# Polarized Proton Pionic Capture in Deuterium as a Probe of $3N$ Dynamics

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**Abstract**

The proton analyzing power $A_y$ in pion production reaction $\overrightarrow{p}d \to \pi^0\ ^3\text{He}$ has been calculated including one- and two-body meson production mechanisms with a proper treatment of the three-nucleon dynamics and an accurate solution of the $3N$ bound-state problem for phenomenological two-nucleon potentials. In the region around the $\Delta$ resonance, the structure of the analyzing power can be understood once interference effects among amplitudes describing intermediate $\Delta N$ formation in different orbital states are considered along with the additional interference with the $S$-wave pion production amplitudes. Then, the inclusion of three-nucleon dynamics in the initial state produces the structure of the analyzing power that has been observed experimentally.

It has been known for almost twenty years that the experimental structure of the analyzing power $A_y$ for the $\overrightarrow{p}d \to \pi^0\ ^3\text{He}$ reaction (in the region around the $\Delta$ resonance) exhibits a rapid variation at the c.m. polar angle of $\Theta_{\text{c.m.}} = 90°$, with the appeerence of an additional (second) peak and changing the sign as one moves from lower energies to the $\Delta$-resonance region. To our knowledge, this behavior of the proton analyzing power has never been interpreted theoretically before, while the understanding of the underlying mechanisms is of great importance to describe and understand the basic $NN \to NN\pi$ inelasticities in the most simple "complex" nuclear environment, the three-nucleon system. Single pion production represents the first process that needs to be understood in order to consider more complicated hadronic processes that occur at moderately higher energy, such as double-pion production, or heavier meson (e.g., $\eta$) production. In addition, single pion production could also reveal mechanisms involving multinucleon exchanges, thus providing important information that bridges intermediate-energy phenomena to low-energy three-nucleon and multinucleon forces, to be used in the study of low-energy nuclear systems.

Theoretically, pion production in proton-deuteron collisions was studied long time ago with the deuteron model [1], which describes the $pd \to \pi^+t$ reaction in terms of the differential cross section for the more elementary $pp \to \pi^+d$ process. The model has been subsequently refined by Locher and Weber [2], who introduced additional mechanisms

based on the elastic $\pi d \to \pi d$ cross section, and by Fearing [3] who gave phenomenological estimates of the effects introduced by optical-model distortions. The deuteron model has been later extended by Germond and Wilkin [4] who developed a simple spin-dependent analysis in plane waves by which they predicted the deuteron tensor analyzing power $T_{20}$ to approach its geometrical limit $-\sqrt{2}$ at threshold, based on low-energy theorems. Recently, Falk [5] developed further the deuteron model in its spin dependent form and performed a phenomenological analysis of polarization observables near threshold. Calculations including explicitly $\Delta$ dynamics and meson-exchange processes were pioneered in the late '70 [6], but the complexity involved in the computation of the corresponding amplitudes forced the authors to restrict themselves with only exploratory calculations. Laget and Lecolley [7] performed calculations that tried to go beyond one- and two-body pion production mechanisms and included effective three-body mechanisms via meson double rescatterings, showing beneficial effects at higher energies, well beyond the $\Delta$ resonance. Results around the isobar resonance, however, were not quite satisfactory, and especially the proton analyzing power $A_y$, i.e., the asymmetry of the reaction yield for the opposite proton spin directions,

$$A_y = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}, \tag{1}$$

was in poor accord with the data [8].

The approach involving the $3N$ dynamics has been pushed forward recently in a series of papers [9], where explicit $\Delta$ excitation mechanisms were treated in the three-nucleon system using the few-body techniques [10]. Intermediate $\Delta$ propagation and rescattering were described phenomenologically using an isobar complex-mass scaling that has been tested and parametrized on the $pp \to \pi^+ d$ reaction [11]. Such test calculations have shown that, for the description of the proton pionic capture by protons, the interplay between ($\Delta$-mediated) $P$-wave mechanisms and $S$-wave two-body rescattering mechanisms (in the isoscalar and isovector channels) is a fundamental feature that has to be taken into account for the overall reproduction of both spin-averaged and spin-dependent observables. Antisymmetrization prescriptions due to the identity of the nucleons were fully taken into account.

