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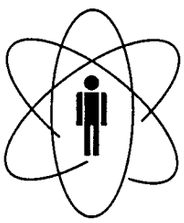

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Electronic bound states in parity-preserving QED_3 applied to high-T_c cuprate superconductors

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We consider a parity-preserving QED_3 model with spontaneous breaking of the gauge symmetry as a framework for the evaluation of the electron-electron interaction potential underlying high-T_c superconductivity. The fact that the resulting potential, $-C_s K_0(Mr)$, is non-confining and "weak" (in the sense of Kato) strongly suggests the mechanism of pair-condensation. This potential, compatible with an s-wave order parameter, is then applied to the Schrödinger equation for the sake of numerical calculations, thereby enforcing the existence of bound states. The results worked out by means of our theoretical framework are checked by considering a number of phenomenological data extracted from different copper oxide superconductors. The agreement may motivate a deeper analysis of our model viewing an application to quasi-planar cuprate superconductors. The data analyzed here suggest an energy scale of 1-10meV for the breaking of the $U(1)$-symmetry.

Key-words: Pair condensation; QED_3; Parity-preserving; Superconductivity.

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I. INTRODUCTION

The planar QED (QED_3) has shown to be an appropriate theoretical framework for discussing issues of contemporary physics, particularly in connection with Condensed Matter Physics. In the latest years, the raising interest in applications of this theory to high-T_c superconductivity and quantum Hall effect [2] has motivated an enormous production of works in this subject. The relation between QED_3 and superconductivity, phenomenon discovered in 1986 [1], can be traced back to 1987, when Anderson [3] suggested that in some copper oxide superconductors (based on La_2CuO_4) the hypothesized resonant-valence-bond state or quantum-spin-liquid state of Mott (a kind of insulator) could migrate to a superconducting state through a doping process. Soon after, in 1988, Laughlin [4] argued that the excitations of the Anderson resonating-valence-bond model behaved like fractional quantum Hall states (anyons), presenting consequently a fractional or anyonic statistics. Despite the initial success of this model, several difficulties with this idea arose. The main problem concerns the necessity of a massless scalar mode in the spectrum which occurs only when the bare Chern-Simons term cancels with the term generated by one-loop radiative corrections. This cancellation occurs exactly at zero temperature, but does not take place at finite temperature. In this way, one can assert that the anyonic model behaves like a superconductor only at zero temperature [5].

At the same time that the anyonic model was developed, a new approach based upon the QED_3 theoretical framework [6] began to be adopted to explain the formation of electron-electron bound states, provided that the high-T_c superconductors had quasi-planar structure. In the domain of QED_3, there arises the necessity of providing the gauge field with a mass in order to circumvent the appearance of a confining potential associated to the long-range Coulomb interaction. The Maxwell-Chern-Simons model is then adopted so as to generate (topological) mass for the photon, leading to a finite range interaction, to which a binding potential is associated instead of a confining one. In the framework of a Maxwell-Chern-Simons theory, numerical evaluation of electron-electron bound states were first addressed to in Ref. [8], but the assumptions and results of the latter induced some controversy [9,10]. Other authors [11], working in this same context, have also obtained bound states, corresponding to the situation where the magnetic-dipole interaction between the electrons is large enough to overcome the Coulombian repulsion. In this case, however, the attractive interaction only appears when the topological mass of the gauge field is larger than the electron mass ($\kappa > m$). This condition prevents the application of this model to Condensed Matter systems, where one must have $\kappa \ll m$ due to the order of magnitude ($\sim$ meV) of the usual relevant excitations. An attempt to bypass this difficulty consists in considering a Maxwell-Chern-Simons model minimally coupled to fermions and bosons with spontaneous breaking of a local $U(1)$-symmetry as a generating mechanism for the photon mass [12], whose results show the possibility of obtaining bound states whenever the attractive Higgs interaction dominates over the gauge boson interchange. This issue is now under investigation [13], which would be suitable to apply such a model to the cases where there is an evidence of a parity-breaking superconductivity quasi-planar phenomenon [14]. Very recently, has been proposed in Ref. [15], an anisotropic $U(1) \times U(1)$ QED_3 model coupled to both the "Berry" and "Doppler" gauge fields, by arguing that the pseudogap regime in cuprates could be modeled as a phase disordered d-wave superconductor.

In the present work we consider a parity-preserving QED_3 model with spontaneous breaking of the lo-

cal $U(1)$-symmetry accomplished by a sixth-power potential [17]. Our aim here is to carry out numerical calculations in searching for electron-electron bound states, in such a theoretical framework. The breaking mechanism of $U(1)$-symmetry gives rise to a Higgs-type boson and a massive photon avoiding the appearance of a confining logarithmic potential (characteristic of massless interactions in three space-time dimensions). Hence, the Higgs mechanism has the relevant role of contributing to the electron-electron binding while yielding a non-confining potential.

