The size of $\chi \equiv \arg(-V_{ts}V_{tb}^{*}V_{cs}^{*}V_{cb})$ and physics beyond the Standard Model

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Abstract

We analyse the allowed range of values of χ, both in the Standard Model and in models with New Physics, pointing out that a relatively large value of χ, e.g. of order λ, is only possible in models where the unitarity of the 3×3 Cabibbo-Kobayashi-Maskawa matrix is violated through the introduction of extra $Q=2/3$ quarks. We study the interesting case where the extra quark is an isosinglet, determining the allowed range for χ and the effect of a large χ on various low-energy observables, such as CP asymmetries in B meson decays. We also discuss the correlated effects which would be observable at high energy colliders, like decays $t\to cZ$, modifications of the cross section and forward-backward asymmetry in $e^+e^-\to t\bar{t}$ and the direct production of a new quark.

1 Introduction

The experimental determination of the physical CP-violating phases entering the quark mixing matrix is of great importance for the study of CP breaking, providing at the same time stringent tests of the Standard Model (SM). The Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] $V_{3\times3}$ describing the mixing among the known quarks contains nine moduli and four linearly independent rephasing invariant phases, which can be taken as [2, 3]

$$\beta=\arg(-V_{cd}V_{cb}^{*}V_{td}^{*}V_{tb})\,,\quad \gamma=\arg(-V_{ud}V_{ub}^{*}V_{cd}^{*}V_{cb})\,,$$
$$\chi=\arg(-V_{ts}V_{tb}^{*}V_{cs}^{*}V_{cb})\,,\quad \chi'=\arg(-V_{cd}V_{cs}^{*}V_{ud}^{*}V_{us})\,. \tag{1}$$

The phases β and γ appear in the well-known (d,b) unitarity triangle corresponding to the orthogonality of the first and third columns of $V_{3\times3}$, while χ and χ' appear in other less studied unitarity triangles. The phases χ and χ' are fundamental parameters of $V_{3\times 3}$ as important as γ and β, playing a crucial rôle in the orthogonality between the $(2,3)$ and $(1,2)$ rows, respectively [4].

Within the three-generation SM, the nine moduli and four rephasing-invariant phases are connected by unitarity, which leads to a series of relations among these measurable quantities. Such relations provide excellent tests of the SM [5], which complement the usual fit of the unitarity triangle, and have the potential for discovering New Physics. In the context of the SM, the values of χ and χ' are very constrained and therefore the determination of these phases provides, by itself, a good test of the SM.

In SM extensions which enlarge the quark sector, the 3×3 CKM matrix is a submatrix of a larger matrix V. Independently of whether extra quarks are present or not, one can always choose, without loss of generality, a phase convention such that [3]

$$\arg V = \begin{pmatrix} 0 & \chi' & -\gamma & \cdots \\ \pi & 0 & 0 & \cdots \\ -\beta & \pi+\chi & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} , \tag{2}$$

which explicitly shows that in the 3×3 submatrix $V_{3\times 3}$ only the four phases in Eq. (1) are linearly independent. However, when extra quarks are present the 3×3 unitarity relations do not hold, and as a result the range of allowed values for χ and χ' may differ from the range implied by the SM. We will show that even in the case that 3×3 unitarity does not apply, χ' is constrained to be rather small. Therefore, we will concentrate most of our attention on χ, investigating its expected size within the SM as well as in models with New Physics. In Section 2 we use extended unitarity relations to estimate the size of χ, χ' within the SM and its extensions, including both the cases where 3×3 CKM unitarity is respected and where it is violated. In Section 3 a more precise analysis of the range of variation of χ in a model with an extra up singlet is carried out. The effects of a large χ in some low energy observables are examined in Section 4, while the effects at high energy are discussed in Section 5. In Section 6 we draw our conclusions.

2 The size of χ and χ' in the SM and its extensions

It is well known that χ' has to be very small in the context of the SM and its extensions which keep the unitarity of the 3×3 CKM matrix. This can be seen, for example, using the relation [5]

$$\sin\chi' = \frac{|V_{ub}|\,|V_{cb}|}{|V_{us}|\,|V_{cs}|}\sin\gamma\,, \tag{3}$$

which shows that $|\chi'| \lesssim \lambda^4$. Within the SM, the 90% confidence level (CL) interval for χ' is

$$4.95 \times 10^{-4} \leq \chi' \leq 6.99 \times 10^{-4} \quad (\text{SM})\,. \tag{4}$$

This range is obtained with a fit to the measured CKM matrix elements in Table 1, together with ε, the B^0 mass difference and the time-dependent CP asymmetry in $B_d^0 \to \psi\, K_S$, $S_{\psi K_S}$, all collected in Table 2 (see Refs. [6, 7]).

Element	Exp. value
$\lvert V_{ud}\rvert$	0.9734 ± 0.0008
$\lvert V_{us}\rvert$	0.2196 ± 0.0026
$\lvert V_{ub}\rvert$	0.0036 ± 0.0010
$\lvert V_{cd}\rvert$	0.224 ± 0.016
$\lvert V_{cs}\rvert$	0.989 ± 0.014
$\lvert V_{cb}\rvert$	0.0402 ± 0.0019

Table 1: Experimental values of CKM matrix elements.

	Exp. value
ε	$(2.282 \pm 0.017) \times 10^{-3}$
δm_{B_d}	0.489 ± 0.008 ps^{-1}
$S_{\psi K_S}$	0.734 ± 0.054

Table 2: Additional observables required for the fit of the CKM matrix.

