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PROPOSAL FOR A STUDY OF $K^{+}_{\ell 3}$-DECAYS*)

J. Heintze, G. Heinzelmann, P. Igo-Kemenes, R. Mundhenke,
H. Rieseberg, B. Schürlein, H.W. Siebert, V. Soergel,
H. Stelzer, K.P. Streit, A. Wagner, A.H. Walenta

CERN-Heidelberg Ke2 Group

*) This proposal summarizes and amends our memoranda dated 1.4.1971 and 28.2.1972 (PH I/COM-71/14 and 72/10)

INTRODUCTION

The present experimental situation in $K_{\ell 3}$-decays is unsatisfactory in several respects. The points which are mainly unclear are the momentum transfer dependence of the form factor $f_+(q^2)$ and of the form factor ratio $\xi(q^2)$, and the $K\mu 3/Ke3$ branching ratio. The dependence of f_+ on the momentum transfer

$$q^2 = (p_K - p_\pi)^2$$

is usually parametrized in a linear expansion

$$f_+ (q^2) = f_+(o) \cdot \left(1+\lambda_+ \frac{q^2}{m_\pi^2}\right).$$

Experimental values for λ_+ between 0.02 and 0.08 with errors $\lesssim 0.01$ have been reported (see Ref. 1-3 for recent review articles). In order to explain these apparent discrepancies, a quadratic expansion

$$f_+ (q^2) = f_+(o) \cdot \left(1+\lambda_+ \frac{q^2}{m_\pi^2} + \lambda'_+ \left(\frac{q^2}{m_\pi^2}\right)^2 \right)$$

with λ_+ and λ'_+ comparable in magnitude was proposed[2]. The validity of such a quadratic expansion would be rather surprising, since $f_+(q^2)$ is connected, by SU_3, with the electromagnetic pion form factor, which in turn seems to be well described by the vector meson dominance model. In this case, validity of the linear expansion with $\lambda_+ \sim 0.025$ should be expected.

The momentum dependence of $\xi(q^2)$ and its value at $q^2 = o$ has been extensively discussed in the last years. It should be noted, that the model independent experimental basis for the momentum dependence of ξ is very weak, due to lacking statistics. This is shown in Fig. 1. It seems however, rather well established that the average value of ξ is near to -1.

An interesting point which can be studied in $K_{\ell 3}$-decays is the muon-electron universality. Assuming universality, the branching ratio

$$R = \frac{\Gamma\ (K\mu 3)}{\Gamma\ (Ke3)}$$

is a function of ξ and λ_+. There are two experiments which give accurate values for R in K^+ decays[4)5)] which, however, differ by ~ 3 standard deviations. Only one of these experiments (the X_2-experiment) is compatible with $\xi = -1$ and μ-e universality; the Oxford experiment would require $\xi \sim 0$ if μ-e universality is assumed. The present situation is displayed both for linear and quadratic expansions of f_+ in Fig. 2. The branching ratio found in K^0 decays (R= 0.684) is close to the Oxford value (R = 0.667).

In the experiment proposed here, we want to measure

a) the lepton spectra from Ke3 and Kμ3 decays

b) the Ke3 and Kμ3 Dalitz plot populations with precisely known relative efficiency and nearly constant π^0 detection efficiency.

From these data, we want to extract a precise value for the Kμ3/Ke3 branching ratio, model independent values for $f_+(q^2)$ and $\xi(q^2)$, and information concerning the μ-e universality.

All data are taken concurrently. We hope to achieve about 10 times the statistics of previous experiments. At the same time our method will permit numerous consistency checks, such that systematical errors could be recognized and controlled.

APPARATUS

The proposed apparatus is shown in Fig. 3. K^+-Mesons from the K_{12a} beam are stopped in a LiH target. The momenta of charged particles from K^+ decays are measured in a magnetic spectrometer equipped with drift chambers. Behind the spectrometer, the particle energy is measured in an array of large NaI(Tl) crystals and leadglass Cerenkov counters.*) The combined momentum and kinetic energy measurement permits an unambiguous separation of pions, muons and electrons. The gas-Čerenkov counter we used in the Ke2 experiment between target and magnet is therefore not needed. We intend to put in this place a π^o-detector which surrounds the target from all sides, except relatively small holes for beam entrance and exit and for the passage of particles to the spectrometer. This geometry permits π^o detection with nearly constant efficiency in the whole Dalitz plot.

The target arrangement and the spectrometer are essentially the same we used in the Ke2-experiment. The performance of this part of the apparatus is demonstrated by the experimentally observed Kμ2 line shown in Fig. 4. We plan to operate the spectrometer at B=3 kG (half the field used in the Ke2 experiment). At this field, the spectrometer response is flat above 100 MeV/c, and the momentum resolution will be about 2.5% FWHM.

The NaI (Tl)-leadglass array has a sensitive area of 130x45 cm^2. The thickness of the NaI blocks is 16 cm corresponding to 6.2 radiation lengths. This

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*) The NaI (Tl) and leadglass blocks are purchased by our institute for various experiments we plan to execute at the DESY-Storage Rings and at CERN. They will be available at the end of 1972.

thickness is sufficient to stop muons from Kμ3, while a part of the shower produced by Ke3-electrons will escape through the back. According to tests performed at the SC, the muon energy is measured with a resolution σ=4%. The energy resolution for electrons is improved by the leadglass counters behind the NaI. In particular, by use of the NaI-leadglass sandwich a symmetrical electron line shape free of a low pulse height tail is obtained. Fig. 5 shows the line shape observed with a NaI-leadglass counter in a test performed at the Bonn Synchrotron. On the basis of these tests we expect a μ-e discrimination of about 99%.

Below 126 MeV/c, the ratio of π from τ decay to μ from Kμ3 decay is ~1. At those momenta, μ-π discrimination is no problem. At high momenta it becomes more difficult. We therefore plan to measure the small pion contamination expected here by running a few days with a preabsorber in front of the NaI blocks.

The π^0-detector is shown in detail in Fig. 6. It consists of a nearly closed box formed by sandwiches made of lead converters, drift chambers, scintillators and 17 cm thick leadglass counters. The drift chambers are equipped with readout both at the anode wires and at the cathode planes. It has been shown[6)] that such a system can be used for two dimensional readout (space resolution of the cathode readout σ~ 3mm) and at the same time for resolving the x, y ambiguity occuring in double track events. The latter property is due to the fact that the signal at the anode wire is within 2ns in time with the influence signal at the cathode plane, while the two signals from a double track event in a drift chamber are separated by ~100ns in the average.

Due to the particular kinematics of $K^+ l3$ decays, the π^o energy is determined by the momentum of the lepton and by the direction of the two π^o γ-rays plus a very crude measurement of the γ-ray energy. With a space resolution of $\sigma \sim 0.5$ cm in the determination of the decay- and conversion points a π^o energy precision of $\sigma \sim 8$ MeV can be obtained in the apparatus proposed here. This accuracy would be sufficient for our experiment.

The efficiency for π^o detection is given by the probability that both γ-rays hit the lead converter, times the probability that both γ-rays convert and give signals. The first (geometrical) factor can be accurately computed for each point of the Dalitz plot; the second factor (γ-ray detection) depends only on the γ-ray energy. It will be determined experimentally by study of $K\pi 2$ decays. We expect a total π^o detection efficiency of ~15%. A Monte Carlo calculation has shown that this efficiency is essentially constant over the whole Dalitz plot. This important feature is a consequence of the geometry of the π^o-detector. The accurate value of the π^o efficiency will be determined by experiment.

TRIGGER RATE AND EVENT RATE

The trigger will be (K^+_{stop} · H·A·NaI). We expect a rate of ~200 triggers/burst from K^+ decays and a small amount of accidental triggers. $K\mu 2$ decays and a part of $K\pi 2$ decays can be rejected during the burst by a fast momentum cut done by the computer. The remaining ~60 events per burst will then be analysed between bursts and their momentum will be computed. After this, <10 events/burst remain for transfer onto magnetic tape. They contain the useful $K_{\ell 3}$ data in the momentum interval 90 MeV/c< p< 190 MeV/c and a sample of $K\pi 2$ events needed for calibration of the π^o-detector.

