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STRATEGIES FOR FILLING THE LONGITUDINAL PHASI SPACE ("PAINTING") OF THE EHF BOOSTER AT INJECTI

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1 -INTRODUCTION

In this note we describe how the bunches of the EHF Booster should be " into the buckets in the course of the H^- injection process.

"Painting" is a term now generally employed to describe the task of popul: 2-dimensional phase plane with many small spots such as to fill a much larger sı The spots are (small) bunches injected from a linac and the surface to be fi a (larger) bunch in a circular machine with an accelerating RF System. Th of building a bunch structure is appropriate for the charge exchange injection enables us to "beat" the Liouville theorem in the transverse phase space, al (nearly) unlimited superposition of a large number of linac microbunches tı turn. Application of the method to machines with classical multi-turn inj appears not feasible as the ratio of circulating to linac bunch area is genei large number, whereas the number of turns injected is limited for aperture re

2 -PARTICULARITIES OF THE EHF CONFIGURATION

A particular feature of the EHF scenario is that two linac bunches, 45° RF apart and arriving each revolution, have to be distributed over the bucket suc a smooth, stationary bunch of least possible peak linear density is obtained.

As any painter will probably confirm, painting with two rigidly connected l deeply affects the results and renders a number of artistic conceptions simply cutable. The same restriction holds for bunch shaping: whatever strategy is c all two-bunches schemes show some discontinuity in phase space density which lesser though, the projected density or bunch shape.

For this reason, and also because linac builders do not completely exclu so-called "one out of eight" bunch scheme, if sufficiently strong arguments favour were found, we investigated single-bunch painting too. In particular the more ambitious strategies described in Section 3.2 rely on one linac bunch per bucket and turn.

The following parameters of the linac beam, after drift and rebunching in the injection line, have been adopted:

Area (per bunch):	$10^{-4} eVs$	
Length:	$\pm 16^{o}(400MHz)$	$\pm 2^{o}$ RF
Energy spread:	$\geq \pm 1MeV$	

According to[1], the bunch length corresponds to a drift length of about 50 m to the buncher; the bunch is tilted (and could be further compressed in energy, if necessary).

These figures have to be related to the parameters of the EHF Booster and the bunch to be paint:

Area (per bunch):	$0.075eVs$	
Length:	$\pm 108^{o}$ RF	
Bunch eight:	$dp/p = \pm 0.00245$	$(dE = \pm 4.23MeV)$

These figures were found with RAMA[2] for the $h = 90$ RF system ($50.5MHz$), for $V_{RF} = 463kV$, $\gamma_t = 13$ and N=2.5x10^{13} p/p.

As injection extends over 200 turns, a total of 400 linac bunches is injected into one bucket; this corresponds to a total area of $0.04eVs$. Hence even for uniform filling, only 53 % of the bunch area can be covered; the rest will be void.

For reasonable painting strategies these voids should hardly be visible on the phase-space plots. After filamentation, this figure can rather be interpreted as a dilution factor.

3 -FILLING STRATEGIES

3.1 Simple strategies

By "simple" strategies we understand linear motion (in time) of the locus of the injected bunch (we actually consider the "two out of eight" bunch scheme; but the trailing bunch being always at 45 RF degrees lag position, we talk only about the leading one). These case were already looked at and presented at the Frankfurt Workshop.

The most interesting case is what has been dubbed the "no-painting" scheme: injecting the bunches at fixed locus and leave the painting to their own synchrotron motion.

Keeping in mind that the injected bunches are short and rather tall with respect to the shape of the booster bunch one wants to create, and that the synchrotron tune $Q_s \simeq 1/40$, one such bunch will paint an annular domain after 40 turns injected, yielding a characteristic double-hump projection (cf. Fig. 7). With 200 turns injected five such annuli are painted, and one can guess the bunch shape to be expected from a particular slow linear motion of the locus as a superposition of five different double-humped projections. Injection of a pair of bunches complicates this picture with another five projections to be added.

Comparing the results of some simple strategies however convinces quickly that microbunches with highest possible energy spread yield flattest, smoothest and thus most suitable projections. In fact the best result was achivied with the highest possible (within the bunch height wanted) linac bunches, in particular in the "2-out-of-8" scheme. Moreover, for these 'smooth' distributions, space charge hardly alters the results for zero intensity, i.e. the purely kinematic approach is valid. Fig. 1a shows the best case just mentioned - a pair of linac bunches injected at fixed positions. Obviously with the imposed distance of 45 RF degrees between the two bunches, there is an "optimum" bunch size giving the wanted flatness. It happens that this size is close to the one specified for the booster (for completely reasons). The bunching factor, principal figure of merit, attainable with these simple painting schemes appears limited to < 0.4 for a base length of ± 110 RF degrees, corresponding to filling of $\sim 60\%$ of the bucket area.

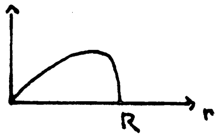
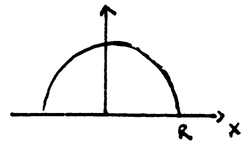
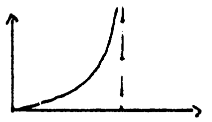
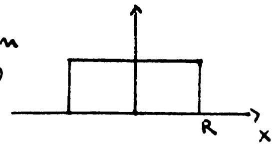
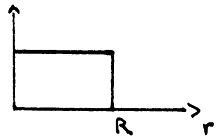
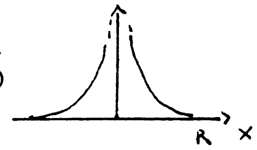
3.2 Strategies aiming at flattopped bunches

It is obvious that - for given bunch length - the square bunch has the lowest possible peak density. For the specified length of ± 110 RF degrees, this theoretical limit to the bunching factor would be 0.6 or more than 50% above the best result obtained with "simple" strategies. Obviously it is hardly possible to paint a particular prescribed phase space distribution with a rigidly coupled pair of linac bunches. The following considerations refer to painting with one single bunch per bucket, and were performed to assess the potential of these strategies in the frame of the EHF scenario. Substantial gains would be an argument for a modification of the present layout of the linac.

Phase space density and its projection (line density) are related by the Abel transform:

$$p(x) = \frac{1}{\pi} \int_x^R \frac{dr n(r)}{\sqrt{r^2 - x^2}}$$

which we have written here for rotationnally symmetric density distributions $P(r)$ and $n(r) = 2\pi r P(r)$ is the radial density. Many transformation pairs are known and tabulated in[3]. A few useful examples are given below.

Radial density $n(r)$	Line density $p(x)$
Elliptic  $n(r) = \frac{3r}{R^3}\sqrt{R^2-r^2}$	Parabolic (natural bunch)  $p(x) = \frac{3}{4R^3}(R^2-x^2)$
 $n(r) = \frac{1}{R}\frac{r}{\sqrt{R^2-r^2}}$	Constant Projection (ideal bunch shape)  $p(x) = \frac{1}{2R}$
Constant  $n(r) = 1/R$	Result of simple (linear painting)  $p(x) = \cosh^{-1}\frac{R}{x}$

With a brush of finite size and/or limited number of spots (turns injected) the formal phase space densities can only be approximately produced.

Although, at least on paper, any trajectory can be programmed for the position of the spot being painted (the "working point"), auto- matching or correctly shaped annular patterns can be produced when the working point moves slowly with respect to the synchrotron motion the injected linac bunches are undergoing. Otherwise the working-point has to be programmed according to a pre-calculated bunch shape in phase space.

If the synchrotron frequency is high enough, one can leave the azimuthal distribution of spots to the synchrotron motion and only slowly move the working point on a radius, in which case not annular but rather spiral patterns will be painted.

Moving the working point on a radius, the increment Δr between consecutive spots (bunches) is simply the reciprocal radial density $n(r)$ corresponding to the desired line density. If $n(r)$ is normalised such that

$$\int_o^R drn(r) = 1$$

then the radial increment is simply

$$\Delta r = \frac{1}{n(r) N_{inj}}$$

where N_{inj} is the number of spots painted, i.e. the number of turns injected. For the azimuthal increment $\Delta\phi$ the only constraints are $\Delta\phi \simeq const$ and $\Delta\phi N_{inj} \gg 2\pi$ to ensure rotational symmetry.

Since $n(r)$ has to be zero at $r = 0$ for all well-behaving distributions, special attention has to be paid to the painting of the very first spots in the vicinity of the origin. As Δr formally growths to infinity, they are calculated individually according to the $n(r)$ and the $\Delta\phi$ chosen, such as to yield a local phase space density as uniform as possible.

As the linac bunches have a shape not at all matched to the bucket, it makes a difference if the radius in phase space is along the abscissa (painting by "phase ramping" between linac and booster RF) or the energy coordinate ("energy ramping"). With phase ramping, the short and tall microbunches produced by the 1.2 GeV linac allow to paint steep slopes of the line density ; unfortunately the limited number of bunches (200) and the $Q_s \simeq 1/40$ of the EHF booster limit the spiral to five spires which are well distinguishable in the projection (Figs. 5b,c) and keep the bunching factor below its theoretical limit. On top of that, the steep slopes of the thin spires produce strong local space- charge forces, which in turn lead to total destruction of the quasi stationary distribution one wants to produce. This is clearly revealed when spatial resolution in computation of the space-charge forces is pushed to its limits, cf. Sec. 5 and Fig. 5e.

Energy ramping (in the EHF case) produces phase space distributions where the micro bunches are placed side by side (cf. Figs. 6a,b) and the complications due to steep gradients do not occur (cf. Fig. 6c), but the gain in bunching factor is marginal too. $B_f < 0.44$ in all cases. It certainly does not justify a modification of the present linac design.

4 -THE TRACKING CODE

4.1 Description.

The optimization of the injection procedure is a rather recent problem arising in the design of high intensity, fast-cycling machines. Moreover, up to now no code exists which tracks particles in the 6-dimensional phase space in presence of space charge. Limiting ourself to paint, in a first approach, only the longitudinal phase space, we have used the code LONG1D[4]. Written at RAL for an IBM computer, we used a VAX version used at TRIUMF. This code is essentially based on the theory reviewed for example by A. Hofmann[5], which considers a coasting beam moving along the axis of a round pipe. This applies to the case of a long beam in a smooth, round vacuum chamber.

A very delicate problem for a code which has to simulate a multi-particle process like space charge force, is statistics. The line density is discretized over a finite number of bins and unphysical results can originate from a poor population of them. On the other hand, to save computing time, one has to simulate the behaviour of say 10^{13} by using about 10^4 "macro-particles". As a consequence some kind of "smoothing" is desirable. In fact, space charge force is computed in LONG1D by performing a Fourier analysis of the line density λ. The number of significant harmonics is obviously limited by the number of points where λ is known (number of bins). To save computing time, in the original version the code automatically truncates the number of harmonics to an "optimum", determined by the r.m.s. bunch length. This truncation seems to us rather arbitrary; in fact, one risks of loosing information and of smoothing out much of the space charge effect. For this reason we decided to switch off the automatic truncation in some of the cases considered (cf. Section 7).

Related to statistics there is the problem of a right choice of the number of bins. This will be discussed in Sec. 5 where a criterion to choose the number of bins is given.

Another critical parameter is the number of RF cavities considered; as a consequence of the discrete energy gain at each accelerating station, trajectories are tilted in the phase space. Results are also affected by the number of RF cavities assumed. We took 3 RF cavities in the cases quoted in Sec. 7.

4.2 Adaptation to the EHF scenario.

The code needed several important modifications in order to match the EHF scenario requirements.

First of all we had to simulate the filling process of the bucket during several injection turns. This has been made in a such way that the length, energy and phase (relative to the machine RF) of the incoming bunches can be varied during injection. In particular special phase and energy ramping algorithms were introduced to reproduce the special density distributions of Sec. 3.2 in the longitudinal phase space. Moreover the filling strategy proposed by E. Colton[6] and implemented in its own code PAINT (cf. Sec. 6) was also included for comparison. During injection the phase Φ of the linac leading bunch is moved according to

$$n - 1 = \alpha \Phi sin(\Phi/2)$$

where n is the is the injection turn. The coefficient α is determined by the area you want to fill.

