,\varepsilon)}{\sigma_p A_p^2 V} \right\rangle . \qquad (2)$$

Here

$$C(k,\varepsilon) = \sqrt{\pi} k\varepsilon \int\limits_{\varepsilon}^{\infty} \chi^{-\frac{3}{2}} \left[\frac{\chi}{\varepsilon} - \frac{1}{2}\ln\frac{\chi}{\varepsilon} - 1\right] \cdot S(\chi,k) d\chi,$$

the modified "loss function" which depends on the parameters k and $\varepsilon = A_p^2/\gamma\sigma_p^2$ with A_p, the "energy aperture" limiting the deviation of the longitudinal momentum from the equilibrium value. The factor "2" in (2) takes into account for the fact that both particles get lost in one event of scattering in CMS. This makes an essential difference in respect to the view of similar formulae given in [1] and [4].

At $k \to \infty$ the expression for C takes the form of the one-dimensional approximation given in [1]:

$$C(\infty,\varepsilon) = \varepsilon \int\limits_{\varepsilon}^{\infty} \chi^{-2} \left[\frac{\chi}{\varepsilon} - \frac{1}{2}\ln\frac{\chi}{\varepsilon} - 1\right] \cdot e^{-\chi} d\chi\,.$$

The same takes a place in the case of asymptotic behaviour of the modified diffusion rate function $B(k,\chi_m)$.

3 Energy aperture

The beam lifetime will be mostly determined by single IBS for the low energy operation at CESR and VEPP-4M. According to (2)

$$\frac{1}{\tau} \propto \left\langle \frac{C(k,\varepsilon_m)}{A_p^2 \sigma_Z \sqrt{\mathcal{E}_X^3 æ \beta_Y (1+\alpha_X^2)}} \right\rangle.$$

In order to calculate the beam lost rate caused by IBS one should know the following beam parameters: $\mathcal{E}_X$, σ_Z, æ, A_p. The horizontal emittance $\mathcal{E}_X$ can be reliably calculated based on the optics with consideration of both SR and IBS effects. In VEPP-4M case, it also may be measured by the SR-based beam size monitor. Coupling parameter æ can be estimated from the vertical beam size monitor's data as well as the specific luminosity measurement. The bunch length σ_Z can be calculated as well as be measured with a streak camera.

The beam loss rate is most sensitive to the energy aperture (as A_p^{-2} if neglecting the weak influence of the factor $C(k,\varepsilon_m)$ [4]). In general case the quantity A_p is $\min\{A_L, A_T\}$ where the limiting half apertures A_L and A_T

describe the boundaries for longitudinal and transverse motions respectively. The limit A_L is determined by the RF bucket height [5]:

$$\left(\frac{A_L}{mc\gamma}\right)^2 = \frac{U_0}{\pi\alpha h E}F(\xi),$$

$$F(\xi) = 2[\sqrt{\xi^2 - 1} - \arccos(1/\xi)].$$

Here U_0 is the radiation loss of particle energy per turn; h is the RF harmonic number; $\xi = eV_{RF}/U_0$ is the RF overvoltage. Taking into account that the synchrotron tune may be written as

$$Q_S = \frac{\alpha h e V_{RF}}{2\pi E}\sqrt{1 - \frac{1}{\xi^2}}$$

and using the reasonable approximation $\xi >> 1$, one can obtain A_L in a most simple form

$$\frac{A_L}{mc\gamma} \approx \frac{2Q_S}{\alpha h}.$$

For CESR $A_L/(mc\gamma) \approx 0.9\%$ at $Q_S = 0.067$ (26 kHz). Ideally, the value A_T for a given azimuth ϑ where the scattering occurs is defined through the geometrical horizontal aperture of the vacuum chamber A_X at the azimuth ϑ^* where the particle is lost:

$$\frac{A_T(\vartheta)}{mc\gamma} = \frac{A_X(\vartheta^*) - X_0(\vartheta^*)}{\eta_X(\vartheta^*) + \sqrt{\beta_X(\vartheta^*)\mathcal{H}(\vartheta)}}.$$

Here the term X_0 describes the closed orbit distortion which may be significant like as the pretzel orbit at CESR. Really, the dynamic aperture $A_{DA} < A_X$ may play the role of the effective aperture instead of the geometrical one ($A_X \to A_{DA}$). It implies also that A_{DA} may depend on the closed orbit distortions (X_0).