The rate of good $K_{\ell 3}$ events is estimated from the data taken in our Ke2-experiment. We observed $8 \cdot 10^6$ uniquely identified Kμ2 events per week. This corresponds to 600 000 Ke3 decays per week. By the smaller solid angle we lose 60%, by more stringent criteria on the track quality we expect to lose 50%, and by momentum cuts 25%.

We therefore expect in the total lepton spectra

90 000 K_{e3} events/week

and 60 000 $K_{\mu 3}$ events/week,

and in the Dalitz plot

13 000 K_{e3} events/week

and 9 000 $K_{\mu 3}$ events/week.

We expect that up to 25% of these Dalitz plot events will be contaminated by random tracks in the π^0 detector.

ANALYSIS OF THE DATA

a) Branching ratio Kμ3/Ke3

The branching ratio Kμ3/Ke3 is obtained directly by the ratio of identified Kμ3 and Ke3 decay events per momentum bin, after subtraction of background. This method is independent of the spectrometer acceptance.

A test at the K_{12} beam has shown, that the accidental background in the NaI has no effect on the energy measurement.

The only background expected in Ke3 is due to Dalitz pairs with escape of all other charged particles. This is expected to be of the order of a percent; it can be measured by running with reversed field.

In the Kμ3 the background situation is more difficult due to decays in flight of pions from Kπ2.

Decays in and behind the spectrometer (regions III and IV in Fig. 7a) can be rejected, because the momentum-energy relation for μ will not be fulfilled.

Decays between the magnet and the first chamber near the target (region II) can be rejected, if their decay kink is big enough (>50mrad). Unrecognized decays in this region will concentrate at small kink angles, i.e. at high and low muon momenta. Decays before and near the first chamber (region I) cannot be rejected at all. The resulting background spectrum is shown in Fig. 7b.

We estimate that the total background from Kπ2 to be subtracted is ~10% and that it can be determined with 10% accuracy. Decays in flight of π from τ and τ' decays is estimated to be ~0.8% for momenta above 90 MeV/c.

Obviously, the application of kink criteria risks to reduce the muon detection efficiency. We will therefore apply the same criteria to the Ke3 decays and to Kμ3 Dalitz plot events below the π^o energy of Kπ2. A further consistency check is obtained by study of the Kμ3 spectral shape.

In order to compute the Kμ3/Ke3 branching ratio, extrapolations to p = 0 and p = 228 MeV/c are needed. This extrapolation depends only very little on the $K_{\ell 3}$ form factors, as the accepted momentum band extends from 90 to 190 MeV/c.

b) Determination of $f_+(q^2)$

The determination of $f_+(q^2)$ will be made as usual by measurement of the π^o spectrum of Ke3:

$$\frac{dn}{dE_{\pi o}} = f_+^2\,(q^2)\;\cdot\;(E_{\pi o}^2 - m_{\pi o}^2\,)^{3/2}$$

$$q^2 = (p_K - p_{\pi o})^2 = m_K^2 + m_{\pi o}^2 - 2\,m_K\,E_{\pi o}$$

The crucial point is the knowledge of the π^o detection efficiency, $\varepsilon_{\pi o}$. We want to determine it by computation and by experiment using Kπ2 decays. A difficulty comes from the fact, that π^o's from Kπ2 decay are collinear, whereas the π^o-lepton angle varies between 0^o and 180^o (Fig. 8). Therefore, homogeneity of the γ-ray detection efficiency is to be assumed in the determination of $\varepsilon_{\pi o}$ by this method. This homogeneity is certainly only approximately fulfilled. The necessary corrections can be determined by study of the shape of electron spectra at fixed π^o energies. Our experiment should contain sufficient statistics for this purpose (Table I). A consistency check is obtained by the condition that the Dalitz plot integrated over the π^o energy should result in an electron spectrum consistent with the spectrum measured without recording the π^o. Assuming that we can determine the variation of $\varepsilon_{\pi o}$ over the Dalitz plot with 5% accuracy, we expected to determine independent values of λ_+ in different q^2 intervals between q^2 = o and $q^2 = 7\,m_\pi^2$ with an error of $\Delta\lambda_+ \sim 0.01$. This accuracy would allow a reasonable statement about the value and possible variations of λ_+ with momentum transfer.

The shape of the Ke3 spectrum recorded without π^o detection contains information on radiative corrections and λ_+ (see Fig. 9). λ_+ can be determined with a precision of $\sim$0.01, if correctness of the radiative corrections is assumed.

c) Determination of $\xi(q^2)$

Assuming muon electron universality, a model independent determination of $\xi(q^2)$ is possible which is also independent of precise knowledge of the π^0 detection efficiency. The total number of $K\mu3$ events at fixed π^0 energy (i.e. fixed q^2) is divided by the number of Ke3 events at the same π^0 energy. This ratio depends only on $\xi(q^2)$. The sensitivity is displayed in Fig. 10. The $K\mu3$ rates in different strips of the Dalitz plot, expected for $\xi = -1$, are included in Tab. I. We hope to obtain values for $\xi(q^2)$ in 5 different q^2 bins with errors $\Delta\xi \simeq 0.15$.

At low values of q^2 (high values of E_{π^0}) background from $K\pi2$, $\pi \to \mu\nu$ decays will be present. We have not yet studied the reconstruction properties of these background events.

d) Investigation of μ-e-universality

In the method proposed above for the determination of $\xi(q^2)$, any violation of μ-e universality would be hidden by assignment of a suitable (but wrong) value of ξ. μ-e universality could be tested by comparing the values of ξ obtained in this analysis with values of ξ determined independently of the Ke3 Dalitz plot.

Such a value can be found by a study of μ-spectra at fixed π^0 energy. At high q^2, the shape of the muon spectrum is reasonably sensitive to ξ (Fig. 11). For example, we should be able to extract an independent value of ξ at $q^2 = 5\ m_\pi^2$ with an error of $\Delta\xi = \pm 0.3$.

We also can compare our values for $\xi(q^2)$ with values of ξ determined independently from the Ke3 Dalitz plot in other experiments (Fig. 1).

MACHINE TIME AND COMPUTER TIME REQUEST

For the execution of this experiment at the K_{12a} beam we need one and a half 3 week period for setting up and running in, two 3 week periods for data taking and half a period for various calibration measurements. This machine time request assumes that the useful time and the beam intensity on our target within a 3 week period will be the same as in 1972.

For preparation and evaluation of this experiment we ask for 400 hours of CD 6600 equivalent time over a period of 2 years. Part of the tape handling will be done on our PDP-9, especially a data condensation by an estimated factor of 3. The planned Heidelberg University computer center, which we could use for our evaluation, will not be operational for the next two years. Part of the preparation can be done on the IBM 360/65 of the Heidelberg Institut für Hochenergiephysik.

We would be ready to start this experiment in spring 1973.

q^2/m_π^2	N (Kμ3)	N (Ke3)
0-1	2460	33550
1-2	21890	24860
2-3	18440	19150
3-4	14100	13440
4-5	9710	7840
5-6	5230	3410
6-7	1330	750

TABLE I Kμ3 and Ke3 rates in different bins of q^2, normalized to 10^5 Ke3 events. Constant form-factors and $\xi = -1$; momentum limits: 90 MeV/c $< p <$ 190 MeV/c.

REFERENCES

1) M.K. Gaillard, L.M. Chounet, CERN 70-14 (1970)
L.M. Chounet, M.K. Gaillard, Phys. Letters 32 B, 505 (1970)

2) L.M. Chounet, J.-M. Gaillard, M.K. Gaillard, to be published in Physics Reports

3) E. Dally et al., UCLA 1064 (1972)

4) D.R. Botterill et al., Phys.Rev. 174, 1661 (1968)

5) The X_2 collaboration, Phys. Letters 27B, 586 (1968)

6) G. Charpak, D. Rahm, H. Steiner, Nucl. Inst.Meth. 80, 13 (1970)

G. Fischer, J. Plch, Nucl. Instr. Meth. 100, 515 (1972)

A.H. Walenta, thesis, Heidelberg 1972, and submitted to Nucl. Instr. Meth.

7) D. Cutts et al., Phys. Rev. Letters 20, 955 (1968)

8) The X_2 collaboration, Phys. Rev. D3, 10 (1971)

9) The X_2 collaboration, Nuovo Cim. LVIA, 1107 (1968)

10) C.Y. Chien et al., Phys. Letters 33B, 627 (1970)

FIGURE CAPTIONS

Fig. 1: Results of model-independent $\xi(q^2)$ analysis

a) $K\mu3$ polarization measurements, Ref. 7,8.

b) $K\mu3$ Dalitz plot measurements, Ref. 9,10.