The computation of the bunching factor, which is an important figure of merit, was introduced into the code; it is computed directly by the definition as the ratio of the average number of particles per bin to the number of particles in the most populated bin.

As the energy of the injected particles can be varied during injection and off-energy injection has also to be foreseen, we had to decouple the injection energy from the reference one.

Injection is supposed to take place at constant frequency on a relatively flat bottom of the guiding field, which is assumed to be a harmonic function of time: $B(t) = B_0 - B_1 cos(ft)$. Synchronous phase is then computed by

$$sin\phi_s = \frac{2\pi R\rho}{\eta\gamma_t^2 V_o}\frac{dB}{dt}$$

which is obtained combining[7]

$$\frac{dB}{B} = \frac{\gamma^2 - \gamma_t^2}{\gamma^2}\frac{dp}{p}$$

with

$$\frac{dp}{dt} = \frac{eV_o sin\phi_s}{2\pi R} \tag{4.1}$$

Closed orbit radius changes during injection and, in general, to save vacuum chamber dimensions, the beam will be not on the geometrical centre of the pipe when injection ends. Due to the jump of the synchronous phase, dipolar instabilities arise when switching abruptly to the constant radius acceleration regime. Then, after injection, synchronous phase is calculated by

$$sin\phi_s = \frac{2\pi R\beta E}{ceV_o}[\frac{1}{B}\frac{dB}{dt} + \frac{\gamma_t^2}{R}\frac{dR}{dt}]$$

which comes from[7]

$$\frac{dp}{p} = \frac{dB}{B} + \gamma_t^2\frac{dR}{R} \tag{4.2}$$

and eq.(4.1). Let say t_o the time when injection ends and R_o and R_c the synchronous radius at $t = t_o$ and the reference radius respectively; imposing in eq.(4.2)

$$R(t) = A\frac{(t - t_0)}{\tau}e^{\frac{-(t-t_0)}{\tau}} + Ce^{\frac{-(t-t_0)}{\tau}} + D$$

with A, C and D given by

$$A = R_o - R_c + \frac{\tau R_o}{(\gamma^2 - \gamma_t^2)}\frac{1}{B}\frac{dB}{dt}(t_o)$$

$$C = R_o - R_c$$

$$D = R_c$$

the beam will move continuosly towards the vacuum chamber centre R_c.

Finally, the space charge effect was included in the calculation of the separatrix according to[8], but no significant deformations were observed in the cases under study.

5 -COMMENT ON MODIFICATION OF THE CODE AND LIMIT TO SIMULATION

When tracking results of the modified LONG1D code were compared with those of E. Colton's code PAINT - cf. Sec. 6 -, it turned out that the match between the two depends crucially on parameters related to spatial resolution, namely the number of bins and the number of Fourier harmonics used in the computation of the space charge force.

There is an obvious tendency to reduce the number of bins in the presentation of the line density (and in the computation of the bunching factor). In fact, if the binning is finer than actual structure details, fluctuation due to poor statistics may yield a pessimistic result.

On the other hand, pushing the number of Fourier harmonics H and the number of bins $N_{bins} \geq 2H$ to their reasonable limits changes the results dramatically in those cases where the line density shows pronounced peak structure (cf. results for spiral filling, Figs. 5a-e). Fig.5a, produced with $N_{bins} = 36$ and H truncated by the original automatics, suggests a bunching factor of 0.507. High resolution ($N_{bins} = 240$) in Fig. 5b but H still limited as above, makes this figure drop to $B_f = 0.373$! Pushing H higher, results change more and more and the spiral structure is completely destroyed for $H = 50$ and heavily deformed with $H = 120$ (Figs. 5d,e).

These results raise the question what the "reasonable limits" to the spatial resolution (expressed in terms of H or N_{bins}) be.

Two effects compete :

(i) Very short-wave length modulation of line density will not be "resolved" at the walls of the pipe and the depth of the potential will not follow this variation. Formally, the g_o-factor $g_o = 1+2ln(b/a)$ rolls off at short wave length. The expression can be found in[9]:

$$g_o = \frac{4}{x^2} - \frac{4}{x}[K_1(x) + I_1(x)\frac{K_o(xb/a)}{I_o(xb/a)}]$$

$$x = \frac{2\pi a}{\gamma\lambda}$$

with a, b beam and pipe radius respectively, and λ wavelength of the line density modulation.

λ can be expressed by $N_{bins} = 2H$:

$$\lambda = \frac{\lambda_{RF}}{N_{bins}/2} = \frac{4\pi R}{h N_{bins}},$$

or

$$x = \frac{h a N_{bins}/2}{R\gamma}$$

where h is the RF harmonic number. The expression for g_o is plotted in Fig. 10 for b/a = 3, and from the characteristic parameter of the EHF booster indicated on the plot, we conclude that the effect is not dramatic for the number of bins envisaged.

ii) Another critical parameter is the cut-off frequency of the waveguide, i.e. when the lowest TE mode begins to propagate inside the pipe and one may expect any perturbation of the line density of this or shorter wavelength being "radiated away". This limit was also quoted by J.A. MacLachlan[10], who developed a tracking code similar to LONG1D. The cutoff for the TE11 mode of a circular waveguide of radius b is: $\lambda_c = 3.413b$. The equivalent number of bins for a pipe of $b = 3$ cm is

$$N_{bins} = 2\gamma \frac{\lambda_{RF}}{\lambda_c} = \frac{4\pi R\gamma}{3.41bh} = 240$$

for the EHF Booster and that is the figure that has been chosen for testing painting strategies with high-resolution tracking (normally the number of bins is 100). An immediate consequence is that the number of superparticles has to be increased in order to avoid excessive fluctuation due to the poorer statistics : as fluctuations are in general proportional to the square root of the number of particles per bin, the total number of superparticles has to be scaled with the square of N_{bins}; in our test runs with $N_{bins} = 240$, the number of superparticles was chosen to be 60 000. This number limits the time the ensemble being trackable to the painting process proper. Tracking of early acceleration (3 ms) was performed with standard 100 bin resolution only.

Contrary to the wild bunch deflagration found for spiral filling (see above and Figs. 5d, 5e) the 2-bunch, "no-paint" strategy chosen for longitudinal painting of the EHF Booster bucket is not sensitive to the choice of spatial resolution in space-charge computation : Figs. 4a, b, c representing no space charge, automatic truncation of Fourier harmonics related to r.m.s. bunch dimensions, and $H = 120$ (maximum resolution), respectively, look very much the same, also the bunching factors achievable. This result gives confidence that the distribution painted this way will be well-behaving and no major problem is expected from this side.

6 -COMPARISON WITH THE CODE "PAINT"

The code PAINT was independently developed by E. Colton. In absence of an user guide describing this code, our comparisons could be incomplete.

We know that in PAINT the space charge force is directly computed making the difference between populations of adiacent bins. In our opinion, this enhances the space charge effect, because of the unavoidable poor statistics. Then we expect that any regular structure in the phase space distribution will be destroyed. This is confirmed by results.

Another difference consists in the way in which bunching factor is evaluated. It is not calculated directly by its definition, but using a formula which is in fact valid only for gaussian bunches and gives more favorable values.

Thus attention has to be payed in comparing results obtained by using these two codes.

The filling strategy implemented in PAINT was described in Sec. 4.2. Figs. 9a,b show particle distribution in longitudinal phase-space and line density at the end of the injection, switching off and on respectively the space charge term. These results have been obtained by our version of LONG1D, matching PAINT code conditions and for $\alpha = 284$, which gives a bunch extending from -115^o to $+115^o$ about. In the first case (without space charge) results look very similar to PAINT ones (Fig. 8a), but the regular distribution is conserved when switching on space charge. Using 45 harmonics (the maximum number of harmonics allowed by 90 bins), space charge becomes more influent; the regular structure is destroyed (Fig. 9c) and, as expected, results are in agreement with PAINT ones (Fig. 8b). Bunching factors computed by us are in any case worse and probably make this filling strategy less interesting than expected.

7 -RESULTS OF TRACKING STUDY

7.1 Painting proper

For the EHF scenario as published in the proposal, the simplest possible (no-paint) scheme using the 2-out-of-8 scheme of the linac, gives bunching factors slightly below 0.4 for a bunch ± 110 RF degrees long, which also appears to be well-behaving over the injection interval ($360\mu s$). This our choice is depicted in Fig. 1a. More refined strategies aiming at a "square bunch" do not give significantly better results in the EHF emvironment with slim, tall linac bunches and only five "spires" or synchrotron periods during the whole painting (Figs. 5a-e, 6a-c). The fine structure of these spirals bears the risk that bunches painted this way will not remain unaltered (cf. Figs. 5d, e) and will be prone to losses. Thus these strategies cannot be recommended for EHF. However, for a very long injection time, as e.g. in the TRIUMF KAON Accumulator, they may be more promising and are in fact studied there[11].

7.2 Early Acceleration

The first 3 ms of the total 30 ms acceleration cycle have been tracked for the chosen filling strategy (Fig. 1b). Three tracking modes have been compared :

(i) Automatic truncation of the Fourier representation of the line density to a resolution related to r.m.s. bunch length : Figs. 1a-c.

(ii) Highest possible resolution possible for 100 bins : Fourier harmonics up to 50 : Figs. 2a-c.

(iii) No space charge : Figs. 3a-c.

The somewhat surprising result is that (i) shows the development of some azimuthal asymmety of dipolar character - an effect also observed in tracking studies of the filling of the TRIUMF KAON Accumulator and dubbed bunch "sloshing"[11]. This effect is however hardly visible in (ii), where also the finestructure of the space charge force is processed and completely absent in (iii), i.e. without space charge. We interprete this result as a hint that the spatial resolution in computation of space charge forces should be pushed as far as possible. Nevertheless Figs. 2b and 2c point out an effect which should incite us to be watchful : note that there are particles leaking out of the origin bunch area ; in fact bunch length at the base has grown, if compared with 1c or 3c. This growth deserves further attention an tracking through longer interval. On the other hand, "bunch sloshing" does not necessarily entail loss : The ISIS synchroton at RAL shows this effect throughout the whole cycle at high intensity without suffering from loss[12].

8 -CONCLUSION AND OUTLOOK

The painting strategies compared in this study can be divided into two groups:

(i) Painting with two linac bunches (corresponding to the present linac specs). Best results (bunching factor $\simeq 0.38$) have been achieved with stationary injection position in phase space ("no-paint" approach).

(ii) More ambitious strategies aiming to approach a square-shaped linear density, requiring painting with a single bunch. For the limited number of turns injected in the EHF Booster, these "spiral filling" schemes, however, do not yield substantially better bunching factors, the best results being comparable to (i) when tracked with higher spatial resolution. This not exclude that these strategies may prove superior for machines with many turns injected, like the TRIUMF KAON Accumulator.

Another distinctive feature between the two approches is the sensitivity of group (ii) to tracking parameters, mainly to the spatial resolution involved in the computation of the space charge force; this sensitivity is absent in the simple painting strategies (i).

A criterion for the resolution required is given and all relevant painting strategies have been checked with this resolution. The favorable behaviour of the simple strategies led to the recommendation of one of those for EHF (Figs. 1a, 4a).

Figs. 1a-c and 2a-c also show the early 3 ms of acceleration, with low and medium spatial resolution respectively. Highest spatial resolution requires forbiddingly long CPU time.

No dramatic events happen, but some sensitivity to spatial resolution becomes apparent: bunch "sloshing" (Figs. 1) or some diffusion into tails in the case of higher resolution (Figs. 2).

This effect and the whole acceleration cycle deserves further studies, which go beyond the scope of this note. There is some evidence that the presently used code is not fully mature, in particularly in the treatment of fast acceleration. Some doubts about the way the space charge force is taken into account have also been raised on a recent workshop at TRIUMF. We recommend to wait for further development in this field before starting the tracking study of acceleration in the EHF Booster.