Even in models where $V_{3\times3}$ is not unitary, but part of a larger unitary matrix V, χ' is constrained to be rather small [3]. From orthogonality of the first two columns of V, one readily obtains

$$\cos\chi' \geq \frac{|V_{ud}|^2 + |V_{cs}|^2 + |V_{us}|^2 + |V_{cd}|^2 - |V_{ud}|^2|V_{cs}|^2 - |V_{us}|^2|V_{cd}|^2 - 1}{2|V_{ud}||V_{us}||V_{cd}||V_{cs}|}\,, \tag{5}$$

implying $\cos\chi' \geq 0.9983$ and

$$|\chi'| \leq 0.0579 \tag{6}$$

at 90% CL. This limit is robust in the presence of New Physics, since the moduli involved are obtained from experiment through tree-level decays, where the SM is expected to give the dominant contribution. From the strict bound of Eq. (6) it is clear that it will be very difficult to obtain a direct measurement of χ'. Therefore, in the remaining of this work we will focus our attention on χ.

Within the SM and any extension where $V_{3\times3}$ is unitary, like supersymmetric or multi Higgs doublet models, we have the relation

$$\sin\chi = \frac{|V_{ub}||V_{us}|}{|V_{cb}||V_{cs}|} \sin(\gamma + \chi' - \chi)\,, \tag{7}$$

which shows that $|\chi| \lesssim \lambda^2$ in any model where 3×3 CKM unitarity holds. In particular, within the SM one obtains at 90% CL

$$0.015 \leq \chi \leq 0.022 \quad (\mathrm{SM})\,. \tag{8}$$

The only models in which χ can be significantly larger than λ^2 are those in which $V_{3\times3}$ is not unitary, what can only be achieved by enlarging the quark sector. The most simple way of doing this is with the introduction of new quark singlets [8, 9].[1] Quark singlets often arise in grand unified theories [10, 11] and models with extra dimensions at the electroweak scale [12]. They have both their left- and right-handed components transforming as singlets under $\mathrm{SU}(2)_L$, thus their addition to the SM particle content does not spoil the cancellation of triangle anomalies. In these models, the charged and neutral current terms of the Lagrangian in the mass eigenstate basis are

$$\begin{aligned} \mathcal{L}_W &= -\frac{g}{\sqrt{2}}\, \bar u_L \gamma^\mu V d_L\, W_\mu^+ + \mathrm{h.c.}\,, \\ \mathcal{L}_Z &= -\frac{g}{2c_W} \left(\bar u_L \gamma^\mu X u_L - \bar d_L \gamma^\mu U d_L - 2 s_W^2 J_{\mathrm{EM}}^\mu \right) Z_\mu\,, \end{aligned} \tag{9}$$

where $u = (u, c, t, T, \dots)$ and $d = (d, s, b, B, \dots)$, V denotes the extended CKM matrix and $X = VV^\dagger$, $U = V^\dagger V$ are hermitian matrices. X and U are not necessarily diagonal and thus flavour-changing neutral (FCN) couplings exist at the tree level, although

[1] The addition of a sequential fourth generation is another possibility, but it is disfavoured by two facts: (i) the experimental value of the oblique correction parameters only leave a small range for the masses of the new quarks; (ii) anomaly cancellation requires the introduction of a new lepton doublet, in which the new neutrino should be very heavy, in contrast with the small masses of the presently known neutrinos.

they are naturally suppressed by the ratio of the standard quark over the heavy singlet masses [8]. Moreover, the diagonal Zqq couplings, which are given by the diagonal entries of X and U plus a charge-dependent term, are also modified. Within the SM $X_{uu} = X_{cc} = X_{tt} = 1$, $X_{qq'} = 0$ for $q \neq q'$, $U_{dd} = U_{ss} = U_{bb} = 1$ and $U_{qq'} = 0$ for $q \neq q'$. The addition of up-type $Q = 2/3$ singlets modifies the first two of these equalities, while the addition of down-type $Q = -1/3$ ones modifies the last two. For our purposes, it is sufficient to consider that either up- or down-type singlets are added to the SM particle content. We analyse in turn these two possibilities.

2.1 Models with down-type singlets

In this case, and assuming that there are n_d extra down singlets, the CKM matrix V is a $3 \times (3+n_d)$ matrix consisting of the first three rows of a $(3+n_d) \times (3+n_d)$ unitary matrix, and $X = 1_{3\times 3}$. From orthogonality of the second and third columns of V, one obtains the generalisation of Eq. (7),

$$\sin\chi = \frac{|V_{ub}||V_{us}|}{|V_{cb}||V_{cs}|} \sin(\gamma + \chi' - \chi) - \frac{\mathrm{Im}\,(U_{bs} e^{-i\chi})}{|V_{cb}||V_{cs}|} \,. \tag{10}$$

From the present bound on $b \to s\ell^+\ell^-$, one obtains that at most $|U_{bs}| \simeq 10^{-3} \sim \lambda^4$ [13,14], thus implying that in this class of models χ cannot be significantly larger than λ^2.

