

# D-particles and the localization limit in quantum gravity

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## ABSTRACT

Some recent studies of the properties of D-particles suggest that in string theory a rather conventional description of spacetime might be available up to scales that are significantly smaller than the Planck length. We test this expectation by analyzing the localization of a space-time event marked by the collision of two D-particles. We find that a spatial coordinate of the event can indeed be determined with better-than-Planckian accuracy, at the price of a rather large uncertainty in the time coordinate. We then explore the implications of these results for the popular quantum-gravity intuition which assigns to the Planck length the role of absolute limit on localization.

# 1 Introduction and summary

The idea of an absolute limit on localization has a very long tradition in quantum-gravity research [1]. Some representative studies of various realizations of this idea can be found in Refs. [2, 3, 4, 5, 6, 7, 8, 9]. As a result, much of the research in quantum gravity is guided by the intuition that the localization of a spacetime event should be fundamentally limited by (at least [6, 7]) the Planck length,  $L_P \sim 10^{-33} \text{cm}$ , and that the length of a spacetime interval could at best be measured with Planck-length accuracy. The “pre-D-brane” perturbative string-theory literature provides support for these expectations, since the analysis of closed-string scattering indicates [10] a corresponding role for the string length  $L_s$  in measurability limits (which is compatible with the Planck-length limit, since in perturbative string theory  $L_P < L_s$ ). However, some more recent studies [11, 12, 13, 14] have obtained results in favour of the possibility that the collision region, for collisions of D-particles, could have substringy and subplanckian size. This is based on the observation [14] that D-particles with appropriately small relative velocity can be basically treated as ordinary very-weakly-interacting point particles up to distances as small as  $g_s^{1/3} L_s$ , without encountering comparatively large relative-position quantum uncertainties, in a framework where  $g_s \ll 1$  and  $L_P \sim g_s^{1/4} L_s$  (so that  $g_s^{1/3} L_s \ll L_P \ll L_s$ ).

These string-theory results have not affected the intuition of those working in other areas of quantum-gravity research. The Planck length is still assumed to set the absolute limit on localization and on length measurement. The D-particle results are apparently perceived as some sort of peculiarity of string theory, which should not affect the intuition of those approaching the quantum-gravity problem from a different perspective. We intend to show here that, on the contrary, the recent string-theory results on localization are a manifestation of a more general weakness of the arguments that were used to suggest that the Planck length sets the absolute limit on localization and on length measurement.

We observe that the D-particle results on localization exploit the fact that D-particles carry other charges in addition to the gravitational charge/mass. The traditional Planck-scale-limit quantum-gravity intuition was based on some analyses which implicitly assumed that the “target particle” (the particle whose position is of interest) and the “probe particle” (the particle used in the measurement procedure) interact only gravitationally. The type of subplanckian accuracy achievable with D-particles is instead the result of a combination of interactions. We conclude that the result of subplanckian accuracy obtained with D-particles exposes the fact that previous quantum-gravity analyses had assumed, without justification, that the particles participating in the localization procedure should only interact gravitationally. Even in other approaches to the quantum-gravity problem, possibly very different from string theory, it would not be surprising to find an analogous result of subplanckian accuracy. However we also find that the example of D-particles suggests that one might be able to achieve subplanckian accuracy in measurement of space position only at the cost of a rather sizeable uncertainty on time measurement.

After some preliminaries (Section 2) on the Bohr-Rosenfeld approach to measurability analysis in physics and on the standard Heisenberg microscope, in Section 3 we analyze localization via a Heisenberg-microscope procedure in the spirit of the quantum-gravity arguments [2, 3] which assume that the probe should be a massless neutral (but interacting gravitationally) particle and lead to the traditional expectation of separate Planck-length uncertainties for the measurement of space coordinates,  $\delta x \geq L_P$ , and time<sup>a</sup>,  $\delta t \geq L_P$ , of an event.

Examining this result we argue that a possible cause of concern is the fact that the associated quantum-gravity intuition is based on measurement analyses all assuming that a massless neutral probe should be used. While in practice localization procedures typically do rely on massless (or anyway relativistic) probes, in order to establish a fundamental measurability limit one should consider the problem in very general terms. It is possible that the probes that turn out to be useful for practical reasons are not the ones conceptually best suited for the task of localization. In order to establish a localization limit of more general validity one should in particular consider both relativistic and non-relativistic probes; moreover, one should consider both the case of probes which only interact gravitationally (“neutral probes”) and the case of probes which (in addition to gravitational charge/mass) also carry other charges (“charged probes”). In Section 4 we start by repeating the

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<sup>a</sup>We use conventions in which  $\hbar = c = 1$ .

same analysis of localization via a Heisenberg-microscope procedure, already considered in Section 3, but replacing the massless neutral probe with a nonrelativistic (massive) neutral probe. However, the analysis rapidly suggests that the use of a nonrelativistic neutral probe cannot improve on the localization limit established in the case of a relativistic neutral probe. The end result is that measurements using a nonrelativistic neutral probe can only achieve localization at the level  $\delta x \geq L_P/\sqrt{V_P^3}$ ,  $\delta t \geq L_P/\sqrt{V_P^5}$ , where  $V_P$  is the velocity of the nonrelativistic probe ( $V_P \ll 1$ ).

In Section 5 we consider the case of D-particles, as an example of probes which do not interact exclusively gravitationally. Our analysis does not require any of the most technical details of the string-theory framework which supports the presence of D-particles. We only take into account the fact that the underlying supersymmetry of the theoretical framework imposes that D-particles, besides carrying gravitational charge, also carry another charge associated (in an appropriate sense) to the gravitational charge through supersymmetry. As a result of a compensation between these two interactions [14] as long as the distance  $d$  between two D-particles is greater than  $\sqrt{v}L_s$  (denoting with  $v$  the relative velocity and  $L_s$  the string length) the energy stored in a two-D-particle system can be described as  $U \sim -L_s^6 v^4/d^7$  (up to an overall numerical factor of order 1). This energy law replaces the corresponding Newton energy law that applies to “neutral” particles. Another property of D-particles which is used in our analysis is the fact that their mass can be expressed as  $g_s^{-1}L_s^{-1}$  in terms of the string coupling and the string length. Using these properties of D-particles we find that the analysis of a Heisenberg-microscope procedure of localization of a collision between D-particles can indeed achieve accuracy  $\delta x \geq g_s^{1/3}L_s \sim g_s^{1/12}L_P$ . We also observe that such a level of spatial localization can only be achieved at the cost of a rather poor level of temporal localization. In fact, we find that the event is temporally localized with uncertainty  $\delta t \sim g_s^{-1/3}L_s \sim g_s^{-7/12}L_P$  (and, since  $g_s \ll 1$ , this amounts to an uncertainty limit which is significantly larger than the usually expected Planck-scale limit).

In Section 6 we stress that our observations for a localization measurement procedure are also applicable (as one would expect) to procedures for the measurement of the length of a spacetime interval.

The closing section (Section 7) is devoted to some comments on the implications of our analysis for the intuition that should guide quantum-gravity research. The example of D-particles suggests that we should not necessarily assume that both  $\delta x \geq L_P$  and  $\delta t \geq L_P$  hold independently. Our findings however provide support for the idea of a combined limit on position/time measurement, something which could perhaps be schematized with a relation of the type  $\delta x \delta t \geq L_P^2$ .

