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Combined OPAL measurements of $\Gamma(Z^0 \rightarrow b\bar{b})/\Gamma(Z^0 \rightarrow \text{hadrons})$

The OPAL Collaboration

Abstract

Four of the five measurements of $\Gamma_{b\bar{b}}/\Gamma_{\text{had}}$, the fractional hadronic decay width of the decay $Z^0 \rightarrow b\bar{b}$, recently made by OPAL have been combined to give an averaged value. Two of the measurements used the yield of inclusive electrons and muons in hadronic Z^0 decays respectively, and the other two used the number of double-tagged events using leptons and using secondary vertices respectively. Combining these results, a value of

$$\frac{\Gamma_{b\bar{b}}}{\Gamma_{\text{had}}} = 0.222 \pm 0.005 \pm 0.005 \pm 0.006$$

was obtained. The first error reflects the data statistics, the second the systematic uncertainties specific to these analyses, and the third comes from uncertainties concerning b and c quark fragmentation and decay processes. This result is consistent with the result of the fifth measurement, $\Gamma_{b\bar{b}}/\Gamma_{\text{had}} = 0.215 \pm 0.006 \pm 0.007 \pm 0.007$, obtained based on a 'mixed-tag' technique.

The data presented in this note are PRELIMINARY.
This note is only for the use of members of the OPAL collaboration and other persons who have been given explicit consent by OPAL.

1 Introduction

The fractional hadronic decay width of the decay $Z^0 \rightarrow b\bar{b}$ has been measured recently by the OPAL collaboration using five methods. The first two methods used the yield of electrons and muons respectively [1], and are referred to as 'single-electron' and 'single-muon' measurements. The next two methods used combinations of the numbers of double-tagged and single-tagged events where, for tagging, leptons [2] and secondary vertices [3] were used respectively. These analyses are referred to as 'double-lepton' and 'double-lifetime' measurements. The last measurement used all possible combinations of double-tagged and single-tagged events by leptons and by the number of tracks with large impact parameter, and is referred to as 'mixed-tag' measurement [4]. Using the 1990 and 1991 OPAL data samples, the values obtained from these analyses were

$$\frac{\Gamma_{b\bar{b}}}{\Gamma_{\rm had}} = \begin{cases} 0.216 \pm 0.003 \pm 0.007 \pm 0.012 & \text{for single-electron,} \\ 0.224 \pm 0.003 \pm 0.010 \pm 0.011 & \text{for single-muon,} \\ 0.223 \pm 0.012 \pm 0.007 \pm 0.009 & \text{for double-lepton,} \\ 0.223 \pm 0.010 \pm 0.007 \pm 0.004 & \text{for double-lifetime,} \\ 0.215 \pm 0.006 \pm 0.007 \pm 0.007 & \text{for mixed-tag,} \end{cases}$$

where the first errors are statistical, the second come from systematic uncertainties of the detector performance and the analysis methods, and the third from the uncertainties of b and c quark fragmentation and decay properties. These results are consistent within the uncorrelated parts of their errors.

In this note, a combined result of the first four measurements in the above list is presented. The measurements are combined to yield the lowest possible total measurement error by taking a weighted mean of their results. The result of the mixed-tag measurement is, however, not included at the moment because the correlation between the errors of the mixed-tag and the other measurements is not fully understood. Since the mixed-tag measurement has the smallest overall error by itself, it can serve as a good crosscheck of the combined result of the other four measurements.

Although these analyses used the same data sample, the statistical errors of the first four measurements are dominated by largely different parts of the sample. The single-electron and single-muon analyses used the numbers of identified electrons $N_{\rm e}^{\rm tag}$ and muons $N_\mu^{\rm tag}$, which are almost exclusive. The double-lepton analysis used the numbers of double-tagged events $N_{\ell\ell}$ and tagged hemispheres N_ℓ using lepton tag, but the statistical error of the measurement is dominated by the former. The number of double-tagged events $N_{\ell\ell}$ is partly included in $N_{\rm e}^{\rm tag}$ and in $N_\mu^{\rm tag}$, but its contribution is small ($N_{\ell\ell} = 414$ events compared with $N_{\rm e}^{\rm tag} = 6721$ and $N_\mu^{\rm tag} = 7249$). The statistical error of the double-lifetime analysis is dominated by the number of double-tagged events N_{vv} using a secondary vertex tag, which has little overlap with $N_{\rm e}^{\rm tag}$, $N_\mu^{\rm tag}$ and $N_{\ell\ell}$. It is therefore possible to combine these four results simply taking a weighted mean of them, assuming no correlation between their statistical errors. The effect of the residual statistical correlation is discussed in more detail later.