2.2 Models with up-type singlets

In these models the quark mixing matrix is a $(3+n_u) \times 3$ matrix, with n_u the number of extra singlets, and $U = 1_{3\times 3}$. Almost all the effects discussed in this paper can be already obtained in the minimal extension with $n_u = 1$, in which case the quark mixing matrix has dimension 4×3. From orthogonality of the second and third columns one obtains the generalisation of Eq. (7) for this model,

$$\sin\chi = \frac{|V_{ub}||V_{us}|}{|V_{cb}||V_{cs}|} \sin(\gamma + \chi' - \chi) + \frac{|V_{Tb}||V_{Ts}|}{|V_{cb}||V_{cs}|} \sin(\sigma - \chi) \,, \tag{11}$$

where $\sigma \equiv \arg(V_{Ts} V_{Tb}^* V_{cs}^* V_{cb})$. χ may be of order λ if $V_{Ts} \sim \lambda^2$ and $V_{Tb} \sim \lambda$, but the possible constraints from FCN currents in the up sector must also be kept in mind. From orthogonality of the second and third rows of V, one gets

$$\sin\chi = \frac{\mathrm{Im}\; X_{ct}}{|V_{cs}||V_{ts}|} + O(\lambda^2) \,. \tag{12}$$

In contrast with models containing down-type singlets, where the size of all FCN couplings is very restricted by experiment, present limits on X_{ct} are rather weak. The most stringent one, $|X_{ct}| \leq 0.41$ with a 95% CL, is derived from the non-observation of single top production at LEP, in the process $e^+e^- \to t\bar{c}$ and its charge conjugate [15]. This bound does not presently provide an additional restriction on the size of χ. In models with extra up singlets $|X_{ct}|$ can be of order λ^3 [14], yielding $\chi \sim \lambda$.

From Eq. (12) one derives some important phenomenological consequences. First, we observe that a sizeable χ is associated to a FCN coupling $X_{ct} \sim 10^{-2}$, which leads to FCN decays $t \to cZ$ at rates observable at LHC. In addition, the modulus of X_{ct} obeys the inequality [16]

$$|X_{ct}|^2 \leq (1 - X_{cc})(1 - X_{tt}) \,, \tag{13}$$

which is verified in any SM extension with any number of up- and/or down-type quark singlets (in particular, with only one $Q = 2/3$ singlet the equality holds). We note that within the SM, $X_{cc} = X_{tt} = 1$ and hence $X_{ct} = 0$. This relation shows that necessary conditions (and sufficient for the case of only one singlet) for achieving $X_{ct} \sim 10^{-2}$ are to have a small deviation $O(\lambda^4)$ of X_{cc} from unity (which is allowed by the measurement of R_c and $A_{\mathrm{FB}}^{0,c}$) and a deviation of X_{tt} from unity of order λ^2. The latter could be measured in $t\bar{t}$ production at a future e^+e^- linear collider like TESLA. There is also a decrease of $|V_{tb}|$ from its SM value $|V_{tb}| \simeq 0.999$, which is however harder to detect experimentally, because the expected precision in the measurement of this quantity at LHC is around ± 0.05 [17]. Last, but not least, this deviation of X_{tt} from unity is only possible if the new quark has a mass below 1 TeV, in which case it would be directly produced and observed at LHC.

3 Detailed analysis of the range of χ with an extra up singlet

The analysis of the previous section has shown that χ can in principle be of order λ in models with up quark singlets. In order to determine its precise range of variation, it is mandatory to perform an analysis including constraints from a variety of processes for which the predictions are affected by the inclusion of an extra up quark. We summarise here the most relevant ones.

1. The presence of the new quark and the deviation of $|V_{tb}|$ and $X_{tt} \simeq |V_{tb}|^2$ from

the SM predictions yield new contributions to the oblique parameters S, T and U. The most important one corresponds to the T parameter, approximately

$$\Delta T = \frac{N_c}{16\pi s_W^2 c_W^2} (1 - X_{tt}) \left[-18.4 + 7.8 \log y_T \right] , \tag{14}$$

with $N_c = 3$ the number of colours and $y_T = (\bar{m}_T/M_Z)^2$. The present experimental measurement $\Delta T = -0.02 \pm 0.13$ sets stronger limits on V_{tb} and X_{tt} than the S, U parameters or the forward-backward asymmetry $A_{\mathrm{FB}}^{(0,b)}$.

2. The deviation of X_{cc} from unity modifies the Zcc couplings and thus the prediction for R_c and the forward-backward asymmetry $A_{\mathrm{FB}}^{(0,c)}$. The precise measurement of these quantities sets a stringent constraint on X_{cc}, with a direct influence on χ, as shown by Eqs. (12), (13).

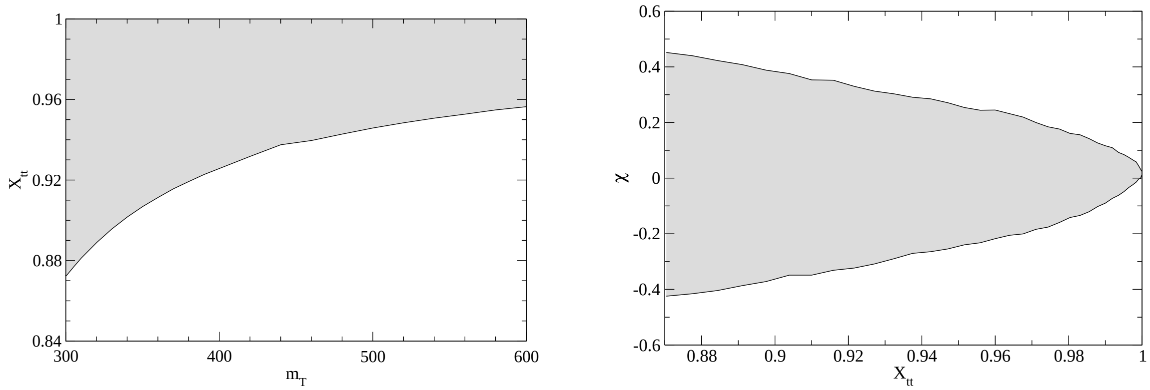
3. The FCN coupling X_{uc} mediates a tree-level contribution to $D^0 - \bar{D}^0$ mixing, which is kept within experimental limits for $X_{uc} \lesssim 5 \times 10^{-4}$.