Thereafter, the consideration of the Møller scattering mediated by both the vector and scalar bosons results in the establishment of an attractive electron-electron potential, independent of the spin polarization state. The potential stemming from the Møller scattering corresponds to a modified Bessel function of zeroth order, $-C_s K_0(Mr)$, that besides being non-confining, assures the semi-boundedness of the system (the so-called weak Kato condition). Once we have proven that the K_0-type potential satisfies the necessary conditions to allow the existence of bound states, a numerical calculation of the ground state energy of the Schrödinger equation is carried out. Incidentally, by virtue of the radial symmetry of the potential, we are bound to only consider the s-wave solutions. An application of these numerical calculations to high-T_c superconductivity is then implemented by fitting some phenomenological data available for the following cuprate superconducting materials: $YBa_2Cu_3O_7$, $Tl_2Ba_2CaCu_2O_{10}$, $Bi_2Sr_2CaCu_2O_8$ and $HgBa_2Ca_2Cu_3O_8$. Our procedure reveals to be successful in the sense that it is always possible to fit the energy gap energy and the correlation length of the samples through the indication of a specific scalar vacuum expectation value (v.e.v.).

The outline of this paper is the following. In Section II, we present the model. Next, in Section III, we address the relevant Schrödinger equation taking into account the properties of the condensate wave-function. Some aspects of the trial function are discussed so that it turns to be suitable to the variational method. In Section IV, we digress on some aspects of copper oxide superconductors, as the order parameter, the pairing mechanism and the effective coupling constant. Section V is devoted to the identification of free pure theoretical parameters with phenomenological ones, and finally, in Section VI, we perform a numerical calculation of the energy gap and correlation length for four cuprate high-T_c superconducting samples.

II. BRIEF SURVEY ON THE PARITY-PRESERVING QED$_3$

The action for the parity-preserving QED$_3$** with spontaneous symmetry breaking of a local $U(1)$-symmetry is given by [16–18]:

$$S_{\rm QED} = \int d^3x \Big\{ -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\overline{\psi}_+ \not{D}\psi_+ + i\overline{\psi}_- \not{D}\psi_- - m_e(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-) - y(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)\varphi^*\varphi + \\ + D^\mu\varphi^* D_\mu\varphi - V(\varphi^*\varphi) \Big\} , \qquad (1)$$

with the potential $V(\varphi^*\varphi)$ taken as

**The metric is given by $\eta_{\mu\nu} = (+,-,-)$; μ,ν=(0,1,2) and the γ-matrices are taken as $\gamma^\mu = (\sigma_x, i\sigma_y, -i\sigma_z)$.

$$V(\varphi^*\varphi) = \mu^2\varphi^*\varphi + \frac{\zeta}{2}(\varphi^*\varphi)^2 + \frac{\lambda}{3}(\varphi^*\varphi)^3 \ , \tag{2}$$

where the mass dimensions of the parameters μ, ζ, λ and y are respectively 1, 1, 0 and 0. The sixth-power potential, $V(\varphi^*\varphi)$, is the responsible for breaking the electromagnetic $U(1)$-symmetry.

The covariant derivatives are defined as follows:

$$\not{D}\psi_\pm \equiv (\partial + ie\not{A})\psi_\pm \quad \text{and} \quad D_\mu\varphi \equiv (\partial_\mu + ieA_\mu)\varphi \ , \tag{3}$$

where e is a coupling constant with dimension of $(\text{mass})^{\frac{1}{2}}$. In the action (1), $F_{\mu\nu}$ is the usual field strength for A_μ, ψ_+ and ψ_- are two kinds of fermions (the $\pm$ subscripts refer to their spin sign [16,17,20]) and φ is a complex scalar. The $U(1)$-symmetry gauged by A_μ is interpreted as the electromagnetic one, so that A_μ is meant to describe the photon.

The action given by Eq.(1) is invariant under the discrete symmetry, P, whose action is fixed below:

$$\begin{aligned}
&x_\mu \xrightarrow{P} x_\mu^P = (x_0, -x_1, x_2), \\
&\psi_\pm \xrightarrow{P} \psi_\pm^P = -i\gamma^1\psi_\mp \ , \ \overline{\psi}_\pm \xrightarrow{P} \overline{\psi}_\pm^P = i\overline{\psi}_\mp\gamma^1 \ , \\
&A_\mu \xrightarrow{P} A_\mu^P = (A_0, -A_1, A_2) \ , \\
&\varphi \xrightarrow{P} \varphi^P = \varphi \ .
\end{aligned} \tag{4}$$

Analyzing the potential (2), and imposing that it is bounded from below and yields only stable vacua (metastability is ruled out), the following conditions on the parameters μ, ζ, λ must be set:

$$\lambda > 0 \ , \ \zeta < 0 \quad \text{and} \quad \mu^2 \leq \frac{3}{16}\frac{\zeta^2}{\lambda} \ . \tag{5}$$

We denote $\langle\varphi\rangle = v$ and the v.e.v. for the $\varphi^*\varphi$-product, v^2, is chosen as

$$\langle\varphi^*\varphi\rangle = v^2 = -\frac{\zeta}{2\lambda} + \left[\left(\frac{\zeta}{2\lambda}\right)^2 - \frac{\mu^2}{\lambda}\right]^{\frac{1}{2}} \ , \tag{6}$$

the condition for minimum being read as

$$\mu^2 + \zeta v^2 + \lambda v^4 = 0 \ . \tag{7}$$

The complex scalar, φ, is parametrized by

$$\varphi = v + H + i\theta \ , \tag{8}$$

where θ is the would-be Goldstone boson and H is the Higgs scalar, both with vanishing v.e.v.'s.