The method used in combining the results are discussed in the next section. The statistical errors of the four measurements and their correlation are discussed in Section 2.1. The systematic errors of the four analyses are treated consistently and their correlation is taken into account, as will be discussed in Sections 2.2 and 2.3.

2 Combination of the Measurements

The result of the four analyses are combined by taking a weighted mean of them as

$$\left.\frac{\Gamma_{b\bar{b}}}{\Gamma_{\rm had}}\right|_{\rm combined} \equiv \sum_i w_i \left(\frac{\Gamma_{b\bar{b}}}{\Gamma_{\rm had}}\right)_i , \qquad (1)$$

where the index i indicates each of the four measurements, and w_i is the weight for the result i satisfying

$$\sum_i w_i = 1. \tag{2}$$

The errors of the measurements are combined as

$$\left.\frac{\Delta(\Gamma_{b\bar{b}}/\Gamma_{\mathrm{had}})}{\Gamma_{b\bar{b}}/\Gamma_{\mathrm{had}}}\right|_{\mathrm{combined}} \equiv \sqrt{\sum_j \sum_i (w_i \Delta_i^j)^2 + \sum_k \Big(\sum_i w_i \Delta_i^k\Big)^2}, \tag{3}$$

where the index j indicates sources of uncorrelated measurement errors, k indicates sources of correlated measurement errors, and $\Delta_i^{j(k)}$ is the fractional error in the result of the analysis i due to the error source $j(k)$.

The fractional errors of the four measurements are summarized in Table 3. The weights w_i are determined by minimizing the total combined error, including both statistical and systematic errors. The weights are also determined considering only statistical errors of the measurements, but the large difference in statistics between single-tag and double-tag measurements gave a much degraded result, as will be shown in a later section.

2.1 Statistical Errors

The statistical error of the measurements are considered to be uncorrelated as discussed in Section 1. This assumption is not entirely true, but can be justified by the previous discussion and a few additional reasons.[1] First, the data samples used by the double-lepton and double-lifetime analyses were smaller than those used by the single-electron and single-muon analyses because of their different event selections and the requirement of silicon microvertex detector information for the double-lifetime analysis. Second, the lepton selection criteria used by the double-lepton analysis were different from those used by the single-electron and single-muon analyses, reflecting different requirement on the statistics and systematics of the lepton sample in each analysis. For the correlations between the double-lepton and the single-electron or single-muon measurements, there is additional suppression which is described below.

To check the validity of the above discussion, the residual statistical correlations were calculated by counting the numbers of events tagged as $b\bar{b}$ signals in more than one analysis. The numbers of tagged events relevant for the four analyses, and their statistical correlation due to overlapping samples are summarized in Table 1. Some of the correlation coefficients, especially those between N_e^{tag} and N_ℓ, and between N_μ^{tag} and N_ℓ, are relatively large. However this doesn't necessarily mean the measured results are strongly correlated. The result of the double-lepton mesurement can approximately written as

$$\frac{\Gamma_{b\bar{b}}}{\Gamma_{\mathrm{had}}} = \frac{N_\ell^2}{4N_{\mathrm{had}}N_{\ell\ell}}, \tag{4}$$

where N_{had} is the number of the hadronic events used in the double-lepton measurement. Since the result depends quadratically on N_ℓ and inversely on $N_{\ell\ell}$, the correlations due to them tend to cancel each other. Resultant correlation coefficients of the four measurements' results are listed in Table 2. The exact formula to obtain $\Gamma_{b\bar{b}}/\Gamma_{\mathrm{had}}$ from measured quantities was used for each measurement and effects of background subtraction and efficiency correction were taken into account. The calculated correlation coefficients are small; the largest correlation is 3.5% between single-electron and single-muon measurements. The measurements were combined with or without this correlation matrix taken into account. The changes of the combined result and its statistical error were $< 1\times10^{-5}$ and 5×10^{-5} respectively, and are negligible.

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[1] See References [1–3] for details of the event selection and lepton selection criteria.

