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# ACOL TUNE SHIFTS AND DISPERSION CHANGE ASSOCIATED WITH QUADRUPOLE CURRENT VARIATIONS

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## 1. Introduction

In the ACOL machine, three trim power supplies will allow separate control of wide quadrupole strength to adjust the betatron tunes $Q_H$, $Q_V$ and the dispersion D. This paper derives the matrix which relates the tune shifts $\Delta Q_{H,V}$ and the dispersion change $\Delta D$ to quadrupole current variations $\Delta I_{1,2,3}$.

## 2. Fundamental relations

Changes in betatron tunes and horizontal orbit dispersion are associated with changes in quadrupole strength by the relations [1]

$$\Delta Q_{H,V} = \frac{1}{4\pi} \int_s^{s+C} \beta_{H,V}(\sigma)\, \Delta K_{H,V}(\sigma)\, d\sigma \tag{1}$$

$$\Delta D(s) = \frac{-\sqrt{\beta_H(s)}}{2\sin \pi Q_H} \int_s^{s+c} \sqrt{\beta_H(\sigma)}\, D(\sigma)\, \Delta K_H(\sigma) \cos(-\pi Q_H + \mu_H(\sigma) - \mu_H(\sigma))\, d\sigma \tag{2}$$

where C is the machine circumference.

Equation (1) can be replaced by a sum over the quadrupoles

$$\Delta Q_{H,V} = \frac{1}{4\pi} \sum_{j=1}^{n} \bar{\beta}_{H,Vj}\, l_j\, \Delta K_{H,Vj} \tag{3}$$

$\bar{\beta}_{H,Vj}$ and $l_j$ being respectively the average value at $\beta_{H,V}$ and the length of quadrupole j.

Equation (2) can also be transformed into a sum over the quadrupoles, defined by the values $\beta_{He}$, $\alpha_{He}$, $\mu_{He}$ at their entrance[2].

$$\Delta D(s) = -\frac{\sqrt{\beta_H(s)}}{2\sin\pi Q_H}\sum_{j=1}^{n}\Delta K_{Hj}\left\{C_{1j}\sqrt{\beta_{Hej}}\cos\varphi_j(s) - \frac{C_{2j}}{2\sqrt{\beta_{Hej}}}[(\alpha_{Hej}\cos\varphi_j(s)+\sin\varphi_j(s)\right\}$$

where

$$\varphi_i(s) = -\pi Q_H + \mu_{Hej} - \mu_H(s)$$

and

$$C_{1j} = \int_0^{l_i} a_j^{11}(\sigma)\, D_j(\sigma)\, d\sigma \qquad\qquad C_{1j} = \int_0^{l_i} a_j^{12}(\sigma)\, D_j(\sigma)\, d\sigma$$

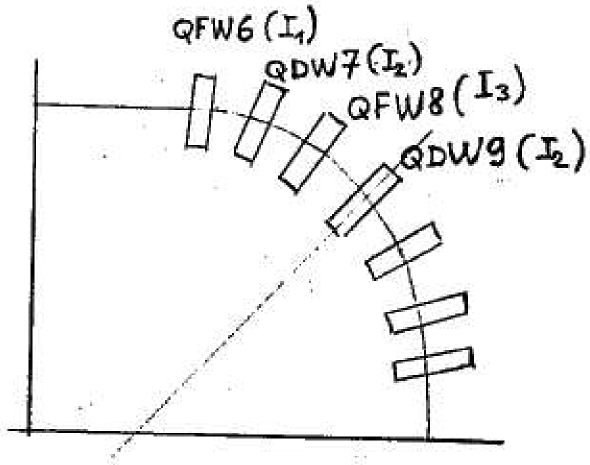
the $a_j^{11}$ and $a_j^{12}$ being the transfer matrix coefficients of the quadrupoles.