By replacing the parametrization (8) for the complex scalar, φ, into the action (1), the following free action comes out:

$$\begin{aligned}
S_{\text{QED}}^{\text{free}} = \int d^3x \Big\{ &-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}M_A^2 A^\mu A_\mu + \overline{\psi}_+(i\partial - m_{\text{eff}})\psi_+ + \overline{\psi}_-(i\partial + m_{\text{eff}})\psi_- + \\
&+ \partial^\mu H\partial_\mu H - M_H^2 H^2 + \partial^\mu\theta\partial_\mu\theta + 2veA^\mu\partial_\mu\theta \Big\} \ ,
\end{aligned} \tag{9}$$

where the parameters M_A^2, m_{eff} and M_H^2 are given by

$$M_A^2 = 2v^2e^2 \ , \ m_{\rm eff} = m_e + yv^2 \quad \text{and} \quad M_H^2 = 2v^2(\zeta + 2\lambda v^2) \ . \tag{10}$$

The conditions (5) and (7) imply the following lower-bound (see Eq.(10)) for the Higgs mass:

$$M_H^2 \geq \frac{3}{4}\frac{\zeta^2}{\lambda} \ . \tag{11}$$

Therefore, a *massless* Higgs is out of the model we consider here, it would be present in the spectrum if $\mu^2 > 3\zeta^2/16\lambda$. However, in such a situation, the minima realizing the spontaneous symmetry breaking would not be absolute ones, corresponding therefore to metastable ground states, that we avoid here. One-particle states would decay with a short decay-rate if compared to an absolute minimum ground state.

In order to preserve the manifest renormalizability of the model, the 't Hooft gauge [21] is adopted:

$$\hat{S}^{\rm gf}_{R_\xi} = \int d^3x \Big\{ -\frac{1}{2\xi}\left(\partial^\mu A_\mu - \sqrt{2}\xi M_A \theta\right)^2 \Big\} \ , \tag{12}$$

where ξ is a dimensionless gauge parameter.

By replacing the parametrization (8) into the action (1), and adding up the 't Hooft gauge (12), it can be directly found the following complete parity-preserving action:

$$\begin{aligned} S^{\rm SSB}_{\rm QED} = \int d^3x \Big\{ & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}M_A^2 A^\mu A_\mu + \overline{\psi}_+(i\partial\!\!\!/ - m_{\rm eff})\psi_+ + \overline{\psi}_-(i\partial\!\!\!/ + m_{\rm eff})\psi_- + \\ & + \partial^\mu H \partial_\mu H - M_H^2 H^2 + \partial^\mu\theta\partial_\mu\theta - M_\theta^2\theta^2 - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 + \\ & - e\overline{\psi}_+ A\!\!\!/\psi_+ - e\overline{\psi}_- A\!\!\!/\psi_- - y(\overline{\psi}_+\psi_+ - \overline{\psi}_-\psi_-)(2vH + H^2 + \theta^2) + \\ & + e^2 A^\mu A_\mu(2vH + H^2 + \theta^2) + 2eA^\mu(H\partial_\mu\theta - \theta\partial_\mu H) + \\ & - c_3H^3 - c_4H^4 - c_5H^5 - c_6H^6 - c_7\theta^4 - c_8\theta^6 - c_9H\theta^2 - c_{10}H^2\theta^2 + \\ & - c_{11}H^3\theta^2 - c_{12}H^4\theta^2 - c_{13}H\theta^4 - c_{14}H^2\theta^4 \Big\} \ , \end{aligned} \tag{13}$$

where the constants M_θ^2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}, c_{13} and c_{14} are defined by

$$\begin{aligned} & M_\theta^2 = \xi M_A^2 \ , \ c_3 = 2v(\zeta + \frac{10}{3}\lambda v^2) \ , \ c_4 = \frac{\zeta}{2} + 5\lambda v^2 \ , \ c_5 = 2\lambda v \ , \\ & c_6 = \frac{\lambda}{3} \ , \ c_7 = \frac{\zeta}{2} + \lambda v^2 \ , \ c_8 = \frac{\lambda}{3} \ , \ c_9 = 2v(\zeta + 2\lambda v^2) \ , \\ & c_{10} = \zeta + 6\lambda v^2 \ , \ c_{11} = 4\lambda v \ , \ c_{12} = \lambda \ , \ c_{13} = 2\lambda v \quad \text{and} \quad c_{14} = \lambda \ . \end{aligned} \tag{14}$$

Working in the 't Hooft gauge, one evaluates, in the non-relativistic limit, the Møller scattering potential mediated by the Higgs quasi-particle and the photon in the center-of-mass frame. In the Born approximation the potential is nothing but the Fourier transform of the total Møller scattering amplitude, yielding, therefore, to the following the electron-electron scattering potential:

$$V(r) = -\frac{1}{2\pi}\left[2y^2v^2K_0(M_H r) - e^2K_0(M_A r)\right] \ . \tag{15}$$

However, the potential thus obtained is attractive provided the attraction caused by the Higgs quasi-particle mediation overcomes the repulsion resulted from the photon mediation. In fact, as one shall present later, the quantity, $C_s = 2y^2v^2 - e^2$, varies from $3,01$ to $7,22$meV for the copper oxides superconductors analyzed here.