References

[1] K. Bongardt, M. Pabst, M. Wagner, EHF-86-50

[2] R. Baartman, H. Schoenauer, TRI-DN-86-15, TRIUMF Design Note

[3] P. Krempl, MPS/Int. BR/74-1, Internal Report, CERN

[4] S.R.H. Koscielniak, RAL T 043, 1987

[5] A. Hofmann, CERN 77-13, pag.139

[6] E. Colton, AHF Technical Note 87-010

[7] C. Bovet et al., CERN/MPS-SI/Int. DL/70/4, pag.9

[8] D. Edwards, MPS/BR Note/74-2

[9] W. Hardt, 9th Int. Conf. on High Energy Particle Acc., Stanford 1974, pag. 434

[10] J.A. MacLachlan, FN-446, FNAL 1987

[11] S.R. Koscielniak, pers. Comm.

[12] G. Rees, Ch. Planner, pers. Comm.

Figure Captions

Fig. 1: Result of painting (a) and evolution during early acceleration, strobed at 1.5 ms (b) and 3 ms (c), for the recommended constant injection locus strategy. Position and size of linac bunches are drawn in Fig. 1a. Beam parameters are as nominal for EHF Booster. Spatial resolution in space charge calculation is limited to typical r.m.s. bunch length by automatic truncation of higher Fourier harmonics in the code. 20000 superparticles, 100 bins.

Fig. 2: Same case as in Fig. 1, but space charge forces evaluated with maximum spatial resolution possible with 100 bins. Note that the dipolar asymmetry ("sloshing") visible in Fig. 1 is much less pronunced, but some blow-up has occurred and bunch edges are smeared out after 3 ms.

Fig. 3: Same case as Figs. 1, 2, but without space charge, for comparison.

Fig. 4: Same case as Figs. 1-3,a (i.e. at the end of the injection), displayed with high resolution (240 bins); without space charge (a), automatic truncation of Fourier harmonics according to r.m.s. bunch size (b), and maximum resolution (c). Some regular structure can be seen in (c), but is likely to be smeared out during subsequent acceleration.

Fig. 5: Results of attempts to paint a square bunch by phase ramping in the EHF Booster (with one linac bunch), for different display resolution and treatment of space charge:
(a): 36 bins, truncation of Fourier harmonics
(b): 240 bins, truncation of Fourier harmonics
(c): 240 bins, no space charge
(d): 100 bins, 50 Fourier harmonics
(e): 240 bins, 120 Fourier harmonics
Note the strong dependance of the results upon the processing parameters!

Fig. 6: Attempt to paint a square bunch by energy ramping:
(a): 36 bins, truncation of Fourier harmonics
(b): 240 bins, truncation of Fourier harmonics
(c): 240 bins, 120 Fourier harmonics
Due to absence of spires, results are fairly comparable regardless of the way space charge is evaluated.

Fig. 7: Painting an annular area with one linac bunch injected at a fixed locus. 40 turns injected, $Q_s \simeq 1/40$.

Fig. 8: Filling scheme of E. Colton, for two linac bunches. Figures reproduced from[6], without space charge (a) and with space charge (b) evaluated by numerical differentation of line density binned in 90 bins (code PAINT).

Fig. 9: Same case as in Fig. 8, using LONG1D:

(a): no space charge, 100 bins
(b): 100 bins, truncation of Fourier harmonics
(c): 90 bins, 45 harmonics
Results tend to agree only if the same spatial resolution is used in space c
calculations regardless of the method employed.