Fig. 2: Comparison of experimental results in form factor analysis.

a) analysis in terms of $\xi(0)$ and λ_+

1= combined results of $K\mu3$ Dalitz plot and polarization measurements, cf. fig. 21a of Ref. 2

2,3 = measured branching ratios of the Oxford and X_2 groups, Ref. 4, 5.

b) analysis in terms of $\xi(0)$ and λ_-.

$\lambda_+ = 0.012 \pm 0.005$, $\lambda'_+ = 0.0052 \pm 0.0013$

assumed from Ke3 data.

1, 2, 3 as for a), cf. fig. 21c of Ref. 2.

Fig. 3: Apparatus.

DC1 to DC6 are drift-chambers

The trajectory is for a 120 MeV/c particle with the magnetic field at 1/2 of the value used for the Ke2 experiment.

For the π^o-detector surrounding the target cf. Fig. 6.

Fig. 4: $K_{\mu2}$ line (236 MeV/c) measured during the K_{e2} experiment.

Fig. 5: Measured energy losses for particles at 200 MeV/c.

1. μ-energy measured in NaI, FWHM = 10%.

2. e-energy measured in NaI-leadglass sandwich, FWHM = 20%.

Fig. 6: π^{o}-detector around target.
γ-rays from π^{o} are converted in a lead plate and the conversion point is recorded with drift chambers. The γ-energy is measured in leadglass counters.

Fig. 7: a) $K\pi 2$ decay regions.

I region around target, where π-μ decays are not identified.

II region between target and magnet, where π-μ decays with π-μ angles >50mrad will be identified.

III region inside magnetic field, where π-μ decays will affect the momentum analysis and will be recognized by a momentum-energy relation not fitting for muons.

IV region behind the magnet, where π-μ decay events will have the π-momentum and the μ-energy.

b) 1. $K\mu 3$ momentum spectrum for $\xi = -1$.

2. μ-spectrum from unidentified $K\pi 2$, $\pi \rightarrow \mu\nu$ decays in region I.

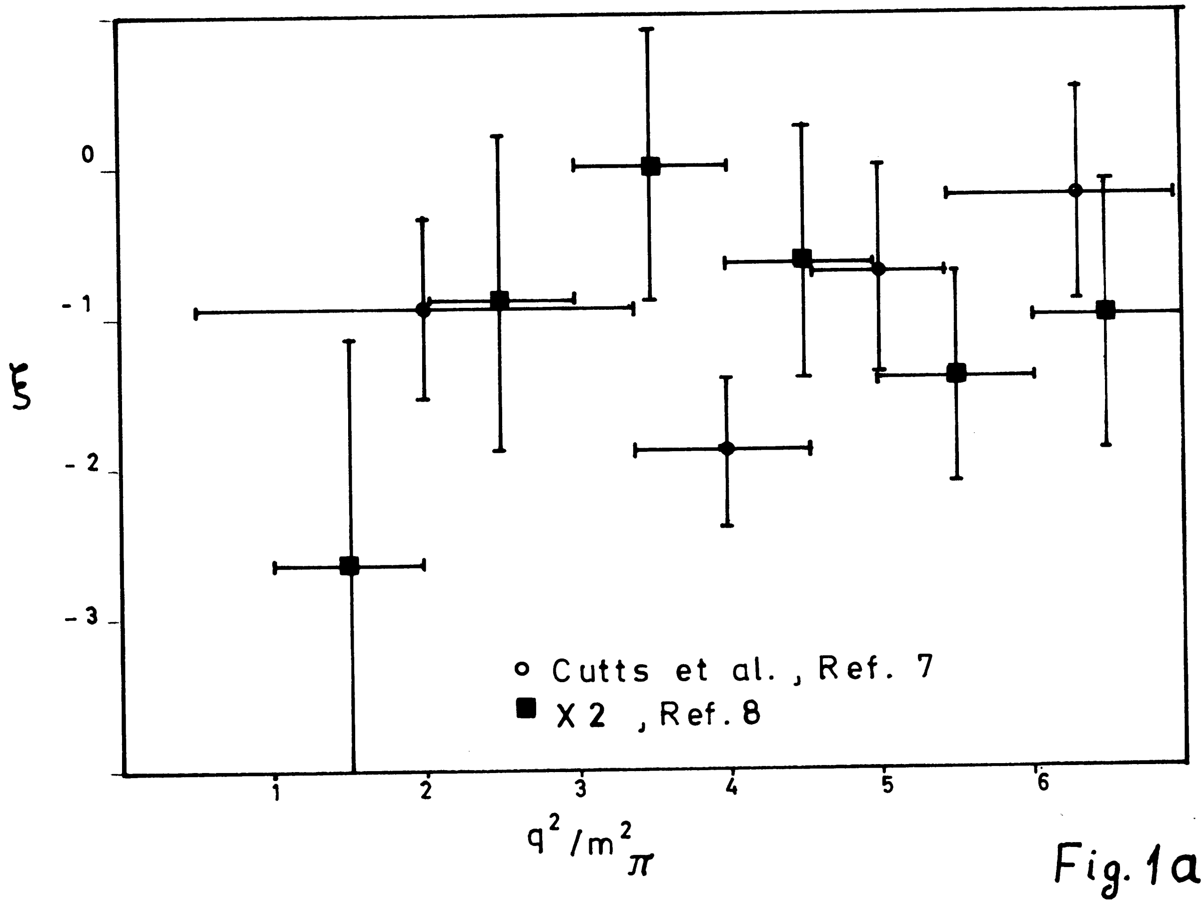
3. μ-spectrum from unidentified $K\pi 2$, $\pi \rightarrow \mu\nu$ decays in region II with π-μ angles <50mrad.

Fig. 8: Ke3 and $K\mu 3$ Dalitz plots, showing the pion-lepton angle. The $K\pi 2$ particle energies are also indicated.

Fig. 9: Shape factors in the Ke3 electron spectrum, relative to the spectrum for $\lambda_{+}=0$ without radiative corrections.
The external bremsstrahlung correction is calculated for electrons from the target center, which traverse ≈ 0.025 radiation lengths.

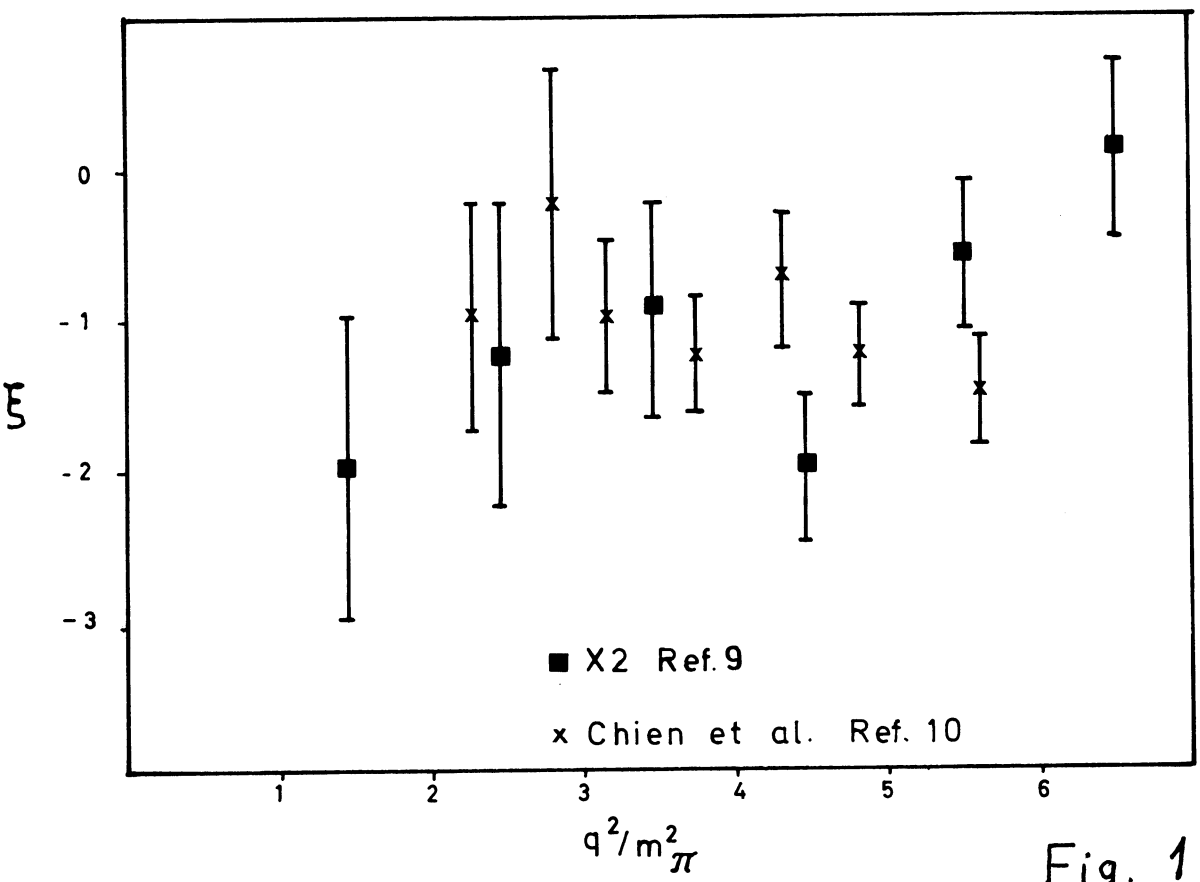
Fig. 10: Ratio of muons and electrons between 90 and 190 MeV/c at fixed pion energies (momentum transfers) for different values of ξ.
Error bars are statistical errors based on 10^5 events in the Ke3 Dalitz plot for an evaluation in 6 different q^2-bins.