Nowadays, the calculations can be performed with more accurate knowledge of the nuclear wave functions (WF), and with including a much larger number of intermediate three-nucleon states. Furthermore, the nucleon-deuteron initial-state interaction (ISI) can be calculated through, e.g, the Alt-Grassberger-Sandhas (AGS) scheme [12], with realistic nucleon-nucleon transition matrices. The complete calculation of the reaction represents a very complicated theoretical task, and involves, due to the variety of the few-body intermediate states that have to be expanded in partial waves, computation of an exorbitant number of multidimensional integrals.

The amplitude of the $pd \to \pi^0\,{}^3$He reaction (or the inverse pion absorption process) is determined by the matrix element

$$A_{if} = {}^{(-)}{}_S\langle \mathbf{q}, \psi_d | A | \psi_t, \mathbf{P}_0^\pi \rangle, \tag{2}$$

where $\psi_t$ represents the three-nucleon bound-state and ${}^{(-)}{}_S\langle \mathbf{q}, \psi_d|$ refers to a properly antisymmetrized interacting nucleon-deuteron state in the initial channel. Diagrammatically, this amplitude is illustrated by Fig. 1. Within the model discussed here, it includes the proton-deuteron ISI, the bound-state two- (deuteron) and three-nucleon ($^3$He) WF,
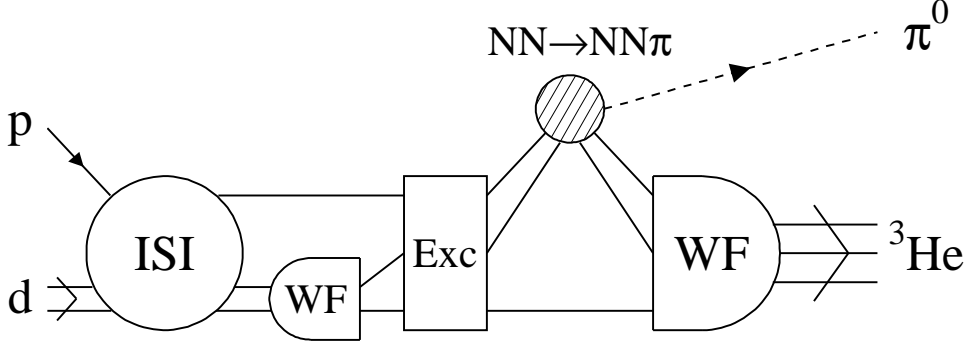
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Figure 1: Diagrammatic representation of the pion production amplitude.

the matrix element of a $3N$ permutation operator that gives rise to the exchange contribution to the amplitude, and an operator of pion production on a pair of nucleons. Note that in formulation of the AGS formalism employed in our calculations, the $3N$ break-up process is effectively included in the ISI block in Fig. 1.

The meson production mechanisms are given by the two-body (Fig. 2a,b,c) and one-
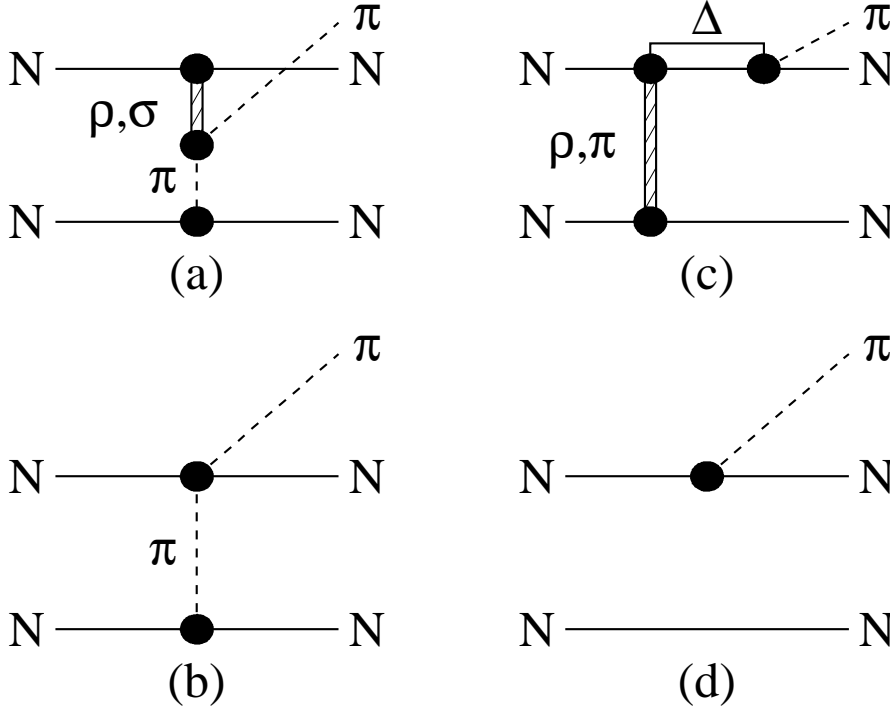