## 2 Preliminaries

### 2.1 The Heisenberg microscope

In Section 3, 4 and 5 we illustrate our point on localization in quantum gravity within the familiar framework of the Heisenberg-microscope measurement procedure. In order to render our discussion self-contained (and in order to introduce some notation which will be useful in the following sections) we provide here a brief review of the original (ordinary quantum mechanics, neglecting gravity) formulation of the Heisenberg-microscope measurement procedure [15, 16]. This original formulation of the microscope is used to explore the implications of the uncertainty relations for the accuracy with which it is possible to measure simultaneously the position and the momentum of an electron. The position measurement is achieved by means of a microscope, represented by a box with an opening which allows incoming light to hit a photographic plate. The electron is assumed to be located somewhere in the  $xy$  plane (see figure). For simplicity we consider the localization in the  $x$  coordinate only. A photon scatters off the electron via Compton scattering and is then collected in the microscope. Were it possible to keep the electron fixed in the same point of the  $xy$  plane during the experiment, one could then scatter a large number of photons off the electron, obtaining a diffraction pattern on the photographic plate of the microscope. The width of the first peak of the diffraction pattern would be given by

$$l' \simeq \frac{\lambda'}{\sin \beta} \simeq \frac{\lambda'}{\tan \beta} = \frac{\lambda' b}{L} \quad , \quad (1)$$

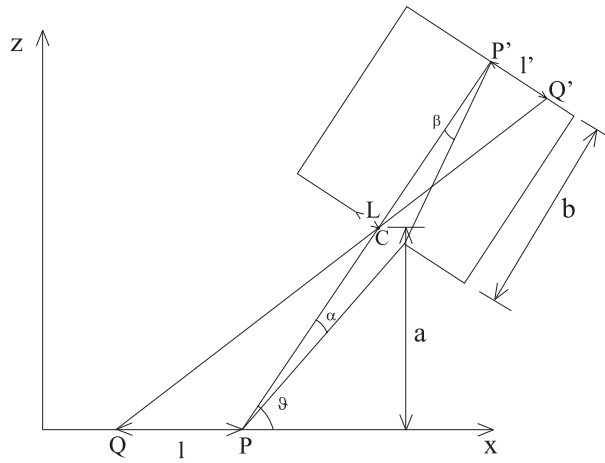


Figure 1: In the Heisenberg-microscope setup, if the target could be kept fixed in  $P$ , one could scatter a large number of photons off the target, obtaining a diffraction pattern centered in  $P'$  on the photographic plate of the microscope. If only one photon scatters off the target it is likely that it reaches the photographic plate at some point  $Q'$  within the first peak (of width  $l'$ ) of the expected diffraction pattern. Then  $Q'$  is most likely within a distance  $l'$  from  $P'$ . The projection  $Q$  of  $Q'$  through the centre  $C$  of the opening of the microscope is most likely within a distance  $l$  from  $P$ , which represents the actual position of the target.  $l$  is then the uncertainty in the measurement of the position of the target.

where  $\lambda'$  is the wavelength of the scattered photon, and  $\beta$ ,  $L$  and  $b$  are shown in the figure. By establishing the position of the center of the diffraction pattern one could then infer the position of the electron in the  $xy$  plane. However, already the first photon-electron collision would cause the electron to (acquire some momentum and) move away from its original position, which is the position that one is here attempting to measure. Thus the position measurement must be based on a single point on the photographic plate of the microscope, rather than on a whole diffraction pattern. This leads to an uncertainty in the measurement of the ( $x$ -component of the) position of the electron which can be estimated assuming that the single point on the photographic plate of the microscope must have appeared somewhere in the region which would have been occupied by the first peak of the diffraction pattern. The uncertainty is estimated as the width of the peak projected on the  $xy$  plane, and it is easy to verify that

$$\delta x \gtrsim \frac{a}{b} l' \simeq \frac{\lambda' a}{L} . \quad (2)$$

In order to also estimate the uncertainty on the  $x$ -component of the momentum of the electron,  $p_x$ , one can use the fact that in the photon-electron collision a part of the initial momentum of the photon is transferred to the electron, and after the collision  $p_x$  is given by

$$p_x \simeq p_x^0 + \frac{1}{\lambda} - p'_x , \quad (3)$$

where  $p_x^0$  is the  $x$ -component momentum of the electron before the scattering, and  $p'_x$  is the  $x$ -component of the momentum of the photon after the scattering. The direction taken by the photon after the scattering can be deduced from the fact that it was collected by the microscope. This allows us to estimate the uncertainty on the  $x$ -component of the momentum of the photon:

$$\delta p'_x \sim \frac{1}{\lambda'} \sin \alpha \sin \theta \gtrsim \frac{1}{\lambda'} \frac{L}{a} , \quad (4)$$

where  $\alpha$  is shown in figure.

The uncertainty on the  $x$ -component of the momentum of the electron will be of the same order:

$$\delta p_x \simeq \delta p'_x \gtrsim \frac{1}{\lambda'} \frac{L}{a} . \quad (5)$$

Finally, combining this equation with equation (2) one finds a limit on the simultaneous measurement of the  $x$ -components of the position and momentum of the electron,

$$\delta x \delta p_x \gtrsim 1 , \quad (6)$$

consistently with Heisenberg's uncertainty relation. Of course, a similar conclusion can be reached for the  $y$ - and  $z$ -directions, if needed, after a rotation of the apparatus.

The result (6) limits the simultaneous measurability of two observables,  $x$  and  $p_x$ , but does not constrain in any way the measurability of a single one of them. In this framework there is no in-principle reason (although it would not be feasible experimentally) that prevents one from measuring  $x$  exactly,  $\delta x = 0$ , although this should come at the cost of renouncing to all information on  $p_x$ .

In quantum gravity, as we will discuss in the following sections, it is instead expected that there should be an in-principle obstruction for achieving  $\delta x = 0$ .

## 2.2 Bohr-Rosenfeld probes

In the following sections we will analyze the Heisenberg-microscope procedure from a quantum-gravity perspective. In the original Heisenberg microscope one measures the position of an electron at rest and uses photons as probes, but we find that a key issue from the quantum-gravity perspective is the one of selecting the particles to be used as probes and the particles whose position is to be determined via the Heisenberg-microscope procedure. Clearly an electron at rest would not be a very sharp way to mark a spacetime point in quantum gravity, since already in relativistic quantum mechanics the localization of a particle of mass  $M$  at rest is limited by its Compton wavelength, and therefore it appears that large-mass particles should be preferable. On the other hand, as we shall discuss in greater detail later, gravity introduces a source of uncertainty that grows with the particle's mass. A balance of this competing sources of uncertainty must be achieved in order to obtain the true absolute limit on localization in quantum gravity.

