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Abstract

The pion-nucleus scattering around 200 MeV and just above 1200 MeV is dominated by strong, broad $\Delta(3,3)$ and weak resonances in the $\pi^{\pm}N$ interaction. The interaction to a first approximation can be described as diffraction process. Since it is well known that the strength of the $\pi^{+}N$ and $\pi^{-}N$ interactions are quite different from each other at the resonances, the analyses of differential cross section for $\pi^{+}N$ and $\pi^{-}N$ elastic scattering data in the region of low-lying pion-nucleus resonances will be a good test of different strengths. In the present work we analyze pions scattering from nuclei ^{9}Be, ^{28}Si, ^{58}Ni, ^{89}Y and ^{208}Pb at incident pion energies between 50 and 291 MeV within the framework of the three parameter version of the Strong Absorption Model of Frahn and Venter. All the oscillations in the elastic scattering experimental data and for the experimental angular distribution leading to 2^{+} and 3^{-} collective states could be well reproduced by the model. The best fit parameter values for T, Δ and μ are determined. They are respectively the cut-off angular momentum, rounding parameter and the real nuclear phase shift parameters of the model. The interaction radius 'R', the surface diffuseness 'd' and the reaction cross-section σ_r have been determined from the derived parameter values. The standard nuclear radius r_0 and the surface diffuseness 'd' are fairly constant. The deformation parameters β_L have been determined from the normalization constant of the theory to the experiment without making any change in the elastic scattering parameters. The β_2 and β_3 values so extracted are in good agreement with other works. It is observed that there is hardly any difference between the values of β_2 (π^{+}) and β_2 (π^{-}) and that between β_3 (π^{+}) and β_3(π^{-}) values.

I. Introduction

Pion-Nucleus Scattering is a promising tool for obtaining information about the proton and neutron radii and densities in nuclei. The pion is a spinless boson which exists in all three charge states, viz. positive, negative and neutral. Thus it has the right properties to behave as an Yukawa particle. Pions can initiate elastic, inelastic scattering and nuclear reactions including single and double charge exchange reactions. The pion incident on a nucleon forms a resonance as $\pi^+ + p \rightarrow N^* (=\Delta^{++}) \rightarrow \pi^+ + p$ at pion incident energy at around 200 MeV. This is called Δ^{++} resonance in an $l_p = 1$ state with J=3/2. Its width is ~ 125 MeV and its life span is ~ 5×10^{-24} sec., which is too small to be measured directly. Around the $\Delta(3,3)$ resonance and beyond it, the pion-nucleon interaction is strongly absorptive and the pions thus have wave lengths shorter than the characteristic dimension of the target nucleon/nucleus (< 0.5 fm or so for E_π = 277 MeV). This strong absorption phenomena greatly simplify the theoretical formalism in analyzing the elastic scattering problem without requiring any knowledge of the absorption mechanism and naturally leads to analytic expressions for the elastic scattering amplitude.

II. Mathematical Preliminaries

We start with the scattering amplitude of a spin-zero pion incident on a spin-zero target nucleus in the form of partial wave expansion:

$$f(\theta) = (i/2k)\sum_{l=0}^{\infty}(2l+1)[1-\eta_l(l)]P_l(\cos\theta) \qquad (1)$$

where η_l is a sub matrix of the full scattering matrix S(l). The condition $|\eta_l(l)| \ll 1$ or $\eta_l = 0$ for $l < l'$ holds for the strong absorption of the incident pions. And the condition $|\eta_l| \geq 1$ or $\eta_l = 1$ for $l \geq l'$ refers to the scattering of the incident pions. The conditions finally demand for critical or cut-off angular momentum $T^{\pm}$ ($=l\pm1/2$) semi-classically. In other words this leads to the fact that the nuclear interior is black to the high energy pions due to large nuclear matter and strong interaction. Secondly the surface region of the nucleus contributes to the elastic scattering.