Figure 2: Meson production mechanisms considered.

body (plain $\pi NN$ vertex in Fig. 2d) contributions. The $\Delta$ excitation mechanism (Fig. 2c) has the structure

$$\mathcal{A}_\Delta = V_{NN-N\Delta} G_\Delta F_{\Delta N\pi},\qquad(3)$$

where $V_{NN-N\Delta}$ is the $NN \to N\Delta$ transition potential generated by $\pi$- and $\rho$-exchanges, $G_\Delta$ effectively describes the intermediate $\Delta NN$ propagation, and $F_{\Delta N\pi}$ is the $\Delta N\pi$ transition vertex function. All the corresponding details including the extended formulas

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representing the mechanisms in the three-nucleon partial waves have been published elsewhere [9, 10]. The asymptotic pion wave is not decomposed in partial waves but treated directly in three dimensions. An important quantity for the present calculation is the maximum orbital momentum $l_\Delta$ in the $\Delta N$ subsystem. We found that $l_\Delta = 0$ is not sufficient to describe polarization observables, and one has to go up to $l_\Delta = 2$, at least.

In the $\Delta$ resonance region, the spin structure of the reaction amplitude is dependent strongly also on the interference effects with the other mechanisms, particularly the $S$-wave two-body pion production contributions shown in Fig. 2a,b. Each of these mechanisms can be schematically described as

$$\mathcal{A}_\pi = F_{NN\pi} G_\pi \lambda_{N\pi-N\pi} + \text{time reordering} , \tag{4}$$

where $F_{NN\pi}$ is the $\pi NN$ vertex, $G_\pi$ is the pion propagator, and $\lambda_{N\pi-N\pi}$ is the corresponding contribution to the half-off-shell $\pi N$ scattering amplitude. By "time reordering" we denote a term coming from "backward in time" propagation of the intermediate-state pion , in which case $\lambda_{N\pi-N\pi}$ is replaced by a $N \to 2\pi N$ vertex amplitude for double-pion production.

The starting point for the construction of $\lambda_{N\pi-N\pi}$ is the phenomenological low-energy ($S$-wave) pion-nucleon interaction Hamiltonian [13]

$$\mathcal{H}_{\text{int}} = \frac{\lambda_V}{m_\pi^2} \bar{\psi} \gamma^\mu \boldsymbol{\tau} \psi \cdot [\boldsymbol{\phi} \times \partial_\mu \boldsymbol{\phi}] + \frac{\lambda_S}{m_\pi} \bar{\psi} \psi [\boldsymbol{\phi} \cdot \boldsymbol{\phi}] . \tag{5}$$

The calculations herein illustrated have been performed with the set of parameters used in the previous study [9]. Namely, for the isovector coupling constant in Eq. (5), we have $\lambda_V/4\pi = 0.045$, whereas the $\pi NN$ and $\pi N\Delta$ coupling constants are equal to $f_{\pi NN}^2/4\pi = 0.0735$ and $f_{\pi N\Delta}^2/4\pi = 0.32$, respectively. Our present treatment is based on Ref. [14]. The isospin-odd contribution is given in terms of a $\rho$-exchange model, while the isospin-even term is described as the combined effect of a phenomenological short-range (SR) process (Fig. 2b) and an effective scalar-meson ($\sigma$) exchange of Fig. 2a. The two latter mechanisms act in opposite directions and almost cancel each other in case of the free pion-nucleon scattering. However, an off-shell enhancement of the probability amplitude takes place in the scalar-isoscalar channel for pion production off a bound system. A question still remains open, if this form mimics other physical effects, such as the $2N$ SR-exchange contributions proposed in Ref. [15] (see also Ref. [16], where the current status of theory and phenomenology of pion production in $NN$ collisions close to threshold is reviewed).