We want to stress here that it is not uncommon that the choice of the particles used in the measurement procedure turns out to be a key point of the analysis of a measurability limit. The best example is provided by attempts (in the 1930s) to establish whether quantum electrodynamics sets a measurability limit for the electromagnetic fields. Various arguments had suggested that there might be a logical inconsistency in quantum electrodynamics, since on the one hand the formalism predicts no absolute limit on the measurability of the electromagnetic fields (if one is willing to lose all information on some conjugate fields), whereas the analysis of several gedanken measurement procedures had provided evidence of an absolute measurability limit. The situation was clarified in a study by Bohr and Rosenfeld [17], who proposed a gedanken measurement procedure using probes of charge  $Q$  and inertial mass  $M$  in such a way to obtain an uncertainty on the measurement of an electromagnetic field that is proportional to the ratio  $Q/M$ . Considering the limit  $Q/M \rightarrow 0$  the measurement procedure reproduced the result expected on the basis of the formal structure of quantum electrodynamics, *i.e.* an uncertainty-free measurement of the relevant electromagnetic field.

In the Bohr-Rosenfeld analysis clearly the nature of the probes used in the measurement procedure plays a key role, in light of the important dependence on  $Q/M$ . This provides some guidance for the study we are here reporting: we should be open to the possibility that the localization of a spacetime point may depend significantly on the particles used in giving operative meaning to that point.

The Bohr-Rosenfeld analysis also provides insight on the nature of the particles to be used as probes in measurement analysis. In fact, it is noteworthy that not only the measurability limit may be different if we use different "known particles" (the ratio  $Q/M$  has a different value for electrons and muons), but it appears that we should consider all particles that are allowed by the formalism, even when these particles have not been observed in Nature. This is the line of analysis advocated by Bohr and Rosenfeld when they contemplate the limit  $Q/M \rightarrow 0$ . If one restricts the analysis to known particles it would of course not be possible to gain access to the  $Q/M \rightarrow 0$  limit and therefore the (uncertainty-free) result expected on the basis of the formal structure of quantum electrodynamics could not be achieved by the measurement procedure. However, Bohr and Rosenfeld [17] stress that quantum electrodynamics does not predict its constituents (*e.g.* it predicts how electrons interact

but it does not predict the existence of electrons) and its intrinsic structure should not be assumed to depend on elements external to the theory, such as indeed the types of probes that happen to be available in Nature. In actual measurements the measurability of an electromagnetic field will be limited due to various practical facts, including the fact that Nature does not make available to us particles with arbitrarily small ratio  $Q/M$ , but quantum electrodynamics does predict in a logically consistent way the absence of an in-principle limitation of the measurability of electromagnetic fields, which finds its support in measurement theory upon considering the limit of particles with arbitrarily small ratio  $Q/M$ .

### 3 Localization of an event marked by the collision between a photon and a neutral massive particle

As mentioned, most approaches to quantum gravity are guided by the intuition that the Planck length should set absolute limits on the measurability of spatial distances,  $\delta x \geq L_P$ , and time intervals,  $\delta t \geq L_P$ . The emergence of these measurability limits has been suggested by various types of analyses [2, 3, 8], and can provide some guidance in proposing schemes for spacetime noncommutativity [4, 5, 18] and certain types of spacetime discretization [19]. While in different measurement procedures the details of the analysis can be rather different, this result is basically inevitable if quantum mechanics and general relativity are combined straightforwardly. In fact, as we stressed earlier, quantum mechanics introduces a ‘‘Compton-wavelength uncertainty’’ in localization, which decreases with the particle mass, and for a particle of mass of order  $L_P^{-1}$  leads to a localization uncertainty of order  $L_P$ . On the other hand general relativity introduces a ‘‘Schwarzschild-radius uncertainty’’ in localization, which increases with the particle mass, and for a particle of mass of order  $L_P^{-1}$  leads to a localization uncertainty which is also of order  $L_P$ . So, as long as quantum mechanics and general relativity are the only ingredients of the analysis, the absolute limit on localization is necessarily of order  $L_P$  (and is achieved in the measurement of the position of particles with mass of order  $L_P^{-1}$ ).

In order to illustrate the derivation of this limit within a complete analysis of a localization measurement procedure, in this section we consider, in the spirit of Refs. [2, 3], a reformulation of the Heisenberg microscope gedanken experiment that takes into account gravitational effects. For simplicity we describe the gravitational interaction between the probe, a photon, and the target, a massive neutral particle at rest, using a semi-Newtonian framework<sup>b</sup>. In Refs. [2, 3, 8] the mass of the particle whose position is being measured (which we will sometimes call ‘‘target particle’’) is not specified, but of course, as we stressed above, its mass cannot be smaller than  $L_P^{-1}$  (otherwise its Compton wavelength would be larger than the sought position accuracy  $L_P$ ), and its mass cannot be larger than  $L_P^{-1}$  (otherwise the measurement procedure should bring the photon inside the Schwarzschild radius of the particle, since the probe-target distance must be at some point of the order of  $L_P$  if the position measurement procedure must achieve  $L_P$  accuracy). We will therefore implicitly assume that the ‘‘target particle’’ is of Planckian mass.

The line of reasoning that we adopt here is slightly different from that of the analysis of the Heisenberg-microscope procedure in Subsection 2.1. While in ordinary quantum mechanics (with its Galileo-Newton spacetime background) one is exclusively interested in space-position localization, from a quantum-gravity perspective one is of course interested primarily in the spacetime localization of an event. Therefore our attention is here shifted from the measurement of the position in space of a target particle, to the measurement of the spacetime coordinates of the event of collision between the probe and the target. Whereas in the original Heisenberg-microscope analysis one considers the simultaneous measurement of a coordinate and of the corresponding component of the momentum of the target particle, here one is interested primarily in the measurement of two coordinates, one space coordinate and the time coordinate of the spacetime event of collision.

We must also stress that in principle the analysis of such a measurement procedure from a quantum-gravity perspective should take into account a very large number of potential sources of

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<sup>b</sup>As shown in Ref. [2], the estimates obtained using this semi-Newtonian gravity turn out to be correct (using general relativity one obtains the same estimates, after a somewhat more tedious analysis).

contributions to the overall uncertainty. We do not claim to consider all of these possible sources of uncertainty, but we focus on some which appear to be most significant for the quantum gravity analysis. The uncertainties we do consider lead to an absolute measurability limit. Other sources of uncertainty (that may be present but are not considered in our analysis) could in principle lead to a stricter limit, but the limit we obtain is absolute (cannot be violated, since the presence of other sources of uncertainty of course could not lead to an improved accuracy).

The possibility to lower/violate our absolute limit could be contemplated however for other measurement procedures. The Heisenberg-microscope procedure is one of the possible ways to localize a spacetime event and is clearly affected by the absolute localization limit we describe. Although this seems unlikely to us (and the analysis of a few alternative measurement procedures will quickly lead the reader to an analogous intuition), one cannot exclude the possibility that some other localization procedure may not be affected by such a localization limit. We will take as working assumption that this is not the case, but it cannot be excluded in principle.

We denote with  $X$  the spatial coordinate and with  $T$  the time coordinate<sup>c</sup> which are to be measured. As in the traditional Heisenberg-microscope procedure the observable  $X$  is here still obtained by observing the position of arrival of the probe on the photographic plate of the microscope, while  $T$  can be obtained as

$$T \simeq t_i + X \quad , \quad (7)$$

where  $t_i$  marks the instant when the experiment begins (the instant when the probe is fired toward the target).