In the smooth approximation

$$\frac{A_T}{mc\gamma} \sim \frac{\min|A_X - X_0|}{2\bar{\eta}_X},$$

$\bar{\eta}_X$ is the characteristic value of dispersion in arcs. According to this crude estimation $A_T/(mc\gamma) \sim 0.5\%$ for the CESR operation regime at the energy $E = 1843$ MeV. This is of the same order like the value A_L obtained above and, consequently, must be refined on. For more accurate determination of both limits A_L and A_T for CESR we use the particle tracking simulation in the 6D phase space.

In contrast to CESR, there are not significant special distortions of the closed orbit in VEPP-4M introduced in regular experiments. Therefore the energy aperture in VEPP-4M is definitely based on the RF voltage and makes up $A_p = A_L \approx 0.6\%$ at $E = 1.84$ GeV ($eV_{RF} \approx 400$ keV).

4 Simulation code

We have developed the special code to calculate the beam energy spread, emittance and lifetime taking into account for the IBS processes. It is based on the modules of BMAD (Lattice Language Standard for CESR based on MAD) [6] and subroutines for computing expressions from Section 2. Main input data include the information about the design magnetic structure with adding the calculated and measured nonlinearities; the coupling parameter (æ); the pretzel orbit scale; the energy aperture (A_p); the beam current and length. Setting of the coupling parameter found from the vertical beam size measurement allows to incorporate an influence of all real perturbations on the vertical emittance. The beam length is also set since it depends on the RF voltage.

At the first stage, the Twiss parameters and the beam parameters determined by SR processes are obtained. At the second stage, the total effect of SR and IBS processes in beam sizes, energy spread is calculated using (1). Finally, the Touschek beam lifetime is computed from (2) based on results of the second stage.

For calculation of the energy aperture determined by the dynamic one we have developed the separate module with particle tracking. Tracking always starts at the same azimuth (IP). Initial conditions depends on the azimuth (ϑ) where the IBS acta occurs resulting in the instant change of the particle's energy of $\epsilon = \delta E/E$:

$$X(IP) = X_0(IP) - \epsilon \cdot (T_{11}\eta_X(\vartheta) + T_{12}\eta'_X(\vartheta)),$$

$$X'(IP) = X'_0(IP) - \epsilon \cdot (T_{21}\eta_X(\vartheta) + T_{22}\eta'_X(\vartheta)).$$

Here T_{ij} are the elements of the transport matrix for transformation from ϑ to IP within one turn. Evolution of the particle's trajectory in the 6D phase space during a few thousands turns is calculated using the special method for tracking in non-linear magnetic fields. The initial energy deviation ϵ is varied for a given azimuth which is linked to each element of the magnetic structure.

Particle is considered to be lost and the appropriate value ϵ is a limit (ϵ_*) if the phase trajectory reaches the design vacuum chamber boundaries anywhere or comes out the RF separatrix. The effective energy aperture is found by averaging ϵ_* over the ring: $A_p/(mc\gamma) =< \epsilon_* >$.

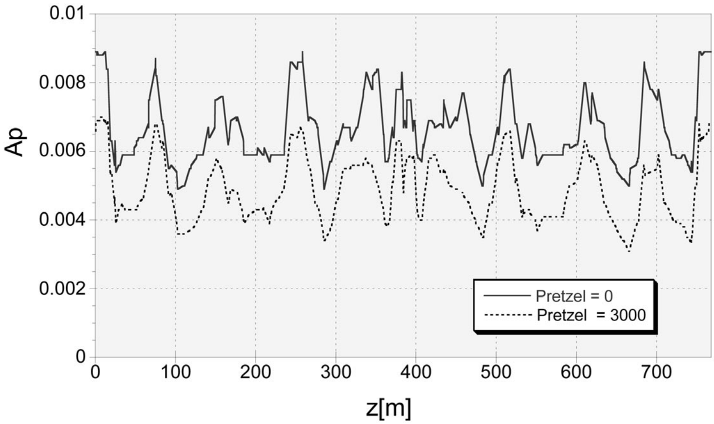

Figure 1: Energy aperture, A_p, as a function of azimuth around ring. Main interaction point is located at $z = 0$. Solid line, $Pretzel = 0$, represents energy aperture for flatten orbit. Dashed line, $Pretzel = 3000$, is for ± 15 mm horizontal orbit distortion.

