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Comparison of Exact Results for the Virtual Corrections to Bremsstrahlung in e^+e^- Annihilation at High Energies†

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Abstract

We have compared the virtual corrections to $e^+e^- \to f\overline{f} + \gamma$ as calculated by S. Jadach, M. Melles, B.F.L. Ward and S.A. Yost to several other expressions. The most recent of these comparisons is to the leptonic tensor calculated by J.H. Kühn and G. Rodrigo for radiative return. Agreement is found to within 10^{-5} or better, as a fraction of the Born cross section.

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High precision studies of the Standard Model at proposed linear colliders will require per-mil level control of both the theoretical and experimental uncertainties in many critical processes to be measured. One important contribution is the virtual photon correction to the single hard bremsstrahlung in e^+e^- annihilations [1–4]. Relevant Feynman diagrams are shown in Fig. 1.

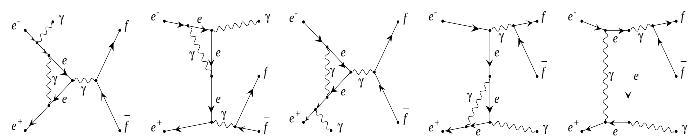

Figure 1: Feynman diagrams for the virtual $\mathcal{O}(\alpha^2)$ correction to the process $e^+e^- \to f\overline{f} + \gamma$.

The $\mathcal{O}(\alpha^2)$ virtual correction to single hard bremsstrahlung can be expressed in terms of a form factor multiplying the $\mathcal{O}(\alpha)$ tree level matrix element [3]:

$$\mathcal{M}_1^{\mathrm{ISR}(1)} = \frac{\alpha}{4\pi}(f_0 + f_1 I_1 + f_2 I_2)\mathcal{M}_1^{\mathrm{ISR}(0)} \tag{1}$$

where $\mathcal{M}_1^{\mathrm{ISR}(0)}$ is the tree-level hard bremsstrahlung matrix element, $\mathcal{M}_1^{\mathrm{ISR}(1)}$ includes an additional virtual photon, and (without mass corrections)

$$\begin{aligned} f_0 &= 2\left\{\ln\left(\frac{s}{m_e^2}\right) - 1 - i\pi\right\} + \frac{r_2}{1-r_2} + \frac{r_2(2+r_1)}{(1-r_1)(1-r_2)}\left\{\ln\left(\frac{r_2}{z}\right) + i\pi\right\} \\ &- \left\{3v + \frac{2r_2}{1-r_2}\right\}\mathrm{Lf}_1(-v) + \frac{v}{(1-r_2)}\,R_1(r_1,r_2) + r_2\,R_1(r_2,r_1), \end{aligned} \tag{2}$$

$$\begin{aligned} f_1 &= \frac{r_1 - r_2}{2(1-r_1)(1-r_2)} + \frac{z(1+z)}{2(1-r_1)^2(1-r_2)}\left\{\ln\left(\frac{r_2}{z}\right) + i\pi\right\} \\ &+ \frac{z}{1-r_2}\left\{\frac{1}{2}R_1(r_1,r_2) + r_2 R_2(r_1,r_2)\right\} \\ &+ \frac{v}{4}\left\{R_1(r_1,r_2)\delta_{\sigma,1} + R_1(r_2,r_1)\delta_{\sigma,-1}\right\}, \end{aligned} \tag{3}$$

$$\begin{aligned} f_2 &= 2 - \frac{1+z}{2(1-r_1)(1-r_2)} + \frac{z(r_2-r_1)}{2(1-r_1)^2(1-r_2)}\left\{\ln\left(\frac{r_2}{z}\right) + i\pi\right\} \\ &+ 2z\,\mathrm{Lf}_2(-v) + \frac{z}{1-r_2}\left\{\frac{1}{2}R_1(r_1,r_2) + (2-r_2)R_2(r_1,r_2)\right\} \\ &+ \frac{r_1-r_2}{4}\left\{R_1(r_1,r_2)\delta_{\sigma,1} + R_1(r_2,r_1)\delta_{\sigma,-1}\right\} \end{aligned} \tag{4}$$

