

Distorting the Hump-backed Plateau of Jets with Dense QCD Matter

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The hump-backed plateau of the single inclusive distribution of hadrons inside a jet provides a standard test of the interplay between probabilistic parton splitting and quantum coherence in QCD. The medium-induced modification of this QCD radiation physics is expected to give access to the properties of the dense medium produced in relativistic heavy ion collisions. Here, we introduce a formulation of medium-induced parton energy loss, which treats all leading and subleading parton branchings equally, and which – for showering in the vacuum – accounts for the observed distribution of soft jet fragments. We show that the strong suppression of single inclusive hadron spectra measured in Au–Au collisions at the Relativistic Heavy Ion Collider (RHIC) implies a characteristic distortion of the hump-backed plateau; we determine, as a function of jet energy, to what extent the soft jet fragments can be measured above some momentum cut. Our study further indicates that the approximate p_T -independence of the measured nuclear modification factor does not exclude a significant Q^2 -dependence of parton energy loss.

Over the last five years, experiments at RHIC have established a phenomenon of strong high- p_T hadron suppression [1, 2], which supports the picture that high- p_T partons produced in the dense matter of a nuclear collision suffer a significant energy degradation prior to hadronization in the vacuum [3]. The microscopic dynamics conjectured to underly high- p_T hadron suppression is medium-induced gluon radiation [4, 5, 6], i.e., a characteristic medium-induced distortion of the standard QCD radiation pattern tested extensively by jet measurements in high energy e^+e^- and pp ($p\bar{p}$) collisions. Modeling this effect accounts for the measured single-inclusive spectra and leading back-to-back hadron correlations [3, 6, 7, 8]. However, existing models indicate only rough qualitative features of subleading jet fragments such as their broadening and softening due to medium-effects. This is so mainly, since medium-induced parton splitting is included for the leading partons only, but not consistently for the subleading splitting processes [6, 9], and since energy momentum conservation is taken into account globally, but not ensured locally for each parton splitting [6, 8]. Here, we propose a formulation which overcomes these limitations.

Subleading jet fragments are known to provide many fundamental tests of QCD radiation physics. In particular, for soft particle momentum fractions $x = p/E_{\text{jet}}$ inside a quark- or gluon- ($i = q, g$) initiated jet of energy and virtuality $Q \sim E_{\text{jet}}$, the single inclusive distribution $D_i(x, Q^2)$ is dominated by multiparton destructive interference, and thus tests quantitatively the understanding of QCD coherence [10, 11]. Remarkably, to double and single logarithmic accuracy in $\xi \equiv \ln[1/x]$ and $\tau \equiv \ln[Q/\Lambda_{\text{eff}}]$, $\Lambda_{\text{eff}} = O(\Lambda_{\text{QCD}})$, the effects of this destructive quantum interference can be accounted for by an angular ordering prescription of a probabilistic parton cascade with leading order (LO) splitting functions. The so-called modified leading logarithmic

approximation (MLLA) resums these effects and accounts for the large $\sqrt{\alpha_s}$ next-to-leading corrections of $D_i(x, Q^2)$ [10, 11, 12]. The MLLA leads to an evolution equation for the ν -th Mellin moments $M_i(\nu, \tau) = \int_0^\infty d\xi e^{-\nu\xi} x D_i(x, Q^2)$ [11, 12, 13],

$$\frac{\partial}{\partial \tau} \begin{pmatrix} M_q(\nu, \tau) \\ M_g(\nu, \tau) \end{pmatrix} = \begin{bmatrix} \Phi_{qq} \left(\nu + \frac{\partial}{\partial \tau} \right) & \Phi_{qg} \left(\nu + \frac{\partial}{\partial \tau} \right) \\ \Phi_{gq} \left(\nu + \frac{\partial}{\partial \tau} \right) & \Phi_{gg} \left(\nu + \frac{\partial}{\partial \tau} \right) \end{bmatrix} \times \frac{\alpha_s(\tau)}{2\pi} \begin{pmatrix} M_q(\nu, \tau) \\ M_g(\nu, \tau) \end{pmatrix}. \quad (1)$$

Here, Φ_{ij} denote combinations of particular moments of leading order splitting functions, for example

$$\Phi_{qq}(\nu) = 2 \int_0^1 dz P_{qq}(z) (z^\nu - 1). \quad (2)$$

The shift $(\nu + \frac{\partial}{\partial \tau})$ in (1) accounts for angular ordering. For a parton fragmentation which starts at high initial scale τ and ends at some hadronic scale τ_0 , the solution of (1) has to fulfill the initial conditions $M(x, \tau_0) = \delta(1-x)$ and $\frac{\partial}{\partial \tau} M(x, \tau = \tau_0) = 0$, since the parton must not evolve if produced at the hadronic scale.