4. The new quark T gives additional loop contributions to K and B oscillations and rare decays $K^+ \to \pi^+ \nu \bar{\nu}$, $K_L \to \mu^+ \mu^-$, $b \to s\gamma$ and $b \to s l^+ l^-$. The new terms are similar to the top ones, but proportional to some combination of the CKM matrix elements V_{Td}, V_{Ts}, V_{Tb} and with the corresponding Inami-Lim functions evaluated at $x_T = (\bar{m}_T/M_Z)^2$. For the unrealistic case $x_T \simeq x_t$ the Inami-Lim functions for the t, T quarks take similar values, and the sum of both terms may be very similar to the top SM contribution. Therefore, in this situation the constraints on V_{Td}, V_{Ts}, V_{Tb} are rather loose. However, for $m_T \gtrsim 300$ GeV these observables provide important constraints on V_{Td} and V_{Ts}, forcing also V_{td} and V_{ts} to lie in their SM range.

These and other less important constraints like ε'/ε have been taken into account in our analysis [14]. It is important to note that the most recent bound on the CP asymmetry in $b \to s\gamma$ [18] is still not relevant. Using an appropriate generalisation of the formulas in Refs. [19] for the present case, we always find $|A_{\mathrm{CP}}^{b \to s\gamma}| \lesssim 0.02$, to be compared with the experimental 90% CL interval $-0.06 \leq A_{\mathrm{CP}}^{b \to s\gamma} \leq 0.11$.

We will conservatively assume that the mass of the new quark T is of 300 GeV or larger. Present Tevatron Run II measurements seem to exclude the existence of a new quark with a mass around 200 GeV and decaying to Wb [20]. However, we will briefly comment on the situation if the new quark is lighter than 300 GeV. We remark that the allowed range of χ only depends on the mass of the new quark through the

m_T dependence of X_{tt}. The possible values of X_{tt} are constrained mainly by the T parameter, and are shown in Fig. 1a as a function of m_T. For a fixed X_{tt}, the interval in which χ can vary turns out to be independent of m_T. The allowed range of χ as a function of X_{tt} is plotted in Fig. 1b. We observe that, as anticipated in the previous section, a deviation of X_{tt} from unity is necessary in order to have χ large. For $X_{tt} = 1$ the range of χ reduces to the SM interval (see Fig. 1b).



(a) (b)

Figure 1: (a) Allowed interval of X_{tt} (shaded area) as a function of the mass of the new quark (adapted from Ref. [14]). (b) Allowed interval of χ (shaded area) as a function of X_{tt}.

We present two examples of matrices V for $m_T = 300$ GeV which give large $|\chi|$ with positive and negative sign, respectively. We have not chosen examples which maximise $|\chi|$ but have instead selected two matrices which yield theoretical predictions for presently known observables in very good agreement with experiment, while showing significant departures in χ from the SM expectation. We write the full 4×4 unitary matrices, although only the 4×3 submatrices enter the charged current interactions. We choose the phase parameterisation in Eq. (2), in which the values of the four phases in Eq. (1) are easy to read directly from the matrices. The first example is

$$\left| V_{300}^{(+)} \right| = \begin{pmatrix} 0.9748 & 0.2229 & 0.0038 & 0.0097 \\ 0.2230 & 0.9733 & 0.0406 & 0.0362 \\ 0.0072 & 0.0355 & 0.9422 & 0.3332 \\ 0.0009 & 0.0419 & 0.3327 & 0.9421 \end{pmatrix} ,$$

$$\arg V_{300}^{(+)} = \begin{pmatrix} 0 & 6.92 \times 10^{-4} & -0.8222 & -0.1046 \\ \pi & 0 & 0 & 0 \\ -0.4099 & \pi + 0.3513 & 0 & 1.940 \\ 0 & 2.346 & 0.1001 & -1.106 \end{pmatrix} . \quad (15)$$

This matrix has $\beta = 23.5°$, $\gamma = 47.1°$ in the (d,b) unitarity triangle. While β is close to the SM prediction, $\chi = 0.35$ presents a large deviation from the SM value. For this matrix $S_{\psi K_S} = 0.70$, with ϵ, δm_B, $\mathrm{Br}(b \to s\gamma)$, $\mathrm{Br}(b \to s l^+ l^-)$ and the rest of observables considered in Ref. [14] also in good agreement with experiment. The second example is

$$\left| V_{300}^{(-)} \right| = \begin{pmatrix} 0.9748 & 0.2229 & 0.0038 & 0.0090 \\ 0.2230 & 0.9733 & 0.0419 & 0.0347 \\ 0.0077 & 0.0406 & 0.9571 & 0.2865 \\ 0.0024 & 0.0366 & 0.2864 & 0.9574 \end{pmatrix} ,$$

$$\arg V_{300}^{(-)} = \begin{pmatrix} 0 & 5.17 \times 10^{-4} & -1.020 & 0.0700 \\ \pi & 0 & 0 & 0 \\ -0.3608 & \pi - 0.2382 & 0 & -1.576 \\ 0 & -1.026 & 0.8784 & 2.449 \end{pmatrix} . \quad (16)$$

For this matrix $\beta = 20.7°$, $\gamma = 58.4°$ in the (d,b) unitarity triangle and $\chi = -0.24$, in clear contrast with the SM prediction. We find that $S_{\psi K_S} = 0.74$, with the other observables agreeing with experimental data. In both examples we observe that $X_{ct} = -V_{c4} V_{t4}^*$ has a large imaginary part (in this phase convention), as required for a large χ according to Eq. (12). The values obtained for χ are of the same order as the estimates given in the previous section. We stress that χ can be of order λ while keeping $S_{\psi K_S}$ close to its experimental value. Hence, a future improvement of this measurement (e.g. a reduction of the statistical error by a factor of two) has little effect on our results.

