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Abstract

The pion-nucleon interaction above the Δ (1232) resonance and in the region of low-lying pion-nucleon resonances is studied. $\pi^{\pm}$ p elastic scattering at $T_{\pi^{\pm}}$ = 277 - 640 MeV characterized by diffraction maxima and minima has been analyzed through the strong absorption model due to Frahn and Venter. The proton radius is determined from the best fit values of the cut-off angular momentum to be 0.85 fm with a spread of 0.15 fm. The higher energy pions scan a lower value while the lower energy pions yield a higher value for the size of the proton. The energy averaged radius of the proton size of 0.85 fm obtained in the present analysis is in excellent agreement with proton charge radius of 0.86 fm quoted in the literature.

1. Introduction

The pion has long been considered a promising probe for obtaining information about the proton and neutron radii, densities of nuclei and to unravel the complicated structure of nuclei. It is well known that the π^+p and π^-n elastic scattering amplitudes are three times the corresponding amplitudes for π^+n and π^-p scattering in the region of the pion-nucleon $\Delta_{3,3}$ (J=3/2, T=3/2) resonance. Investigation of π^+p elastic scattering is of fundamental importance for understanding the nature of strong interaction. This has gained additional interest in the context of quark dynamics. In the region of Δ (3,3) resonance and beyond, the pion-nucleon interaction is strongly absorptive and the pions have shorter wave lengths than the characteristic dimension of the target nucleus/nucleon (≤ 0.5 fm or so around the Δ (3,3) resonance and it is further less beyond i.e. $\lambdabar = 0.25$ fm at 640 MeV). The pion-nucleus scattering around 200 MeV and just above 1200 MeV is dominated by strong, broad Δ (3,3) and weak resonances in the $\pi^{\pm}N$ interaction. This interaction to a first approximation can be considered as diffraction process. This is evidenced as diffraction maxima and minima in the angular distributions (i.e. in $\frac{d\sigma}{d\Omega}$ versus $\theta_{c.m.}$ curve). Since the strengths of π^+N and π^-N interactions are quite different from each other at the resonances, the π^+p and π^-p elastic scattering data analyses in the low lying region of pion-nucleon resonances will be a good test for different strengths.

We present the analysis of the experimental data [1] of π^+p and π^-p elastic scattering at $T_{\pi^{\pm}} = 277$-640 MeV by the generalized diffraction model developed by Frahn and Venter [2].

2. Mathematical preliminaries

The scattering amplitudes expressed in terms of the partial wave have the form:

$$f(\theta) = \frac{i}{2k}\sum_{\ell=0}^{\infty}(2\ell+1)[1-S(\ell)]P_{\ell}\ (\cos\theta) \qquad (1)$$

The complex quantities $S(l)$, the amplitude of the ℓ-th outgoing partial wave is called the scattering function (S – function) in which all the observable quantities are contained and which acts as a link between the actual nuclear properties and the measured nuclear data. $S(l)$ is assumed to have non-zero real and imaginary components. We take advantage of the facts:

i) the nuclear interior is approximately black to the high energy pions due to large nuclear matter density and strong interaction

and

ii) the nuclear surface region of the nucleus contributes to the elastic scattering

Therefore the appropriate choice for the S- function is:

$$S(\ell) = g(\ell) + i\mu[\frac{dg(\ell)}{d\ell}] \qquad (2)$$

The real part of $S(\ell)$ or the quantity $[1 - g(\ell)]$ with ' Woods – Saxon form for $g(\ell)$, approaches nuclear density distribution:

$$g(\ell) = [1 + \exp\{(R - r)/d\}]^{-1} \qquad (3)$$

where $r = \lambda\!\!\!^{-}(\ell + 1/2)$ is the impact parameter and $R = \lambda\!\!\!^{-}(L + 1/2)$ is the interaction radius of the target nucleus. With the help of strong absorption formulation, $S(\ell)$ semi-classically leads to

$$\begin{aligned} S(\ell) &= 0, \qquad \ell \leq L - ½ \qquad (4) \\ &= 1, \qquad \ell > L - ½ \end{aligned}$$

which paves the way for the existence of a cut-off angular momentum. The imaginary part of the S function is chosen so that it adjusts the minima with the correct choice of the parameter μ.