5 Calculation results

Using the simulation code developed we have calculated the azimuthal dependence of the CESR energy aperture for two main pretzel orbit scales (in technical units): Pretzel=0 and Pretzel=3000 (see Figure 1). The scale $Pr = 1$ (in arbitrary units) corresponds to the nominal pretzel wave amplitude of $\approx \pm 15$ mm with the vacuum chamber's radial aperture of ± 45 mm. The tracking simulation shows that at $Pr = 1$ the energy aperture is determined through DA for any azimuth where the scattering event occurs. The situation for $Pr = 0$ is approximately the same: the RF limit is essential for events only at IR (the interaction region) where the dispersion is zero. Fig.2 represents the energy aperture averaged over the azimuth as dependent on the pretzel orbit scale Pr. The behaviour of A_p with varying the betatron tunes is of more complicated character as it is seen in Fig.3. The minimum of this dependence is caused by the influence of the resonance $2Q_X - Q_S = 21$.

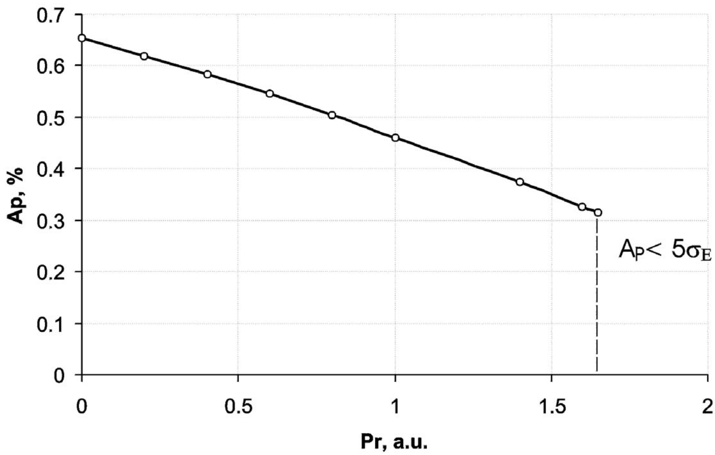

Figure 2: IBS Energy aperture vs. pretzel orbit scale at CESR.

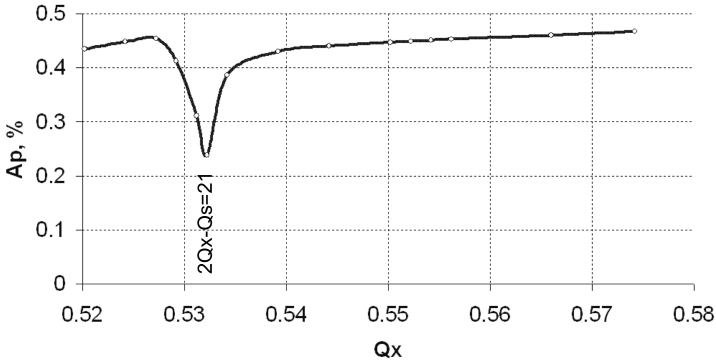

Figure 3: IBS energy aperture vs. horizontal betatron tune at CESR.

The calculation have indicated that for ~ 1.8 GeV CESR operation the contribution of multi-IBS processes to the diffusion rate is negligible in compare with diffusion caused by synchrotron radiation. As it was shown in the experiment and calculations [4, 7], IBS starts to play a distinct role in the beam emittances formation at VEPP-4M only below 1.3 GeV. The energy scalings of the IBS effect in the beam emittances and lifetime in the CESR

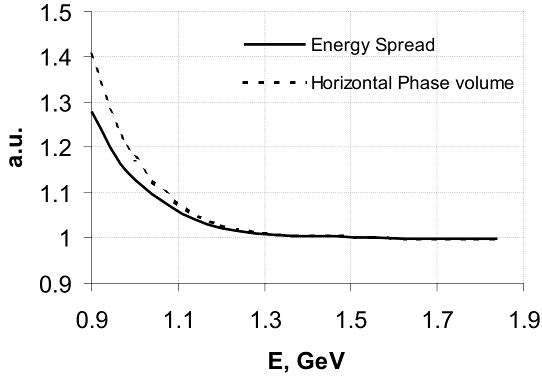

Figure 4: CESR's horizontal phase volume and energy spread in units of their values on quantum fluctuations vs. energy at bunch current is 4 mA.