4 Low energy observables sensitive to χ

The decay $B_d^0 \to \phi K_S$ is an interesting example in which CP-violating effects sensitive to χ may be found, with the advantage that B_d^0 mesons can be produced at present B factories. The time-dependent CP asymmetry is given by:

$$S_{\phi K_S} = \frac{2 \, \mathrm{Im} \, \lambda_{\phi K_S}}{1 + |\lambda_{\phi K_S}|^2} \,, \tag{17}$$

where

$$\lambda_{\phi K_S} = \left(\frac{q}{p} \right)_{B_d^0} \frac{A(\bar B_d^0 \to \phi \, \bar K^0)}{A(B_d^0 \to \phi \, K^0)} \left(\frac{q}{p} \right)_{K^0} \,. \tag{18}$$

The q/p factors come from B_d^0 and K^0 mixing. The SM decay amplitudes are, to a very good approximation,

$$\begin{aligned} A(\bar B_d^0 \to \phi \, \bar K^0) &= a(x_t) V_{tb} V_{ts}^* \,, \\ A(B_d^0 \to \phi \, K^0) &= a(x_t) V_{tb}^* V_{ts} \,, \end{aligned} \tag{19}$$

with $a(x_t)$ a function of $x_t = (\overline{m_t}/M_W)^2$, to be specified later. In the SM, or in any model without New Physics in the decay amplitudes, $\lambda_{\phi K_S}$ can be related to its analogous in the ψK_S decay channel,

$$\lambda_{\psi K_S} = \left(\frac{q}{p} \right)_{B_d^0} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \left(\frac{q}{p} \right)_{K^0} \,. \tag{20}$$

Bearing in mind the definition of χ we can write

$$\lambda_{\phi K_S} = \lambda_{\psi K_S} e^{-2 i \chi} \,, \tag{21}$$

so that defining $\bar \beta$ by $\lambda_{\psi K_S} = - e^{-2 i \bar \beta}$ ($\bar \beta = \beta$ in the SM, but these two angles may differ if there are new contributions to the mixing) we have

$$S_{\phi K_S} = \sin (2 \bar \beta + 2 \chi) \,. \tag{22}$$

Therefore, if a substantial departure from the approximate SM prediction $S_{\phi K_S} \simeq S_{\psi K_S}$ is confirmed, it cannot be explained in models with 3×3 CKM unitarity and without new contributions to the decay amplitudes.

The best place to measure χ is in CP asymmetres in $B_s^0 - \bar B_s^0$ oscillations and decay. In the SM the B_s^0 mixing factor is

$$\left(\frac{q}{p} \right)_{B_s^0} = \frac{M_{12}^{B_s}}{|M_{12}^{B_s}|} = \frac{(V_{ts} V_{tb}^*)^2}{|V_{ts} V_{tb}^*|^2} = e^{2 i \chi} \,. \tag{23}$$

In any channel without a weak phase in the decay amplitude, for example in the $D_s^+ D_s^-$ and $\psi\,\phi$ channels, the time dependent CP asymmetry is

$$S_{D_s^+ D_s^-} = \sin 2\chi\,, \tag{24}$$

which in the SM is of order $2\lambda^2$.

4.1 $b \to s\bar{s}s$ with an extra up singlet

In these models Eq.(19) is replaced by

$$\begin{aligned} A(\bar{B}_d^0 \to \phi\,\bar{K}^0) &= a(x_t) V_{tb} V_{ts}^* + a(x_T) V_{Tb} V_{Ts}^*\,, \\ A(B_d^0 \to \phi\,K^0) &= a(x_t) V_{tb}^* V_{ts} + a(x_T) V_{Tb}^* V_{Ts}\,, \end{aligned} \tag{25}$$

with $x_T = (\overline{m_T}/M_W)^2$, due to the additional exchange of the T quark. Similarly, Eq. (21) is generalised to

$$\lambda_{\phi K_S} = -e^{-i(2\bar{\beta}+2\chi)} \left(\frac{1 + f(x_T, x_t) V_{Tb} V_{Ts}^* / V_{tb} V_{ts}^*}{1 + f(x_T, x_t) V_{Tb}^* V_{Ts} / V_{tb}^* V_{ts}} \right)\,, \tag{26}$$

with $f(x_T, x_t) = a(x_T)/a(x_t)$. Using the fact that $2|\lambda_{\phi K_S}|/(1 + |\lambda_{\phi K_S}|^2) \simeq 1$ to a very good approximation, we obtain

$$S_{\phi K_S} = \sin(2\bar{\beta} + 2\bar{\chi})\,, \tag{27}$$

where the "effective" $\bar{\chi}$ for this process is defined as

$$\bar{\chi} = \chi - \frac{1}{2} \arg \left(\frac{1 + f(x_T, x_t) V_{Tb} V_{Ts}^* / V_{tb} V_{ts}^*}{1 + f(x_T, x_t) V_{Tb}^* V_{Ts} / V_{tb}^* V_{ts}} \right)\,. \tag{28}$$

The geometrical interpretation of the effective phase $\bar{\chi}$ can be seen in Fig. 2, for different values of f. It is also useful to define $\chi_{\rm SM}$ as:

$$\chi_{\rm SM} = \arg[V_{cb} V_{cs}^* (V_{cs} V_{cb}^* + V_{us} V_{ub}^*)] = \arg \left(1 + \frac{V_{us} V_{ub}^*}{V_{cs} V_{cb}^*} \right) \tag{29}$$

which equals χ in any model with 3×3 unitarity. Since $\sin \chi_{\rm SM} \leq |V_{us} V_{ub}|/|V_{cs} V_{cb}|$, $\chi_{\rm SM} \sim \lambda^2$ even when 3×3 unitarity does not hold (see Fig. 2).