Further details (in particular, the treatment of the fully antisymmetrized matrix elements with respect to the Faddeev three-nucleon wave function) are not discussed here and can be found in the papers [9, 10, 11]. The three-body dynamics in the initial state has been described following the treatment [17] of the low-energy proton-deuteron scattering, with the set of the AGS equations being reduced through application of the spline-interpolation technique to a linear equation system.

In Fig. 3, the results for the proton analyzing power are compared with what has been observed in experiments. For the three different phenomenological nucleon-nucleon potentials, one observes an additional peak at $\Theta_{\text{c.m.}} \simeq 100°$ when going from $E_{\text{lab}} = 350$ MeV towards higher energies. This behaviour is fully explained by the model calculation, once the ISI involving the three-nucleon dynamics is taken into account. Otherwise, if
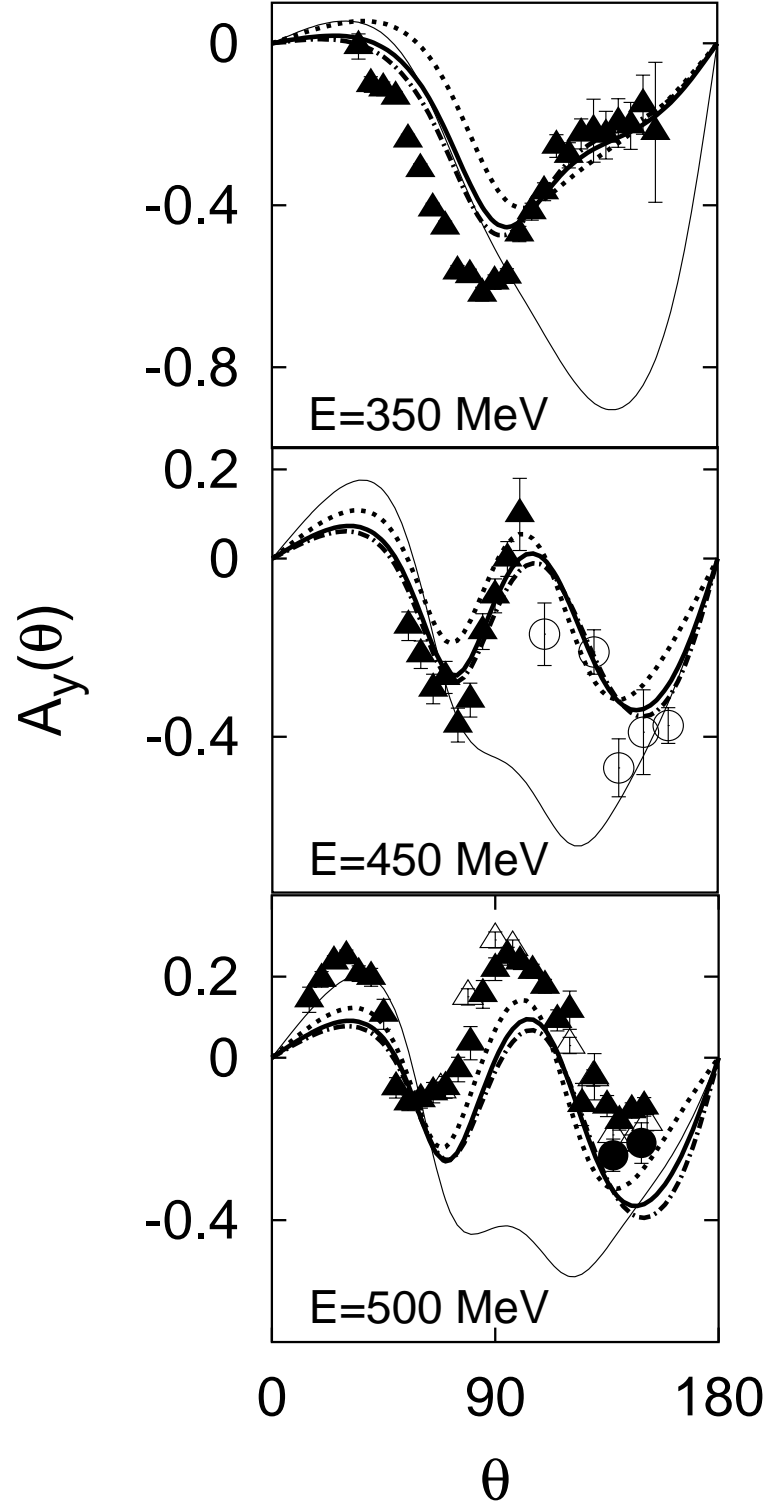
Figure 3: Proton analyzing power for three different proton lab energies. Solid, dash-dotted and dotted lines denote our complete calculations for Bonn-B, Bonn-A, and Paris $NN$ potentials, respectively, while thin solid line corresponds to the plane-wave calculation (without the $pd$ ISI). Full triangles, blank circles, blank triangles and full circles mark data of Refs. [8, 18, 19] and [20], correspondingly.