The lowest Mellin moments $\nu \sim 0$ determine the main characteristics of $D_i(x, Q^2)$. For an approximate solution of (1), one can thus expand the matrix in (1) to next-to-leading order in $(\nu + \frac{\partial}{\partial \tau})$ and diagonalize it. Its eigenvalue with leading $1/(\nu + \frac{\partial}{\partial \tau})$ -term yields a differential equation of the confluent hypergeometric type [11]. This leads to an analytic expression for $D(x, Q^2)$, whose shape does not distinguish between quark and gluon parents, since the multiplicity is dominated in both cases by gluon branching. For the hadronic multiplicity distribution $dN^h/d\xi$, one assumes that at the scale τ_0 , a parton is mapped locally onto a hadron with proportionality factor $K^h \sim O(1)$ ("local parton hadron duality", LPHD)

$$\frac{dN^h}{d\xi} = K^h D \left(x, \tau = \ln \left[\frac{Q = E}{\Lambda_{\text{eff}}} \right] \right). \quad (3)$$

Comparisons of (3) to data have been performed repeatedly [13, 14, 15, 16, 17] over a logarithmically wide kinematic regime $7 < E_{\text{jet}} < 150$ GeV in both e^+e^- and $pp/p\bar{p}$ collisions. To illustrate the degree of agreement, we reproduce in Fig. 1 two sets of data [15, 16] together with the curves obtained from (3). The parameters K^h and Λ_{eff} entering (3) were chosen as in Refs. [15, 16], $\Lambda_{\text{eff}} = 254$ MeV, $K^h = 1.15$ for $E_{\text{jet}} = 100$ GeV, $K^h = 1.46$ for $E_{\text{jet}} = 7$ GeV. Following Ref. [16], we use $N_f = 3$. From Fig. 1, we conclude that Eq.(3) accounts reasonably well for the jet multiplicity distribution in the kinematic range accessible in heavy ion collisions at RHIC ($E_{\text{jet}} \sim 10$ GeV) and at the LHC ($E_{\text{jet}} \sim 100$ GeV). Corrections not included in (3) are of relative order $1/\tau$, which at face value corresponds to a 30% (15%) uncertainty at typical RHIC (LHC) jet energies. Also, the MLLA resums large ξ , $\tau \sim \xi$, but is expected to be less accurate for hard jet fragments, where other improvements are currently sought for [18]. Thus, the agreement of (3) to data for the entire ξ -range is surprisingly good. At least from a pragmatic point of view, (3) can serve as a baseline on top of which one can search for medium effects.

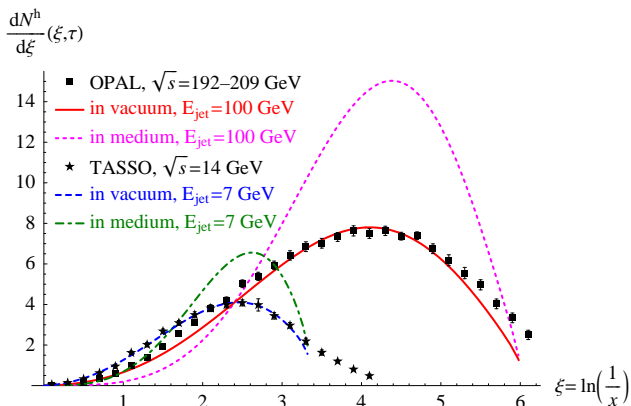


FIG. 1: The single inclusive hadron distribution as a function of $\xi = \ln[E_{\text{jet}}/p]$. Data taken from e^+e^- collision experiments TASSO [15] and OPAL [16], $E_{\text{jet}} = \sqrt{s}/2$. Lines through data obtained from the MLLA result (3). Dashed and dash-dotted curves labeled "in medium" are calculated with a medium-modification $f_{\text{med}} = 0.8$ of the LO splitting functions.