Using equations 1-4, including strong absorption conditions such as $\lambda < R$ i.e. $2\pi KR >>$ and $\pi/2(\Delta/kR) << 1$ and making room for consistent approximation procedure, one gets the analytic expression for the elastic scattering amplitude [3,4]:

$$f(\theta) = R[\theta/\sin\theta]^{1/2}[\frac{\pi\Delta\theta}{\sinh(\pi\Delta\theta)}][\frac{ij_1(KR_0\theta)}{\theta} - (\frac{2n}{KR\theta^2} - \mu)J_0(KR\theta)] \qquad (5)$$

with $\Delta = Kd$.

3. Results and discussions

We calculate the elastic scattering of 277 – 640 MeV positive and negative pions from proton targets, using the analytic expression (5).

The experimental data has been taken from ref. [1], which has statistical errors of 2 to 5% only as against 10% and in some cases up to 30% errors of the previous experimental data in the region up to 700 MeV [5-7]. The parameters that enter in the calculation are the interaction radius R, the surface thickness d and the real nuclear phase shift μ. These are adjusted within reasonable limits to reproduce the experimental data. The theoretically predicted differential cross section for the elastic scattering of pions from proton targets are presented in the figs. 1-3 along with those measured experimentally. The relevant values of the input parameters and the derived quantities are tabulated in tables I and II. It is reasonable that the SAM being a first order process, can reproduce the qualitative features of the experimental data up to large angle $\sim 175^0$ such that the position of the maxima and minima are fairly well reproduced by the model SAM although not in exact magnitude. It may be mentioned here that the experimental cross section at a minimum is usually over a wider acceptance angle and is averaged over Gaussian distribution.

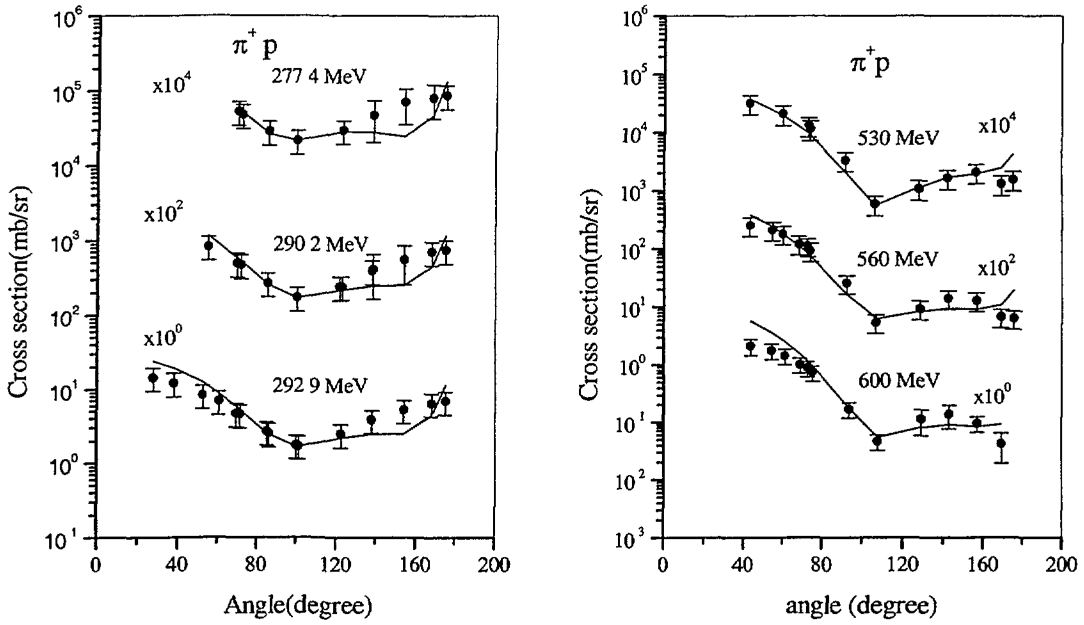

Fig. 1. SAM analysis of elastic scattering of π^+ from proton at different energies.