The uncertainties on the measure of  $X$  and  $T$ ,  $\delta X$  and  $\delta T$  respectively, can be described in this way<sup>d</sup>:

$$\delta X \gtrsim \delta X_c + \delta x_0 + d_{\min} + \delta x_p + \delta x_t \quad (8)$$

$$\delta T \gtrsim \delta t_i + \delta X$$

where:

$\delta X_c$  is the uncertainty due to the width of the opening of the microscope;

$\delta x_0$  is the uncertainty on the initial position of the probe (which quantum mechanics relates to the uncertainty in the initial momentum of the probe);

$d_{\min}$  is the minimum distance between probe and target reached during the measurement procedure;

$\delta x_p$  is the uncertainty on the position of the probe at the moment of the collision;

$\delta x_t$  is the uncertainty on the position of the target at the moment of the collision;

$\delta t_i$  is the uncertainty on the instant when the probe is fired toward the target.

We start by noticing that  $\delta X_c$  is an uncertainty of classical-mechanics origin, and it can be reduced at will by varying the width of the opening of the microscope. Therefore  $\delta X_c$  cannot be significant in establishing an absolute limit on localization.

Concerning  $\delta x_t$  we must stress that clearly (on the basis of our considerations concerning the competing contributions to the  $\delta x_t$  uncertainty due to the ‘‘Compton-wavelength uncertainty’’ and the ‘‘Schwarzschild-radius uncertainty’’) one must necessarily find  $\delta x_t \geq L_P$ . Therefore the best we can hope for is a localization at the level  $\delta X \sim L_P$ . We intend to show, through an analysis of the other sources of uncertainty, that  $\delta X \sim L_P$  can be achieved. We will in general denote with  $\delta x^*$  the

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<sup>c</sup>These are the  $X$  coordinate and the time coordinate ‘‘of the event’’. The careful reader might notice that the spacetime point marked by the probe-target collision is not even sharply defined, since the probe and the target have ‘‘finite size’’ (intrinsic position uncertainty) and there is no instant in the collision at which the probe and the target have the same spacetime coordinates. This, however, is of merely academic concern: in a measurement procedure that aims for space/time accuracies of order  $L_P$  it is sufficient to introduce all aspects of the analysis with corresponding accuracy, and indeed we will make sure that the setup is such that the spacetime point marked by the collision is specified with  $L_P$  accuracy.

<sup>d</sup>Our analysis looks only for an estimate of orders of magnitude, and therefore we will discard all numerical factors of order 1.

accuracy for which one aims in the measurement procedure. In this case, with  $\delta x^* \sim L_P$ , consistently we must take

$$\begin{aligned}\delta x_0 &\lesssim \delta x^* \sim L_P \\ d_{\min} &\lesssim \delta x^* \sim L_P\end{aligned}\tag{9}$$

The fact that, as stressed above, our analysis assumes that the mass of the target is of the order of the Planck scale,  $m_t \sim L_P^{-1}$ , is compatible with the choice  $d_{\min} \sim L_P$  (we remind the reader that the minimum probe-target distance  $d_{\min}$  must not be smaller than the Schwarzschild radius of the target particle,  $d_{\min} \gtrsim Gm_t$ ).

From the fact that  $\delta x_0 \lesssim L_P$  it follows that

$$\delta p_p^0 \sim \frac{1}{\delta x_0} \gtrsim \frac{1}{L_P}\tag{10}$$

where  $p_p^0$  is the initial momentum of the probe, and  $\delta p_p^0$  is the corresponding uncertainty. From the fact that of course we must demand that  $p_p^0 \gtrsim \delta p_p^0$ , one finds that

$$p_p^0 \gtrsim \delta p_p^0 \sim \delta x_0^{-1} \gtrsim E_P \quad ,\tag{11}$$

where  $E_P$  is the Planck energy.

Next we should consider the contribution to the uncertainties that originates from the uncertainty in the energy stored in the probe-target system in the course of the collision process. For our purposes (since we are only looking for an order-of-magnitude estimate) it is sufficient to consider this issue only for the stage of the collision in which the probe-target distance is of order  $d_{\min}$ . At such short probe-target distances there is a rather strong gravitational field, which is significantly affected by the uncertainty  $\delta p_p^0$ . However, this strong gravitational field is only present when indeed the probe-target distance is small. At early and late times in the measurement procedure nearly all the energy of the system is stored as kinetic energy, and we can expect that the kinetic energy of the probe at early and late times will be of the same order<sup>e</sup>.

In order to estimate  $(\delta p_p)_x$ , the uncertainty on the  $x$ -component of the probe's momentum, we can proceed as in the preceding section, relying on the observation that the measurement procedure requires that the probe reaches the photographic plate of the microscope. Therefore

$$(\delta p_p)_x \lesssim p'_p \sin \alpha \simeq p'_p \tan \alpha = p'_p \frac{L}{a} \sim p_p \frac{L}{a} \quad ,\tag{12}$$

where  $p'_p$  is the momentum of the probe after the scattering,  $\alpha$  is defined in the figure, and we also used the observation that  $p'_p \sim p_p$ .

From this it follows that

$$\delta x_p \gtrsim \frac{1}{p_p \frac{L}{a}} \quad .\tag{13}$$

This  $\delta x_p$  describes the uncertainty on the probe's  $x$  coordinate at the moment in which the probe reaches the opening of the microscope. This same  $\delta x_p$  represents a good estimate of the uncertainty

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<sup>e</sup>In the collision the target (which was initially at rest) will only take away from the probe a small (negligible) fraction of the kinetic energy, even though the target acquires a nonnegligible momentum. This is due to the large mass of the target (as compared to the massless particle used as probe). In general, for a photon scattered along the direction characterized by scattering angle  $\theta$  the relation between the momentum of the probe before the collision,  $p_p$ , and the momentum of the probe after the collision,  $p'_p$ , is set by the formula  $p'_p = p_p/[1 + p_p(1 - \cos \theta)/m_t]$ , where  $m_t$  denotes again the mass of the "target" particle.



on the probe's  $x$  coordinate at the moment of the collision (all distance scales in the measurement procedure are certainly small enough that the spread of the probe's wave packet is negligible [20]).

Finally we need an estimate for  $\delta x_t$ , the uncertainty on the target's  $x$  coordinate at the moment of the collision. A key contribution to this uncertainty (in addition to the Compton-wavelength uncertainty, which is under control with our choice of target-particle mass) originates from the gravitational interaction between probe and target. By momentum conservation we have that the gravitational interaction must induce correlated variations of the target's momentum and of the probe's momentum,

$$\Delta p_t(t) \simeq \Delta p_p(t) \simeq \frac{G p_p m_t}{d(t)} \quad , \quad (14)$$

so that

$$(\delta v_t)_x(t) = \frac{(\delta p_t)_x(t)}{m_t} \simeq \frac{\Delta p_p(t)}{m_t} \frac{L}{a} \quad (15)$$

(Note that in this equation we also used the fact that, because of the large mass of the target, while the target's momentum is nonnegligible, its velocity  $v_t$  is small.)