An analytic expression for the parameterized S-matrix model of η_l in angular momentum space. i.e. in l-space is accomplished, if one assumes the scattering amplitude to split into real and imaginary parts:

$$\eta_l(t) = g(t) + i\mu\,[\,dg(t)/dt] \qquad (2)$$

The g(t)'s above having any functional form, are characterized by the parameters like $T^{\pm}(L\pm1/2)$ and rounding parameter $\Delta^{\pm}$ around $T^{\pm}$ in l-space. The analytic expression for the elastic scattering amplitude $f(\theta)$ is obtained after performing consistent approximations as in Refs.[1-4] starting from eqns.(1) and (2):

$$f(\theta) = T/K\,(\theta/\sin\theta)^{1/2}[\pi\Delta\theta/\sinh(\pi\Delta\theta)][iJ_1(T\theta)/\theta + \mu\, J_0(T\theta)] \qquad (3)$$

Once the elastic scattering cross section is obtained in terms of the model parameters like T,Δ and μ, the inelastic scattering amplitude can be expressed as the first derivative of the elastic scattering amplitude [5,6] . The inelastic scattering cross section has the analytic form:

$$d\sigma/d\Omega \quad (0\rightarrow L)=|C_L|^2(2L+1)(T^2/16\pi)(\theta/\sin\theta)\{(H_-+H_+)^2$$
$$\sum[\alpha_{LM}(\theta)J_{|M|}(T\theta)-\beta_{LM}(\theta)J_{|M|-1}(T\theta)]^2 +$$
$$(H_--H_+)^2\sum[\alpha_L(\theta)J_{|M|}(T\theta)+\beta_{LM}(\theta)J_{|M|}/T(\theta)]^2\} \quad (4)$$

The functions $\alpha_{LM}(\theta)$ and $\beta_{LM}(\theta)$ are the elements of the rotation matrix and has the property:

$$\alpha_{LM}(\theta)=0; \qquad \text{if } (L+M) \text{ is odd.}$$
$$\beta_{LM}(\theta)=0: \qquad \text{if } (L+M) \text{ is even.} \quad (5)$$

The coefficient C_L is the reduced nuclear matrix elements. For single excitation these are expressed in terms of the deformation distances δ_L by

$$C_L = \delta_L/(2L+1)^{1/2} \quad (6)$$

The discrete summation in eqn. (4) runs from $M=-L$ to $+L$. The functions $\alpha_{2M}(\theta)$ and $\beta_{2M}(\theta)$ corresponding to $L=2$ state have the following forms:

$$\alpha_{2,\pm2}(\theta) = 1/4\,(3/2)^{1/2}(1+\cos\theta), \quad \beta_{2,\pm2}(\theta)=0,$$

$$\alpha_{2,\pm1}(\theta)=0, \quad \beta_{2,\pm1}(\theta)=1/2\,(3/2)^{1/2}\sin\theta,$$

$$\alpha_{2,0}(\theta)=1/4\,(3\cos\theta-1), \quad \beta_{2,0}=0$$

Eqn. (4) for the inelastic scattering cross section for the $L=2$ state becomes:

$$d\sigma/d\Omega(0\rightarrow2)=\delta_2^2\,(T^2/64\pi)(\theta/\sin\theta)\{(H_-+H_+)^2[1/4\,(3\cos\theta-1)^2+3\sin^2\theta]$$
$$J_0^2(T\theta)+3/4\,(1+\cos\theta)^2 J_2^2(T\theta)\}+4(H_--H_+)^2J_1^2(T\theta)] \quad (7)$$