Measurements			Correlation coefficients N_e^{tag}	 N_μ^{tag}	 N_ℓ	 $N_{\ell\ell}$	 N_v^{fold}	 N_{vv}^{fold}
Single-electron	N_e^{tag}	(=6721)	1	0.035	0.460	0.172	0.054	0.014
Single-muon	N_μ^{tag}	(=7249)	0.035	1	0.472	0.185	0.061	0.014
Double-lepton	N_ℓ	(=11621)	0.460	0.472	1	0.365	0.093	0.025
	$N_{\ell\ell}$	(=414)	0.172	0.185	0.365	1	0.020	0.002
Double-lifetime	N_v^{fold}	(=7378)	0.054	0.061	0.093	0.020	1	0.460
	N_{vv}^{fold}	(=488)	0.014	0.014	0.025	0.002	0.460	1

Table 1: Numbers of tagged leptons, hemispheres and double-tagged events relevant for the four analyses and their correlation coefficients. The quantities N_v^{fold} and N_{vv}^{fold} are defined by $N_v^{fold} \equiv N_v - N_{\bar{v}}$ and $N_{vv}^{fold} \equiv N_{vv} - N_{v\bar{v}} + N_{\bar{v}\bar{v}}$. See References [1–3] for the definitions of N_e^{tag}, N_μ^{tag}, N_ℓ, $N_{\ell\ell}$, N_v, $N_{\bar{v}}$, N_{vv}, $N_{v\bar{v}}$ and $N_{\bar{v}\bar{v}}$.

Measurements	Correlation coefficients Single-electron	 Single-muon	 Double-lepton	 Double-lifetime
Single-electron	1	0.035	0.026	0.018
Single-muon	0.035	1	0.018	0.022
Double-lepton	0.026	0.018	1	0.003
Double-lifetime	0.018	0.022	0.003	1

Table 2: Correlation coefficients between the four measured results of $\Gamma_{b\bar{b}}/\Gamma_{had}$.

2.2 Uncorrelated Systematic Errors

The errors arising from the statistics of the Monte Carlo samples are also assumed to be uncorrelated. As described in References [1–3], the four measurements used mainly the same sample of Monte Carlo simulated data. However, their errors due to Monte Carlo statistics are again dominated by different parts of the sample. The errors of the single-electron and single-muon analyses are dominated by the statistics of the simulated $b\bar{b}$ events including electrons and muons, respectively, while the errors of the double-lepton and double-lifetime measurement are primarily due to the statistics of the simulated $c\bar{c}$ events tagged by leptons and secondary vertices, respectively.

Possibilities of efficiency correlation between the two hemispheres of an event are important sources of systematics in the double-lepton and the double-lifetime measurements. The systematic errors due to such effects are also uncorrelated between the two analyses.

2.3 Correlated Systematic Errors

The systematic errors of the four measurements, except for the Monte Carlo statistics and efficiency correlation, were considered to be fully correlated for each source of uncertainties. This treatment is slightly conservative; for example, the electron identification algorithms of the single-electron and double-lepton analyses are not identical and their uncertainties are therefore not fully correlated. The range of the systematic uncertainties of various parameters were taken consistently among the measurements with the few exceptions described below.

The single-electron and single-muon analyses used the Standard Model prediction of $\Gamma_{c\bar{c}}/\Gamma_{had} = 0.171$ with a fractional error of 22% according to the most precise published measurement [5]. The double-lepton and double-lifetime analyses used the Standard Model prediction with a different parametrization:

$$\lambda_{\mathrm{c}} \equiv \frac{\Gamma_{\mathrm{c\bar{c}}}}{\Gamma_{\mathrm{c\bar{c}}} + \Gamma_{\mathrm{u\bar{u}}} + \Gamma_{\mathrm{d\bar{d}}} + \Gamma_{\mathrm{s\bar{s}}}} = 0.218, \tag{5}$$

and assigned a fractional error of 16% according to the currently best measurement, described in an OPAL Physics Note [6]. The difference in the parametrization causes practically no problem as far as the result of the measurement is consistent with the Standard Model prediction. Using the result of the two measurements, $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}} = 0.223$, the above value of λ_{c} corresponds to $\Gamma_{\mathrm{c\bar{c}}}/\Gamma_{\mathrm{had}} = 0.169$. The fractional difference of $\Gamma_{\mathrm{c\bar{c}}}/\Gamma_{\mathrm{had}}$ between the two parametrizations is less than 1% and is negligible. The fractional error of $\Gamma_{\mathrm{c\bar{c}}}/\Gamma_{\mathrm{had}}$, or, equivalently, of λ_{c}, is taken to be 16% in this note. The errors given in Table 3 have already been corrected for this effect.

The fragmentation function form suggested by Peterson et al. [7] was used for the c and b quark fragmentation modelling in the single-electron, single-muon and double-lepton analyses. The fragmentation parameter for the c quark was taken to be $\epsilon_{\mathrm{c}} = 0.05 \pm 0.02$, corresponding to the mean energy of the primary charmed hadrons $\langle x_E \rangle_{\mathrm{c}} = 0.51 \pm 0.02$. In the double-lifetime analysis, the Lund fragmentation was used to obtain the central result with the parameters optimized by OPAL [8]. The corresponding value of $\langle x_E \rangle_{\mathrm{c}}$ is 0.497. The systematic error was evaluated by reweighting Monte Carlo events to reproduce the Peterson function with $\epsilon_{\mathrm{c}} = 0.03$–$0.08$. Since this range covers the range of the ϵ_{c} used in the other three analyses, the effect of the difference in the choice of the fragmentation function used to obtain the central result is expected to be covered by the quoted systematic error. The systematic errors due to the uncertainties in the c quark fragmentation are treated as fully correlated among the four analyses.