From Eq. (28) it can be seen that in the limit $m_T = m_t$ the effective χ entering the CP asymmetry reduces to $\chi_{\rm SM}$,

$$\lim_{m_T \to m_t} \bar{\chi} = \chi_{\rm SM}\,, \tag{30}$$

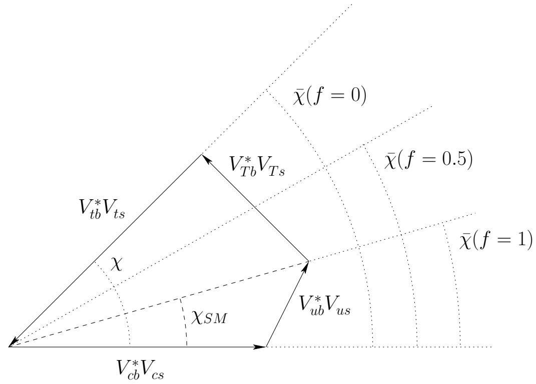
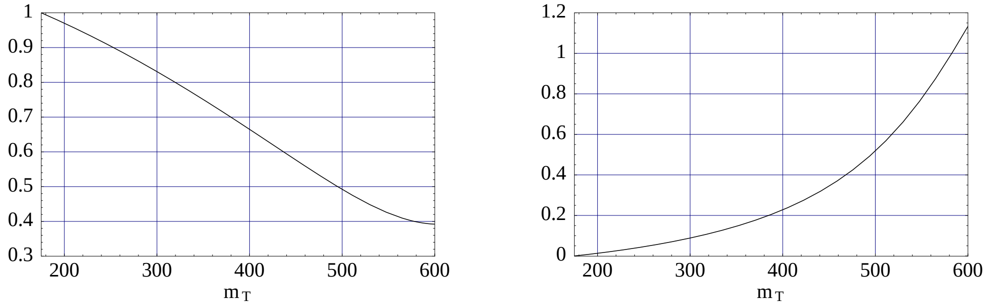


Figure 2: Different values of $\bar{\chi}$ and its geometrical meaning. The relative lengths of the sides of the quadrangle are illustrative.

independently of the value of χ. This "screening" property implies that, despite the fact that the actual value of χ may be very different from the SM prediction, the effective $\bar{\chi}$ that enters the CP asymmetry is $O(\lambda^2)$ when m_T tends to m_t. For larger m_T, the degree of screening depends on the value of $f(x_T, x_t)$: for $f = 0$ there is no screening, and the screening is maximal for $f = 1$. We calculate $a(x)$ using the QCD factorisation result of Refs. [21], obtaining

$$
\begin{aligned}
a(x) \;=\;& -0.036880 - 0.012896\, i - 0.005829\, B_0(x) + 0.004137\, C_0(x) \\
& -0.000438\, \tilde{D}_0(x) + 0.016376\, E_0'(x) + 0.004074\, \tilde{E}_0(x)\,. \qquad (31)
\end{aligned}
$$

The Inami-Lim [22] functions B_0, C_0, etc. can be found in Ref. [23]. The function $f(x_T, x_t)$ is plotted in Fig. 3 for fixed x_t. The screening is important for low m_T, becoming milder as m_T grows. In contrast, χ can be almost arbitrary for $m_T \sim m_t$, while its size is more restricted for a heavier T, as can be observed in Fig. 1. With both effects working in opposite directions, we find that $S_{\phi K_S}$ is always inside the interval $[0.57, 0.93]$, approaching the extremes for heavier T. Since the screening is present in any $b \to s\bar{s}s$ transition, we expect a similar behaviour for all other strong penguin dominated processes.



(a) (b)

Figure 3: Modulus (a) and argument (b) of f as a function of m_T, for fixed x_t.

4.2 $B_s^0 - \bar{B}_s^0$ mixing with an extra up singlet

With the addition of a $Q = 2/3$ singlet, the element M_{12} of the $B_s^0 - \bar{B}_s^0$ mixing matrix can be written as

$$M_{12}^{B_s} = K \sum_{i,j=t,T} (V_{is}^* V_{ib})(V_{js}^* V_{jb}) S(x_i, x_j) = K S(x_t, x_t) |V_{ts}|^2 |V_{tb}|^2 r_s^2 e^{-2i\chi_{\text{eff}}} \,, \tag{32}$$

with K a constant factor, S the usual Inami-Lim box function and

$$\begin{aligned} r_s^2 e^{-2i\chi_{\text{eff}}} &= e^{-2i\chi} \left\{ \left[1 + \frac{S(x_t, x_T) V_{Ts}^* V_{Tb}}{S(x_t, x_t) V_{ts}^* V_{tb})} \right]^2 \right. \\ &\left. + \left[\frac{S(x_T, x_T)}{S(x_t, x_t)} - \left(\frac{S(x_t, x_T)}{S(x_t, x_t)} \right)^2 \right] \left(\frac{V_{Ts}^* V_{Tb}}{V_{ts}^* V_{tb}} \right)^2 \right\} . \end{aligned} \tag{33}$$