Our estimate of  $\delta x_t$  is based on the fact that it must be greater than (but comparable to) the corresponding uncertainty that develops in a small time interval, of size  $d_{\min}$ , around  $T$ :

$$\delta x_t \geq \int_{T-d_{\min}}^T |(\delta v_t)_x| dt \simeq \int_{T-d_{\min}}^T \left| \frac{\Delta p_p(t)}{m_t} \right| \frac{L}{a} dt \sim \left. \frac{\Delta p_p(t)}{m_t} \right|_{d_{\min}} d_{\min} \frac{L}{a} \simeq G p_p \frac{L}{a} \quad . \quad (16)$$

We are finally ready to combine all the contributions to  $\delta X$  listed in (8), obtaining

$$\delta X \gtrsim \delta X_c + \delta x_0 + d_{\min} + \delta x_p + \delta x_t \gtrsim \delta X_c + \delta x_0 + d_{\min} + \frac{1}{p_p \frac{L}{a}} + G p_p \frac{L}{a} \quad . \quad (17)$$

Not all of these contributions are equally significant from our quantum-gravity perspective. We already stressed that  $\delta X_c$  depends on the structure of the microscope, and can be reduced at will, while  $\delta x_0$  and  $d_{\min}$  depend on the characteristics of the probe and the target, and are chosen precisely in order to reach the intended  $\delta x^* \sim L_P$  accuracy goal. One easily sees that, in order to verify that this sought accuracy can be actually reached, it is necessary to examine the contributions  $\delta x_p$  and  $\delta x_t$ , since one of them decreases as  $p_p$  is increased while the other increases<sup>f</sup> with  $p_p$  (and therefore they cannot be both made small at will). The minimum-uncertainty case corresponds to

$$\min(\delta x_p + \delta x_t) = \sqrt{\frac{1}{p_p \frac{L}{a}} G p_p \frac{L}{a}} = L_P \quad . \quad (18)$$

Thus, we find (as expected) that it is indeed possible to reach the intended accuracy of the order of the Planck length:

$$\min(\delta X) \sim L_P \quad . \quad (19)$$

The analysis of the  $\delta x_p$  and  $\delta x_t$  contributions also reassures us that indeed it would have not been possible to aim for anything better than Planck-length accuracy: even choosing  $\delta x_0$  and  $d_{\min}$  smaller

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<sup>f</sup>This is a key point: in ordinary quantum mechanics one achieves as good a localization as desired by increasing the momentum of the probe, whereas by taking into account gravitational effects an increase in the momentum of the probe is not always beneficial for localization. A higher frequency photon, while resolving smaller lengths, brings about a more intense gravitational field, which ends up introducing a bigger uncertainty in the target's position.

than the Planck length one would still inevitably find a Planck-length uncertainty due to the combined contribution from  $\delta x_p$  and  $\delta x_t$ .

For the uncertainty on  $T$  we find

$$\delta T \gtrsim \delta t_i + \delta X \gtrsim L_P \quad , \quad (20)$$

where

$$\delta t_i \sim \frac{1}{\delta E_p} \sim \frac{1}{\delta p_p} \sim \delta x_0 \sim L_P \quad . \quad (21)$$

The result (19) for  $\delta X$  is widely accepted [2, 3, 8], and can be obtained on the basis of several types of measurement analyses, of which our Heisenberg-microscope procedure is only one example. Studies of the limit on  $\delta T$  are somewhat less numerous, but our result (20) also finds support in all the related literature [2, 3, 8].

As mentioned, the intuition emerging from these analyses is also consistent with some results obtained in the “pre-D-brane” perturbative string-theory literature, since the analysis of closed-string scattering [10] provides support for  $\delta X \geq L_s$  (where  $L_s$  is the string length, and in the relevant framework  $L_P < L_s$ ).

## 4 Localization of an event marked by the collision between two neutral massive particles

The localization limit  $\delta X \geq L_P$ ,  $\delta T \geq L_P$  is widely accepted within the quantum-gravity community. We feel that it is often overlooked that the derivation of this localization limit relies on two crucial ingredients: the nature and strength of the gravitational interactions and the fact that a massless particle with energy uncertainty  $\delta E$  has position uncertainty  $1/\delta E$ . As stressed earlier, while in practice localization procedures typically do rely on massless (or anyway relativistic) probes, according to the Bohr-Rosenfeld line of analysis [17] it is necessary to wonder whether the probes that turn out to be useful for practical reasons are the ones conceptually best suited for the task of localization. In order to establish a localization limit of more general validity one should in particular consider both relativistic and non-relativistic probes; moreover, one should consider both the case of probes which only interact gravitationally (“neutral probes”) and probes which, in addition to gravitational charge/mass, also carry other charges (“charged probes”). In this Section we start by repeating the same analysis of localization via a Heisenberg-microscope procedure, already considered in the previous Section, but replacing the massless neutral probe with a nonrelativistic (massive) neutral probe.

We denote by  $m_p$  and  $m_t$  respectively the masses of the probe and the target. Our analysis can follow the same steps already discussed in the previous section. The only key differences originate from the fact that here the speed of the probe is taken to be much smaller than 1. Again the  $X$  coordinate of the collision event can be obtained by observing the position of arrival of the probe on the photographic plate of the microscope, while  $T$  can be here estimated from

$$T \simeq t_i + \frac{X}{v_p} \quad , \quad (22)$$

where  $v_p$  is the speed of the probe.

$\delta X$  and  $\delta T$  are again a combination of various contributions:

$$\begin{aligned} \delta X &\gtrsim \delta x_0 + d_{\min} + \delta x_p + \delta x_t \\ \delta T &\gtrsim \delta t_i + \frac{\delta X}{v_p} + \frac{X}{v_p^2} \delta v_p \quad , \end{aligned} \quad (23)$$

where  $\delta x_0$ ,  $d_{\min}$ ,  $\delta x_p$ ,  $\delta x_t$ ,  $\delta t_i$  have been defined in the preceding Section 3, while  $\delta v_p$  is the uncertainty on  $v_p$ . (Of course, also in this case there is a  $\delta X_c$  contribution, which we are omitting since, as clarified in the previous section, it is of classical-mechanics origin and can therefore be reduced at will.)

Our objective here is to show that, using a nonrelativistic probe, one finds an absolute limit on localization with minimum uncertainty larger than  $L_P$ . We therefore set up the measurement procedure aiming for Planck-length accuracy

$$\delta x_0 \lesssim \delta x^* \sim L_P \quad (24)$$

$$d_{\min} \lesssim \delta x^* \sim L_P \quad . \quad (25)$$

and show that there are some contributions to  $\delta X$  and  $\delta T$  which lead to a worse-than-Planckian result.

Again we observe that from (25) one obtains

$$\delta p_p^0 \sim \frac{1}{\delta x_0} \gtrsim \frac{1}{L_P} \quad (26)$$

and that from the requirement  $p_p^0 \gtrsim \delta p_p^0$  it then follows that

$$p_p^0 \gtrsim \delta p_p^0 \sim \delta x_0^{-1} \gtrsim E_P \quad , \quad (27)$$

(we are again denoting with  $p_p^0$  the initial momentum of the probe and with  $\delta p_p^0$  the corresponding uncertainty).