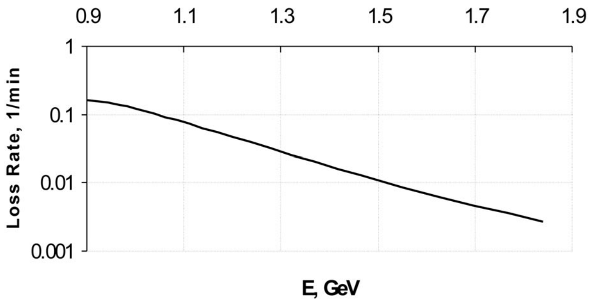

Figure 5: Particle loss rate due to IBS versus CESR's beam energy at bunch current is 4 mA, Energy aperture is 0.44%.

case are represented in Fig.4 and Fig.5 respectively. For simplicity, the scalings are calculated in the assumption that the strength of each CESR magnet varies proportionally to the energy value.

Calculation results for the CESR specific particle loss rate (SLR is the loss rate normalized to one mA of the beam intensity) are given in Table for two pretzel scales and the typical value of the coupling parameter: æ = 0.01.

The similar calculation result for 1.84 GeV VEPP-4M is represented in Fig.9 by the straight line (æ = 0.01, $A_p = 0.56$).

Table. Experimental and calculation data for IBS Loss Rate at CESR

	Pretzel	Energy aperture, A_p, %	Coupling	Specific loss rate, 1/(min·mA)
Calculation	0	0.654	0.01	$0.595 \cdot 10^{-3}$
	3000	0.461	0.01	$1.23 \cdot 10^{-3}$
Experiment	0	–	~ 0.01	$1.05 \cdot 10^{-3} \pm 10^{-4}$
	3000	–	~ 0.01	$2.42 \cdot 10^{-3} \pm 3 \cdot 10^{-5}$

6 Experimental data

The beam lost rate (inverse beam life time) was measured as a function of beam intensity. The measurements made at CESR with one bunch on two different dates, 10/10/02 and 11/8/02, are represented in Fig.6. In given experiments beam orbit was flatten. The vertical beam size measured with

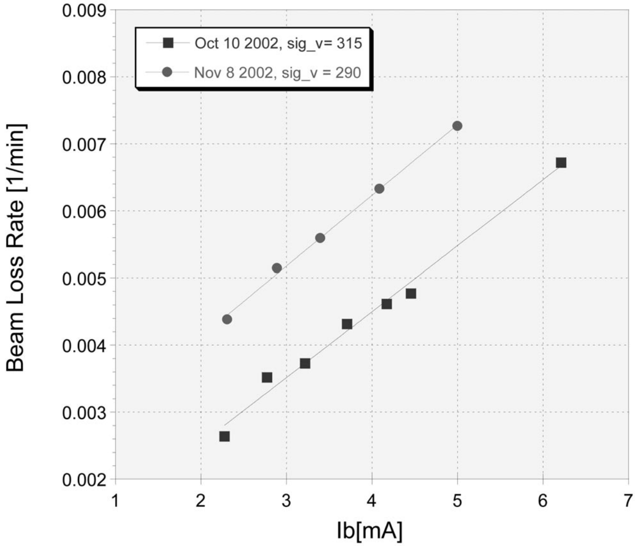

Figure 6: Measured dependence of beam loss rate as a function of the bunch intensity. A linear least-squares fit of the data $1/\tau[1/\min] = m_0 + \alpha \cdot I_b[\text{mA}]$ gives: $m_0 = (0.57 \pm 0.20) \cdot 10^{-3}[1/\min]$, $\alpha = (0.98 \pm 0.05) \cdot 10^{-3}[1/\min/\text{mA}]$ for 10/10/02 data and $m_0 = (2.02 \pm 0.11) \cdot 10^{-3}[1/\min]$, $\alpha = (1.05 \pm 0.03) \cdot 10^{-3}[1/\min/\text{mA}]$ for 11/08/02.

synchrotron light monitor slightly varied (10%). Both dependencies are very close to straight lines with similar slopes but differ in the pedestal level. The latter may be explained by changing residual gas pressure between these measurements.