Fig. 11: Kμ3 muon energy spectra at low pion energies (high q^2) for different values of ξ.



0
- 1
- 2
- 3
ξ
1
2
3
4
5
6
q^2/m^2_π
o Cutts et al., Ref. 7
■ X 2 , Ref. 8

Fig. 1a



0
- 1
- 2
- 3
ξ
1
2
3
4
5
6
q^2/m^2_π
■ X2 Ref. 9
x Chien et al. Ref. 10

Fig. 1b

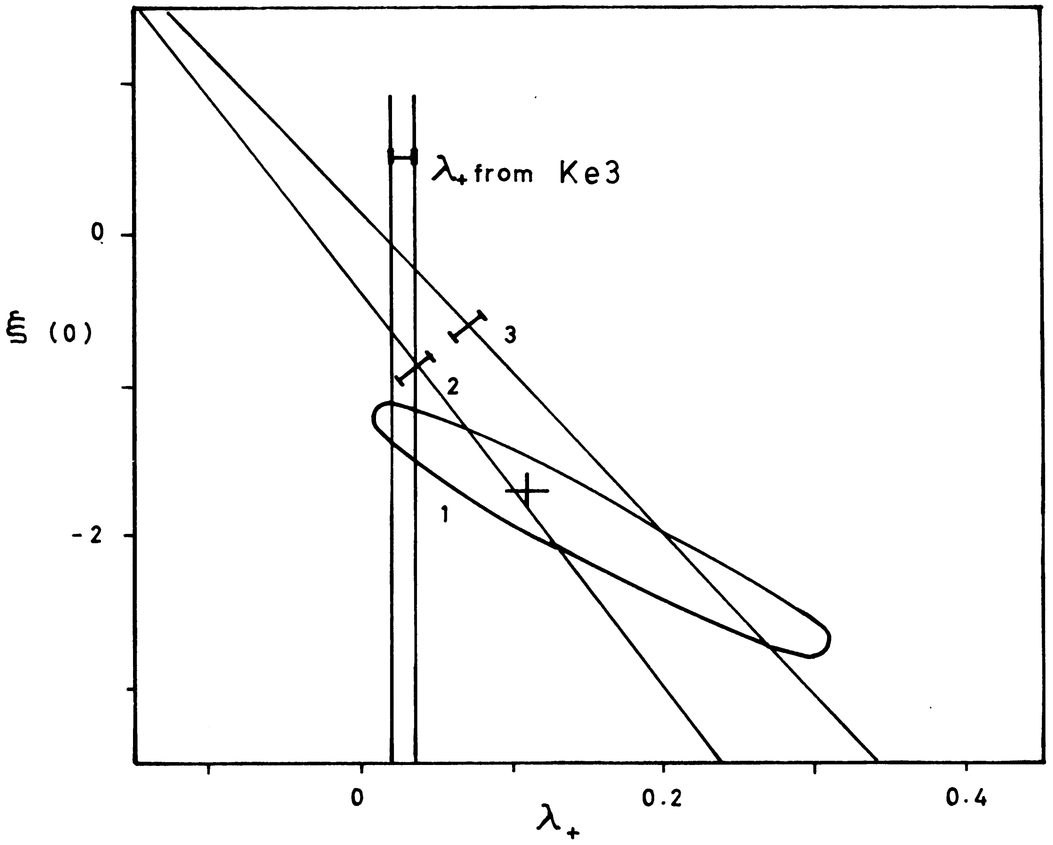


Fig. 2a

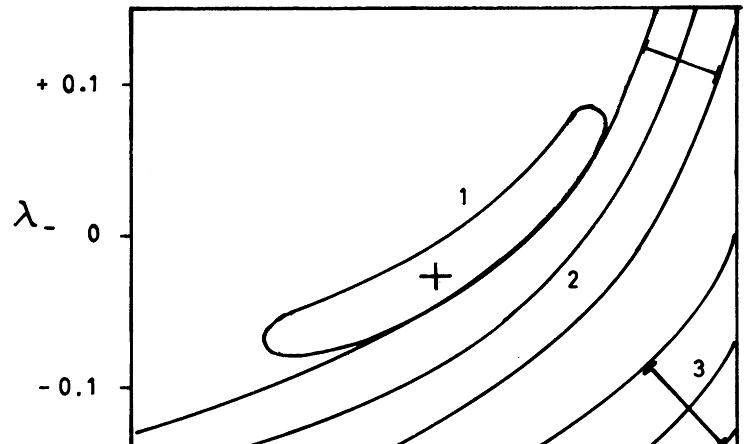
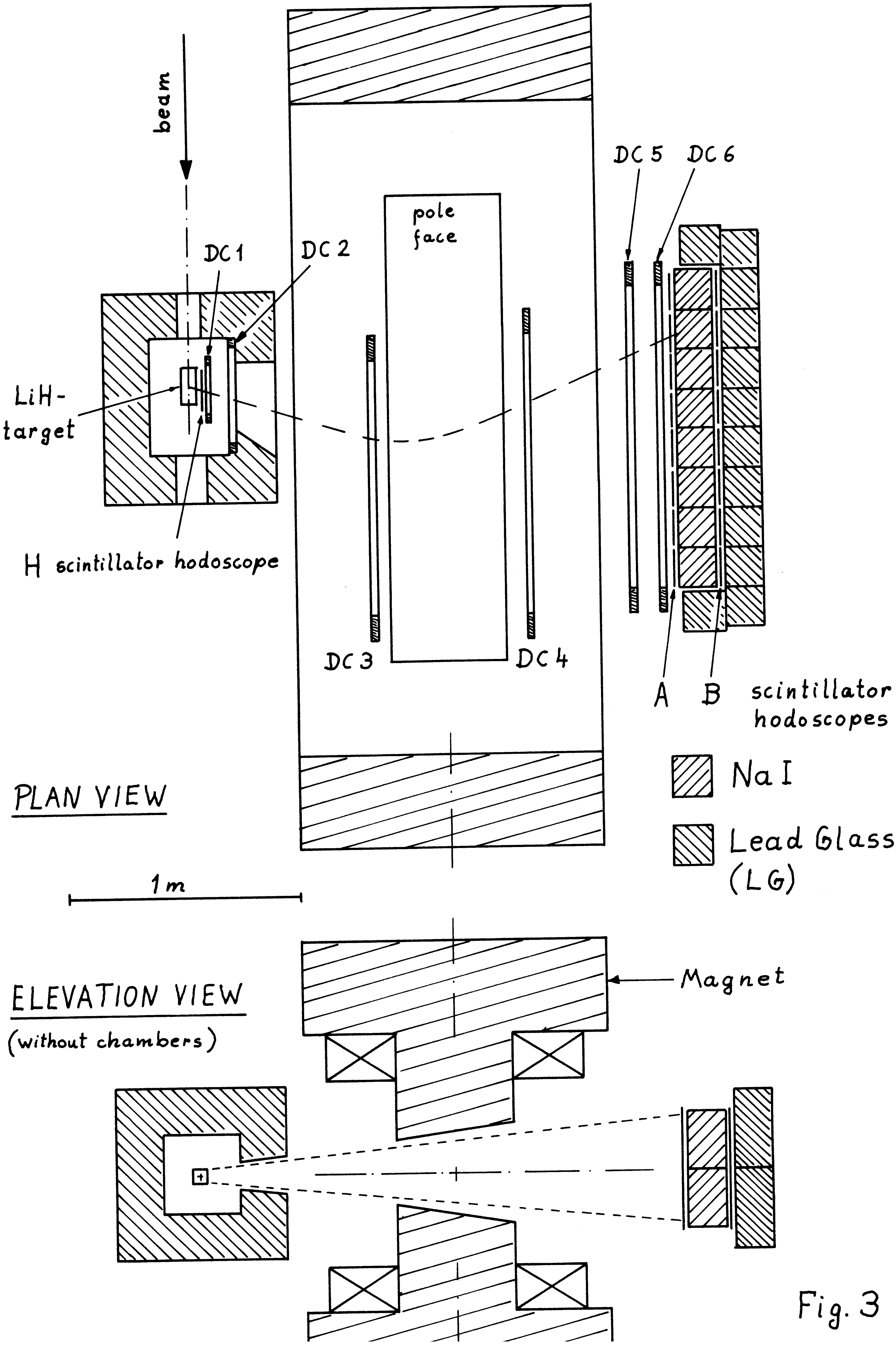