one neglects these three-body effects in the initial channel, the structure of the observable is poorly reproduced.

In generation of this structure, another quantity that plays an important role is the angular momentum of the intermediate $\Delta N$ state. While $l_\Delta = 0$ is sufficient in description of the general behaviour of the excitation function (see Ref. [9]), one must consider higher $\Delta N$ partial waves, at least up to $l_\Delta = 2$, in case of the analyzing power $A_y$. As follows from Fig. 4, there is no need, however, to go beyond this value, since the numerical
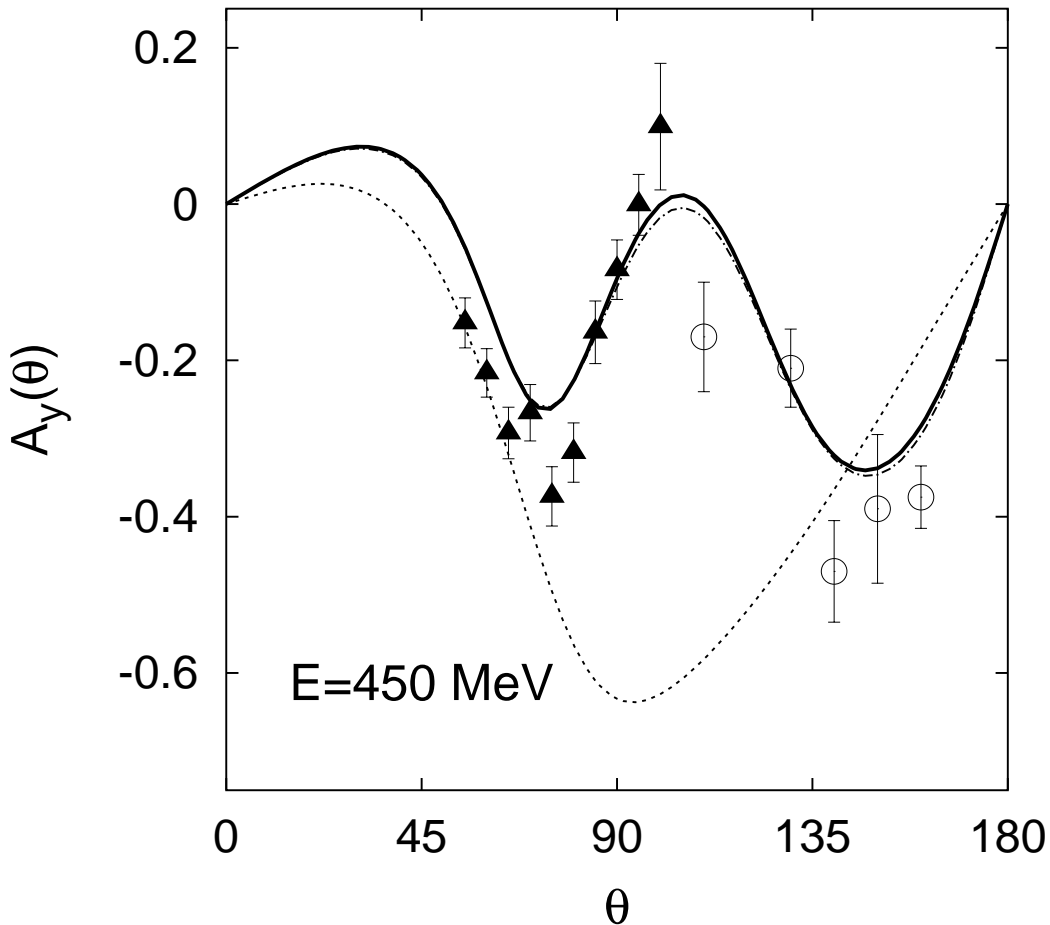