The similar dependence at more larger beam intensities were studied in the experiment with 8 bunches of the total current up to 40 mA. In Figure 7 one of the lines demonstrates this dependence versus the beam current per one bunch. For comparison, the result of the "one-bunch" experiment is

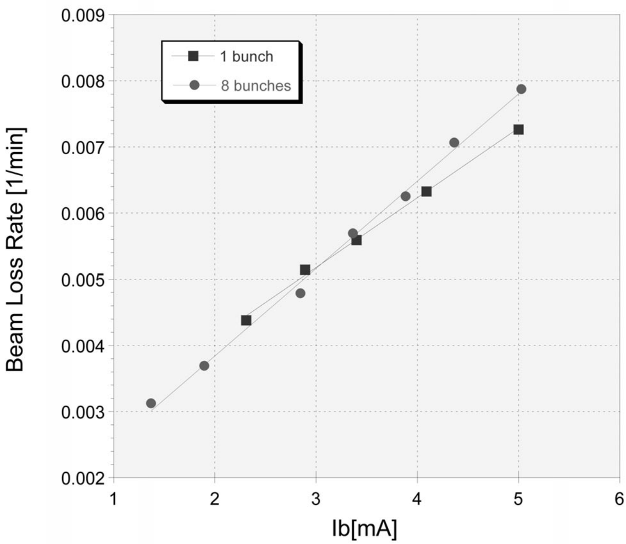

Figure 7: Dependence of beam loss rate as a function of the bunch intensity for a single (squares) and for 8 bunches (circles). A least-squares fit of a straight line through the data: $1/\tau[1/\min] = m_0 + \alpha \cdot I_b[\mathrm{mA}]$ gives $m_0 = (2.0 \pm 0.1) \cdot 10^{-3}[1/\min]$, $\alpha = (1.05 \pm 0.03) \cdot 10^{-3}[1/\min/\mathrm{mA}]$ for a single bunch and $m_0 = (1.2 \pm 0.1) \cdot 10^{-3}[1/\min]$, $\alpha = (1.32 \pm 0.03) \cdot 10^{-3}[1/\min/\mathrm{mA}]$ for 8 bunches.

shown here too. The slope of the multi-bunch line is slightly larger due to gas desorbtion induced by the intense beam. But taking into account for the significant difference in the total beam intensity one can suggest that the basic mechanism of particle losses remains the same in both cases and does not connect to beam-gas scattering.

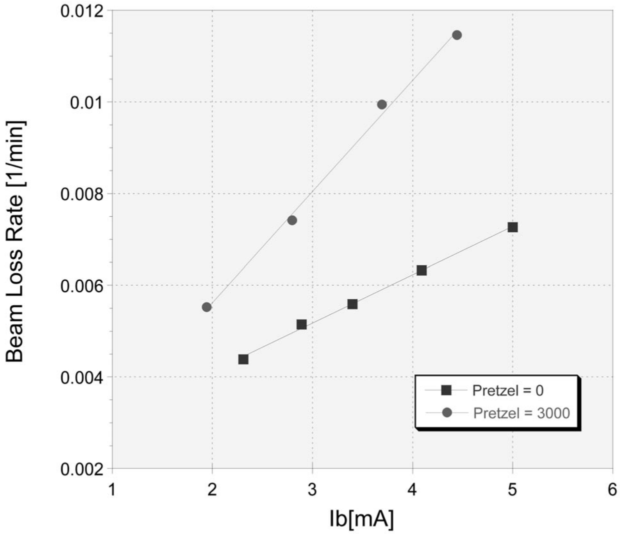

Figure 8: The beam loss rate measured as a function of the bunch intensity for flatten orbit, $Pr1 = 0$, (circles) and for ± 15 mm horizontal orbit distortion, $Pr1 = 3000$, (squares). A least-squares linear fit $1/\tau[1/\min] = m_0 + \alpha \cdot I_b[\text{mA}]$ gives $m_0 = (2.0 \pm 0.1) \cdot 10^{-3}[1/\min], \alpha = (1.05 \pm 0.03) \cdot 10^{-3}[1/\min/\text{mA}]$ for flatten orbit and $m_0 = (0.8 \pm 0.3) \cdot 10^{-3}[1/\min]$, $\alpha = (2.42 \pm 0.10) \cdot 10^{-3}[1/\min/\text{mA}]$ for distorted orbit.

Figure 8 shows some results of measuring the pretzel orbit influence on the beam lifetime. The experiment was performed at minimal initial closed orbit distortions for two states of the pretzel orbit ($Pr = 0$ and $Pr = 1$). The vertical beam size was remaining unchanged and minimal. Its value corresponds to æ ≈ 0.01 that was sustained by the specific luminosity measurement. Generalized data on SLR measurement depending on the pretzel orbit scale are summarized in Table.