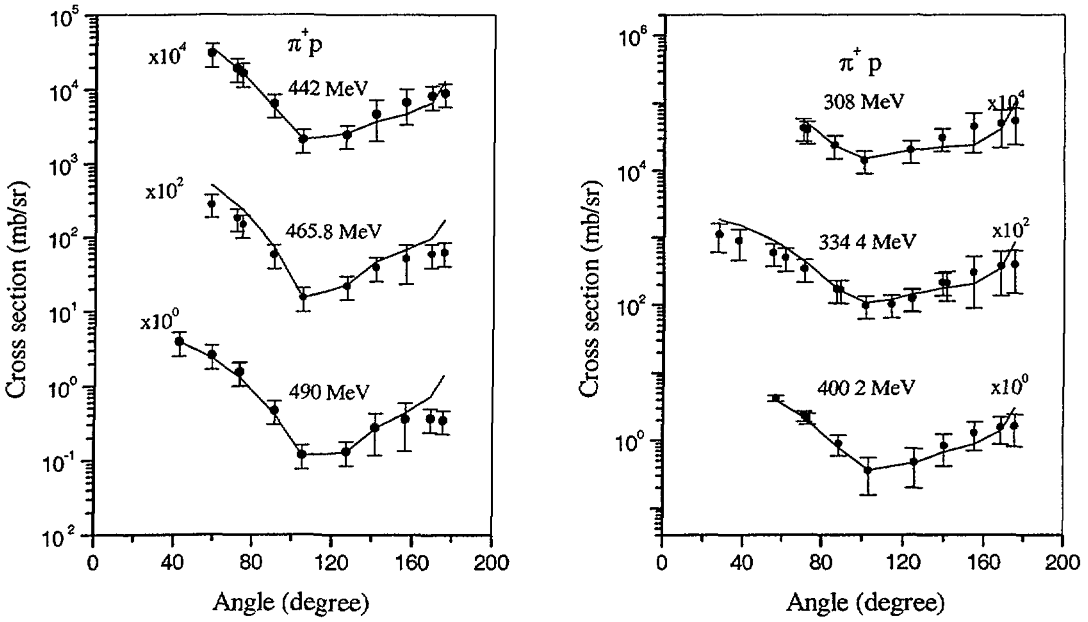

Fig. 2. SAM analysis of elastic scattering of π^+ from proton at different energies.