Fig. 2b



beam
DC 5
DC 6
pole face
DC 1
DC 2
LiH-target
H scintillator hodoscope
DC 3
DC 4
A
B
scintillator hodoscopes
PLAN VIEW
Na I
Lead Glass (LG)
1 m
Magnet
ELEVATION VIEW
(without chambers)

Fig. 3

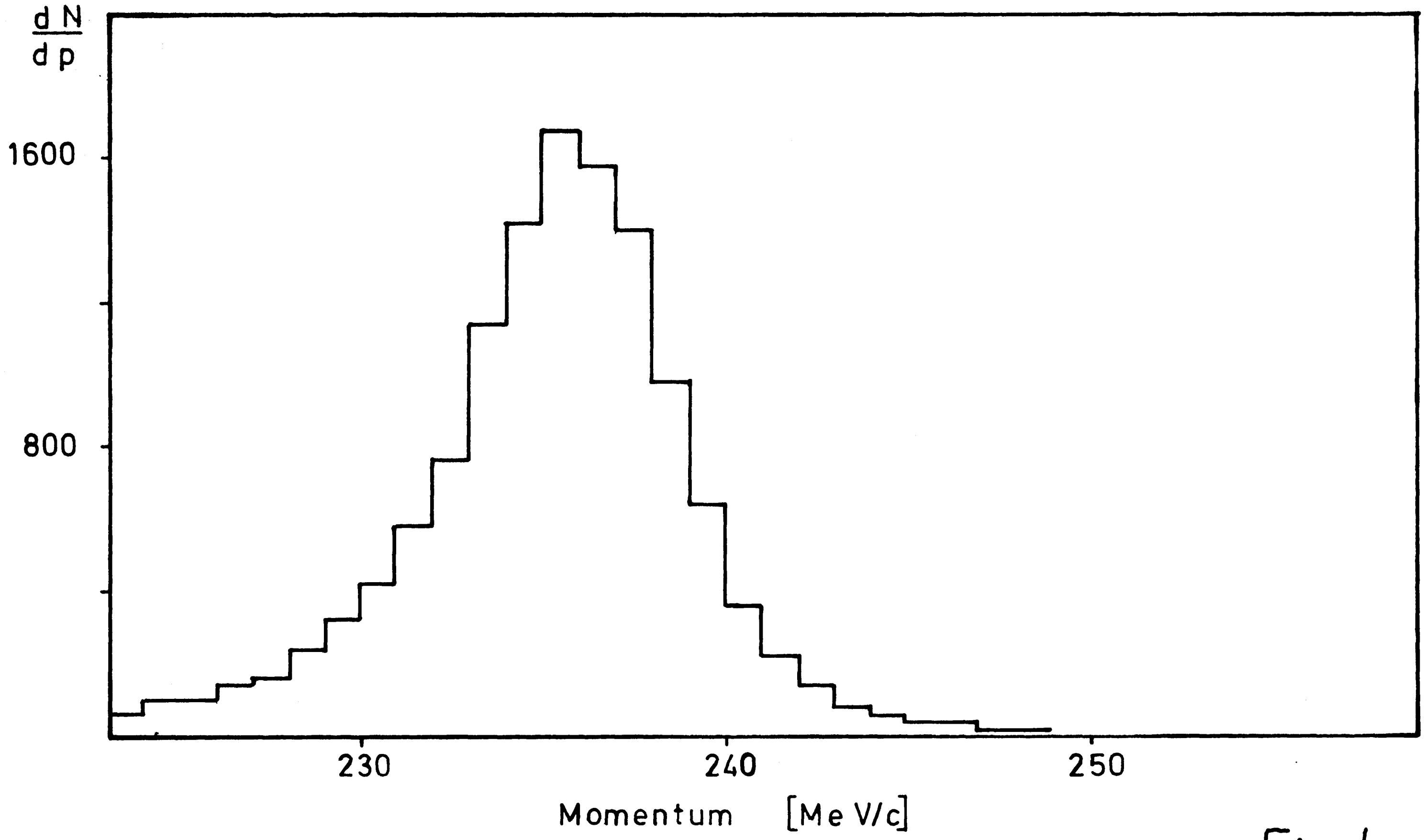

dN/dp
1600
800
230
240
250
Momentum [MeV/c]