Figure 4: Results for different values of the maximum $\Delta N$ orbital momentum. Dotted, solid and dashed-dotted curve refer to calculations with $l_\Delta = 0$, 2, and 4, respectively, performed with the Bonn-B potential and inclusion of the $3N$ ISI.

convergence is already reached with the $\Delta N$ $D$-waves.

Apart from the analyzing power $A_y$, also the angular differential cross-section has always been a challenge for the theory. Recent COSY data [21] provided new information about the shape of the cross-section, being at the same time consistent with earlier TRIUMF measurements [8]. All these data compare fairly well with our calculations, as shown in Fig. 5. Indeed, almost all of the data points fall in the strip determined by the calculations performed with different $NN$ potentials. It is worth noting that for higher energies and backward angles, the data seem to favor more the Paris potential, while in the lower part of the energy range considered, the Bonn potentials (Bonn-B, in particular)
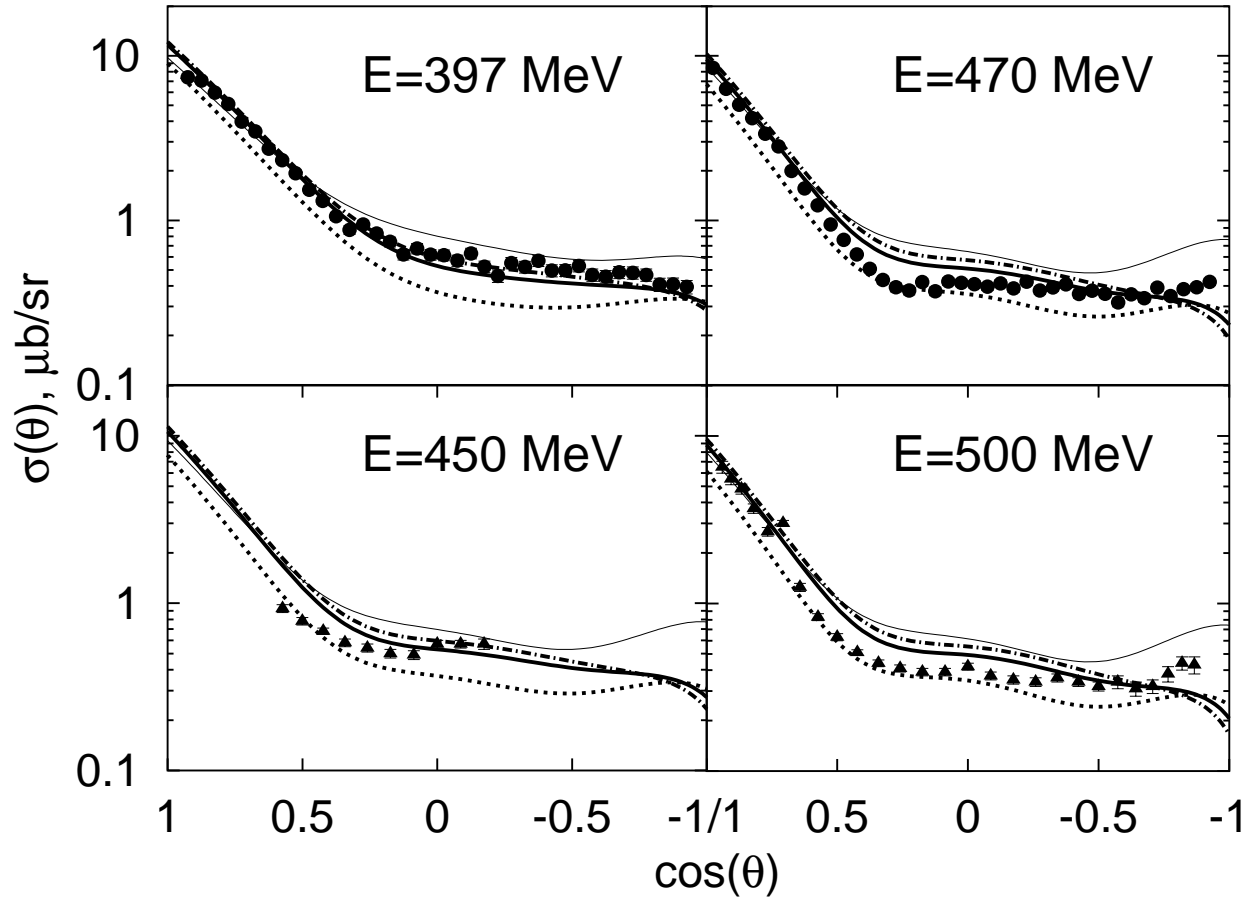
Figure 5: Differential cross-section of the $pd \to \pi^0$ $^3$He reaction. Data of Refs. [21] and [8] are shown by full circles and triangles, respectively. Notations for the curves are the same as in Fig. 3.