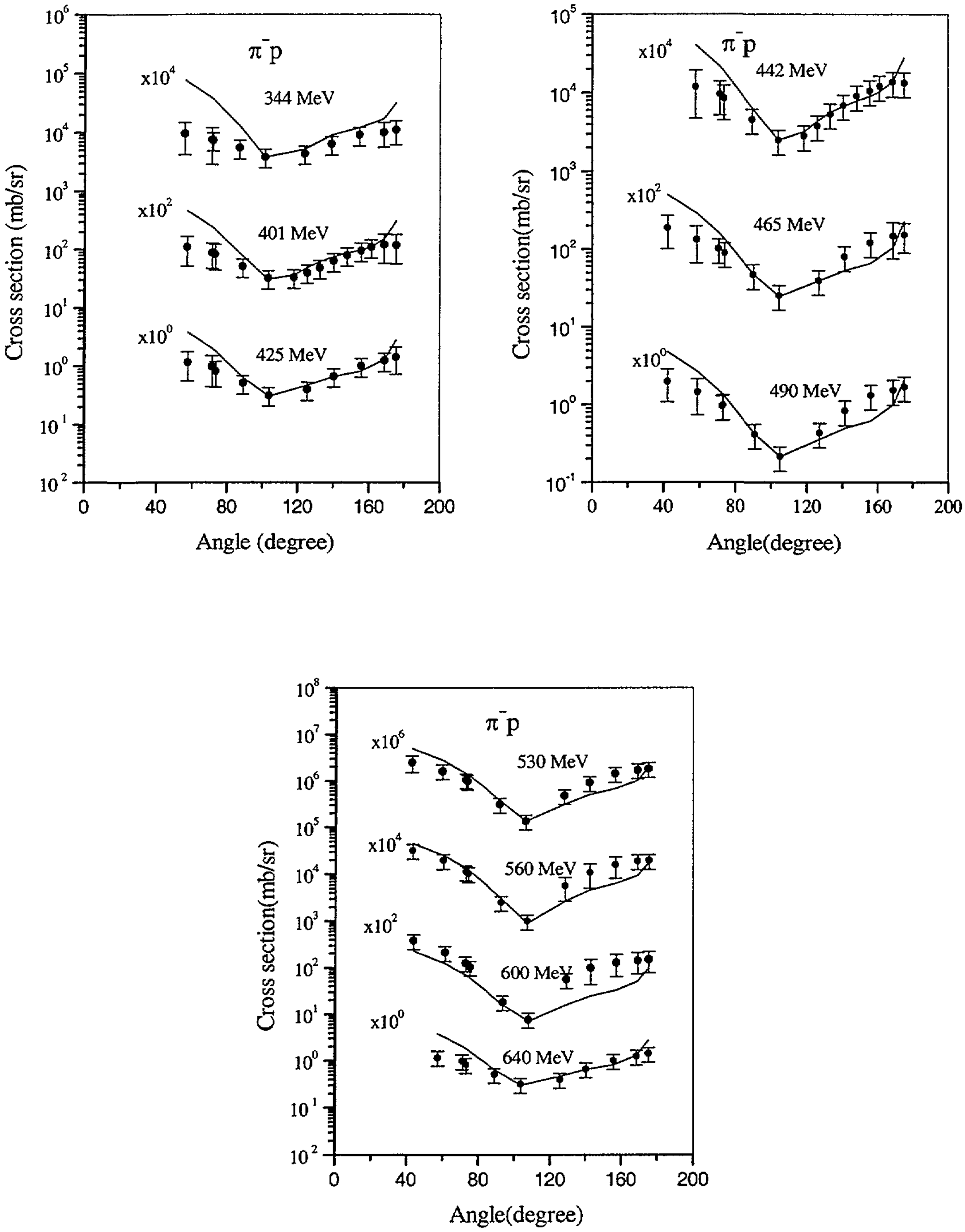

Fig. 3. SAM analysis of elastic scattering of $\pi^-$from proton at different energies.

Table I. - Strong Absorption Model (SAM) parameters for π^+ at different beam energies.

Target Nucleus	Beam Energy (MeV)	SAM parameters			Derived parameters		
		T	Δ	μ	R (fm)	d (fm)	r_0 (fm)
Proton	277.4	2.0	0.04	0.53	1.637	0.033	1.07
P	290.2	2.0	0.05	0.47	1.601	0.04	1.05
P	292.9	2.0	0.05	0.47	1.593	0.04	1.04
P	308	2.0	0.05	0.45	1.554	0.04	1.02
P	334.4	2.0	0.08	0.39	1.491	0.04	0.97
P	400.2	1.95	0.1	0.25	1.329	0.07	0.87
P	442	1.95	0.2	0.23	1.264	0.13	0.83
P	465.8	1.95	0.2	0.17	1.231	0.13	0.81
P	490	1.90	0.15	0.14	1.17	0.092	0.76
P	530	2.0	0.25	0.15	1.184	0.15	0.77
P	560	2.0	0.33	0.21	1.152	0.19	0.75
P	600	2.0	0.38	0.24	1.113	0.21	0.73

Table II. - Strong Absorption Model (SAM) parameters for π^- at different beam energies.