Fig. 4

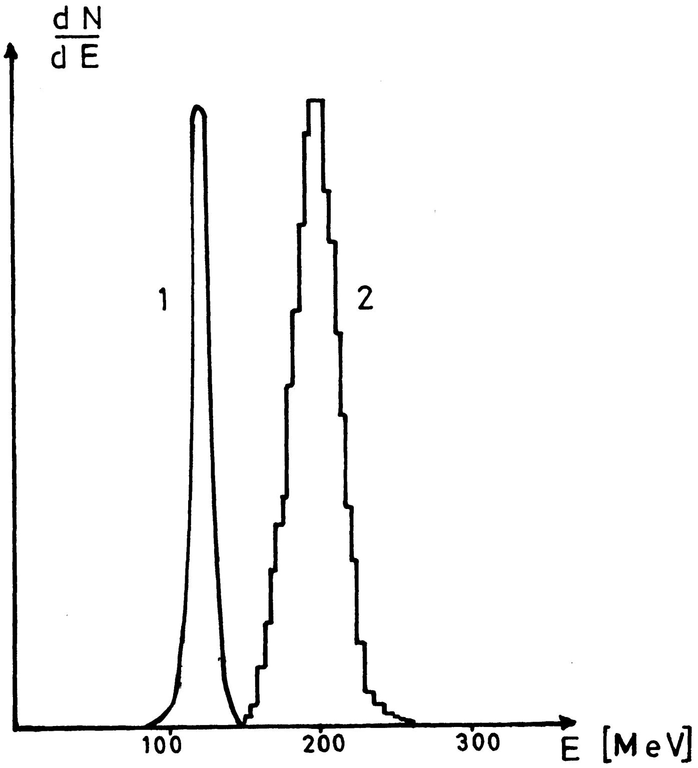
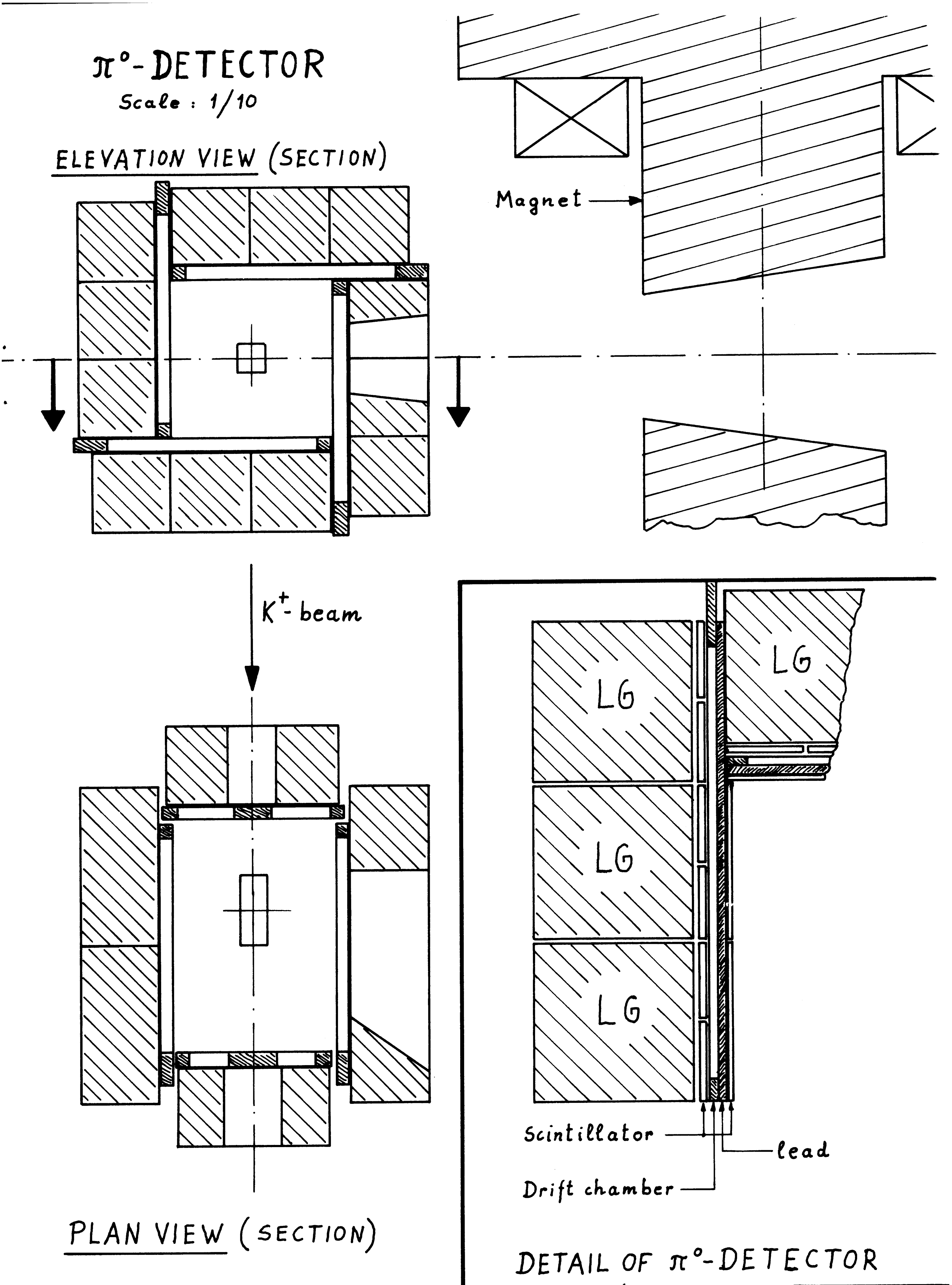

dN/dE
1
2
100
200
300
E [MeV]

Fig. 5



π°-DETECTOR
Scale : 1/10
ELEVATION VIEW (SECTION)
Magnet
K+- beam
PLAN VIEW (SECTION)
LG
LG
LG
LG
Scintillator
lead
Drift chamber
DETAIL OF π°-DETECTOR
Scale : 1/5