III. THE COMPOSITE WAVE-FUNCTION AND THE SCHRÖDINGER EQUATION

Before studying the Schrödinger equation, it is instructive to analyze the behavior of the total wave-function (Ψ) of a two-electron system in light of the Pauli exclusion principle. By exchanging both fermions, one knows that Ψ must be antisymmetric with respect to a permutation between them

$$\Psi(\rho_1, s_1, \rho_2, s_2) = -\Psi(\rho_2, s_2, \rho_1, s_1). \tag{16}$$

Assuming that no significant spin-orbit interaction takes place, this function can be written in terms of the following three independent functions:

$$\Psi(\rho_1, s_1, \rho_2, s_2) = \psi(\mathbf{R})\varphi(\mathbf{r}_1, \mathbf{r}_2)\chi(s_1, s_2) \ , \tag{17}$$

which represent, respectively, the center-of-mass wave function, the relative one, and the spin wave function ($\mathbf{R}$ and s being the center of mass and spin coordinates respectively, while $\mathbf{r}_1$ and $\mathbf{r}_2$ are the electrons coordinates relative to $\mathbf{R}$).

Each of these functions contain information on the mechanism underlying superconductivity. Flux quantization results from the boundary conditions on ψ, and from this it can be deduced that the charge carriers are pairs of particles [19]. The other two functions tell us about other features of the condensates. For instance, the radial component of φ has information on the spatial extent of the pairs, and the rest of the factors determine whether they are in an s, p or d state, or even whether the system is in a singlet or triplet spin-state.

For the spin singlet ($S = 0$), the spin-function, χ, is antisymmetric, while for the spin triplet ($S = 1$) it is symmetric. Consequently, the space-function $\varphi(r)$ associated with a spin triplet must be even, and the one associated with a spin singlet must be odd:

$$\begin{aligned} \Psi^{S=1} &= \varphi_{\text{odd}}(\mathbf{r}_1, \mathbf{r}_2)\chi^{S=1}_{\text{even}}(s_1, s_2) \ , \\ \Psi^{S=0} &= \varphi_{\text{even}}(\mathbf{r}_1, \mathbf{r}_2)\chi^{S=0}_{\text{odd}}(s_1, s_2) \ . \end{aligned} \tag{18}$$

Thus, by only considering the Pauli exclusion principle, one concludes that the total wave-function will be composed by an even angular momentum state (s-wave, d-wave) and a s-spin state, or by an odd angular momentum state (p-wave, f-wave) and a p-spin state.

Consider now the planar Schrödinger equation for the relative wave-function, $\varphi(r)$, representing an electron-electron system, with relative radial coordinate r:

$$\frac{\partial^2\varphi(r)}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi(r)}{\partial r} - \frac{l^2}{r^2}\varphi(r) + 2\mu_{\text{eff}}[E - v(r)]\varphi(r) = 0 \ , \tag{19}$$

where $V(r)$ represents the interaction potential between the two electrons, Eq.(15), and μ_{eff} the effective reduced mass of the system

$$\mu_{\text{eff}} = \frac{1}{2}(m_e + yv^2) \ . \tag{20}$$

By means of the following transformation:

$$\varphi(r) = \frac{1}{\sqrt{r}}\, g(r) \ , \tag{21}$$

one has

$$\frac{\partial^2 g(r)}{\partial r^2} - \frac{l^2 - \frac{1}{4}}{r^2} g(r) + 2\mu_{\text{eff}}[E - V(r)]g(r) = 0 \ . \tag{22}$$

Looking at this equation, it is easy to identify the effective potential for the electron-electron system as:

$$V_{\text{eff}}(r) = \frac{l^2 - \frac{1}{4}}{2\mu_{\text{eff}} r^2} - C_s K_0(M_H r) \ , \ C_s = \frac{1}{2\pi}(2y^2 v^2 - e^2) \ , \tag{23}$$

where, for the sake of simplicity, we considered equal masses for the scalar and vector excitations in order to check the possibility of bound states in the model. However, we should stress that the assumption of equal masses might lead to conflicts with phenomenological data. Later, as we shall discuss, this will not be the case; in any case, it is a sensible attitude to reassess our results by taking $M_H \neq M_A$.