Fig. 10: Roll-off of the g_o factor at high mode numbers (short wavelengt

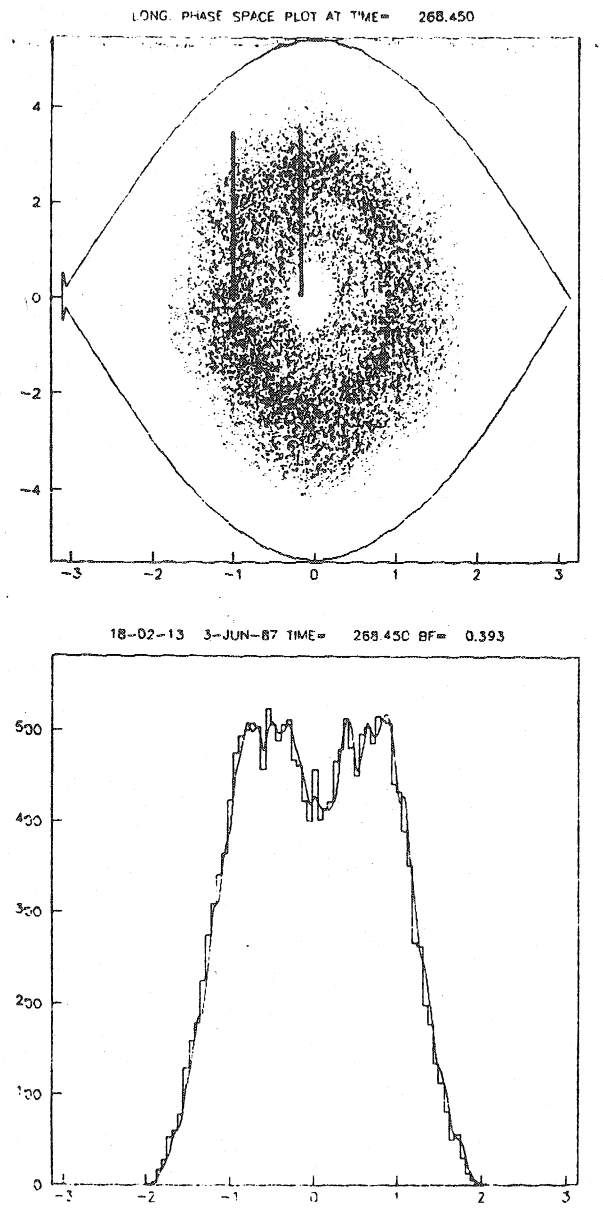


Fig. 1a

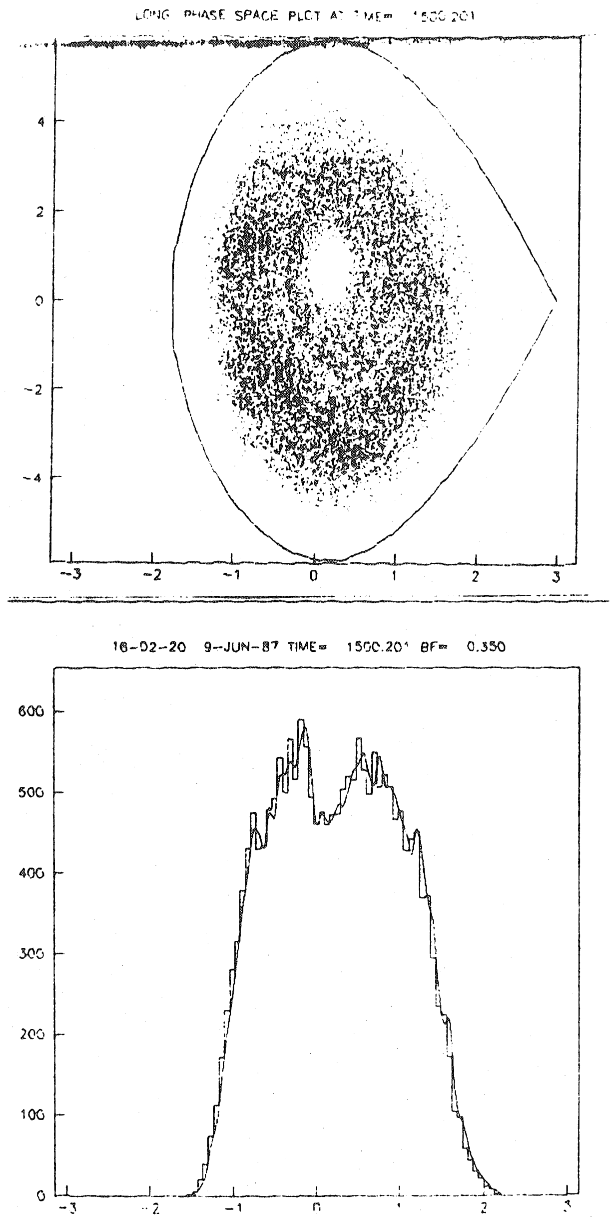


Fig. 1b

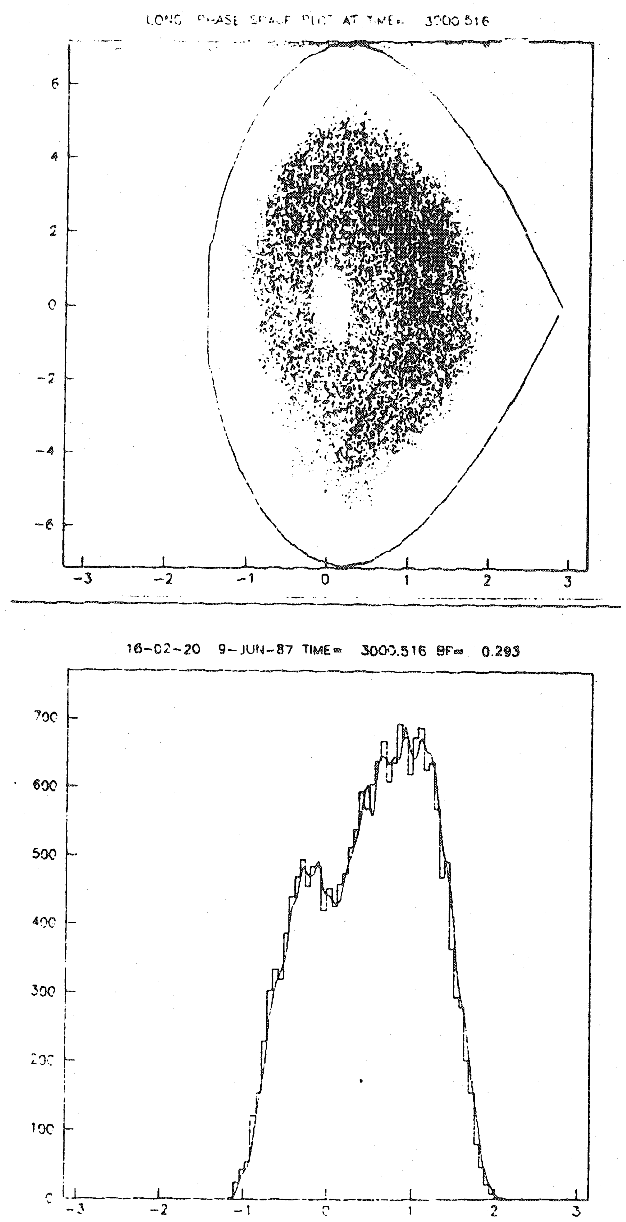


Fig. 1c

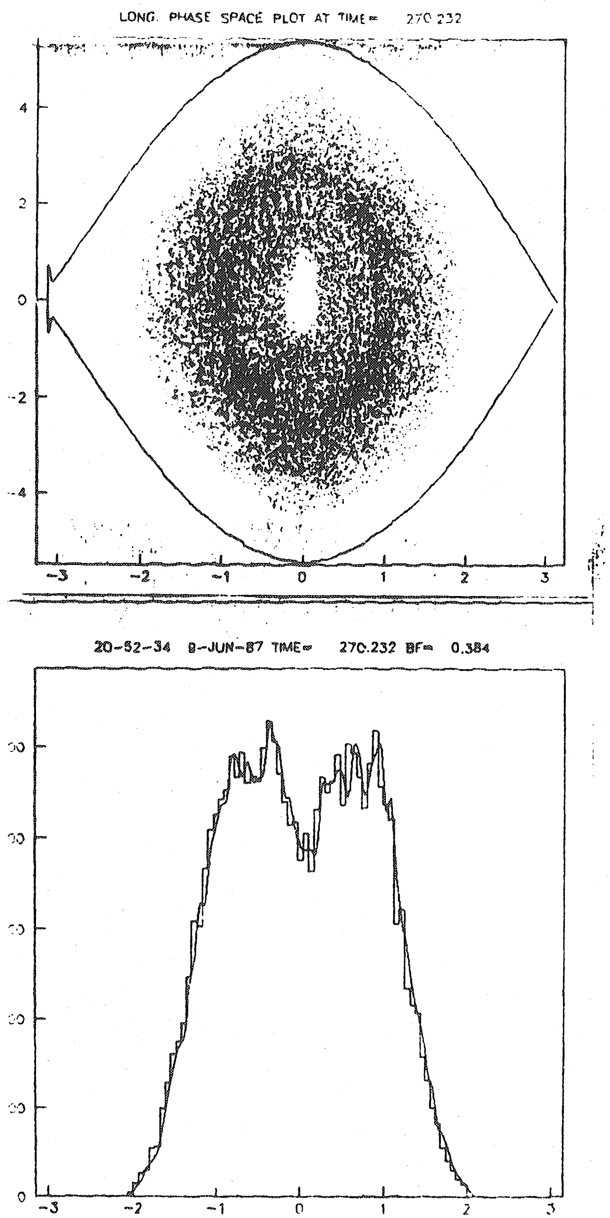


Fig. 2a

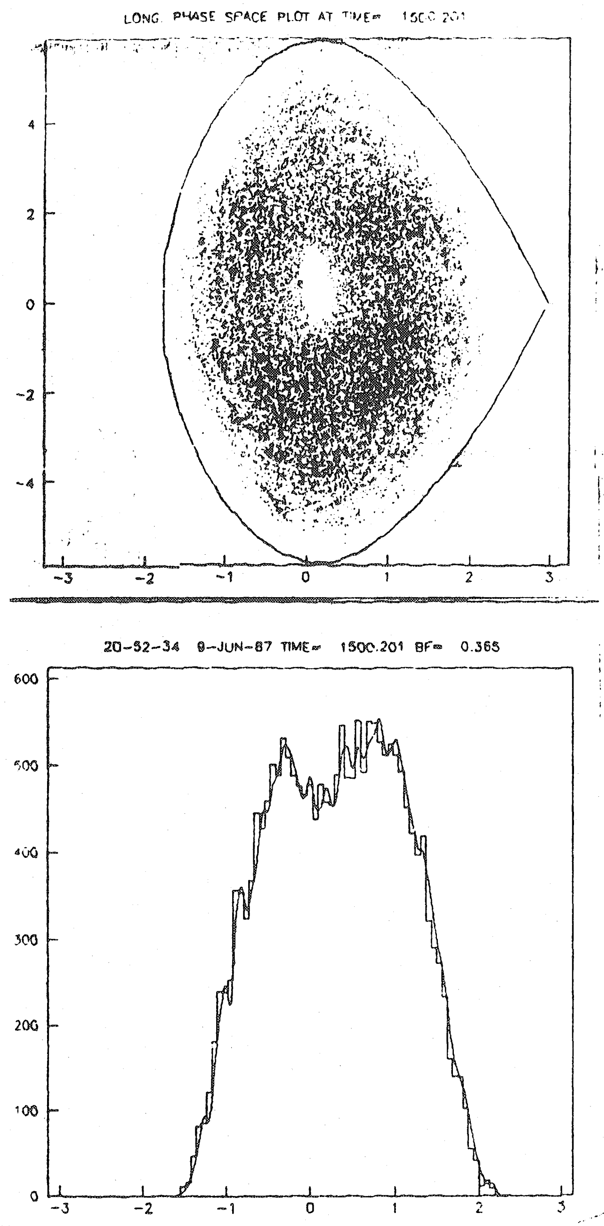


Fig. 2b

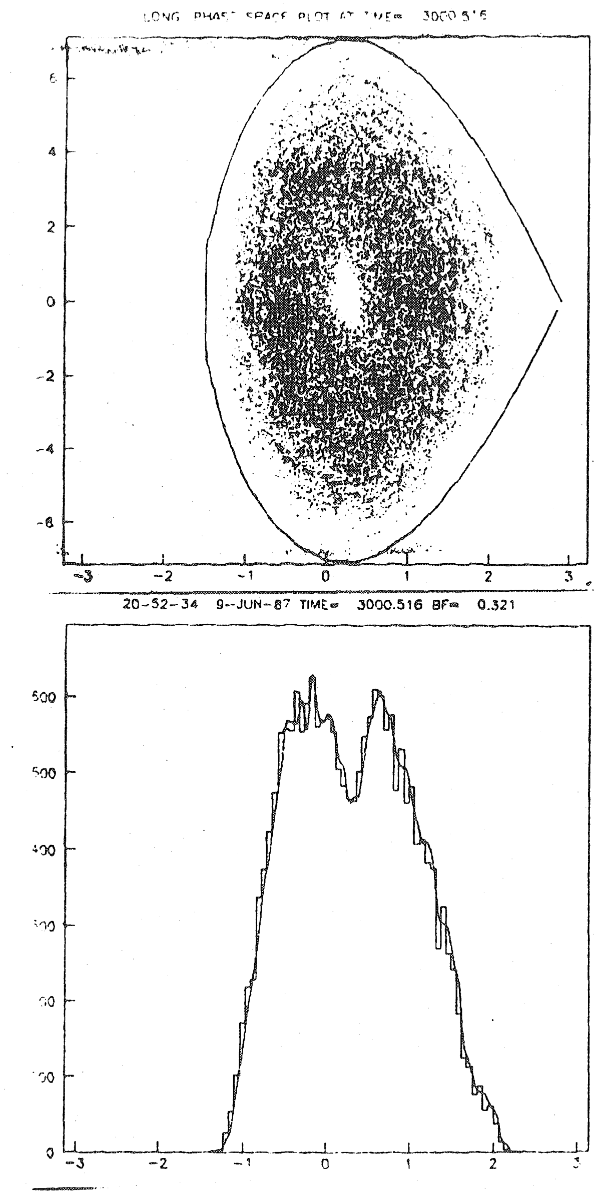