Fig. 6

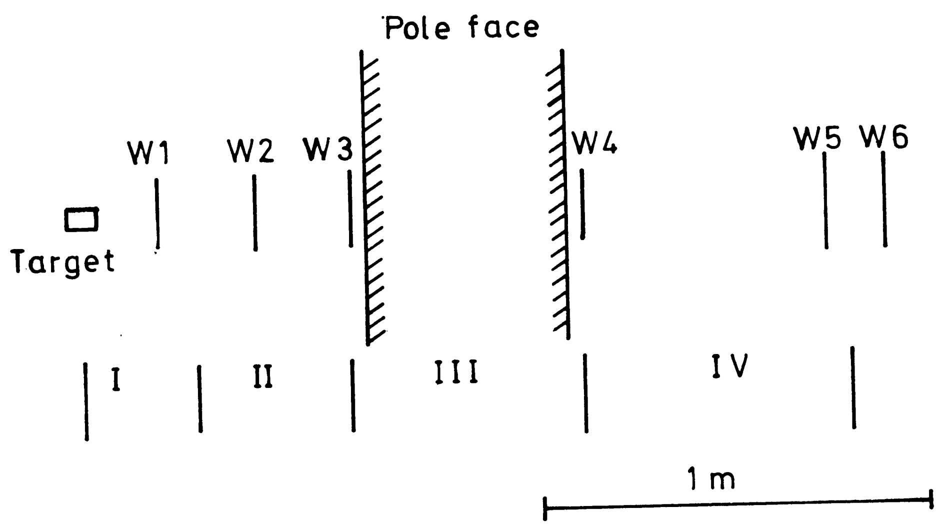

Pole face
W1
W2
W3
W4
W5
W6
Target
I
II
III
IV
1m

Fig. 7a

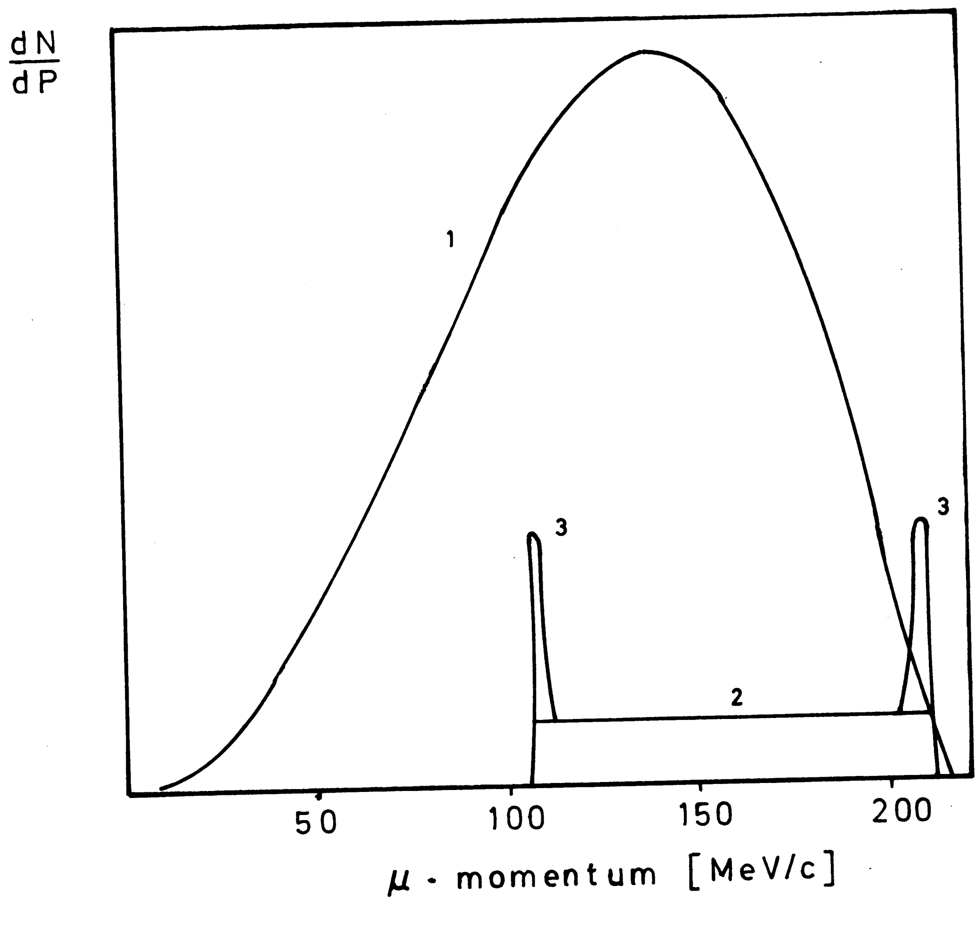
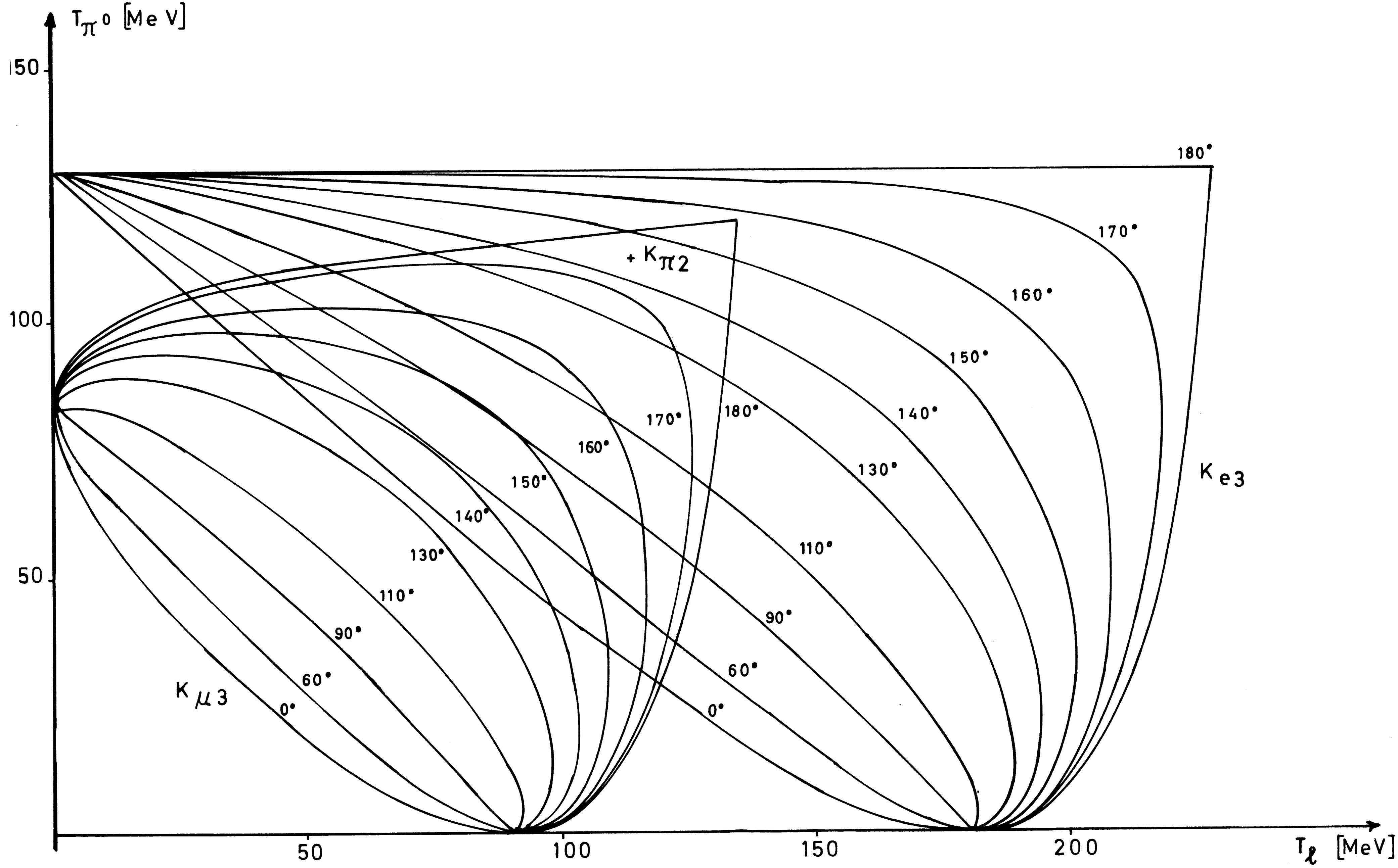

dN/dP
1
2
3
3
50
100
150
200
μ - momentum [MeV/c]

Fig. 7b



T_{π^0} [MeV]
150
100
50
50
100
150
200
T_ℓ [MeV]
180°
170°
160°
150°
140°
130°
110°
90°
60°
0°
K_{e3}
+ $K_{\pi 2}$
180°
170°
160°
150°
140°
130°
110°
90°
60°
0°
$K_{\mu 3}$