A. The choice of the trial function

The variational method is used for the approximate determination of the ground state energy level, and consists in determining the wave-function $\varphi(r)$ that provides the largest (absolute) binding energy value. This method is applied mainly in situations where the wave function for the system is unknown, depending on the choice of a trial function. The closer the trial function is to the true solution of the system, the better the energy value numerically obtained will be. The definition of a trial function must observe some conditions, such as the asymptotic behavior at infinity, the analysis of its free version and its behavior at the origin. For a zero angular momentum ($l = 0$) state, Eq.(22) becomes

$$\left\{ \frac{\partial^2}{\partial r^2} + \frac{1}{4r^2} + 2\mu_{\text{eff}}[E + C_s K_0(M_H r)] \right\} g(r) = 0 \ . \tag{24}$$

The free version, $V(r) = 0$, of the last equation is given merely by

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{4r^2} + k^2 \right] u(r) = 0 \ , \tag{25}$$

whose solution is

$$u(r) = B_1 \sqrt{r} J_0(kr) + B_2 \sqrt{r} Y_0(kr) \ , \tag{26}$$

where B_1 and B_2 are arbitrary constants and $k = \sqrt{2\mu_{\text{eff}} E}$. In the limit $r \to 0$, Eq.(26) goes simply as

$$u(r) \longrightarrow \sqrt{r} + \lambda \sqrt{r} \ln(r) \ . \tag{27}$$

Since the second term in Eq.(25) behaves like an attractive potential, $-1/4r^2$, this implies the possibility of obtaining a bound state ($E < 0$) even for $V(r) = 0$ [22,23]. This is not physically acceptable, leading to a restriction on the needed self-adjoint extension of the differential operator $-d^2/dr^2 - 1/4r^2$. Among the infinite number of self-adjoint extensions of this symmetric operator, the only physical choice corresponds to the Friedrichs extension ($B_2 = 0$), which behaves like $\sqrt{r}$ at the origin. The choice of the Friedrich extension thus circumvents the non-physical possibility of a bound state solution for a free potential equation, and for this reason, it is the only acceptable one. For the complete equation, $V(r) \neq 0$, one must start from this particular extension of the free Hamiltonian and then add a potential. This does not

alter the self-adjointness, provided the potential is "weak" in the sense of Kato. The reason is that as the system is in the eminence of a bound state, adding any attractive potential to the Friedrich extension, no matter independently how weak it could be, provides at least one bound state [23]. The potential to be considered, must therefore preserve the self-adjointness of the differential operator, according with the following Kato condition:

$$\int_0^\infty r(1+|\ln(r)|)|V(r)|dr < \infty \ . \tag{28}$$

Provided the interaction potential, $V(r) = -C_s K_0(M_H r)$, satisfies the Kato condition, the self-adjointness of the total Hamiltonian is assured. The Kato condition is also decisive to establish a finite number of bound states (discrete spectrum) and the semi-boundness of the complete Hamiltonian. In conclusion, the physical asymptotic solution of Eq.(24) is given only by $\sqrt{r}$. In this way the behavior of the trial function at the origin is completely determined.

On the other hand, at infinity, the trial function must vanish asymptotically in order to fulfill square integrability. Therefore, a good and suitable trial function choice (for zero angular momentum) can then be given by

$$\varphi(r) = \sqrt{r}\exp(-\beta r) \ , \tag{29}$$

where β is a free parameter whose variation approximately determines an energy minimum.

An analogous procedure can be undertaken to determine the behavior of a trial function when the angular momentum is different from zero ($l \neq 0$). In this case, and in the limit $r \to 0$, Eq.(22) reduces to

$$\left[\frac{\partial^2}{\partial r^2} - \frac{l^2 - \frac{1}{4}}{r^2} + k^2\right] u(r) = 0 \ , \tag{30}$$

whose general solution reads

$$u(r) = B_1 r^{(l+1/2)} + B_2 r^{(-l+1/2)} \ . \tag{31}$$

For $l > 0$, the choice $r^{(l+1/2)}$ assures a trial function well-behaved at the origin. Since the Schrödinger equation depends only on l^2, any of the choices, $l > 0$ or $l < 0$, is enough for providing the energy values of the physical states and one gets

$$\varphi(r) = r^{1/2+l}\exp(-\beta r) \ , \tag{32}$$

where β again is a spanning parameter to be numerically fixed in order to maximize the binding energy. Though this last result is mathematically correct, we should point out that the discussion regarding non-zero angular momentum states here is merely for the sake of completeness. The true wave-function in this case should actually stem from the complete differential equation, for which one should include the angular components which remain precluded in this approach. We shall further comment about this question at the end of Section IV.

IV. DIGRESSION ON THE HIGH-T_C ORDER PARAMETER, PAIRING MECHANISM AND EFFECTIVE COUPLING CONSTANT

Experimental results have revealed that high-T_c superconductivity, as well as the BCS theory, are related to the existence of electron-electron bound states. Indeed, there are strong evidences that the charge carriers are pairs of electrons, for instance in the experiments of quantization of magnetic flux by Gough *et al.* [24] and the observation of voltage steps (Shapiro steps) in the current-voltage curves inside Josephson junctions by Niemeyer *et al.* [24]. These facts, among others, indicate that the order parameter of a reliable theory for high-T_c superconductivity must consist of a wave-function representing an electron pair. Now, there arise two fundamental questions: i) the determination of the type of wave-function-pairing (s-wave, as in the case of the usual BCS superconductors; p-wave, as it is observed in the superfluid state of ^{3}He; or d-wave, as in the case of the heavy-fermion superconductors); ii) the investigation of the physical mechanism underlying the electron-electron attraction, and its contribution to the effective coupling constant.