Here, with the nonrelativistic probe, the fact that  $\delta p_p^0 \gtrsim 1/L_P$  implies  $\delta v_p \gtrsim 1/(m_p L_P)$ , a rather large velocity uncertainty. Since we will anyway find that the use of the nonrelativistic probe leads to worse-than-Planckian localization, we allow ourself a “rather optimistic” attitude<sup>g</sup> concerning the possibility of satisfying the requirements  $v_p \gtrsim \delta v_p$  and  $v_p < 1$ .

In order to estimate  $\delta x_t$  we must consider the effects induced by the gravitational probe-target interaction at least over a small time interval, of size  $d_{\min}$ , around  $T$ . We observe that also in this case it is legitimate to assume  $p_p' \sim p_p$ , *i.e.* the momentum of the probe at early and late times is of the same order<sup>h</sup>. This allows us to proceed just like in Eqs. (12)-(13) of the previous section, finding

$$\delta x_p \gtrsim \frac{1}{p_p \frac{L}{a}} \quad . \quad (28)$$

and finding for  $\delta x_t$  (again proceeding as in the previous section)

$$\delta x_t \geq \int_{T-\frac{d_{\min}}{v_p}}^T |(\delta v_t)_x| dt \simeq \int_{T-\frac{d_{\min}}{v_p}}^T \left| \frac{\Delta p_p(t)}{m_t} \right| \frac{L}{a} dt \sim \frac{\Delta p_p(t)}{m_t} \Big|_{d_{\min}} \frac{d_{\min}}{v_p} \frac{L}{a} \quad . \quad (29)$$

where  $\Delta p_p(t)$  is the variation of the momentum of the probe induced by the gravitational interaction at the instant  $t$  (where  $t$  is taken to be close to  $T$ ).

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<sup>g</sup>As mentioned, in order to keep the “Schwarzschild-radius uncertainty” at or below the accuracy goal  $L_P$ , it is necessary to assume  $L_P^{-1} \gtrsim m_p$ . On the other hand one must have  $v_p < 1$  in order to be consistent with the nonrelativistic nature of the probe and one must have  $v_p \geq \delta v_p \gtrsim 1/(m_p L_P)$  in order to be able to aim the probe toward the target. The fact that this requirements cannot be simultaneously implemented is already an indication of the fact that the  $L_P$  accuracy is not within the reach of a measurement procedure using a nonrelativistic neutral probe. We will set this concern aside, and find that it is anyway impossible to achieve  $L_P$  accuracy using a nonrelativistic neutral probe.

<sup>h</sup>This can be easily verified by looking at the formulae that describe nonrelativistic scattering between particles of equal mass, in the case where one of the particles is initially at rest while the other particle initially carries a large momentum. For generic scattering angle one finds that the two particles carry final momenta of the same order of magnitude (which of course is the same order of magnitude of the initial momentum of the probe). One well-known exception is the case of a “central collision”, in which the particle initially at rest ends up carrying all the momentum in the final configuration, but of course this is not a viable option for the setup of our microscope.

To estimate  $\Delta p_p(t)$  we can argue as follows. Considering the probe's total energy  $E_p$  and the gravitational potential energy  $U(t)$  originated by the interaction with the target

$$E_p = m_p + \frac{p_p^2}{2m_p} + U(t) \quad , \quad (30)$$

we can estimate the probe's momentum as

$$p_p = \sqrt{2m_p(E_p - U(t) - m_p)} \quad , \quad (31)$$

and we find that

$$\Delta p_p \simeq -\frac{m_p}{p_p} \Delta U \sim \frac{m_p}{p_p} \frac{Gm_t m_p}{d_{\min}} = \frac{Gm_t m_p}{v_p d_{\min}} \quad . \quad (32)$$

Using (29) this allows to derive

$$\delta x_t \sim \frac{\Delta p_p(t)}{m_t} \Big|_{d_{\min}} \frac{d_{\min} L}{v_p a} \simeq \frac{Gm_p}{v_p d_{\min}} \frac{d_{\min} L}{v_p a} = \frac{Gp_p L}{v_p^3 a} \quad , \quad (33)$$

and finally, combining all these results, we find that the total uncertainty on  $X$  is estimated by

$$\delta X \gtrsim \delta x_0 + d_{\min} + \delta x_p + \delta x_t \gtrsim \delta x_0 + d_{\min} + \frac{1}{p_p \frac{L}{a}} + \frac{Gp_p L}{v_p^3 a} \geq \frac{1}{p_p \frac{L}{a}} + \frac{Gp_p L}{v_p^3 a} \quad , \quad (34)$$

where on the right-hand side we are inviting the reader to focus on two of the contributions. Those two contributions are sufficient for establishing our result that using the nonrelativistic neutral probe only a worse-than-Planckian accuracy is achievable. In fact,

$$\delta X \geq \min(\delta x_p + \delta x_t) = \sqrt{\frac{1}{p_p \frac{L}{a}} \frac{Gp_p L}{v_p^3 a}} = \frac{L_P}{v_p^{\frac{3}{2}}} \quad , \quad (35)$$

which indeed represents worse-than-Planckian accuracy (since the hypothesis of using a nonrelativistic probe requires  $v_p < 1$ ).

The use of a nonrelativistic neutral probe is even more obviously costly for the accuracy in the measurement of  $T$ . In fact, on the basis of the observations reported above one finds

$$\delta T \gtrsim \delta t_i + \frac{\delta X}{v_p} + \frac{X}{v_p^2} \delta v_p \sim \delta t_i + \frac{\delta X}{v_p} + X v_p^{-\frac{1}{2}} \quad , \quad (36)$$

with

$$\delta t_i \sim \frac{1}{\delta E_p} \sim \frac{1}{v_p \delta p_p} \sim \frac{\delta x_0}{v_p} \quad . \quad (37)$$

Since in a reasonable Heisenberg-microscope setup the distance  $X$  (distance between the point where the experimenter introduces the probe and the point where the probe-target collision occurs) is macroscopic, the contribution  $X v_p^{-1/2}$  can be very large. And in any case the contribution  $\delta X/v_p$  implies that  $\delta T$  is clearly larger than  $L_P$ :  $\delta X$  is already larger than  $L_P$  by at least  $v_p^{-3/2}$  and therefore  $\delta T$  is larger than  $L_P$  by at least  $v_p^{-5/2}$ .

## 5 Localization of an event marked by the collision between two D-particles

In our investigation of the expectation that a localization limit  $\delta X \geq L_P$ ,  $\delta T \geq L_P$  should hold in quantum gravity we observed that this limit is essentially based on the analysis of measurement procedures in which a massless neutral particle probes the position of a massive target particle, and we argued that a more robust estimate of the localization limit could be achieved by considering both relativistic and non-relativistic probes, and by considering both the case of neutral probes (probes which only interact gravitationally) and the case of charged probes (probes which, in addition to a gravitational charge/mass, also carry other charges). In the previous section we showed that replacing the massless neutral probe with a nonrelativistic neutral probe one cannot improve on the expected localization limit  $\delta X \geq L_P$ ,  $\delta T \geq L_P$ . In this Section we use the case of D-particles as an illustrative example of the role that charged particles might have in allowing to achieve an improved level of localization.