Fig. 8

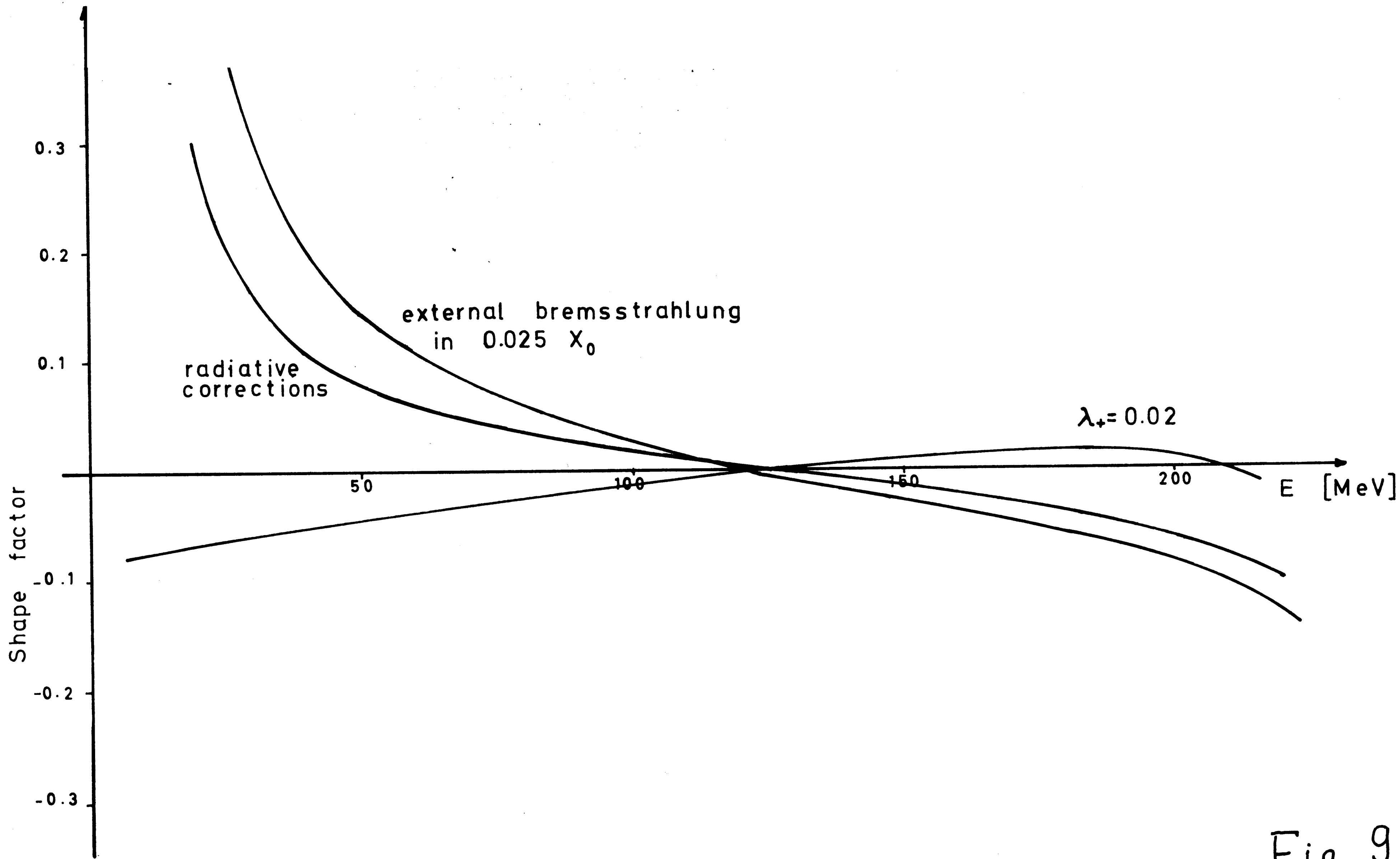
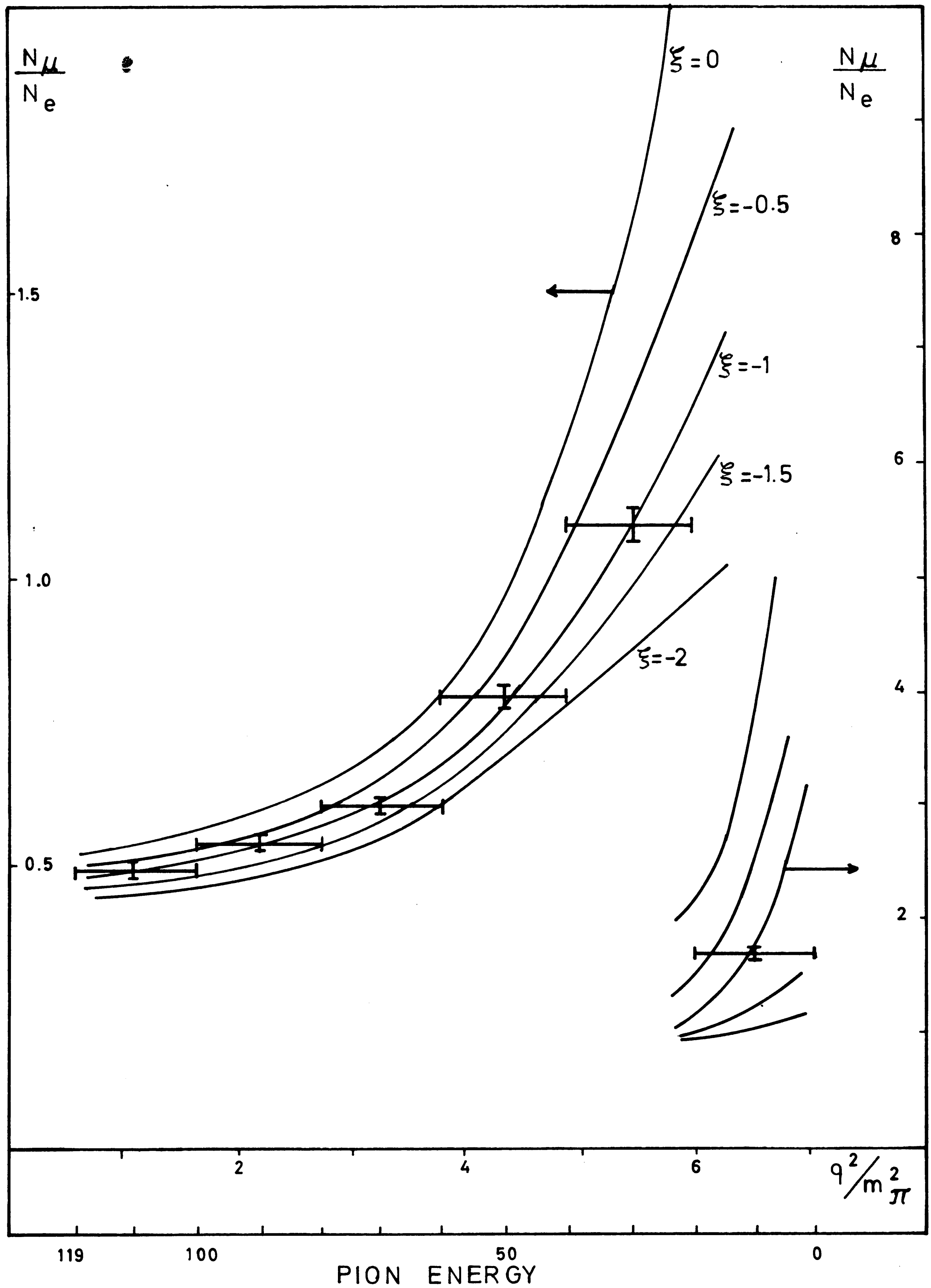

0.3
0.2
0.1
-0.1
-0.2
-0.3
Shape factor
50
100
150
200
E [MeV]
external bremsstrahlung
in 0.025 X_0
radiative
corrections
$\lambda_+ = 0.02$

Fig. 9



Nμ / Ne
ξ=0
ξ=-0.5
ξ=-1
ξ=-1.5
ξ=-2
Nμ / Ne
1.5
1.0
0.5
8
6
4
2
2
4
6
q²/m²π
119
100
50
0
PION ENERGY

Fig. 10

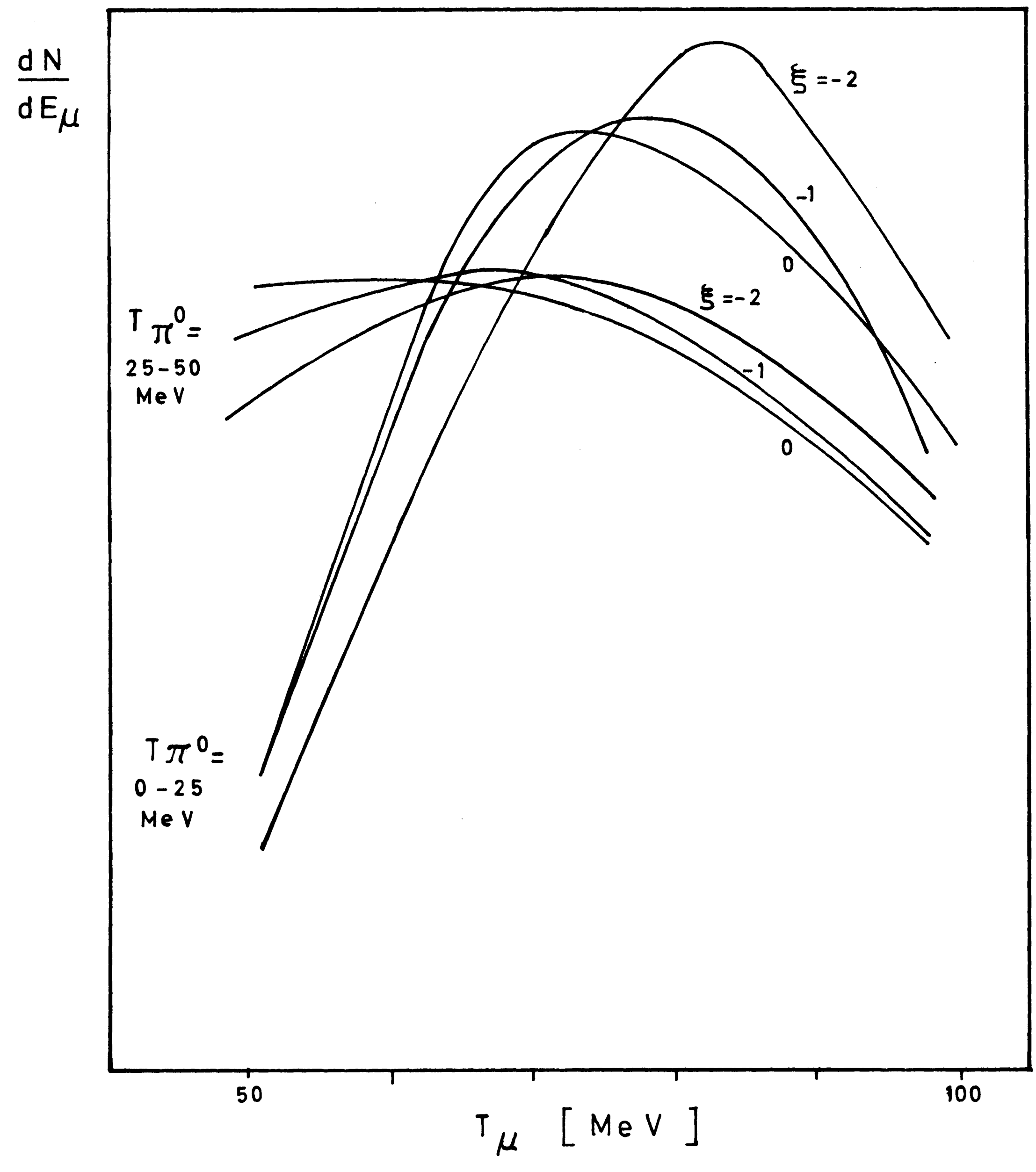


Fig. 11