The multiplicity distribution $dN^h/d\xi$ is dominated by soft gluon bremsstrahlung, $dI^{\text{vac}} \simeq C_R \frac{\alpha_s(k_T^2)}{\pi} \frac{dk_T^2}{k_T^2} \frac{d\omega}{\omega}$, $\omega = zE_{\text{jet}}$, which is described by the singular parts $\sim \frac{1}{z}$, $\sim \frac{1}{(1-z)}$ of the QCD splitting functions entering (2). They determine the leading $\frac{1}{v}$ -terms of the evolution matrix in (1). Remarkably, calculations of the additional medium-induced radiation indicate that $\omega \frac{dI^{\text{med}}}{d\omega}$ is $\sim \frac{1}{\sqrt{\omega}}$ if the medium is modeled by soft multiple momentum transfers [19, 20], and $\sim \frac{1}{\omega}$ if the medium is modeled by a single hard momentum transfer [6, 20]. Thus, parametrically, the additional medium-dependent contributions to

the gluon bremsstrahlung are more singular than dI^{vac} for small ω and may thus be expected to dominate the multiplicity distribution (3). However, destructive interference due to finite in-medium path length is known to regulate the soft ω -divergence [20]. For the relevant range of soft ω , this may be modeled as $\omega \frac{dI^{\text{med}}}{d\omega} \sim f_{\text{med}} = \text{const.}$ A medium-induced gluon bremsstrahlung spectrum, consistent with this ansatz, was also found in [21]. This suggests that medium effects enter (3) by enhancing the singular parts of all LO splitting functions P_{gg} , P_{qg} , P_{qq} by the same factor $(1 + f_{\text{med}})$, such that for example

$$P_{qq}(z) = C_F \left(\frac{2(1 + f_{\text{med}})}{(1-z)_+} - (1+z) \right). \quad (4)$$

We do not modify the non-singular subleading terms. On general grounds, one expects that medium-induced rescattering is a nuclear enhanced higher-twist contribution ($f_{\text{med}} \sim \frac{L}{Q^2}$) [22]. This means that it is subleading in an expansion in Q^2 , while being enhanced compared to other higher twist contributions by a factor proportional to the geometrical extension $\sim L$ of the target. A $1/Q^2$ -dependence of f_{med} is also suggested by the following heuristic argument. A hard parton of virtuality Q has a lifetime $\sim 1/Q$ in its own rest frame, and thus a lifetime (in-medium path length) $t = \frac{1}{Q} \frac{E}{Q}$ before it branches in the rest frame of the dense matter through which it propagates. Medium effects on a parton in between two branching processes should grow proportional to (some power of) the in-medium path length and thus $\propto 1/Q^2$ or higher powers thereof.

In contrast, jet quenching models [3, 6, 7, 8] reproduce inclusive hadron spectra in Au-Au collisions at RHIC by supplementing the standard QCD LO factorized formalism with the probability $P(\Delta E)$ that the produced partons radiate an energy ΔE due to medium effects prior to hadronization in the vacuum [9]

$$P(\Delta E) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^n \int d\omega_i \frac{dI^{\text{med}}(\omega_i)}{d\omega} \right] \delta \left(\Delta E - \sum_{i=1}^n \omega_i \right) \times \exp \left[- \int_0^{\infty} d\omega \frac{dI^{\text{med}}}{d\omega} \right]. \quad (5)$$

This formula is based on a probabilistic iteration of medium-modified parton splittings, but does not keep track of virtuality or angular ordering. The k_T -integrated medium-induced contribution dI^{med} is treated on an equal footing with LO vacuum splitting functions. In this sense, the medium-modified fragmentation function $D_{h/q}^{(\text{med})}(x, Q^2) = \int_0^1 d\epsilon E P(\Delta E) \frac{1}{1-\epsilon} D_{h/q}(\frac{x}{1-\epsilon}, Q^2)$, $\epsilon = \Delta E/E$, entering jet quenching models [3, 6, 7, 8], amounts to a medium-induced Q^2 -independent modification of parton fragmentation.

The single inclusive distribution $D(x, Q^2)$, supplemented by LPHD, is a fragmentation function. Single inclusive hadron spectra, whose parent partons show a

power law spectrum $\propto 1/p_T^{n(p_T)}$, test $D(x, Q^2)$ in the range, in which $x^{n(p_T)-2} D(x, Q^2)$ has significant support. However, for large x , the accuracy of the MLLA result for $D(x, Q^2)$ becomes questionable [13, 18]. To understand to what extent the MLLA result may still be used from a practical point of view, we have compared it to the KKP parametrization [24] of fragmentation functions. In the range of Q^2 and x relevant for single inclusive spectra ($0.4 < x < 0.9$), we observe that the KKP and MLLA fragmentation functions both drop by 2 orders of magnitude. They do show somewhat different shapes but – after adjusting the overall normalization – they differ for all x -values by maximally $\sim 30\%$ or significantly less (data not shown). For the nuclear modification factor R_{AA} , which is the ratio of modified and unmodified single inclusive hadron spectra, and which does not depend on the overall normalization of $D(x, Q^2)$, this is a relatively small uncertainty, if one aims at characterizing a factor 5 suppression. We thus conclude that the MLLA fragmentation function obtained from (1) can be used to calculate R_{AA} .