The effective phase entering $B_s^0 - \bar{B}_s^0$ mixing is in this case χ_{eff}, defined from the above equation. In the limit $x_T \to x_t$, the second term in the curly brackets goes to zero and we get

$$\lim_{x_T \to x_t} \chi_{\text{eff}} = \chi_{\text{SM}} \tag{34}$$

as in the previous process. However, in contrast with the function $f(x_T, x_t)$ which determines the screening in the $b \to s\bar{s}s$ transitions, the ratio $S(x_t, x_T)/S(x_t, x_t)$ in the first term of Eq.(33) is an increasing function of x_T. This means that, although for $x_T \to x_t$ the screening operates (as can be read from Eq. (34)), for large x_T we can have some enhancement of χ_{eff} with respect to χ. The range of variation of the asymmetry $S_{D_s^+ D_s^-} = \sin 2\chi_{\text{eff}}$ is shown in Fig. 4. Although for heavier T the allowed interval for χ is narrower, the enhancement above mentioned makes the asymmetry be between -0.4 and 0.4 for the T masses considered (this range of variation is quite different from the one predicted by the SM). Such asymmetry could be easily be measured at LHCb, where the expected precision in the $\psi\,\phi$ channel is around 0.066 for one year of running [24].

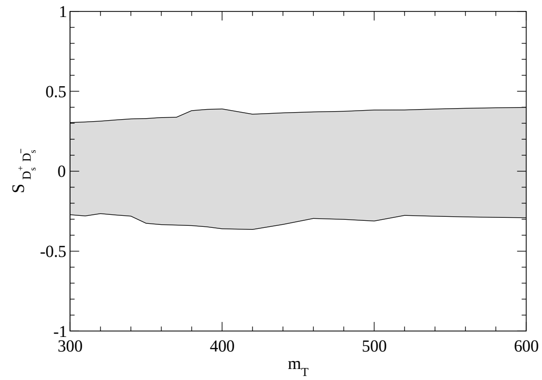


Figure 4: Range of variation of the asymmetry $S_{D_s^+ D_s^-}$ (adapted from Ref. [14]).

4.3 Unitarity and $D^0 - \bar{D}^0$ mixing

The present experimental values of CKM matrix elements in the first row seem to have a discrepancy of $2.2 - 2.7$ standard deviations [25] with respect to the SM unitarity prediction $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$.[2] It is then worthwhile to question whether such apparent unitarity deviation could be explained in scenarios with a large χ, which also require a sizeable breaking of 3×3 unitarity. We will show that this is not possible in the minimal SM extension studied here. In general, we have the inequality

$$|X_{uc}|^2 \leq (1 - X_{uu})(1 - X_{cc})\,, \tag{35}$$

but for only one extra singlet the equality holds. With $(1 - X_{uu}) \sim 4 \times 10^{-3}$ (implying $|V_{u4}| \simeq 0.06$) from the apparent unitarity deviation in the first row and $(1 - X_{cc}) \sim 10^{-3}$ in order to have large χ, the FCN coupling X_{uc} would give a tree-level contribution to the D^0 mass difference [28, 29] above the present experimental limit $|\delta m_D| \leq 0.07$ ps^{-1} [6]. In models with more than one extra singlet, the equality in Eq. (35) does not hold and this argument is relaxed.

We also point out that, in this minimal extension with only one extra singlet, 4×4 unitarity implies that in case V_{Td} and V_{u4} are both very small X_{ct} is also negligible.

[2]Recent theoretical calculations [26] and experimental results [27] would eliminate this discrepancy.

Since V_{Td} must be small due to constraints from B oscillations (see for instance the matrices in Eqs. (15), (16)), a large χ requires V_{u4} not much smaller than 10^{-2}. Therefore, it is expected that a large χ is associated with a D^0 mass difference not far from the present experimental limit.

5 Effects at high energy colliders

As implied by Eqs. (12) and (13), the fact of having a phase $\chi \sim \lambda$ has consequences in some high energy processes: rare top decays, $t\bar{t}$ production at e^+e^- collisions and the direct production of a new quark at LHC.

5.1 Top decays $t \to cZ$

Top FCN decays are extremely suppressed within the SM and hence they are a clear signal of New Physics, if observed. In SM extensions with $Q = 2/3$ singlets the tree-level FCN couplings X_{ut} and X_{ct} can be large enough to yield measurable top FCN interactions. These vertices lead to rare top decays $t \to uZ, cZ$ and single top production in the processes $gu, gc \to Zt$ (in hadron collisions) and $e^+e^- \to t\bar{u}, t\bar{c}$ (in e^+e^- annihilation), plus the charge conjugate processes (see Ref. [30] for a review). The best sensitivity to a Ztc coupling is provided by top decays $t \to cZ$ at LHC. With a luminosity of 100 fb^{-1}, FCN couplings $|X_{ct}| \simeq 0.015$ can be observed with more than $5\,\sigma$ statistical significance [30]. With a luminosity of 6000 fb^{-1}, achievable in one year with a high luminosity upgrade [31], a $3\,\sigma$ significance can be obtained for $|X_{ct}| \simeq 0.0031$. A moderately small phase, for instance $\chi \simeq 0.15$, requires Im $X_{ct} \simeq 0.006$, which would be observed with more than $5\,\sigma$ significance.