for $\sigma = \lambda_1$, with $r_i = 2p_i \cdot k/s$ for momenta p_1, p_2 of the incoming e^-, e^+, $v = r_1 + r_2$ is the fraction of the beam energy radiated into the hard photon, $z = 1 - v$, and real photon helicity σ. When $\sigma = -\lambda_1$, r_1 and r_2 must be interchanged in eq. (2) – (4). In addition, we will let p_3, p_4 label the outgoing f, $\overline{f}$ momenta, and λ_i label the helicity of a fermion with momentum p_i. The standard YFS soft virtual photon term $4\pi B_{\rm YFS}$ has been subtracted from f_0. We make use of functions

$$
\begin{aligned}
R_1(x,y) &= \mathrm{Lf}_1(-x)\left\{\ln\left(\frac{1-x}{y^2}\right) - 2\pi i\right\} \\
&+ \frac{2(1-x-y)}{1-x}\,\mathrm{Sf}_1\left(\frac{y}{1-x}, \frac{x(1-x-y)}{1-x}\right), \qquad (5) \\
R_2(x,y) &= 1-x-y+\frac{1}{1-x}\left\{\ln\left(\frac{y}{1-x-y}\right)+i\pi\right\} \\
&+ \mathrm{Lf}_2(-x)(\ln y + i\pi) - (1-x-y)\,\mathrm{Lf}_1(-x) - \frac{1}{2}\,\mathrm{Lf}_1^2(-x) \\
&+ \frac{1-x-y}{(x+y)(1-x)}\left\{x\,\mathrm{Lf}_1\left(\frac{-y}{1-x}\right) - y\,\mathrm{Lf}_2\left(\frac{-y}{1-x}\right)\right\} \\
&+ \left(\frac{1-x-y}{1-x}\right)^2 \mathrm{Sf}_2\left(\frac{y}{1-x}, \frac{x(1-x-y)}{1-x}\right), \qquad (6)
\end{aligned}
$$

and $\mathrm{Lf}_n(x)$, $\mathrm{Sf}_n(x,y)$ defined recursively by

$$\mathrm{Lf}_0(x) = \ln(1+x), \qquad \mathrm{Lf}_{n+1}(x) = \frac{1}{x}\left(\mathrm{Lf}_n(x) - \mathrm{Lf}_n(0)\right), \qquad (7)$$

$$\mathrm{Sf}_0(x,y) = \mathrm{Sp}(x+y), \qquad \mathrm{Sf}_{n+1}(x,y) = \frac{1}{y}\left(\mathrm{Sf}_n(x,y) - \mathrm{Sf}_n(x,0)\right). \qquad (8)$$

with $\mathrm{Sp}(x)$ the Spence dilogarithm function. Only the f_0 term contributes to NLL order. The f_1 and f_2 terms contain spinor coefficients

$$
\begin{aligned}
I_1 &= \sigma\lambda_3 s_{\lambda_1}(p_1,k)s_{-\lambda_1}(p_2,k) \\
&\quad\times\frac{s_{\lambda_3}(p_4,p_2)s_{-\lambda_3}(p_2,p_3) - s_{\lambda_3}(p_4,p_1)s_{-\lambda_3}(p_1,p_3)}{s_{-\sigma}(p_1,p_2)s_{-\sigma}(p_3,p_4)s_\sigma^2(p_{21},p_{34})}, \qquad (9) \\
I_2 &= \lambda_1\lambda_3\frac{s_{\lambda_1}(p_1,k)s_{-\lambda_1}(p_2,k)s_{\lambda_3}(p_4,k)s_{-\lambda_3}(p_3,k)}{s_{-\sigma}(p_1,p_2)s_{-\sigma}(p_3,p_4)s_\sigma^2(p_{21},p_{34})}, \qquad (10)
\end{aligned}
$$

where the spinor product is $s_\lambda(p,q) = \bar{u}_{-\lambda}(p)u_\lambda(q)$, and $p_{ij} = p_i$ or p_j when $\sigma = \lambda_i$ or λ_j. The expressions f_i are equivalent to those in Ref. [3], but with improved numerical stability in the collinear limits, while the spinor terms I_i correct misprints in the versions in Ref. [3]. Mass corrections are added following the method of Ref. [5], and we confirmed [3] that all significant mass corrections are included in this manner.