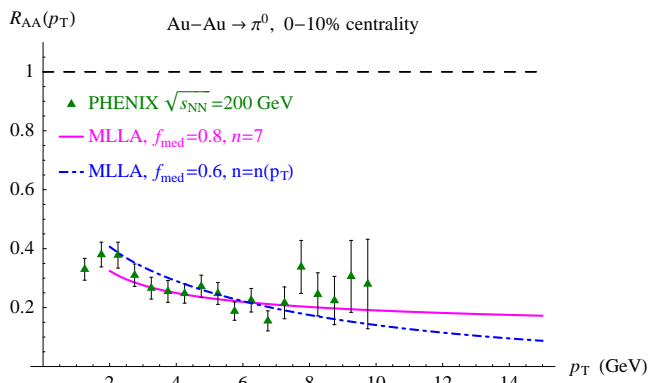


FIG. 2: The p_T -dependence of the nuclear modification factor R_{AA} , calculated for a medium-enhanced parton splitting with f_{med} . Data taken from [23].

To determine R_{AA} , we have parametrized the partonic p_T -spectrum at RHIC energies $\sqrt{s_{NN}} = 200$ GeV by a power law $1/p_T^{n(p_T)}$, $n(p_T) = 7 + 0.003 p_T^2/\text{GeV}^2$, which accounts for kinematic boundary effects at $p_T \sim O(\sqrt{s_{NN}})$. Single inclusive hadron spectra and R_{AA} are calculated by convoluting this spectrum with $D(x, Q^2)$. Medium effects are included through the factor f_{med} in the singular parts of all LO parton splitting functions, see Eq.(4). As seen in Fig. 2, the choice $f_{\text{med}} \simeq 0.6 \div 0.8$ reproduces the size of the suppression of $R_{AA} \sim 0.2$ in central Au-Au collisions at RHIC [23]. Jet quenching models based on (5) yield a slightly increasing p_T -dependence of $R_{AA}(p_T)$ for a power law $n(p_T) = n$, and a rather flat dependence if trigger bias effects due to the p_T -dependence of $n(p_T)$ are included [7, 8]. In contrast, the MLLA result for $R_{AA}(p_T)$ decreases with increasing p_T , and this

tendency is even more pronounced for a realistic shape of the underlying partonic spectrum, see Fig. 2. The reason is that in the MLLA, parent partons of higher p_T have higher initial virtuality $Q \sim p_T$, and undergo more medium-induced splittings; this results in a smaller value of R_{AA} . A proper treatment of nuclear geometry may affect quantitative aspects of Fig. 2, but is unlikely to change this qualitative observation. Hence, the observed flat p_T -dependence of $R_{AA}(p_T)$, one may require (in accordance with the arguments given above) a non-vanishing Q^2 -dependence of f_{med} , which would reduce medium-effects on high- p_T ($p_T \sim Q$) partons.

Motivated by this observation, we have attempted to solve Eq.(1) for a non-trivial Q^2 -dependence of the medium-enhancement f_{med} . We did not find an analytical solution. However, in the absence of medium effects, the analytical solution (3) is reproduced by Monte Carlo (MC) parton showers based on angular ordering [13]. This remains true for non-vanishing f_{med} . The present study can serve to check future MC showers implementing (4), and it can be extended in MC studies to include a non-trivial Q^2 -dependence of f_{med} . We plan such a MC study, mainly to establish to what extent the approximate p_T -independence of R_{AA} up to $p_T \sim 10$ GeV allows for a significant Q^2 -dependence of parton energy loss. The question of whether and on what scale these effects are $1/Q^2$ -suppressed is of obvious importance for heavy ion collisions at the LHC, where medium-modified parton fragmentation can be tested in a logarithmically wide Q^2 -range.