For octupole excitation ($L=3$) the functions $\alpha_{3M}(\theta)$ and $\beta_{3M}(\theta)$ corresponding to $L=3$ states are as follows:

$$\alpha_{3,\pm3}(\theta) = 0.466\cos 3\theta/2 + 0.4191\cos\theta/2, \quad \beta_{3,\pm3}(\theta)=0,$$
$$\alpha_{3,\pm2}(\theta) = 0, \quad \beta_{3,\pm2}(\theta) = 0.1140\sin3\theta/2 + 0.3421\sin\theta/2,$$
$$\alpha_{3,\pm1}(\theta) = 0.1805\cos 3\theta/2 - 0.108\cos\theta/2, \quad \beta_{3,\pm1}(\theta) = 0,$$
$$\alpha_{3,0}(\theta)=0, \quad \beta_{3,0}(\theta) = 0.069\sin3\theta/2 - 0.375\sin\theta/2,$$

Then eqn. (4) for the inelastic scattering cross section for the $L=3$ state reduces to:

$$d\sigma/d\Omega(0\rightarrow3)=\delta_3^2\,(T^2/16\pi)(\theta/\sin\theta)[(H_--H_+)^2(J_0^2(T\theta)\{2\alpha_{31}^2(\theta)+$$
$$\beta^2_{30}(\theta))+J_2^2(T\theta)(2\alpha^2_{33}(\theta)+2\beta^2_{32}(\theta))\}+$$
$$(H_-+H_+)^2\{J_1^2(T\theta)(2\alpha_{31}^2(\theta)+2\beta^2_{32}(\theta)+\beta^2_{30}(\theta)+J_3^2(T\theta)2\alpha^2_{33}(\theta)\}] \quad (8)$$

where $H_{\pm}=[1+\mu(\theta_c\pm\theta)]F[(\Delta(\theta_c\pm\theta)]$.

The expressions for eqns. (7) and (8) depend on four parameters viz. T, Δ, μ and δ_L. The SAM parameters T, Δ, μ are obtained by fitting the angular distribution for elastic scattering and the deformation length, $\delta_L = \beta_L R$ is determined by normalizing the SAM predicted cross section with the experimental ones. The deformation parameter β_L is related to the deformation distance δ_L through the relation:

$$\delta_L=(T/k)\,\beta_L. \qquad (9)$$

The total reaction cross section:

$$\sigma_r = \pi / k^2 \sum_{l=0}^{\infty} (2l+1)(1-|\eta_l|^2)$$

which for spin zero charged particle becomes:

$$\sigma_r=\pi T^2/k^2[1+2\Delta/T+1/3\pi^2(\Delta/T)^2-1/3(\mu/\Delta)^2(\Delta/T)] \qquad (10)$$

III. Results and Discussion

The analytical expressions (3) have been used to calculate the elastic scattering cross sections for $\pi^{\pm}$ incident on the target nuclei ^{9}Be, ^{28}Si and ^{58}Ni at 291 MeV and on the target nucleus ^{208}Pb at 50,80,115.9 and 291 MeV [7,8]. The angular distribution data for the elastic scattering $\pi^{\pm}$ from the target nuclei covered in the present work are available over the angular range 10^0-90^0. The parameters T and Δ of the model, the quantity $\sigma r/4\pi R^2$, the standard nuclear radius r_0 and the deformation parameters are shown/given in Table 1. Typical SAM predicted elastic and inelastic scattering angular distributions for π^+ and π^- probes along with experiment are presented in Figs.1 and 2 and in Figs.3-5, respectively. The fit obtained to the experimental data, is excellent. It is observed that there is no diffraction pattern in $\pi^{\pm}$ - ^{9}Be angular distribution at pion incident energy 291 MeV as well as in $\pi^{\pm}$ - ^{208}Pb angular distributions at pion incident energy 50 MeV. No structure is seen. But surprisingly enough, the theory viz. the Strong Absorption Model has reproduced all the structureless features of the angular distribution, for the target mass as light as ^{9}Be and not to speak of high mass number nucleus ^{208}Pb. The absence of diffracted behavior in the angular distribution in the mentioned cases is obviously an interesting observation, indicating an enhanced transparency. The reproductions are quite satisfactory throughout the whole angular range except some minor discrepancy in the magnitude of the differential cross section between theory and experiment.