In the single-electron, single-muon and double-lepton analyses, the average semileptonic branching fraction of charmed hadrons were estimated using the measured branching fractions of $\mathrm{D}^{\pm}$ and D^0, the ratios of the measured lifetimes of $\mathrm{D}^{\pm}$, D^0, $\mathrm{D}_{\mathrm{s}}^{\pm}$ and $\Lambda_{\mathrm{c}}^{\pm}$, and the relative abundance of the charmed hadrons predicted by the JETSET Monte Carlo program. The estimated branching fraction was $B(\mathrm{c} \rightarrow \ell) = (9.6 \pm 0.9 \pm 0.6)\%$ where the first error is due to the measurement errors of $B(\mathrm{D}^{\pm} \rightarrow \mathrm{e})$ and $B(\mathrm{D}^0 \rightarrow \mathrm{e})$, and the second is due to the uncertainty of the relative abundance. The effect of the measurement errors of the charmed hadron lifetimes is negligible. The lifetimes of charmed hadrons and their relative abundance were also used in the double-lifetime analysis with uncertainties compatible with those used in the other three analyses. The systematic errors due to the uncertainty in the relative abundance of the charmed hadrons are assumed to be fully correlated.

The average charged track multiplicity for charmed hadron decays is considered as a possible source of systematic error in the double-lifetime analysis. The uncertainty in the decay multiplicity is also known to affect the momentum spectrum of leptons from charmed hadron decays, which is a source of the systematic errors in the other three analyses. The systematic errors of these origins are therefore assumed to be fully correlated.

3 Result

The combined result of the four measurements is

$$\left.\frac{\Gamma_{\mathrm{b\bar{b}}}}{\Gamma_{\mathrm{had}}}\right|_{\mathrm{combined}} = 0.222 \pm 0.005 \pm 0.005 \pm 0.006.$$

The first error is statistical and the second is the systematic error specific to these analyses. The third error arises from the systematic uncertainties of external information concerning b and c quarks, namely, b and c quark fragmentation, semileptonic branching fractions, decay modelling, charm production cross section, charmed hadron lifetimes, and b and c quark pair creation in fragmentation. The fractional error of the combined result is 4.0%.

The weights of the four measurements are 19:23:17:42 for single-electron, single-muon, double-lepton and double-lifetime analyses, respectively. The contribution of the double-lifetime measurement is larger than those of the other measurements using leptons, because of its lower systematic error which has largely different origins from the others.

When the weights of the measurements were determined using only their statistical errors, the combined result was highly (93%) dominated by the single-electron and single-muon analyses because of the much larger statistical errors of the double-lepton and double-lifetime analyses. The result was $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}} = 0.220 \pm 0.002 \pm 0.006 \pm 0.010$, where the errors are separated in the same way as above. This result is almost identical to the combined result of the single-electron and single-muon analyses, $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}} = 0.220 \pm 0.002 \pm 0.006 \pm 0.011$, given in Reference [1].

The above result can be compared with the result of the mixed-tag measurement, $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}} = 0.215 \pm 0.006 \pm 0.007 \pm 0.007$. Although the correlation between the errors of the two results are not quantitatively known, it is expected that their statistical errors are largely uncorrelated; the mixed-tag measurement used an impact parameter method for the lifetime-tag which has much higher efficiency than the secondary vertex finding used in the double-lifetime measurement; it also used the information of the mixed-tag events that have a lepton-tag and a lifetime-tag in the opposite hemispheres. Some of the systematic errors are also uncorrelated between two, especially those coming from the fragmentation and the decay properties of bottom quarks (±0.004 of the last error of the combined result). Both results are also consistent with the Standard Model prediction, $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}} = 0.217$, calculated by the program ZFITTER [9] using a top quark mass of $150\,\mathrm{GeV}/c^2$, a Higgs boson mass of $300\,\mathrm{GeV}/c^2$, and $\alpha_s = 0.12$ as input parameters.