Fig. 2c

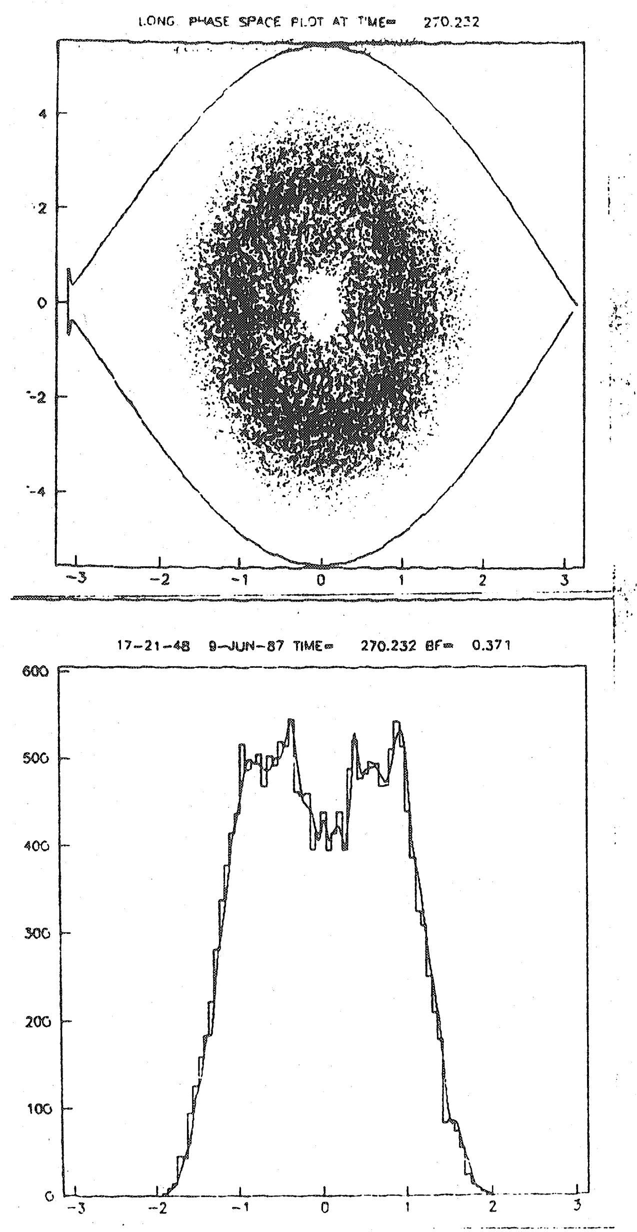


Fig. 3a

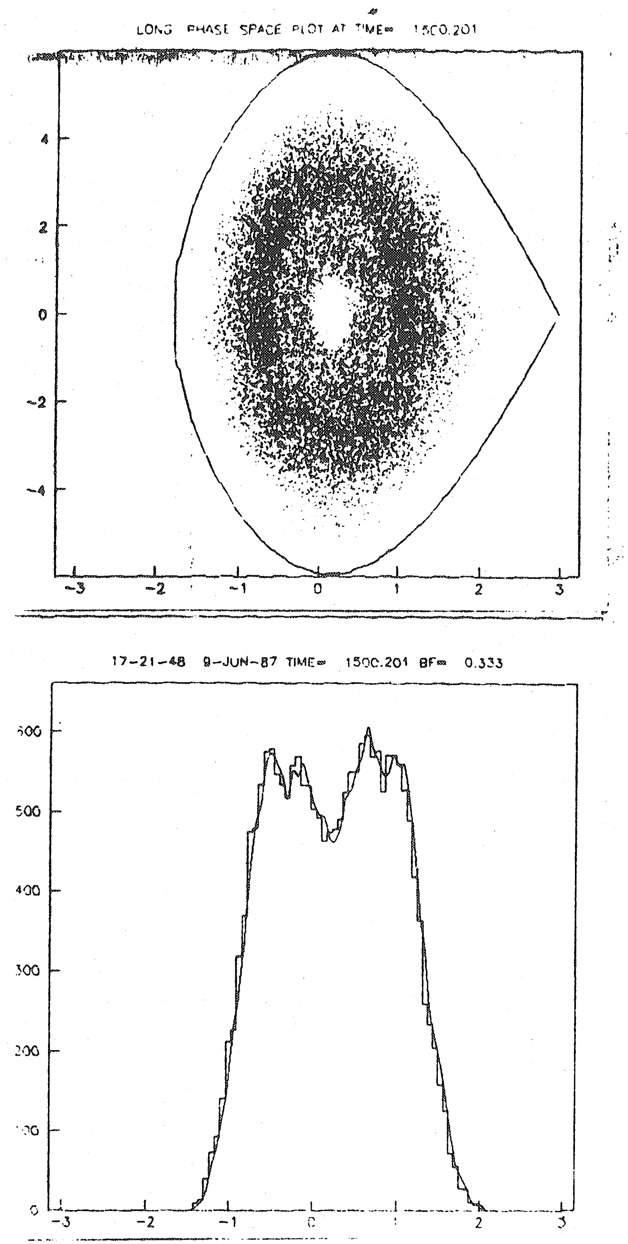


Fig. 3b

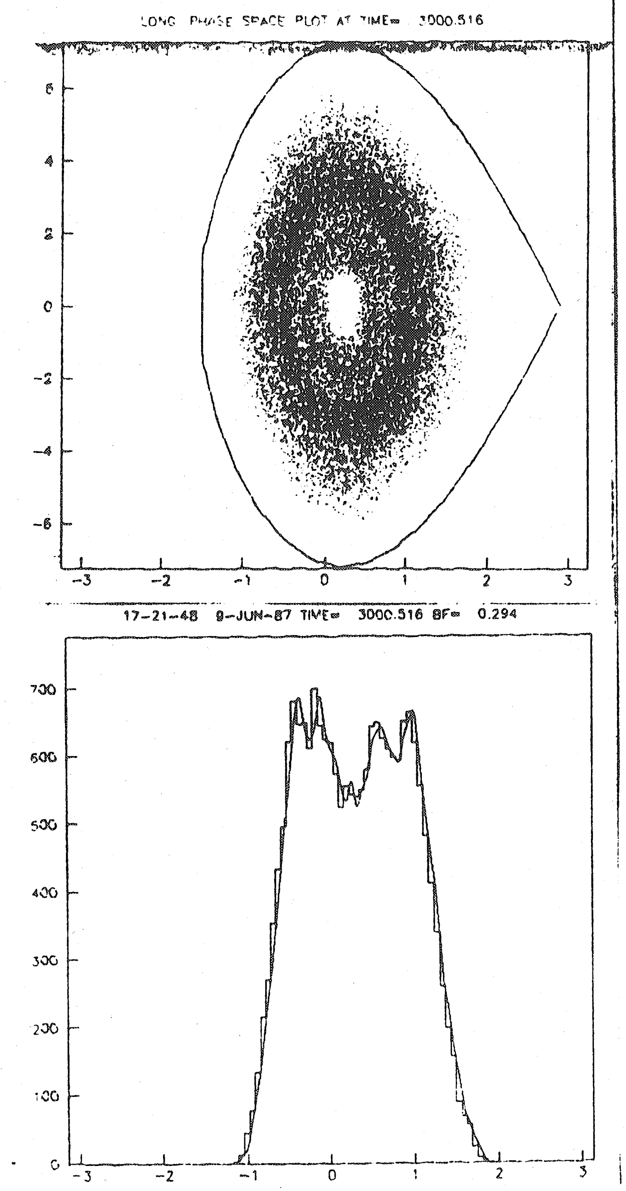


Fig. 3c

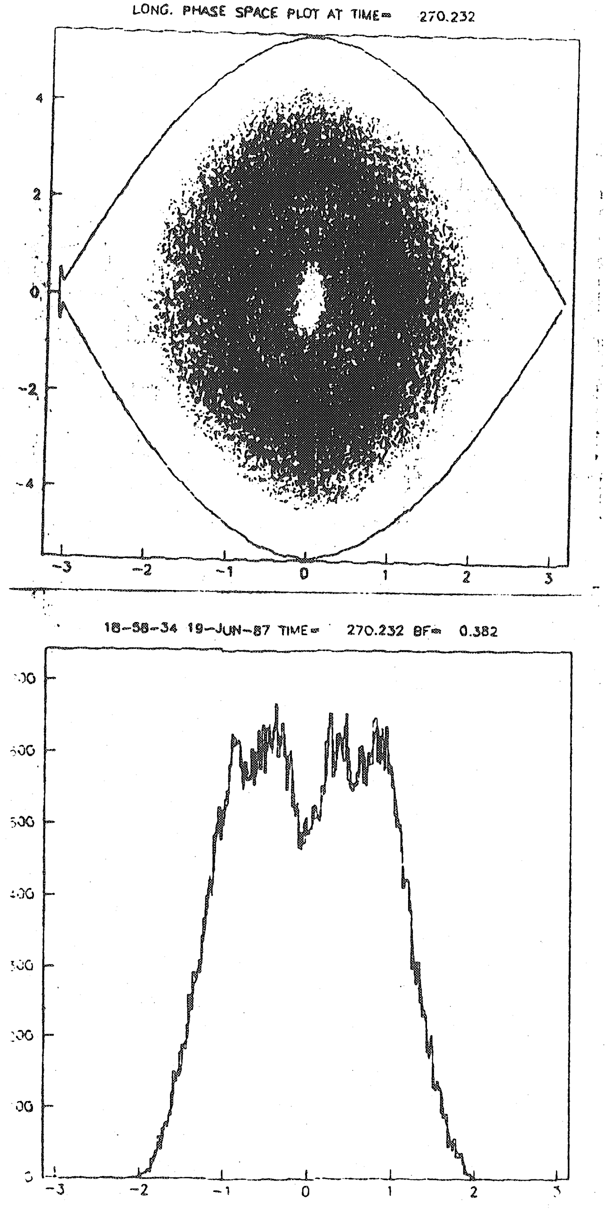


Fig. 4a

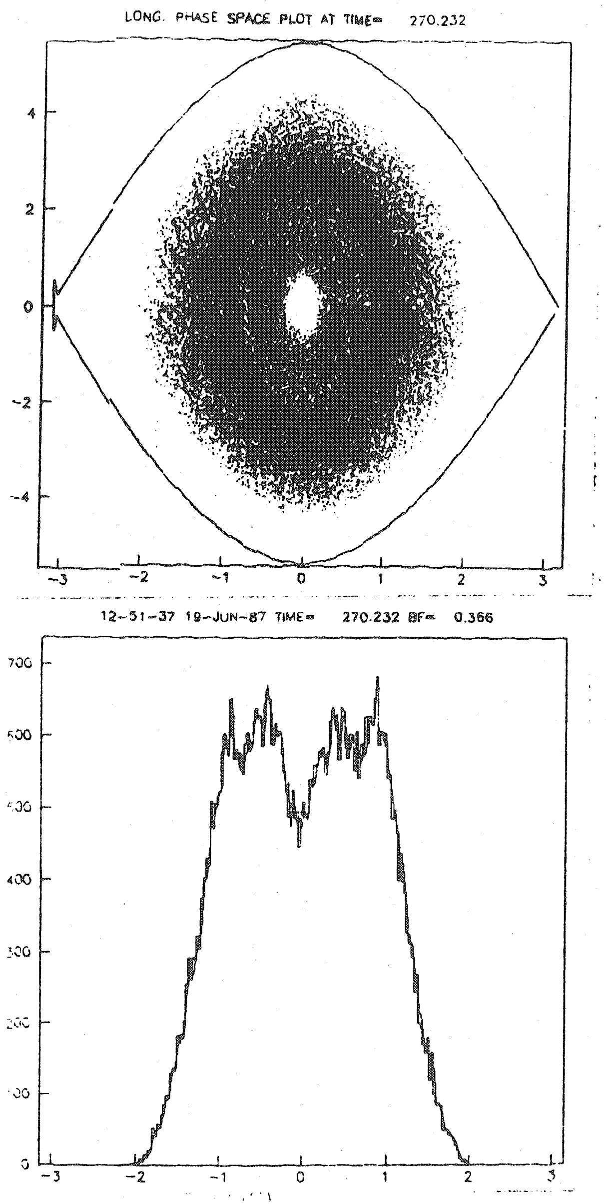


Fig. 4b

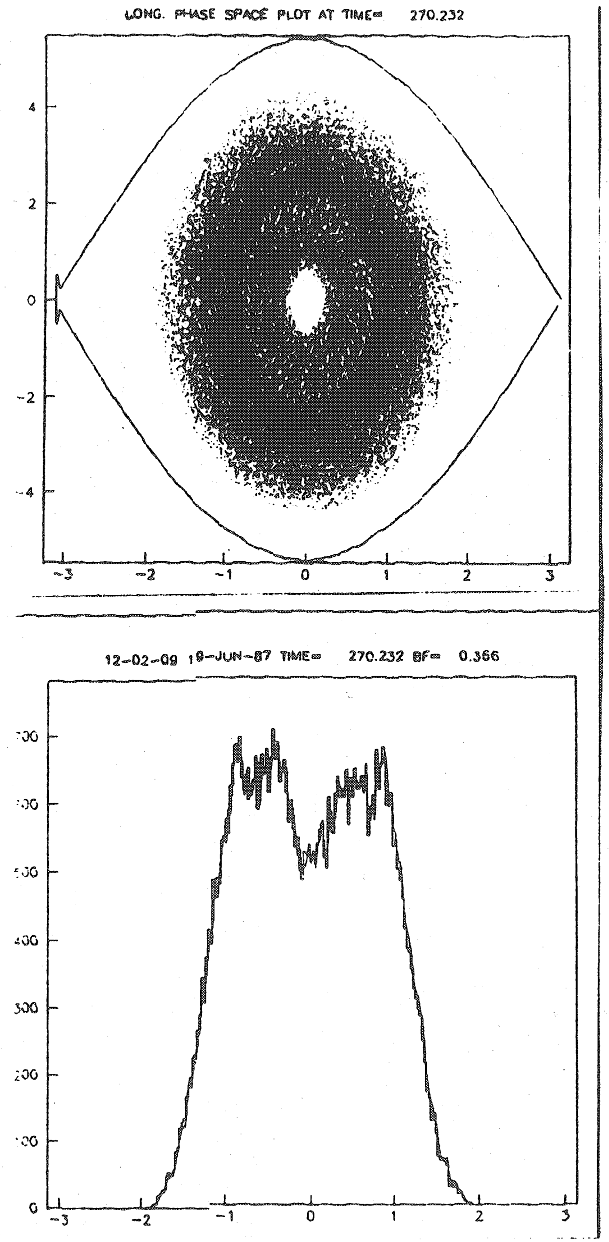
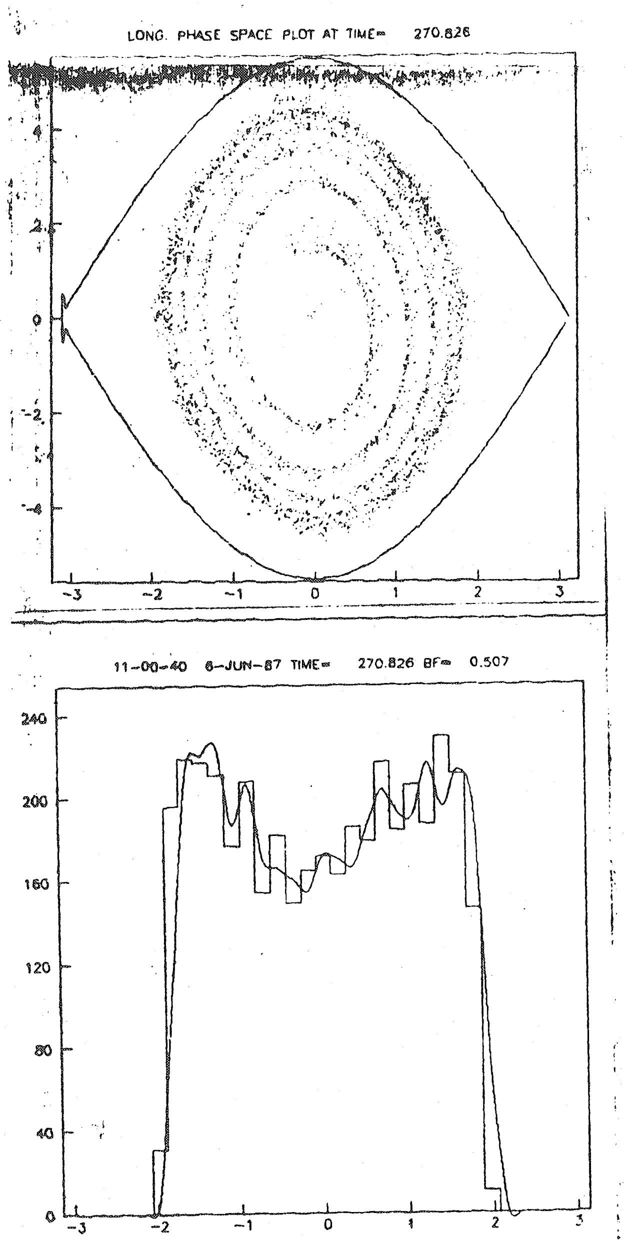