Regarding to the first question, i), we should emphasize that the type of order parameter constitutes a key question for the understanding of high-T_c superconductivity. In the latest 80's, consensus about the s-wave pairing was nearly established due to some pioneer experiments, *e.g.* Josephson tunneling in YBaCuO samples, the temperature dependence of the penetration depth, $\lambda(T)$, and observation of persistent supercurrents in rings [25]. The early Josephson experiments [26] were based on the conviction that the Josephson tunneling was not feasible between paired electrons in two different angular momentum states, unless dissipation occurred in the junction. Experiments with Y123 linked to Pb or Sn point contacts (ordinary BCS superconductors) reported no dissipation, so that Y123 was declared to be in s-pairing state. Despite the observations of $\lambda(T)$ indicating an s-wave order parameter for some planar superconductors [27], experimental verification of a linear behavior for $\lambda(T)$ was afterwards obtained by Hardy *et al.* [29], and theoretically predicted by Annet *et al.* [28]. Making use of the ARPES (angle resolved photoemission spectroscopy) technique, Shen *et al.* [30] reported on the observation of points of very small gap energy along the diagonal direction ($|K_x| = |K_y|$) for the BSCCO and YBCO samples, consistent with a d-wave-function pattern. Other experiments sensitive to phase changes of the order parameter, composed by DC SQUIDs [31], reiterated the $d_{x^2-y^2}$-wave-function model.

Nowadays, the status of the situation moved to a position midway between the two opposing results above discussed. Recently, a modern interpretation of a peculiar Josephson tunneling experiment [32], that measures the tunneling current along the c-axis, has shed light on a new reality concerning the structure of the order parameter. As a matter of fact, the outcomes obtained by Kouznetsov *et al.* [32] in 1997 showed up compatibility only with a *mixed* wave, composed by a d plus an s-wave component, as first noticed by Sun *et al.* [32]. Indeed, several very recent publications [33] have claimed on a s-wave pattern with admixture of d-wave, coming across as a new area of investigation. According to some of these studies, it is verified that the $d_{x^2-y^2}$ order parameter is dominant just for the higher temperatures while at lower ones the order parameter becomes more and more s-like, showing up a mixed symmetry. Actually, the above discussion concerns mainly the order parameter of the usual high-T_c compounds (hole-doped ones). In the case of the electron-doped cuprates, there are strong experimental evidences [34] supporting the conventional s-wave order parameter and suggesting a BCS-like behavior.

Now, regarding to the second question, ii), in the usual superconductors, the isotope effect ($T_c \sim M^{-\alpha}$, $\alpha = 0,5$) was decisive for the establishment of the BCS-theory, which successfully proposed the lattice vibrations (phonons) to explain the electron-electron attraction and a symmetric s-wave-function representing the electron-pair. Beyond the scope of the conventional superconductors, the manifestation of the isotope effect is a rather complex phenomenon dependent on other factors besides the lattice vibrations, as the presence of magnetic impurities. In this regard, the deviations from the BCS reference value ($\alpha \sim 0,5$) observed in many materials, including the high-T_c oxides, cannot be used unequivocally to rule out the phononic mechanisms from the set of the likely excitations that contribute effectively to the pairing [35]. Indeed, there exists the general assumption that the isotope effect and the phonon interaction should be ubiquitous in the cuprates, but not as the only mechanism yielding the pair condensation, which leads to the certainty that other mechanisms must coexist with the phonon one in order to assure the high values of the coupling constant and the large critical temperatures measured. The nature of these mechanisms has been an issue of intensive research, and despite the exhaustive efforts undertaken in this area, no consensus has yet been reached. Among the variety of approaches to this issue, one can mention some exotic attempts (non-phonon ones) [36] pointing to a non-symmetric solution, as the plasmon-wave excitations [37], the magnon interchange model [38], the spin fluctuation interchange model [39], the excitonic pairing model [40], the polarons and bipolarons mechanisms [41].

The fact that the ubiquitous electron-phonon interaction is disguised among other non-phonon mechanisms, creates an identification problem for the corresponding coupling constants. In Condensed Matter terminology, the electron-phonon coupling constant λ_{ep} reflects the effect of collective vibrations (phonons) of the whole lattice on each charge carrier. Besides the phononic interaction, one considers the existence of other mechanism, but up to now nobody knows to determine to which extent the electron-phonon contribution (and the non-phonon ones) participates in the effective electron-electron interaction. The quantification of the contributions of each interaction mechanism to the effective attraction (through the stipulation of values for the coupling constants) is a question that could be answered only if all the mechanisms were well-understood. While the answer is not clear, the option is to work with effective interactions and coupling constants. In this sense, the coupling constant of interest must be an effective one, able to account for the contributions of several similar interactions that, in the case of the present field-theoretic model, will have a scalar character.

According to the phenomenological picture above described, one should accept the evidences pointing to a mixed order parameter composed both by s- and d-waves in the case of hole-doped cuprates and probably pure s in the case of electron-doped materials. The present work will deal with the s-component in view of the microscopic field-theoretic scenario we set up. Our model relies on a mechanism of one-particle exchange (photon and Higgs quasi-particle) in the non-relativistic limit, to account for the attractive electron-electron potential. Should we relax the Born approximation and add up loop corrections to the tree-level amplitudes considered here, an anisotropic potential would come out, so that it could account also for the d-wave contribution as a result of 1-loop effects (this will be analyzed in a separate paper [42]). In any case, the present radial (isotropic) potential is entirely suitable for addressing pure s-wave type systems, as it is the case for the electron-doped high-T_c superconductors.