give rise to the results which are closer to the data.

To summarize, we have described the pionic capture processes in nucleon-deuteron collisions employing elementary production mechanisms obtained from the non-relativistic reduction of the phenomenological $\pi N$ and $\pi \Delta$ Hamiltonians. The corresponding matrix elements between the three-nucleon states have been evaluated within a large-basis space, consisting of 464 three-nucleon partial waves. Total angular momenta of the system up to 7/2 for both the parities have been considered. The prescriptions required by the Pauli principle have been taken into account through the application of the permutation formalism to the three-nucleon system. Finally, these matrix elements have been folded with the nuclear WF to obtain the probability amplitudes, with the complete details of the procedure being outlined in Refs. [9, 10, 11]. In the calculation of the plane-wave part of the amplitude, we mainly follow these papers. The only difference is modification of the standard Jacobi momentum set for the system through the replacement of the pion mass by its total energy, what is more adequate to the kinematics of the present calculations. An extended paper explaining this and some other ingredients of the model is in progress. However, much more important turns out to be our regard of the $3N$ dynamics in the initial proton-deuteron state, which was not extensively investigated in the previous treatment of the reaction.

The results obtained suggest that one achieves a fairly good reproduction of the phenomenology, when including, in addition to the $\Delta$-rescattering mechanism, the $\rho$-exchange process in the $S$-wave, as well as another $S$-wave mechanism that generates a significant amount of interaction in the scalar-isoscalar $\pi N$ channel. On the contrary, the one-body term (the off-energy-shell $\pi NN$ vertex) adds contributions that are significantly smaller. For the scalar-isoscalar component we have considered an off-shell model [14], which has been used before. We do not claim here that this particular way to generate the flux in the scalar-isoscalar channel is more realistic than other alternatives (e.g., the above-mentioned SR-exchange contributions proposed in Ref. [15]). A study on how really the flux is generated in the scalar-isoscalar channel is beyond the scope of the present work, but can represent at this point a subject for future investigations. We hope that studies of spin observables close to threshold of this specific reaction can shed light on that problem.

This work shows that, with a combination of two-body mechanisms, one can explain the structure of the proton analyzing power $A_y$, which has been accurately measured about 20 years ago and represented since then a challenge for theorists, being a difficult observable to interpret. We conclude that it is possible to reproduce qualitatively the nontrivial energy dependence of $A_y$ around the $\Delta$ resonance, provided that the three-body nuclear dynamics in the initial state is taken into account. It is worth mentioning also that we have made use of the techniques that have been developed, in fact, mainly for low-energy few-nucleon applications. It is remarkable that they can be successfully used in this energy regime, where the use of the three-body non-relativistic quantum-mechanical equations and the concept of the standard $NN$ potentials is, in some sense, questionable.

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