D-particles are zero-dimensional (pointlike) D-branes, topological objects on which open strings end. In Refs. [13, 14] the scattering of D-particles in the type IIA string theory was studied in ten spacetime dimensions, and it was found that the minimal size of the collision region could be well below the “ten-dimensional Planck length”<sup>i</sup>.

In the relevant theoretical framework the string coupling constant is small,  $g_s \ll 1$ , and the Planck length is related to the string length,  $L_s$ , by  $L_P \sim g_s^{1/4} L_s$ . One therefore has the following hierarchy of scales:

$$g_s^{1/3} L_s \ll L_P \sim g_s^{1/4} L_s \ll L_s \quad , \quad (38)$$

which will play an important role in our analysis. Also important for our analysis of the Heisenberg-microscope procedure using D-particles is the fact that D-particles have mass higher than the Planck mass [13, 14]:

$$m = \frac{1}{g_s L_s} \sim \frac{1}{g_s^{3/4} L_P} \gg \frac{1}{L_P} \quad , \quad (39)$$

and that in a two-D-particle system, in addition to the gravitational interaction one must take into account a sort of companion interaction (a requirement which primarily follows from the supersymmetry of the theoretical framework) and the two interactions largely compensate each other, leading to a net interaction which is governed by the potential energy

$$U \sim -L_s^6 \frac{v^4}{(r)^7} + \mathcal{O} \left( \frac{v^6 L_s^{10}}{(r)^{11}} \right) \quad , \quad (40)$$

as long as the distance  $r$  between the two D-particles and the relative velocity  $v$  of the two D-particles are such that

$$r \gtrsim \sqrt{v} L_s \quad . \quad (41)$$

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<sup>i</sup>Since the analysis is in a 10-dimensional spacetime of course the relevant length scale is the corresponding Planck length. In presence of “large extra dimensions” the relation between this ten-dimensional Planck length and the Planck length (gravitational coupling constant) we observe in the four spacetime dimension we perceive may be nontrivial. However, the presence of large extra dimensions is not necessary, and in fact it was not assumed in Refs. [13, 14]. Moreover, this issue related to the possible presence of large extra dimensions is irrelevant for our line of analysis: the quantum-gravity intuition in favour of the localization limit  $\delta X \geq L_P$ ,  $\delta T \geq L_P$  applies equally well to the case of a four-dimensional spacetime and to the case of a ten-dimensional spacetime. The point we are trying to investigate is whether (for whatever choice of number of spacetime dimensions) the localization limit  $\delta X \geq L_P$ ,  $\delta T \geq L_P$  can be improved upon. While in studies with different objectives it is sometimes appropriate to denote by  $L_P^{(10)}$  the ten-dimensional Planck length, in order to maintain a distinction from the four-dimensional Planck length, this type of notation is unnecessary in our analysis and we therefore denote simply by  $L_P$  the Planck length, independently of the number of dimensions chosen for spacetime.

(Although it is irrelevant for our analysis, since the localization procedure requires a nonvanishing probe-target relative velocity, it is noteworthy that, in particular, for a system of two D-particles at rest there is no force.)

On the basis of these properties, the analysis of collisions between two D-particles suggests [13, 14] that the minimal dimension of the collision region is given by  $g_s^{1/3} L_s$  which is indeed below the Planck length ( $g_s \ll 1$  implies  $g_s^{1/3} L_s \ll g_s^{1/4} L_s \sim L_P$ ). In light of this result it is natural to wonder whether D-particles can be used for accurate localization of a spacetime point. It is from this perspective that we consider a Heisenberg-microscope procedure in which both the probe and the target that collide are D-particles. As in the other Heisenberg-microscope procedures we analyzed, the  $X$  coordinate can be measured by measuring where the probe hits the “photographic” plate of the microscope, and the  $T$  coordinate can be measured as

$$T \simeq t_i + \frac{X}{v_p} \quad . \quad (42)$$

And once again the uncertainties on the measurement of  $X$  and  $T$  are given by (omitting again a classical-physics contribution of the type  $\delta X_c$  which could anyway be reduced at will)

$$\delta X \gtrsim \delta x_0 + d_{\min} + \delta x_p + \delta x_t \quad (43)$$

$$\delta T \gtrsim \delta t_i + \frac{\delta X}{v_p} + \frac{X}{v_p^2} \delta v_p \quad , \quad (44)$$

using notation already introduced in Eqs. (8) and (23). Here however we assume that  $m_t = m_p = m$ , with  $m$  given by (39) (*i.e.* the target and the probe are D-particles, of mass  $m$ ).

On the basis of the results of Refs. [13, 14] we are encouraged to aim for  $g_s^{1/3} L_s$  accuracy:

$$\delta x^* \sim g_s^{1/3} L_s \quad . \quad (45)$$

and we therefore take

$$\delta x_0 \simeq \delta x^* \sim g_s^{1/3} L_s \quad (46)$$

$$\delta v_p \simeq \frac{1}{m \delta x_0} \sim g_s^{2/3} \quad , \quad (47)$$

where the last equation also takes into account  $p_p \simeq m v_p$  (small  $v_p$ ). Indeed  $v_p$  must be small in order for (40) to be applicable:

$$d_{\min} \simeq \sqrt{v_p} L_s \sim g_s^{1/3} L_s \quad , \quad (48)$$

where we also took into account that for consistency with our accuracy objective we must require  $d_{\min} \sim g_s^{1/3} L_s$ . This leads to a choice of probe velocity of order  $g_s^{2/3}$

$$v_p \sim g_s^{2/3} \quad . \quad (49)$$

The description of  $\delta x_p$ ,

$$\delta x_p \gtrsim \frac{1}{p_p \frac{L}{a}} \quad , \quad (50)$$

maintains the same form as in the previous Section 4 (observing again that the momentum of the probe at early and late times is of the same order of magnitude).

For  $\delta x_t$  the analysis can proceed as in the previous sections but taking into account the different form of the potential energy associated to the probe-target interaction:

$$\delta x_t \sim \frac{\Delta p_p(t)}{m} \Big|_{d_{\min}} \frac{d_{\min} L}{v_p a} \simeq \frac{L_s^6 v_p^4}{p_p d_{\min}^7} \frac{d_{\min} L}{v_p a} = \frac{L_s^6 v_p^2}{m d_{\min}^6} \frac{L}{a} \simeq \frac{1}{m v_p} \frac{L}{a} \quad , \quad (51)$$

where we used again  $p_p \simeq m_p v_p$  and we estimated  $[\Delta p_p(t)]_{d_{\min}}$  using the same type of argument already used in the previous section. Combining all these observations we find that the uncertainty on the measure of  $X$  can be estimated by

$$\delta X \gtrsim \delta x_0 + d_{\min} + \delta x_p + \delta x_t \gtrsim \delta x_0 + d_{\min} + \frac{1}{m v_p} \frac{L}{a} + \frac{1}{m v_p} \frac{L}{a} \quad , \quad (52)$$

where  $\delta x_0$  and  $d_{\min}$  are of the order of the accuracy goal  $g_s^{1/3} L_s$  we are aiming for, and the last two contributions combine to give an overall contribution which can also be reduced to the level  $g_s^{1/3} L_s$ :

$$\min(\delta x_p + \delta x_t) = \sqrt{\frac{1}{m v_p} \frac{L}{a} \frac{1}{m v_p} \frac{L}{a}} = \frac{1}{m v_p} \sim g_s^{1/3} L_s \quad . \quad (53)$$

In summary all contributions to  $\delta X$  can be controlled at the level  $g_s^{1/3} L_s$ , so that it is indeed legitimate to estimate  $\min(\delta X) \sim g_s^{1/3} L_s$ , *i.e.* we find that a space coordinate of the event of collision between two D-particles can be measured with better-than-Planckian accuracy.