		SAM parameters			Derived parameters		
		Target	Beam Energy				
Nucleus	(MeV)	T (fm)	Δ (fm)	μ (fm)	R	d	r_0
Proton	344	2.0	0.15	0.23	1.47	0.11	0.95
P	401	2.0	0.10	0.23	1.36	0.068	0.89
P	425	2.0	0.10	0.24	1.32	0.066	0.86
P	442	2.0	0.10	0.22	1.29	0.065	0.84
P	465	2.0	0.10	0.23	1.26	0.063	0.82
P	490	2.0	0.10	0.22	1.23	0.062	0.80
P	530	2.0	0.10	0.18	1.18	0.059	0.77
P	560	2.0	0.10	0.15	1.15	0.058	0.75
P	600	2.0	0.10	0.14	1.11	0.056	0.73
P	640	2.0	0.10	0.20	1.08	0.054	0.70

It is thus not very physically meaningful to go for exact reproduction of the minima. The quality of angular distribution fit, in the presence of such uncertainties, for π^+ p and π^- p elastic scattering differential cross section is good all over the pion energies just adjusting the cut-off angular momentum as against at least sixteen parameters of the phase shift analysis of the same experimental data. The model SAM does not take into account the degrees of freedom associated with mesons and the excited states of the nucleons and therefore one should not be surprised at the discrepancy in the quality of the fit mentioned above. The deviations between experimental data and theory at large angles are due to processes not included in the SAM formalism. The model describes the scattering process as geometrical effects to a first approximation.

The main information revealed by probing the nucleon by π^+ and π^- probes is that π^- probes can come more close to the nucleon than π^+ probes. These lead to the fact that π^+ yields a higher value of proton size ($r_0 \approx .89$ fm, averaging over all incident energies) as against π^- probe yielding somewhat lower value of the proton size ($r_0 \approx 0.812$ fm, averaging over all energies). This is understood as repulsive Coulomb forces lead to less absorption for π^+ than would be the case for negatively charged pions affected by attractive Coulomb forces. The standard nucleon radius for proton = $R/A^{1/3}$ = 0.85 fm, obtained from the mean of the radii by π^+ and π^- probes in the present work is a consistent value and is in excellent agreement with proton charge radius of 0.86 fm oft quoted value in the literature [8]. The surface thickness 'd' ranges from 0.032 – 0.21 fm, denoting to the fact that the proton has a sharp pointed edge.

Finally, we are of the opinion that the geometrical SAM model is capable of reproducing most of the salient features of $\pi^{\pm}$ elastic scattering data at $T_{\pi^{\pm}}$ = 277 – 640 MeV and furnishing us with valuable information about the proton size, which agrees so very well with the value estimated from the experiment. This obviously speaks of the success of the model used in the present analysis.

3. Conclusion

$\pi^{\pm}p$ interaction atop the Δ-resonance and in the low-lying pion-nucleon resonance having characteristic features of maxima and minima in the angular distributions are analyzed through geometrical model like the generalized diffraction models of Frahn and Venter. The proton radius of 0.85 fm obtained is in excellent agreement with experimental value.

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REFERENCES

[1] GRDEEV V.A., KOPTEV V.P., KRUGLOV S.P., KUZMIN L.A., KULBARDIS A.A., MALOV YU.A., MIKIRTICH'YANTS S.M., STRAKOVSKY I.I. and SCHERBAKOV G.V., N*ucl. Phys. A,* **364** (1981) 428.

[2] FRAHN W.E. and VENTER R.H., *Ann.Phys. (N.Y.),* **24** (1963) 243*;* VENTER R .H., *ibid.* **25** (1963) 405.

[3] HOUDHURY D.C., *Phys. Rev., C,* **22**(1980) 1848.

[4] RAHMAN MD.A., SEN GUPTA H.M. and RAHMAN M., *Phys.Rev. C,* **44** (1991) 2484.

[5] OGDEN P.M., HAGGE D.E., HELLAND J.A., BANNER M., DETOEUF J.-F. and TEIGER J., *Phys. Rev.,* **137** (1965) B1115.

[6] HELLAND J.A., DEVLIN T.J., HAGGE D.E., LONGO M.J., MOYER B.J. and WOD C.D., *Phys. Rev.,***134** (1964)B 1062.

[7] GOODWIN L.K., KENNEY R.W. and PEREZ-MENDEZ, *Phys. Rev.,* **122** (1961) 655.

[8] SIMON G.G. *et al., Nucl. Phys., A, 333* (1980) 381.