Fig. 4c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11-00-40 6-JUN-87 TIME= 270.826 BF= 0.507
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200
160
120
80
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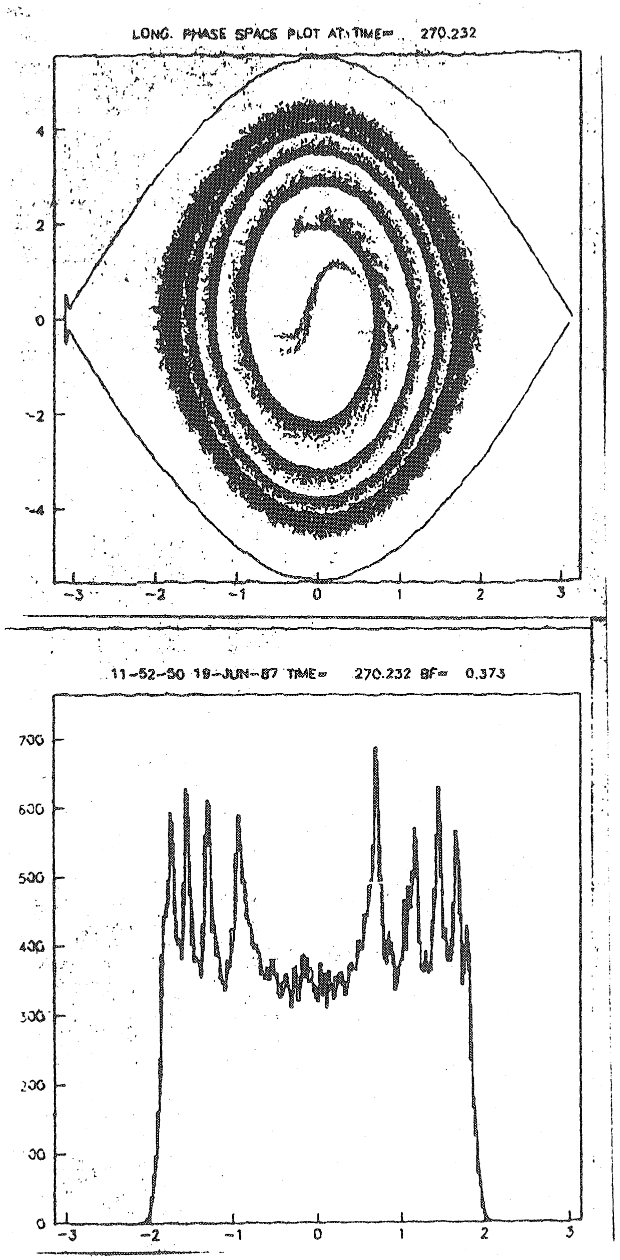
1
2
3

Fig. 5a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11-52-50 19-JUN-87 TIME= 270.232 BF= 0.373
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500
400
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200
100
0
-3
-2
-1
0
1
2
3