V. INTERPLAY BETWEEN HIGH-T_C PHENOMENOLOGY AND PLANAR QED

The evidence of a quasi-planar structure in high-T_c superconductivity is a suitable reason for adopting a planar QED model as a theoretical starting point. However, there arises the necessity of establishing a relationship between the parameters of the model and the experimental data for cuprate high-T_c superconductors. In the present parity-preserving action, there are some free parameters that could be identified with phenomenological observables which are of crucial importance to describe these materials. In Eq. (1), the electron-Higgs coupling, y, is an effective constant that embodies all possible mechanisms of interaction between electrons via Higgs-type excitations. As a result of the scalar character of this mediation, one encloses a large diversity of spinless bosonic interaction mechanisms; namely, the phonons, the plasmons [37], and other collective excitations. This theoretical similarity suggests an identification of the field theory parameter with an effective electron-scalar coupling (instead of an electron-phonon one): $y \rightarrow \lambda_{es}$. It is expected that the values of λ_{es} must be larger than the values of λ_{ep}, in view of the effective character of this new coupling constant, that comprises other interactions besides the phononic case. It must be said that the magnetic models based on antiferromagnetic spin-fluctuations (magnons, spin-polarons, excitons, etc.) support just a d-wave order parameter and suppose an intermediation by 1-spin gauge particles which, if indeed real, obviously does not contribute to λ_{es}.

Another well-known and well-measured high-T_c superconducting parameter is related to the magnetic field penetration depth orthogonally to the Cu-O planes, λ_c. The observation of an orthogonal parameter in a quasi-planar system is an indicative inheritance of a third lost (spatial) dimension. Specifically, in QED_3, the electromagnetic coupling constant squared, e^2, has dimension of mass, rather than the dimensionless character of the usual four-dimensional QED_4 coupling constant. This fact might be understood as a memory (or reminiscence) of the third dimension that appears (into the coupling constant) when one tries to work with a theory intrinsically defined in three space-time dimensions. This dimensional peculiarity could be better implemented through the definition of a new coupling constant in three space-time dimensions [6,7]: $e \rightarrow e_3 = e/\sqrt{l}$, where l represents a distance orthogonal to the planar dimension. This parameter shall be identified with the c-axis magnetic penetration depth (λ_c), whose values will be taken from the phenomenological data set available for the high-T_c cuprate superconductors analyzed here. In this way, one writes, $e_3 = e/\sqrt{\lambda_c}$, where from now on the electromagnetic coupling constant, e, is the actual electron charge. The phenomenological identification of these two parameters will make the planar Schrödinger equation entirely known and, consequently, will allow the application of a numerical method (such as the variational one) for computing its energy bound states.

VI. NUMERICAL EVALUATION

In this Section, examples of quasi-planar copper oxide superconductors are displayed, each one associated to a corresponding energy gap, ($2\Delta(0)$), c-axis magnetic penetration depth (λ_c), electron-scalar coupling (λ_{es}) and the corresponding scalar vacuum expectation value squared (v^2) that provides the gap energy; β is for the value of the parameter that minimizes the energy and C_s the coefficient of the electron-electron scattering potential given by Eq.(15). The numerical procedure is linear; namely, the choice of input data (v^2, λ_c, λ_{es}) determines the coefficient $C_s = 1/2\pi(2\lambda_{es}^2 v^2 - e^2/\lambda_c)$ and the Higgs quasi-particle mass $M_H = \sqrt{2}ve/\sqrt{\lambda_c}$ which is the argument of the Bessel function K_0. All this allows the Schrödinger equation (22) to become totally known. Thereafter, the application of the variational method allows one to find a value of β which provides, up to an uncertainty of $\pm 0,5$meV, the expected gap energy. The quantity ξ_{ab} represents the average size of the wave-function associated to the computed bound state, which might be tantamount to the planar correlation length of the cuprate materials. As a matter of fact, it can be taken as a suitable measure of the correlation length.

It was already explained that the constant λ_{es} constant comprises not only the phonon contribution, but all the scalar ones. There arises the issue of how one may estimates the value of this constant. Regarding λ_{ep}, one knows that the experimental techniques brings to light a great variation of values from a sample to another, and even for the same sample. For example, in $YBa_2Cu_3O_7$ samples, the measurements of λ_{ep} vary from $0,2$ to $2,5$ [43], such an indefinite picture occurs also for other superconductors. In the case of λ_{es}, larger values are expected due to its effective nature, so that in the following Tables I-IV this constant will be spanned from $0,5$ to $4,0$. The Tables I-IV contain data for the zero angular momentum ($l = 0$) and singlet-spin state, in order to account for the s-wave pairing structure of superconductors, where the input data have been collected from the works of Ref. [44] for the following high-T_c cuprate superconductors: $YBa_2Cu_3O_7$, $Tl_2Ba_2CaCu_2O_{10}$, $Bi_2Sr_2CaCu_2O_8$ and $HgBa_2Ca_2Cu_3O_8$.