The value of σ_r increases almost smoothly as the target mass increases, while the beam energy remains the same. It is further observed that σ_r increases almost smoothly as the beam energy increases while the target mass remains the same, as expected. The value of $\sigma_r/4\pi R^2$ remains fairly constant (~0.42) throughout the analyses, as it should be and which is more meaningful than the σ_r itself. The standard nuclear radius 'r_0' is obtained to be constant (~1.5fm) and largely independent of the target nucleus and beam energy.

The inelastic scattering of charged pions leading to the lowest 2^+ collective and 3^- states are theoretically calculated using eqns. (7) and (8) with the SAM parameters fixed from the corresponding elastic scattering analyses. The agreement between the experiment and theory is slightly inferior to the quality of fit obtained in the elastic scattering.

The overall trend of the angular distribution is certainly reproduced by the theory.
The calculated deformation parameters β_2 and β_3 are compared with the values obtained from the DWIA[7)] analysis and with other values obtained by other workers for 2^+ and 3^- states. Our obtained values are usually close to DWIA analyses. A close scrutiny of the values of parameters shows that the present values are fairly close to other values obtained by other workers. It is observed that there is hardly any difference between the values of β_2 (π^+) and β_2 (π^-) and that between β_3 (π^+) and β_3 (π^-) values, within allowed uncertainties, though the strength of the π^+N and π^-N interactions are quite different from each other in the region of low-lying pion-nucleus resonances.

Finally the Strong Absorption Model is a useful model and its advantages, over other sophisticated models, is its simplicity.

IV. Conclusion

The Strong Absorption Model of Frahn and Venter is found to be reasonably successful in analyzing the elastic scattering of pions from ^9Be, ^{28}Si and ^{58}Ni at 291 MeV and from ^{208}Pb at 50,80,115.9 and 291 MeV and a consistent set of elastic SAM parameters are extracted to be used in describing the inelastic scattering phenomena leading to 2^+ and 3^- collective states in nuclei. The deformation parameters so obtained compare themselves reasonably well with other studies. No tangible difference is observed between the values of $\beta_2(\pi^+)$ and $\beta_2(\pi^-)$ and that between $\beta_3(\pi^+)$ and $\beta_3(\pi^-)$ values.

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Table 1 Summary of elastic and inelastic scattering analyses leading to 2^+ and 3^- collective states in target nuclei at pion beam energy 291 MeV.

	T	Δ	$\sigma_r/4\pi R^2$	r_0 (fm)	J_π
$\pi^+ + {}^{28}Si$	7.25	1.00	0.45	1.42	2^+
$\pi^- + {}^{28}Si$	7.25	1.10	0.47	1.40	2^+
$\pi^+ + {}^{58}Ni$	10	1.00	0.4	1.60	2^+
$\pi^- + {}^{58}Ni$	9.70	1.20	0.44	1.50	2^+
$\pi^+ + {}^{28}Si$	7.25	1.00	0.45	1.42	3^-
$\pi^- + {}^{28}Si$	7.25	1.10	0.47	1.40	3^-
$\pi^+ + {}^{58}Ni$	10.00	1.00	0.40	1.60	3^-
$\pi^- + {}^{58}Ni$	9.70	1.20	0.44	1.50	3^-
$\pi^+ + {}^{208}Pb$	14.90	1.10	0.36	1.60	3^-
$\pi^- + {}^{208}Pb$	15.50	1.10	0.37	1.60	3^-