Fig. 5b

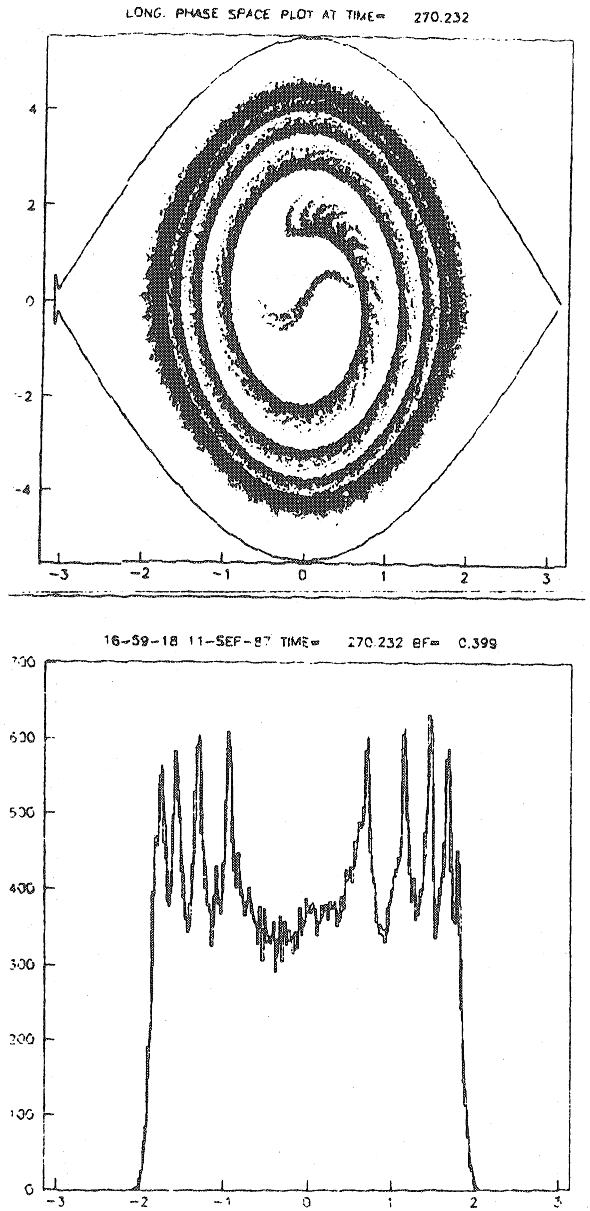

Fig. 5c

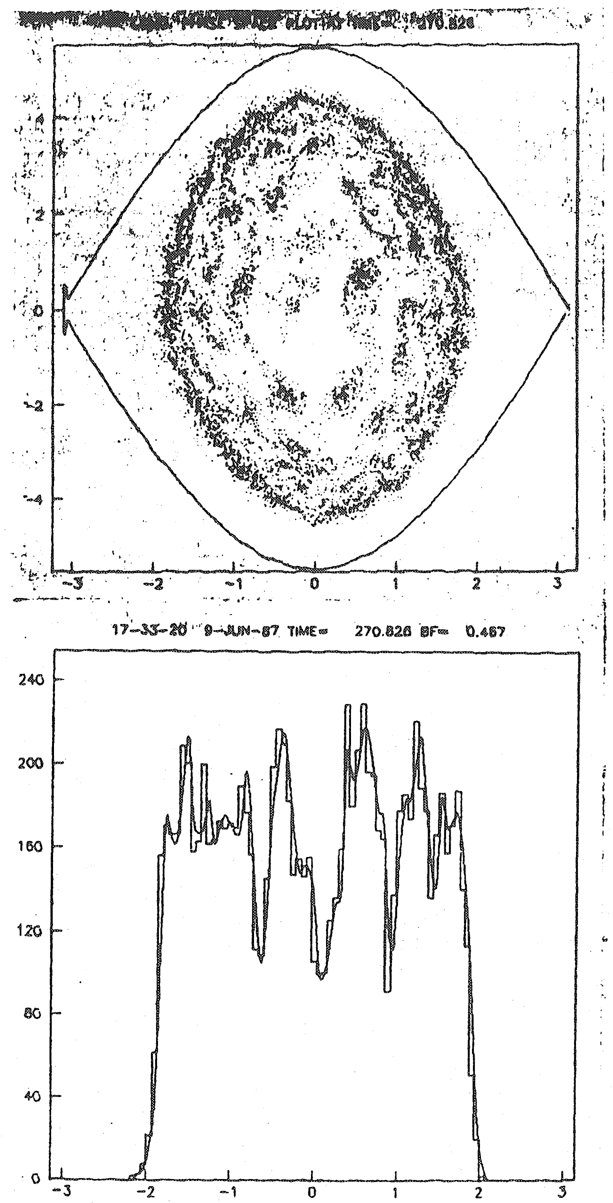

Fig. 5d

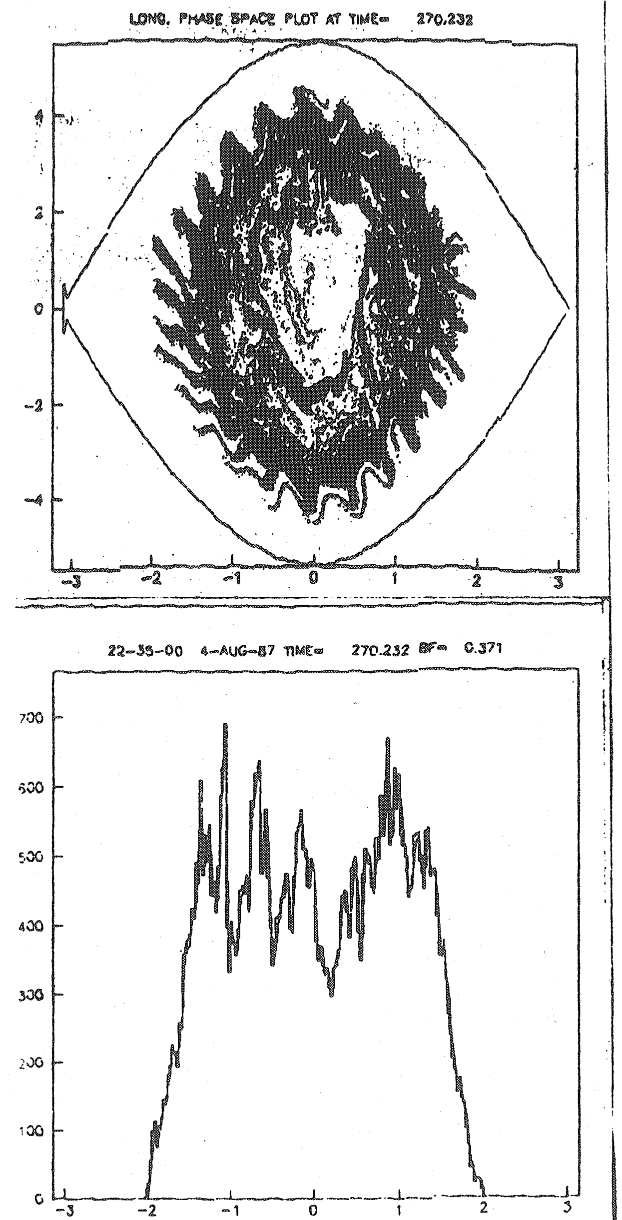

Fig. 5e

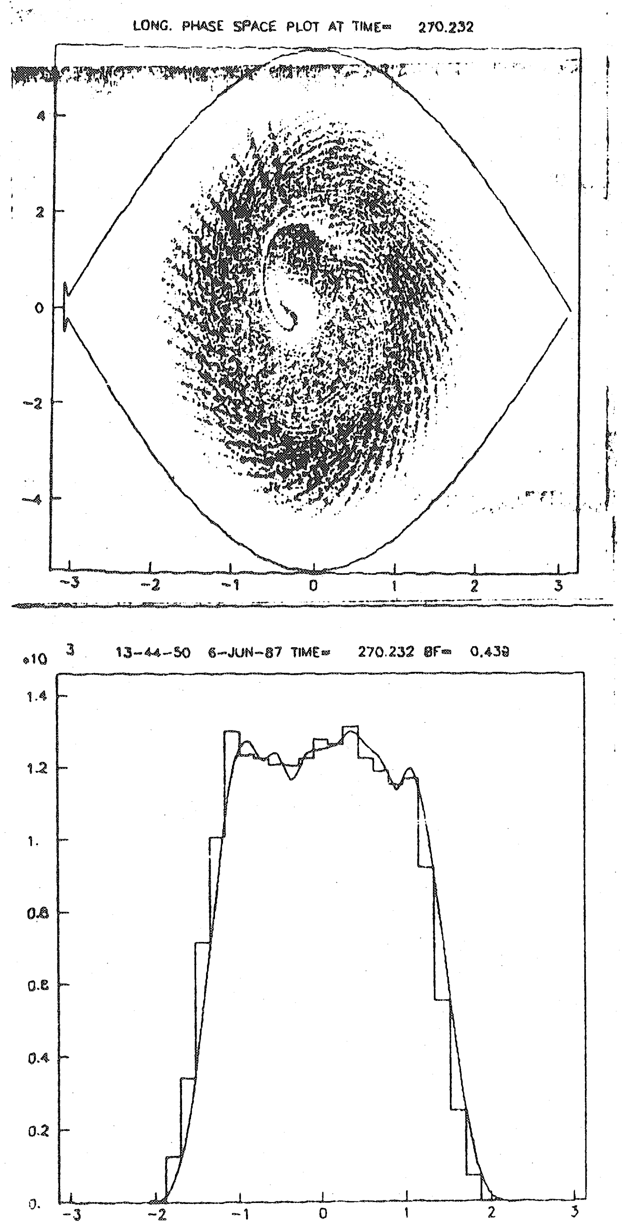

Fig. 6a

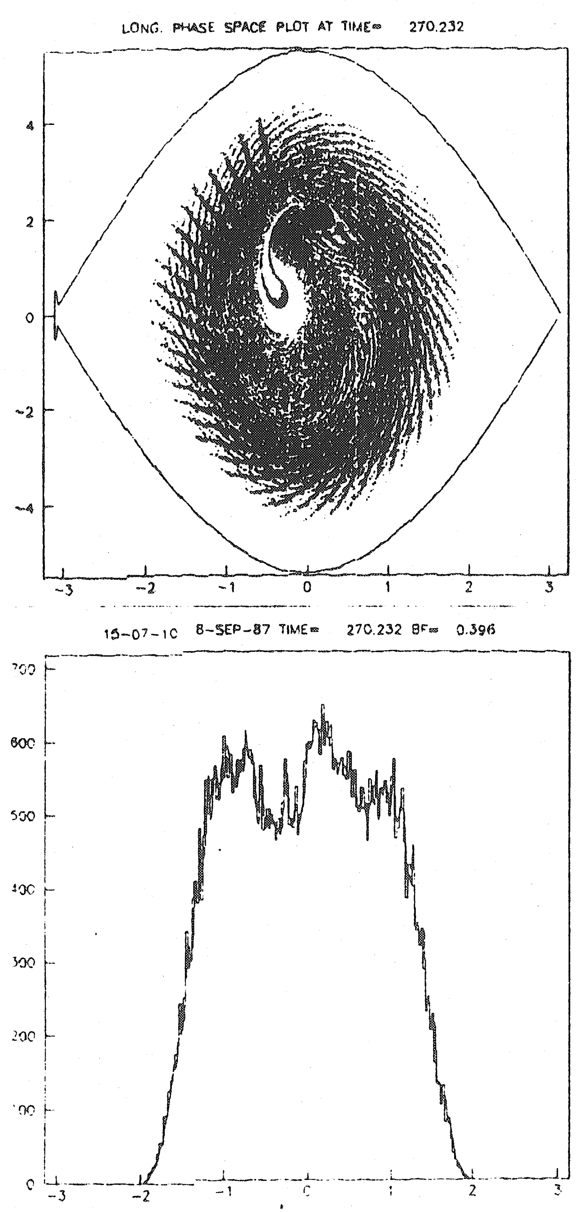

Fig. 6b

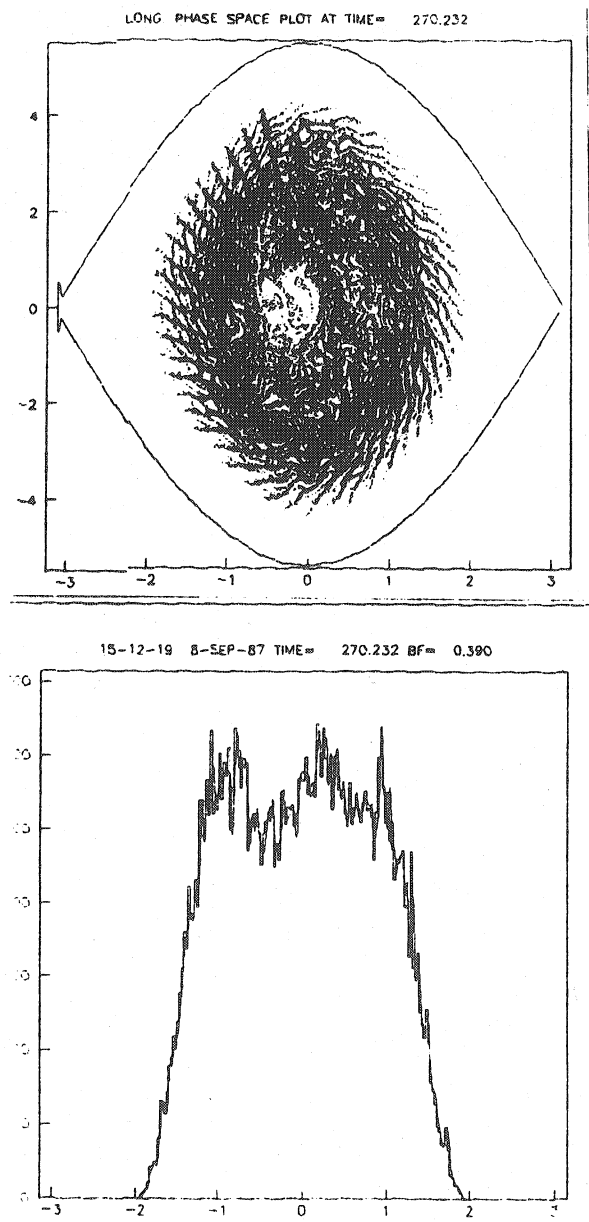

Fig. 6c