In the thin lens approximation, and assuming that the dispersion function may be replaced by its average value $\bar{D}_j$ in each quadrupole, equation (4) becomes

$$\Delta D(s) = -\frac{\sqrt{\beta_H(s)}}{2\sin\pi Q_H}\sum_{j=1}^{n}\Delta K_{Hj}\,\bar{D}_j\, l_j\left\{\sqrt{\beta_{Hej}}\cos\varphi_j(s) - \frac{l_j}{2\sqrt{\beta_{Hej}}}(\alpha_{Hej}\cos\varphi_j(s) + \sin\varphi_j(s))\right\} \qquad (5)$$

## 3. Correction matrix

The AC wide quadrupole (QW) are divided in three families which have a separate trim power supply (currents $I_1, I_2, I_3$). First and third families include each 8 horizontally focusing quadrupoles, the second includes 12 horizontally defocusing quadrupoles (see Figure).




AC first quadrant

The strength variation is produced by a change in quadrupole current

$$\Delta K_{H,V} = \frac{\Delta I}{I} K_{H,V} \qquad (6)$$

Using equations (3) and (5), the changes of betatron tunes and orbit dispersion with the quadrupole currents can be written in a matrix form

$$\begin{pmatrix} \Delta Q_H \\ \Delta Q_V \\ \Delta D \end{pmatrix} = M \begin{pmatrix} \Delta I_1 \\ \Delta I_2 \\ \Delta I_3 \end{pmatrix}$$

The matrix coefficient $m_{jk}$ are

$$m_{1k} = \frac{\partial Q_H}{\partial I_K} \qquad m_{2k} = \frac{\partial Q_V}{\partial I_K} \qquad m_{3k} = \frac{\partial D}{\partial I_K}$$

The analytical expressions being

$$m_{1k} = \frac{1}{I_k} \sum_i \bar{\beta}_{Hi}\, l_i\, K_{Hi} \qquad\qquad m_{2k} = \frac{1}{I_k} \sum_i \bar{\beta}_{Vi}\, l_i\, K_{Vi}$$

$$m_{3k} = \frac{-\sqrt{\beta_H}}{2\, I_k \sin \pi\, Q_H} \sum_i K_H\, \bar{D}_i\, l_i \left\{ \sqrt{\beta_{Hei}} \cos \varphi_i - \frac{l_i}{2\sqrt{\beta_{Hei}}} \left[\alpha_{Hei} \cos \varphi_i + \sin \varphi_i\right] \right\}$$

where $k=1,2,3$ and the sums are performed over all quadrupoles of a given family $I_k$.

The inverse matrix $M^{-1}$, called current corrections matrix, gives the trim current variations needed to achieve a requested correction for one parameter without disturbing the other two.

## 4. Numerical results

The interest is mainly concerned with dispersion correction in nominal zero-dispersion regions where the stochastic cooling systems are located. Consequently, the matrices are evaluated at longitudinal location zero. Numerical calculations using characteristic functions provided by optics program ORBIT[3] gives

$$M = 10^{-3} \begin{pmatrix} 1.36 & -0.62 & 1.43 \\ -0.40 & 1.90 & -0.53 \\ -0.48 & -1.59 & 6.84 \end{pmatrix}$$

and

$$M^{-1} = 10^{2} \begin{pmatrix} 7.19 & 1.15 & -1.41 \\ 1.79 & 5.93 & 0.09 \\ 0.93 & 1.46 & 1.38 \end{pmatrix} \begin{pmatrix} Q_H \\ Q_V \\ D \end{pmatrix}$$

D in m, in QDN01

$$\Delta Q_H = \Delta \cos \pi Q_H \cdot \frac{1}{-\pi \sin \pi Q_H}$$

Annex

Quadrupole characteristics[4]

| | QFW6 | QDW7 | QFW8 | $\overline{QW9}$ |
|---|---|---|---|---|
| l (m) | 0.758 | -0754 | 0.751 | 0.760 |
| $K_H (m^{-2})$ | 0.4406 | -0.4969 | 0.5453 | -0.3894 |
| I (A) | 1830.6 | 1884.8 | 1782.0 | 1884.8 |

Références


1. G. Guignard, Selection of formulae concerning proton storage rings, CERN 77-10 (1977).

2. B. Autin, Lattice perturbations, CERN/PS 84-22 (AA) (1984).

3. B. Autin, private communication, 1986.

4. L. Rinolfi, Distribution des quadripôles ACOL (unpublished note) (1986).