Table 1. Continued

	E_x(MeV)	J_π	Deformation	Parameter	(β_L)
			a	b	c
$\pi^+ + {}^{28}Si$	1.78	2^+	0.60	0.55	0.45
$\pi^- + {}^{28}Si$	1.78	2^+	0.56	0.49	0.44
$\pi^+ + {}^{58}Ni$	1.45	2^+	0.19	0.24	0.20
$\pi^- + {}^{58}Ni$	1.45	2^+	0.17	0.22	0.20
$\pi^+ + {}^{28}Si$	6.88	3^-	0.55	0.44	0.47
$\pi^- + {}^{28}Si$	6.88	3^-	0.60	0.44	0.49
$\pi^+ + {}^{58}Ni$	4.47	3^-	0.16	0.20	0.16
$\pi^- + {}^{58}Ni$	4.47	3^-	0.21	0.21	0.18
$\pi^+ + {}^{208}Pb$	2.61	3^-	0.18	0.17	0.18
$\pi^- + {}^{208}Pb$	2.61	3^-	0.22	0.15	0.13

a: Present work,
b: Reference[7)],
c: Reference[9)],

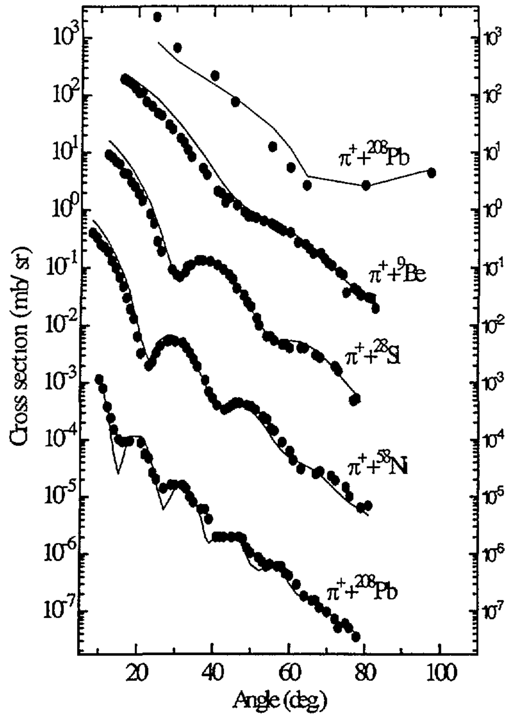

Fig. 1 SAM analyses of the elastic scattering of π^+ from ^{208}Pb at 50 MeV and π^+ from ^{9}Be, ^{28}Si, ^{58}Ni and ^{208}Pb at 291 MeV.

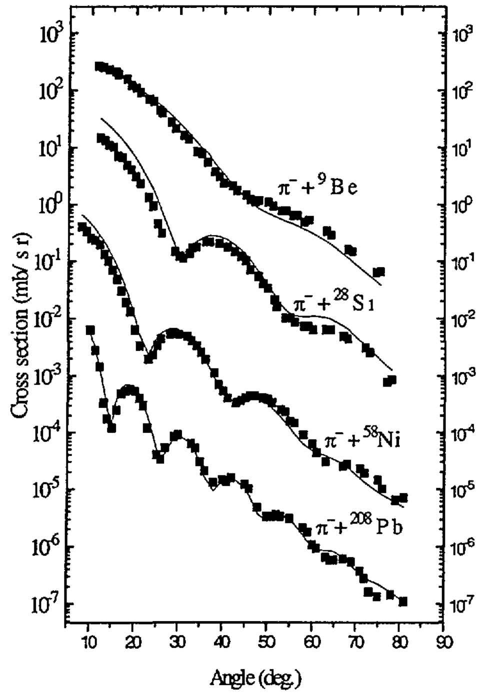

Fig. 2 SAM analyses of the elastic scattering of π^- from ^{9}Be, ^{28}Si, ^{58}Ni and ^{208}Pb at 291 MeV.