v^2(meV)	λ_c(Å)	λ_{es}	C_s(meV)	β	$E_{gap} \pm 0,5$(meV)	ξ_{ab}(Å)
71,33	1800	0,5	4,40	33,52	29,9	29,43
16,65	1800	1,0	4,03	32,07	30,1	30,76
7,10	1800	1,5	3,82	31,21	30,0	31,61
3,90	1800	2,0	3,69	30,71	30,1	32,13
2,44	1800	2,5	3,58	30,23	30,0	32,64
1,67	1800	3,0	3,51	29,97	30,0	32,92
1,22	1800	3,5	3,48	29,84	30,3	33,06
0,92	1800	4,0	3,41	29,52	30,2	33,42

TABLE I. Input (from Hasegawa *et al.* and Gallagher *et al.* [44]) and output data for $YBa_2Cu_3O_7$ ($T_c = 87$K and $2\Delta(0) = 30,0$meV).

v^2(meV)	λ_c(Å)	λ_{es}	C_s(meV)	β	$E_{gap} \pm 0,5$(meV)	ξ_{ab}(Å)
54,00	4800	0,5	3,82	31,23	28,1	31.59
12,50	4800	1,0	3,50	29,94	28,1	32,95
5,30	4800	1,5	3,31	29,12	28,0	33,88
3,10	4800	2,0	3,29	29,03	28,1	33,99
1,92	4800	2,5	3,17	28,42	27,7	34,72
1,32	4800	3,0	3,13	28,28	28,0	34,89
0,95	4800	3,5	3,05	27,92	27,7	35.34
0,72	4800	4,0	3,01	27,73	27.8	35,58

TABLE II. Input (from Hasegawa *et al.* and Thompson *et al.* [44]) and output data for $Tl_2Ba_2CaCu_2O_{10}$ ($T_c = 105K$ and $2\Delta(0) = 28,0meV$).

v^2(meV)	λ_c(Å)	λ_{es}	C_s(meV)	β	$E_{gap} \pm 0,5$(meV)	ξ_{ab}(Å)
96,5	5000	0,5	7,22	43,02	53,4	22,93
22,2	5000	1,0	6,61	41,11	53,4	23,99
9,42	5000	1,5	6,29	40,11	53,4	24,59
5,14	5000	2,0	6,09	39,41	53,4	25,04
3,21	5000	2,5	5,93	38,95	53,3	25,33
2,19	5000	3,0	5,82	38,59	53,4	25,57
1,59	5000	3,5	5,72	38,22	53,4	25,81
1,20	5000	4,0	5,65	38,06	53,5	25,92

TABLE III. Input (from Maeda [44]) and output data for $Bi_2Sr_2CaCu_2O_8$ ($T_c = 109K$ and $2\Delta(0) = 53,4meV$).

v^2(meV)	λ_c(Å)	λ_{es}	C_s(meV)	β	$E_{gap} \pm 0,5$(meV)	ξ_{ab}(Å)
92,00	3500	0,5	6,67	41,28	48,0	23,90
21,20	3500	1,0	6,09	39,48	48,1	24,99
9,00	3500	1,5	5,79	38,49	48,0	25,63
4,90	3500	2,0	5,58	37,78	47,9	26,12
3,07	3500	2,5	5,45	37,31	48,0	26,44
2,10	3500	3,0	5,36	37,03	48,2	26,64
1,51	3500	3,5	5,25	36,61	47,9	26,95
1,15	3500	4,0	5,19	36,41	48,1	27,09

TABLE IV. Input (from Schilling *et al.* [44]) and output data for $HgBa_2Ca_2Cu_3O_8$ ($T_c = 131K$ and $2\Delta(0) = 48,0meV$).

VII. FINAL REMARKS

Starting off from a parity-preserving planar QED model [16–18], this paper sets out to mainly evaluate the energy of the ground state of electron-pairs that interact via photon and Higgs quasi-particle exchange. The numerical results, obtained throughout a variational method, succeeded in fitting some well-known parameters, such as energy gap, for the high-T_c copper oxide superconductors analyzed here, namely, $YBa_2Cu_3O_7$, $Tl_2Ba_2CaCu_2O_{10}$, $Bi_2Sr_2CaCu_2O_8$ and $HgBa_2Ca_2Cu_3O_8$. One has therefore a theoretical model which, supplemented by some experimental data on high-T_c superconductors, may reveal itself suitable for treating quasi-planar superconductivity. An important outcome is that the phenomenological data fix the scale for the breaking of the $U(1)$-symmetry in the superconductors: $v^2 \sim$ 1-10meV, in much the same way as $\sim 10^2$GeV is the scale for the breaking of the electroweak symmetry in the Standard Model. In the described picture, the Higgs mechanism plays an essential role in providing mass for the photons and to ensure a net attractive electron-electron scattering potential through the exchange of photons and Higgs quasi-particles.

The potential resulting from the Møller scattering in the non-relativistic limit, $-C_s K_0(M_H r)$, provides just a symmetric wave-function solution to the order parameter. The search for an anisotropic wave-function (p-wave or d-wave) must pass through the attainment of a potential dependent on the angle variable, in such a way that it may account for the angular variations observed in these non-symmetric states. We will hopefully arrive at the angular dependence by including loop corrections into the scattering potential [42].

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