Experimental data for VEPP-4M at 1.84 GeV are represented in Fig.9. The dependence of PLR on the one-bunch beam current achieves its maximum at ≈ 4.5 mA and then begins to decrease due to the effect of bunch lengthening. This effect plays a notable role at VEPP-4M because of the large inductive impedance of its vacuum chamber [7]. There is the beam-residual gas interaction pedestal in the dependence of about 0.0016 $\min^{-1}$ that is close to CESR data.

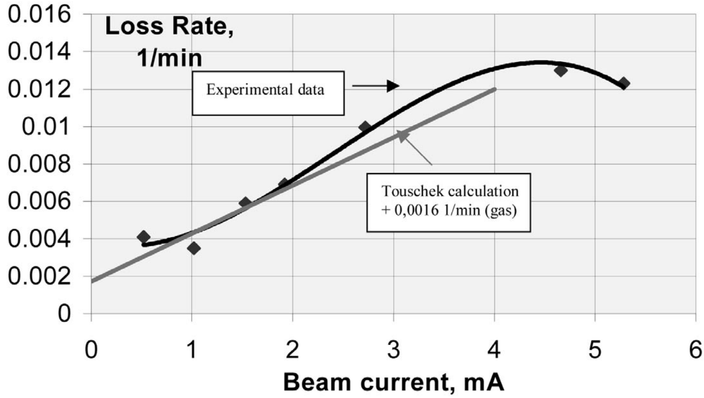

Figure 9: The beam loss rate calculated (the straight line) and measured (the saturated dependence) as a function of the bunch intensity for VEPP-4M (æ ≈ 0.01). $E = 1844$ MeV; the longitudinal size – $2.35\sigma_Z = 340$ ps (10 cm); the vertical beam size – $\sigma_Y = 0.15$ mm; the horizontal beam size – $\sigma_X = 0.86$ mm; the residual gas pressure – $P = 1.1. \cdot 10^{-9}...1.3 \cdot 10^{-9}$ Torr.

7 Discussion

Experimental data for CESR demonstrate the Touschek-like type (linear) behaviour of PLR with changing the beam intensity. Beam-Residual Gas interaction may contribute a small pedestal in this dependence. Gas desorbtion caused by the circulating beam is not significant and its influence on PLR can be neglected. The "PLR vs. the beam intensity " dependence is of a more complicated character in the VEPP-4M case due to the bunch lengthening phenomena. However there exists a wide range of the measured curve where it passes close to the linear law and its slope is in a good agreement with the Touschek PLR calculation. All this proves that IBS is a dominant factor affecting the beam lifetime in both cases.

As one can see from Tab.1, the ratio between calculated SLRs for two states of the pretzel orbit (≈ 2.1) well corresponds to the similar ratio for measured SLRs (≈ 2.3). At the same time the discrepancy between the measured and calculated SLR for CESR makes a factor about two. Since we use the same simulation code for CESR and VEPP-4M one may suppose that one of the possible source of the discrepancy is connected with some uncertainty in the knowledge of the energy aperture which is the most strong factor ($1/\tau \propto A_p^{-2}$). In the CESR case, the value for A_p is found from the DA simulation based on the model distribution of nonlinearities in the guide field over the ring. Really, A_p may be notably smaller. Experimental study of the energy aperture at 1.8 GeV CESR is needed to refine on this parameter that could be useful not only in the viewpoint of the Touschek effect analysis.

Note, our approach is non-relativistic with regard to relative motion of interacting particles in the beam. Relativistic corrections to the IBS cross section are considered below in Appendix. Corresponding estimates are made for CESR at $E = 1843$ MeV whose emittance is a few times larger than VEPP-4M one. CESR transverse momentum spread $\sigma_p \sim 0.2$ MeV/c that yields the transverse velocity in units of the speed of light about $\beta_\perp \sim 0.3$ in CMS. The total difference between the relativistic loss rate and non-relativistic one is found to be characterized by a factor of about 0.97. I.e. the consideration of the relativizm does not change notably the results obtained without relativistic corrections.