Fig. 2 shows a comparison of four expressions for the sub-NLL virtual photon contribution to the $\bar{\beta}_1^{(2)}$ distribution at a CMS energy of 200 GeV, with $f\overline{f} = \mu^-\mu^+$. The NLL contribution calculated in Ref. [3] has been subtracted in each case. The figure compares our exact result JMWY in Ref. [3], the result IN of Ref. [2], the result BVNB of Ref. [1], the new result KR of Ref. [4]. The first two comparisons were included in Ref. [3], where good agreement was found. In fact, both expressions were shown to be analytically identical to ours at NLL order. However, neither of the comparisons in Refs. [1, 2] is fully differential with mass corrections. The result of Ref. [4] is the only comparison which is fully differential and includes mass corrections, allowing a complete test of the sub-NLL terms in eq. (1).

All of the results agree to within 0.4×10^{-5} for cuts below $v_{\rm max} = 0.75$. For cuts between 0.75 and .95, the results agree to within 0.5×10^{-5}, except for the result of Ref. [1]. These results are consistent with a total precision tag of 1.5×10^{-5} for our $\mathcal{O}(\alpha^2)$ correction $\bar{\beta}_1^{(2)}$ for an energy cut below $v_{\rm max} = 0.95$. The NLL effect, which has been implemented in the $\mathcal{KK}$ MC [6], is adequate alone to within 1.5×10^{-5} for cuts below 0.95. More details on the comparisons can be found in Ref. [7].

These comparisons show that we now have a firm handle on the precision tag for an important part of the complete $\mathcal{O}(\alpha^2)$ corrections to the $f\overline{f}$ production process needed for precision studies of such processes in the final LEP2 data analysis, in the radiative return at Φ and B-Factories, and in the future TESLA/LC physics.

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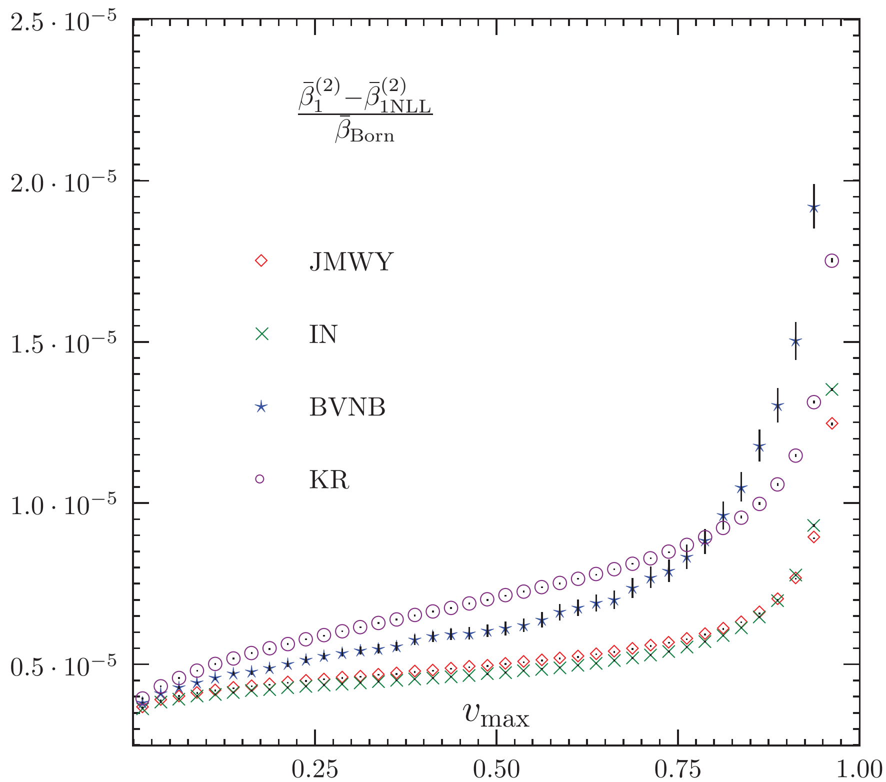

Figure 2: Comparisons of NNLL results in a $\mathcal{KK}$ MC run of 10^8 events.