4 Summary

Four of the five measurements of $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}}$, the fractional hadronic decay width of the decay $\mathrm{Z}^0 \to \mathrm{b\bar{b}}$, recently made by OPAL have been combined to give an averaged value. Two of the measurements used the yield of inclusive electrons and muons in hadronic Z^0 decays respectively, and the other two used the number of double-tagged events using leptons and using secondary vertices respectively. Combining these results, a value of

$$\frac{\Gamma_{\mathrm{b\bar{b}}}}{\Gamma_{\mathrm{had}}} = 0.222 \pm 0.005 \pm 0.005 \pm 0.006$$

was obtained. The first error reflects the data statistics, the second the systematic uncertainties specific to these analyses, and the third comes from uncertainties concerning b and c quark fragmentation and decay processes. This result is consistent with the result of the fifth measurement, $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}} = 0.215 \pm 0.006 \pm 0.007 \pm 0.007$, obtained based on a 'mixed-tag' technique.

The precision of the combined result is limited mainly by three sources of uncertainties: statistics of the double-tagged events, the average branching fraction of the semileptonic decays of bottom hadrons, and the fractional width of the decay $\mathrm{Z}^0 \to \mathrm{c\bar{c}}$. All these errors will improve in future, with increasing statistics at LEP. Significant reduction of the second error source would also require improved measurements at lower energies.

References

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[2] OPAL Collab., OPAL Physics Note PN091, 22 February 1993.

[3] OPAL Collab., OPAL Physics Note PN095, 12 March 1993.

[4] OPAL Collab., OPAL Physics Note PN096, 12 March 1993.

[5] OPAL Collab., G. Alexander et al., Phys. Lett. **B262** (1991) 341.

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[7] C. Peterson et al., Phys. Rev. **D27** (1983) 105.

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[9] D. Bardin et al., *ZFITTER: An Analytical Program for Fermion Pair Production in* $\mathrm{e}^+\mathrm{e}^-$ *Annihilation*, CERN-TH 6443/92, May 1992.

Sources of errors	Single-electron	Single-muon	Double-lepton	Double-lifetime	Combined
Statistical error	1.4%	1.4%	5.5%	4.5%	2.1%
MC statistics	1.1%	1.2%	0.4%	0.8%	0.5%
Efficiency correlation	—	—	1.8%	2.8%	1.2%
Event selection efficiency	0.5%	0.5%	—	—	0.2%
Event selection bias	—	—	0.6%	0.3%	0.2%
Conversions	0.9%	—	1.5%	—	0.4%
Fake electrons	0.2%	—	1.3%	—	0.3%
Fake muons	—	1.7%	1.4%	—	0.6%
Electron id. efficiency	2.9%	—	0.7%	—	0.7%
Muon id. efficiency	—	3.7%	0.3%	—	0.9%
Electron radiation loss	0.8%	—	—	—	0.2%
θ determination	—	0.4%	—	—	0.1%
Track resolution	—	—	—	1.7%	0.7%
$B(\mathrm{b}\rightarrow\ell)$	4.0%	4.2%	—	—	1.7%
$B(\mathrm{b}\rightarrow\mathrm{c}\rightarrow\ell)$	1.9%	1.0%	—	—	0.6%
$B(\mathrm{b}\rightarrow\mathrm{J}/\psi\rightarrow\ell)$	0.5%	0.4%	—	—	0.2%
$B(\mathrm{b}\rightarrow\tau\rightarrow\ell)$	0.6%	0.4%	—	—	0.2%
b decay model	0.5%	0.3%	—	—	0.2%
b fragmentation	1.4%	1.2%	—	—	0.5%
$\Gamma_{\mathrm{c\bar{c}}}/\Gamma_{\mathrm{had}}$	1.4%	0.9%	2.7%	1.1%	1.4%
c fragmentation	0.8%	0.3%	1.5%	0.6%	0.7%
$B(\mathrm{D}^{\pm}/\mathrm{D}^{0}\rightarrow\mathrm{e})$	0.8%	0.5%	1.6%	—	0.5%
$f(\mathrm{D}^{\pm}):f(\mathrm{D}^{0}):f(\mathrm{D}_{\mathrm{s}}^{\pm}):f(\Lambda_{\mathrm{c}}^{\pm})$	0.5%	0.3%	1.0%	0.8%	0.7%
c lifetime	—	—	—	0.4%	0.2%
c decay model	0.9%	0.5%	1.5%	0.6%	0.8%
b/c from fragmentation	0.5%	0.4%	1.2%	0.2%	0.5%
K^{0} production rate	—	—	—	0.3%	0.1%
Total systematics	6.3%	6.5%	5.2%	3.8%	3.4%
Total error	6.5%	6.6%	7.5%	5.9%	4.1%

Table 3: Fractional errors on each and the combined result of the four measurements of $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}}$.