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Appendix

In the general case, the differential cross section of the electron-electron elastic scattering in CMS, averaged over all spin states, may be written as [8]:

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{4\beta_{cm}^4} \frac{(2\gamma_{cm}^2 - 1)^2}{\gamma_{cm}^6} \cdot$$

$$\left[\frac{1 + 3\cos\theta^2}{\sin\theta^4} + \left(\frac{\gamma_{cm}^2 - 1}{2\gamma_{cm}^2 - 1} \right)^2 \cdot \left(1 + \frac{4}{\sin\theta^2} \right) \right].$$

Here β_{cm}, γ_{cm} and θ are respectively the velocity in units of the speed of light, the Lorentz factor and the angle of scattering of the electron in CMS.

Non-relativistic formula for the differential cross section of electron-electron scattering takes a form:

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{4\beta_{cm}^4} \cdot \frac{(1 + 3\cos\theta^2)}{\sin\theta^4}.$$

One can see that the relativistic case differs from the non-relativistic one mainly by the factor

$$\frac{(2\gamma_{cm}^2 - 1)^2}{\gamma_{cm}^6} = \frac{(1 + \beta_{cm}^2)^2}{\gamma_{cm}^2}.$$

This results in the dependence for the cross section at large energies of the form $\propto \gamma_{cm}^{-2}$.

To find the particle loss rate with relativistic corrections one must take into account the relativistic kinematics of relative motion of the interacting particles. The relative velocity in the two-particle interaction is

$$\nu = \frac{2\beta_{cm}}{1 + \beta_{cm}^2}.$$

In the non-relativistic case $\nu = 2\beta_{cm}$. Let σ be the cross section integrated over the angle dependence and corresponding to the event, when the particles with the velocity β_{cm} get loss. The particle loss rate (the beam lifetime) is determined by the product $(\bar{\nu\sigma})_{cm}$ averaged over the particle transverse momentum distribution. Therefore, the total difference between the relativistic loss rate and non-relativistic one may be approximately characterized by the factor

$$\frac{(1 + \beta_{cm}^2)^2}{\gamma_{cm}^2} \cdot \frac{\nu}{2\beta_{cm}} = \frac{(1 + \beta_{cm}^2)}{\gamma_{cm}^2}.$$

The estimation of this factor for the CESRc conditions at $E = 1843$ MeV is about 0.97 ($\beta_{cm} \sim 0.3, \gamma_{cm} \sim 1.06$), i.e. the consideration of the relativizm does not change notably the results obtained without relativistic corrections.

The quantity $(\bar{\nu\sigma})_{cm}$ is an effective volume, averaged over the particle transverse momentum distribution, in which the loss of one particle per one second occurs (in CM). Hence the gain of the rate of such events for dN electrons from the beam volume element dV with the beam particle density ρ will be $d(dN/dt)_{cm} = (\bar{\nu\sigma}\rho dN)_{cm} = (\bar{\nu\sigma}\rho^2 dV)_{cm}$. In the laboratory system (L) it yields the loss rate

$$\frac{1}{\tau} = \frac{2}{\gamma^2 N}(\bar{\nu\sigma})_{cm}\left(\int \rho^2 dV\right)_L = \frac{1}{\tau} = \frac{2N}{\gamma^2 V_L}(\bar{\nu\sigma})_{cm},$$

where N is the total number of beam particles; γ is the Lorentz factor and V_L is the beam volume in L. Note, in contrast to [1], we introduce the factor "2" to account for the fact that two particles get loss in one event.

The quantity σ is determined from

$$\sigma = 2 \int\limits_0^{\arccos\mu} \int\limits_0^{\pi} d\sigma \sin\chi d\chi d\phi,$$

where $d\Omega = \sin\chi d\chi d\phi$; $\cos\theta = \sin\chi\cos\phi$; $\mu = A_p/(\gamma p)$; A_p is the energy aperture; p is the transverse momentum in CM. We obtain

$$\sigma = \frac{\pi r_0^2}{8\beta_{cm}^4}\frac{(2\gamma_{cm}^2 - 1)}{\gamma_{cm}^6}\cdot$$

$$\cdot\left[\frac{1}{\mu^2} - 1 + \ln\mu + 4\left(\frac{\gamma_{cm}^2 - 1}{2\gamma_{cm}^2 - 1}\right)^2 (1 - \mu - 4\ln\mu)\right].$$

First three terms in large brackets describe the non-relativistic part in the angle dependence.

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С.А. Никитин, А.В. Темных

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