5.2 $t\bar{t}$ production in e^+e^- collisions

Top pair production at a 500 GeV linear collider will provide a precise determination of the Ztt coupling through the measurement of the total $t\bar{t}$ cross section and the forward-backward asymmetry. The accuracy of the measurement of X_{tt} is mainly limited by theoretical uncertainties in the prediction of the total cross section. In order to determine the sensitivity to deviations of X_{tt} from unity, a Monte Carlo calculation of this process is necessary [32]. The best results are obtained with beam polarisations $P_{e^+} = 0.6$, $P_{e^-} = -0.8$. We assume that theoretical uncertainties in the total cross section can be reduced to 1% or below, and a luminosity of 1000 fb^{-1}, which can be collected in three years of running. For the SM value $X_{tt} = 1$ the top pair production cross section is $\sigma = 47.9 \pm 0.5$ fb (including theoretical and statistical uncertainties) and the forward-backward asymmetry $A_{\rm FB} = -0.375 \pm 0.004$ (the error quoted is only statistical). For a phase $\chi \simeq 0.15$, X_{tt} must be typically around 0.96, yielding $\sigma = 49.4 \pm 0.5$ fb, $A_{\rm FB} = -0.360 \pm 0.004$, which amount to a combined $4.5\,\sigma$ deviation with respect to the SM prediction. On the other hand, if no deviations from the SM predictions are found, a bound $X_{tt} \geq 0.985$ can be set with a 90% CL, implying that $-0.12 \leq \chi \leq 0.14$, an indirect limit complementing the ones which will be previously available from low energy processes.

5.3 Direct production of $T\bar{T}$ pairs in hadron collisions

The last (but obviously not least important) effect correlated with the presence of a phase $\chi \sim \lambda$ is the direct production of the new quark T. A sizeable deviation of X_{tt} from unity is only possible if the new quark is not very heavy, otherwise the contribution of the new quark to the T parameter, given by Eq. (14), would exceed present experimental limits. With the experimental value $\Delta T = -0.02 \pm 0.13$ and admitting at most a $2\,\sigma$ deviation, a coupling $X_{tt} \simeq 0.96$ (as required by $\chi \simeq 0.15$) is acceptable if the new quark has a mass below approximately 850 GeV. A new quark with this mass can be produced in pairs via strong interactions, with a total tree-level cross section of 170 fb. The observability of the new quark can be estimated as follows. For $m_T = 850$ GeV, $X_{tt} = 0.96$ the new quark decays mainly to Wb and Zt, with branching ratios $\text{Br}(T \to Wb) = 0.7$, $\text{Br}(T \to Zt) = 0.3$. This new quark could be easily seen in its semileptonic decays $T\bar{T} \to l^\pm \nu jjjj$, being the total tree-level cross section of the process $q\bar{q}, gg \to T\bar{T} \to W^+ b W^- \bar{b} \to l^+ \nu jjjj$ (including standard detector cuts) 5.5 fb (the same cross section for the final state $l^- \nu jjjj$) [32]. The $Wjjjj$ background can be greatly reduced with suitable cuts requiring that the events have a kinematics compatible with $T\bar{T}$ production. The tree-level cross sections after cuts for $l^+ \nu jjjj$ and $l^- \bar{\nu} jjjj$ are 75 fb and 45 fb, respectively, calculated with `VECBOS` [33]. Taking into account only statistical uncertainties, with 100 fb^{-1} the $T\bar{T}$ signal could be observed with a significance of $10\,\sigma$.

6 Concluding remarks

We have emphasised that a large value of χ requires physics beyond the SM, in particular violations of 3×3 unitarity of the CKM matrix. It has been shown that if this unitarity breaking arises from the presence of down-type isosinglet quarks, χ is still constrained to be of order λ^2 due to the constraint from the $b \to s l^+ l^-$ decay. On the contrary, it has been pointed out that in the presence of up-type quark singlets a relatively large value of χ can be obtained, without entering into conflict with present experimental data.

The implications of a large χ have been analysed in the context of a minimal model with one $Q = 2/3$ singlet. We have found that a large χ can lead to moderate departures of the SM approximate relation $S_{\phi K_S} \simeq S_{\psi K_S}$, with $S_{\phi K_S}$ approximately in the interval $[0.57, 0.93]$ (the precise range also depends on hadronic matrix elements). On the other hand, the effects on the CP asymmetry $S_{D_s^+ D_s^-}$ (and related channels) are much larger, with these asymmetries ranging in the interval $[-0.4, 0.4]$. These results must be compared with the ones for models with extra down singlets, where large departures of $S_{\phi K_S} \simeq S_{\psi K_S}$ can be accomodated [34] but $S_{D_s^+ D_s^-}$ is small and very close to the SM range [14]. Therefore, we can distinguish three possible New Physics scenarios:

1. If a small departure in the relation $S_{\phi K_S} \simeq S_{\psi K_S}$ and a large (but within $[-0.4, 0.4]$ approximately) $S_{D_s^+ D_s^-}$ are found, they may suggest the presence of a new $Q = 2/3$ singlet.

2. If a large departure in $S_{\phi K_S} \simeq S_{\psi K_S}$ is confirmed, but with $S_{D_s^+ D_s^-}$ very small, it may indicate the presence of a $Q = -1/3$ singlet.

3. In case that $S_{D_s^+ D_s^-}$ is found outside the interval $[-0.4, 0.4]$, or if a large departure in $S_{\phi K_S} \simeq S_{\psi K_S}$ and a large $S_{D_s^+ D_s^-}$ are simultaneously found, they require the presence of New Physics beyond these SM extensions with extra quark singlets, for instance supersymmetric models [35], which in principle could also explain the discrepancies in the two previous scenarios.

If New Physics hints are observed at B factories, its identification may be possible at a large collider, perhaps with the direct production of the new particles. In the SM extensions with extra up-type singlets studied we have found four correlated effects which can be investigated at three different types of colliders: (i) a large phase χ which has consequences on B oscillation phenomena at B factories; (*ii*) a FCN coupling X_{ct} which leads to top decays $t \to cZ$ observable at LHC; (*iii*) a deviation of X_{tt} from unity, which can be measured in $t\bar{t}$ production at TESLA; (*iv*) The direct production of a new quark at LHC. These associated effects, especially the discovery of the new particles, are crucial to establish the origin of New Physics, if observed.

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