What is the distortion of the longitudinal jet multiplicity distribution (3), consistent with the observed factor ~ 5 suppression of R_{AA} ? In contrast to calculations based on (5), the medium-enhanced parton splitting introduced via MLLA conserves energy-momentum exactly at each branching, it treats all secondary branchings of softer gluons equally, and it continues all branchings down to the same hadronic scale. This makes it a qualitatively improved tool for the calculation of longitudinal multiplicity distributions, since it matters obviously for $dN/d\xi$ whether one gluon is radiated into the bin $\xi = \ln[E_{\text{jet}}/p_{\text{gluon}}]$, or – after further splitting $g \rightarrow g(z)g(1-z)$ – two gluons with momentum fractions z and $(1-z)$ into bins $\xi + \ln[1/(1-z)]$, $\xi + \ln[1/z]$, respectively. We have calculated $dN/d\xi$ for a medium-enhanced parton splitting $f_{\text{med}} = 0.8$ consistent with $R_{AA} = 0.2$. Results for jet energies relevant at RHIC and at the LHC are shown in Fig. 1. In general, the multiplicity at large momentum fractions (small ξ) is reduced and the corresponding energy is redistributed into the soft part of the distribution. The maximum of the multiplicity distribution also shifts to a softer value, but this shift is subleading in $\sqrt{\alpha_s}$, $\xi_{\text{max}}/\tau = \frac{1}{2} + a_{\text{med}} \sqrt{\frac{\alpha_s(\tau)}{32 N_c \pi}}$, where $a_{\text{med}} = \frac{1}{3} (11 + 12 f_{\text{med}}) N_c + \frac{2}{3} \frac{N_f}{N_c}$.

Many experimental characterizations of the medium-

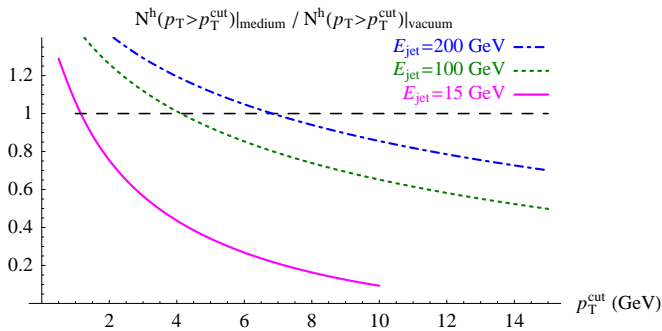


FIG. 3: The change of the total hadronic multiplicity inside jets of different energy E_{jet} , as a function of the soft background cut p_T^{cut} above which this multiplicity is measured. Medium-effects are modeled by $f_{\text{med}} = 0.8$.

modified internal jet structure in heavy ion collisions at RHIC and at the LHC require a soft momentum cut p_T^{cut} to control effects of the high multiplicity background. Can one observe the increase in soft multiplicity shown in Fig. 1, if such a soft background cut p_T^{cut} is applied? To address this issue, we have calculated from (3) the total hadronic multiplicity $N^h(p_T > p_T^{\text{cut}})$ above p_T^{cut} . As seen in Fig. 3, the medium-enhanced component of soft multiplicity lies below a critical transverse momentum cut $p_{T,\text{crit}}^{\text{cut}}$ which increases significantly with E_{jet} . For a typical hard jet at RHIC ($E_{\text{jet}} = 15$ GeV), the additional soft jet multiplicity lies buried in the soft background, $p_{T,\text{crit}}^{\text{cut}} \simeq 1.5$ GeV. For $E_{\text{jet}} = 100$ GeV, accessible at the LHC, $p_{T,\text{crit}}^{\text{cut}} \simeq 4$ GeV lies well above a cut which depletes the background multiplicity by a factor 10. For $E_{\text{jet}} = 200$ GeV, we find $p_{T,\text{crit}}^{\text{cut}} \simeq 7$ GeV. The associated total jet multiplicity $N^h(p_T > p_{T,\text{crit}}^{\text{cut}})$ for these jet energies rises with E_{jet} from $\simeq 4$, to $\simeq 7$. Fig. 3 indicates a qualitative advantage in extending jet measurements in an LHC heavy ion run near design luminosity to significantly above $E_{\text{jet}} \simeq 100$ GeV, where a sizeable kinematic range $2 \div 3$ GeV $< p_T < p_{T,\text{crit}}^{\text{cut}} \simeq 7$ GeV becomes accessible. This may allow a detailed characterization of the enhanced medium-induced radiation above the soft background.

In general, the formulation of parton energy loss within the MLLA formalism allows one to address several fundamental questions, that remain untouched by recent model studies of jet quenching. This concerns in particular the important issue of the Q^2 -dependence of parton energy loss discussed above. Moreover, the use of a probabilistic formulation based on angular ordering can also be viewed as a test of the unproven assumption that the medium-induced destructive interference of multi-parton emission can indeed be accounted for by angular ordering, in close similarity to gluon radiation in the vacuum. We finally note that the formalism introduced here is not limited to a discussion of the hump-backed plateau: e.g. it can be applied to the calculation of two-particle correlations

within jets, which have been studied in the absence of medium effects [12, 13]. It may also apply to transverse jet broadening [12], which for a Q^2 -dependent f_{med} may be significantly reduced since large angle emission would remain essentially unmodified by the medium. We plan to address these open questions in future work.

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