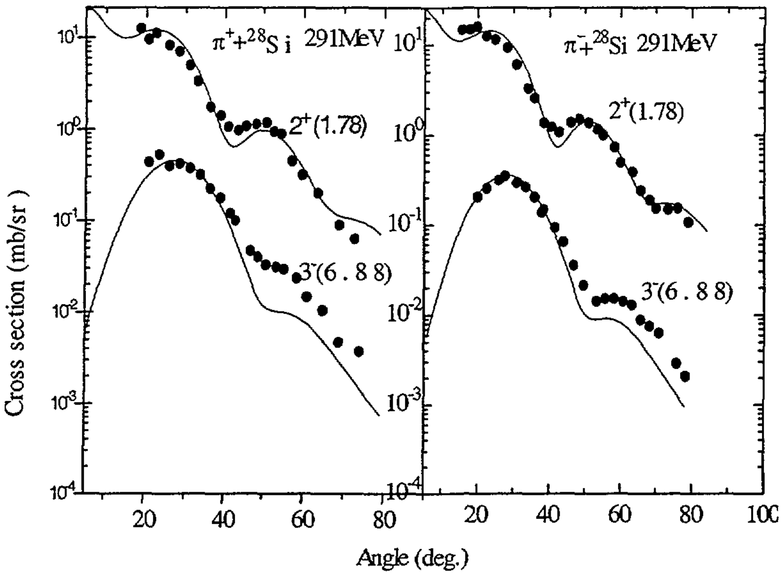

Fig.3 Angular distribution for the inelastic scattering of 291 MeV of π^+ and π^- to the 2^+, 1.78 MeV and 3^-, 6.88 MeV states in ^{28}Si.

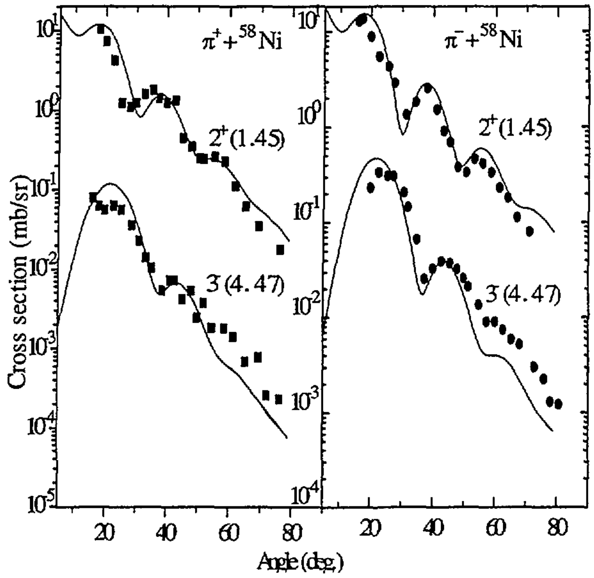

Fig.4 Angular distribution for the inelastic scattering of 291 MeV of π^+ and π^- to the 2^+, 1.45 MeV and 3^-, 4.47 MeV states in ^{58}Ni.

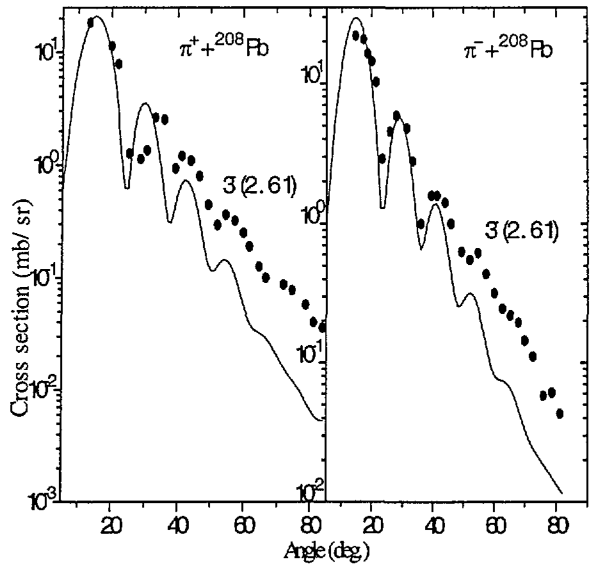

Fig.5 Angular distribution for the inelastic scattering of 291 MeV of π^+ and π^- to the 3^-, 2.61 MeV state in ^{208}Pb.

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