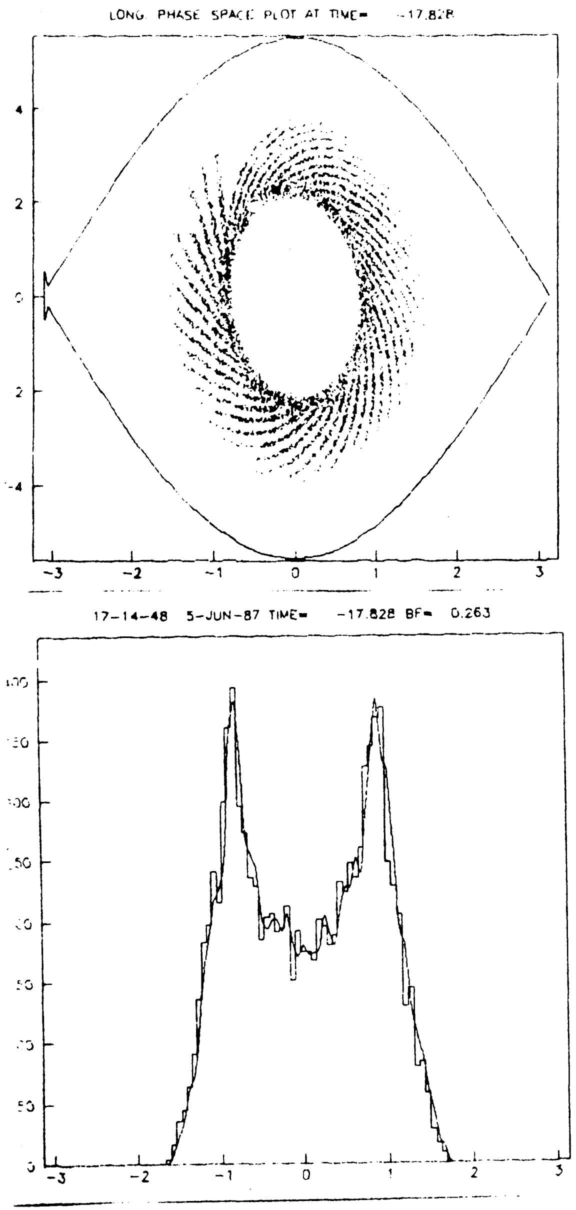

Fig. 7

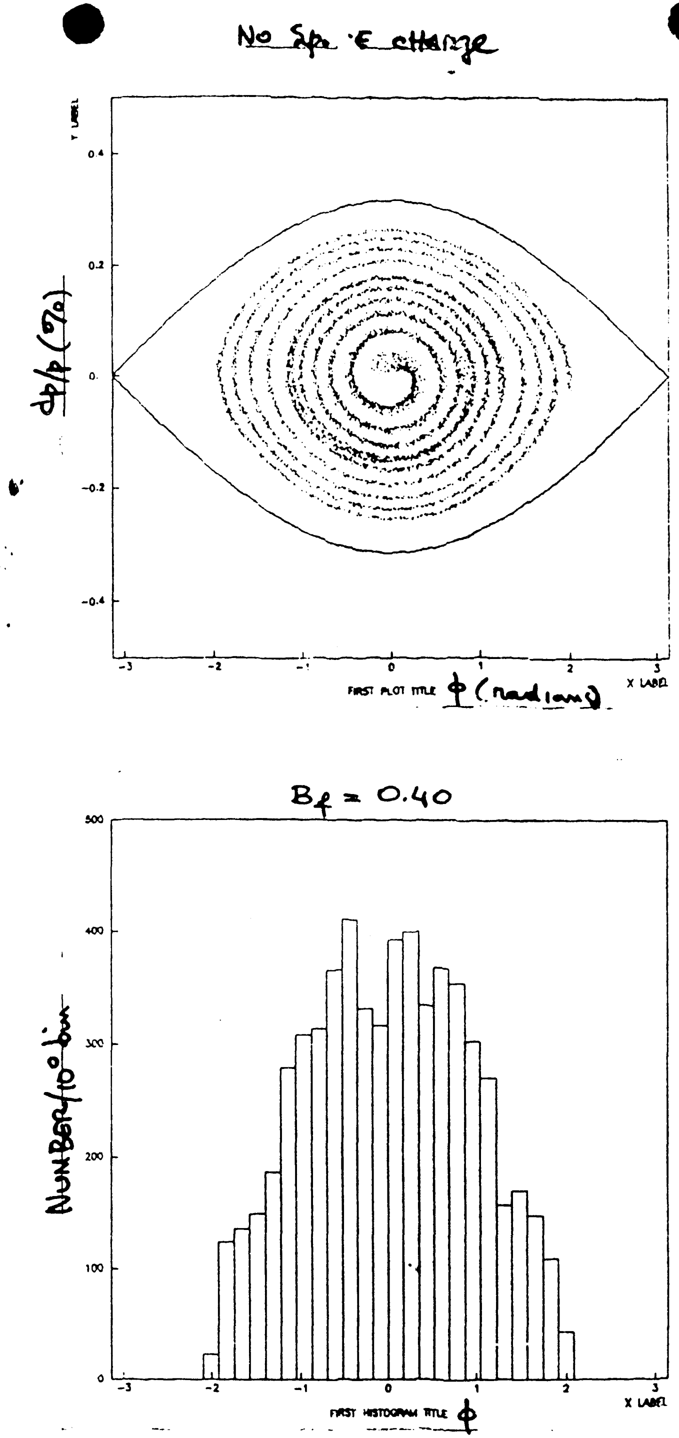

Fig. 8a

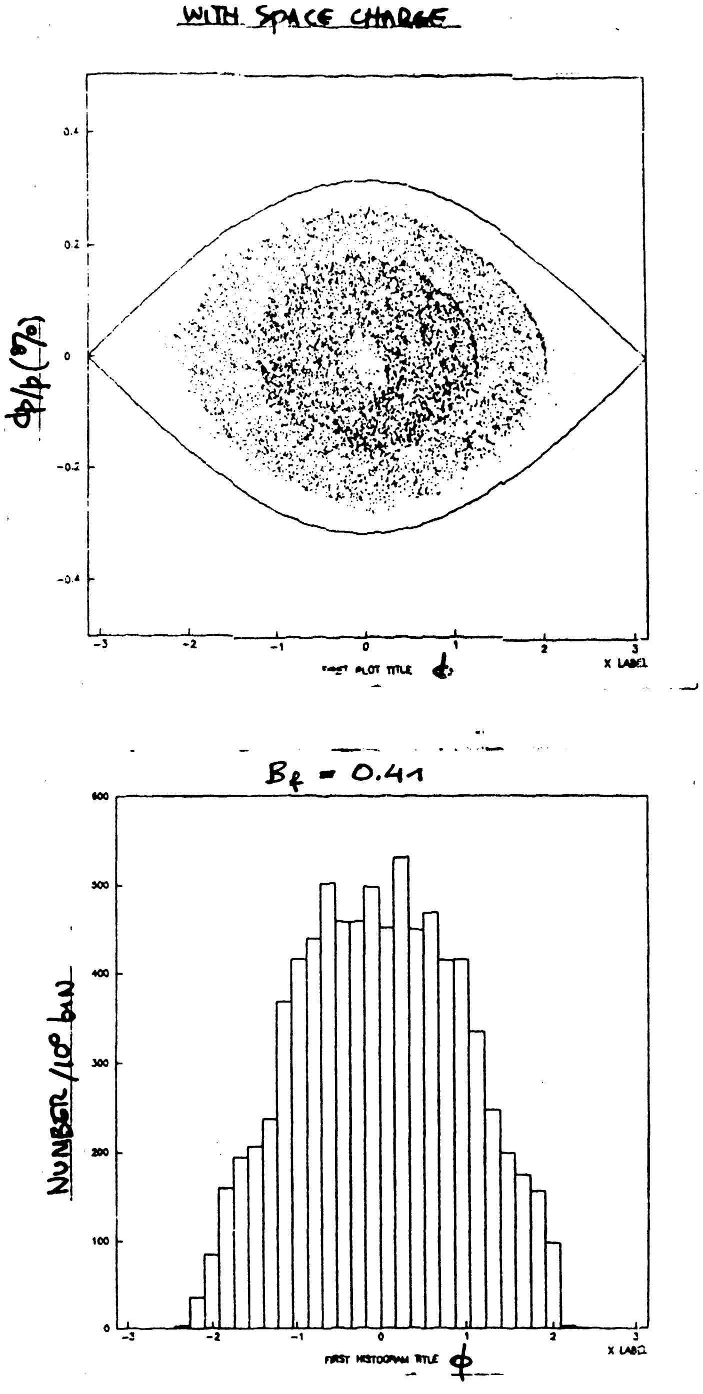

Fig. 8b

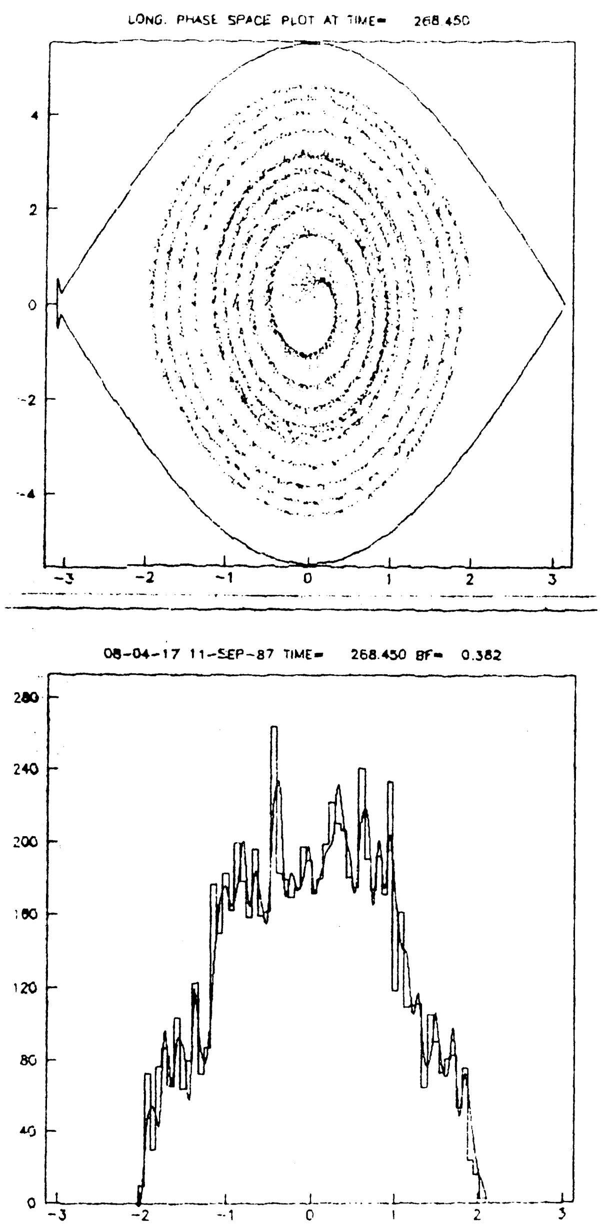

Fig. 9a

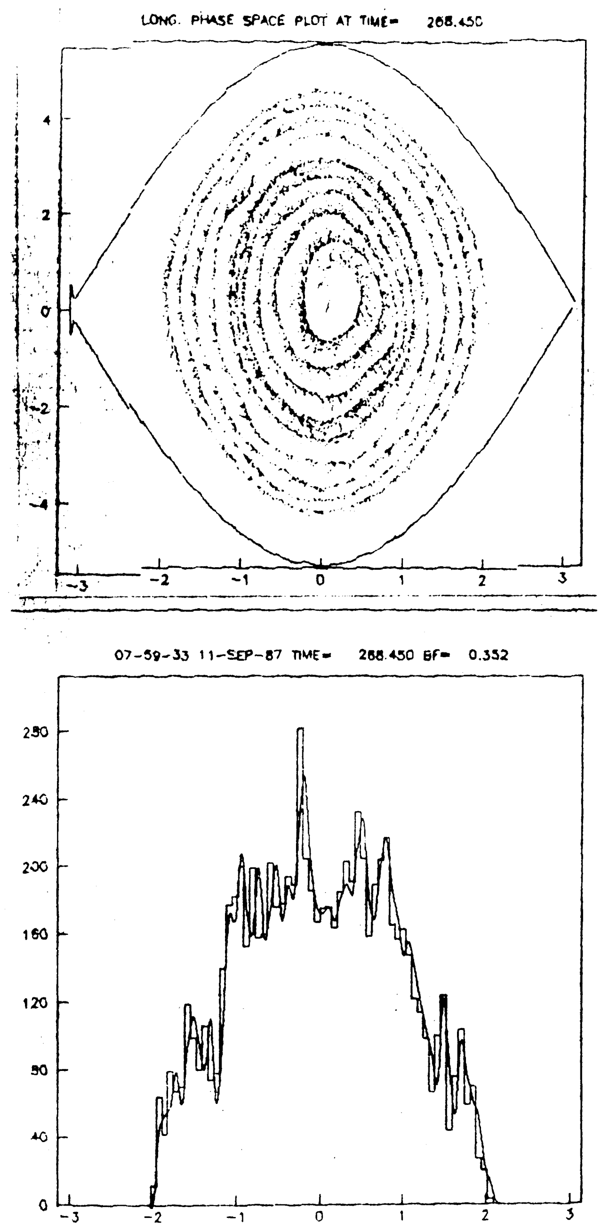

Fig. 9b

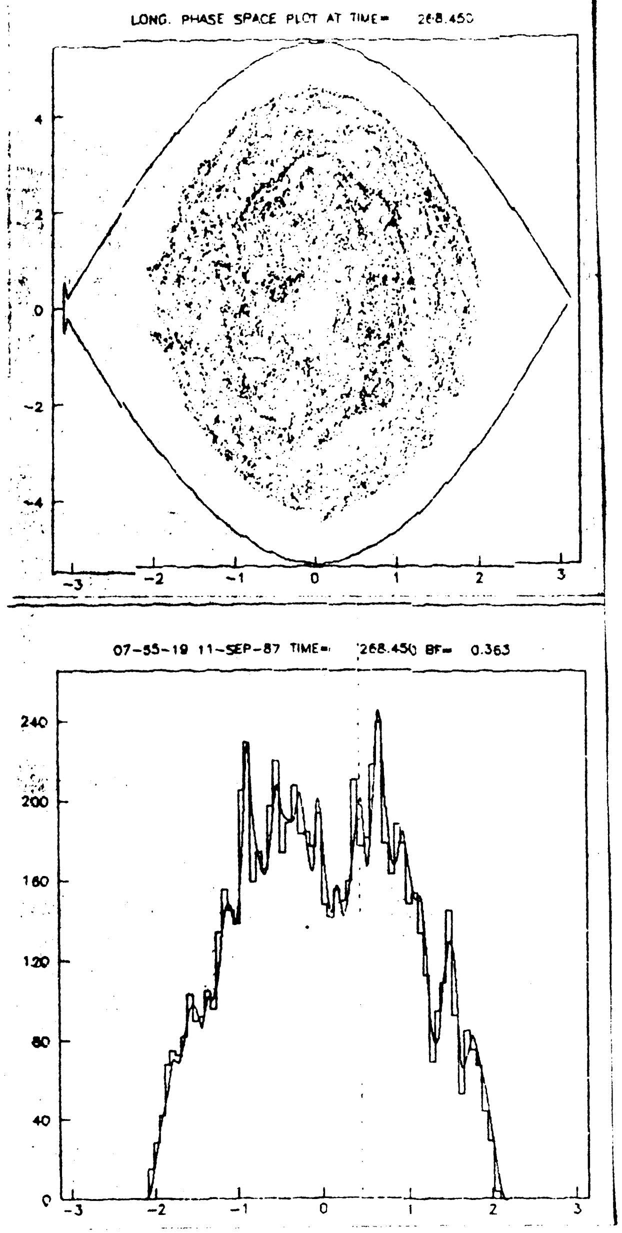

Fig. 9c

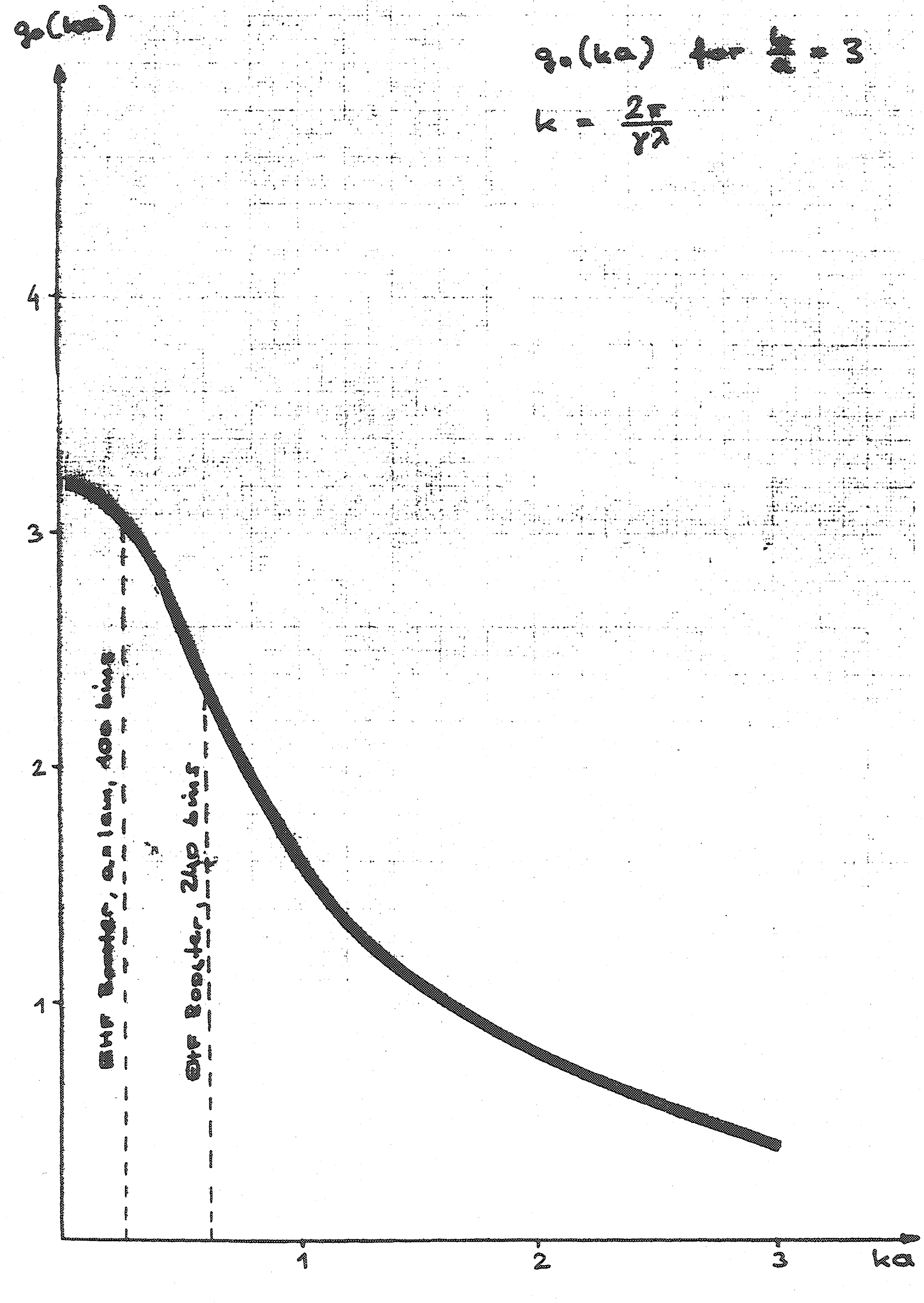

Fig. 10