This comes at the cost of a relatively large uncertainty on the time coordinate of the collision event. In fact, on the basis of the observations reported in this section, we find for  $\delta T$

$$\delta T \gtrsim \delta t_i + \frac{\delta X}{v_p} + \frac{X \delta v_p}{v_p^2} \quad , \quad (54)$$

where

$$\delta t_i \sim \frac{1}{\delta E_p} \sim \frac{E_p}{p_p \delta p_p} \sim \frac{1}{v_p \delta p_p} \sim \frac{\delta x_0}{v_p} \sim g_s^{-1/3} L_s \quad (55)$$

$$\frac{\delta X}{v_p} \sim g_s^{-1/3} L_s \quad . \quad (56)$$

Also in this case (as for the other case in which we considered a nonrelativistic probe) it is noteworthy that in a reasonable Heisenberg-microscope setup the distance  $X$  should be macroscopic, and the contribution  $X v_p^{-1/2}$  should be very large. And in any case the terms  $\delta t_i$  and  $\delta X/v_p$  give contributions of order  $g_s^{-1/3} L_s$ , so that clearly

$$\delta T \gtrsim g_s^{-1/3} L_s \quad . \quad (57)$$

The price for the better-than-Planckian space localization ( $\min(\delta X) \sim g_s^{1/3} L_s < g_s^{1/4} L_s \sim L_P$ ) is therefore a worse-than-Planckian time localization of the event:  $\delta T \gtrsim g_s^{-1/3} L_s \gg g_s^{1/4} L_s \sim L_P$ .

## 6 Aside on distance measurement and the Salecker-Wigner procedure

In our analysis of Heisenberg-microscope measurement procedures we stressed their use in the localization of collision events. It is worth mentioning explicitly the (rather obvious) fact that our

analysis can be viewed also as an analysis of length measurement: the coordinates  $X, T$  also specify the length  $L = X$  of the interval that connects the origin of the coordinate system and the collision event at the time  $T$ . Using D-particles one then finds that, with a large time-measurement uncertainty  $\delta T \gtrsim g_s^{-1/3} L_s$ , the length of the interval can be measured rather sharply

$$\min(\delta L) \sim g_s^{1/3} L_s \quad . \quad (58)$$

This can be easily verified independently using one of the distance/length measurement procedures that are most commonly considered. In particular, some recent studies [6, 7] have considered the Salecker-Wigner [21] measurement procedure. The measurement of the length of, say, a metal bar can be performed by choosing as reference time-like world line “WL1” one extremity of the bar, and marking by “WL2” the world line of the other extremity. The length of the bar can be measured by sending a probe of velocity  $V$  at time  $t = 0$  from WL1 to WL2, and setting up things in such a way that after reaching WL2 the probe is reflected back toward WL1. By measuring the time  $t = t^*$  when the probe finally returns to WL1 one can deduce the length of the bar as  $L = Vt^*/2$ .

The same Salecker-Wigner measurement procedure can of course also be used to localize an event, the spacetime point marked by the arrival of the probe at WL2, with coordinates  $X = L = Vt^*/2$ ,  $T = t^*/2$ .

The careful reader can easily verify, following the steps of the line of analysis presented in the previous section, that, in the case in which both the probe and the particle that marks the WL2 world line are D-particles, the Salecker-Wigner measurement procedure leads to  $\min(\delta L) \sim g_s^{1/3} L_s$  and  $\min(\delta T) \sim g_s^{-1/3} L_s$ .

## 7 Closing remarks

We have argued that, as long as the localization limit is obtained using only ordinary quantum mechanics and general relativity, the Planck length sets absolute limits on the measurability of spatial distances,  $\delta x \geq L_P$ , and time intervals,  $\delta t \geq L_P$ . This is due to the interplay between the “Compton-wavelength uncertainty” and the “Schwarzschild-radius uncertainty”. Essentially D-particles provide us an example of the possibility that the particles that intervene in the localization process carry also some other charges (so that, in addition to quantum mechanics and general relativity, some other structures come into consideration), leading to a weakened “Schwarzschild-radius uncertainty”.

Allowing for D-particles in the measurement procedure, one can find  $\min(\delta x) \sim g_s^{1/12} L_P \ll L_P$  (if  $g_s \ll 1$ ).

D-particles are only an example of charged probes, and with other examples it is plausible that one could manage to further reduce  $\min(\delta x)$ . It therefore seems that one should not insist on simultaneous  $\delta x \geq L_P$  and  $\delta t \geq L_P$  uncertainties, and it is hard to say how low the uncertainties could be in a specific quantum-gravity model. But it is noteworthy that all of our results are consistent with the idea of a general localization limit on the combined measurement of space and time coordinates of a point:  $\delta x \delta t \geq L_P^2$ . In the case of D-particles one finds that the sharp (better-than-Planckian) space-position measurement can only be achieved at the cost of a rather poor time measurement,  $\delta t \sim g_s^{-1/3} L_s$ , and one ends up finding<sup>j</sup>  $\delta x \delta t \geq L_s \gg L_P^2$ . Of course in the case of localization based on massless neutral particles one does find  $\delta x \geq L_P$  and  $\delta t \geq L_P$  which results in  $\delta x \delta t \geq L_P^2$ . Further investigations of the robustness of this  $\delta x \delta t \geq L_P^2$  uncertainty principle could provide an important element of guidance for quantum-gravity research. It may well be that the correct form of the new uncertainty principle is somewhat different, but, in light of our analysis, it appears likely that that

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<sup>j</sup>Some authors, perhaps most notably Yoneya (see, *e.g.* Ref. [22]), have proposed arguments in favour of the general validity in String Theory of an uncertainty relation  $\delta x \delta t \geq L_s^2$ . The arguments adopted by Yoneya do not appear to be fully in the spirit of more traditional measurability analyses (and therefore it would be important to find additional evidence in support of  $\delta x \delta t \geq L_s^2$  in String Theory), but it is nonetheless noteworthy that our D-particles analysis is consistent with this expectation.



it should take the form of a space/time uncertainty principle (an uncertainty principle such that one can improve space localization at the cost of a worse time localization of the event).

Another point that should be explored concerns the covariance of the quantum-gravity uncertainty principles. It appears necessary [23] to verify that the structure of Lorentz transformations (or at least of some suitable deformation of the Lorentz transformations [24]) are compatible with the new uncertainty principle. Especially in light of this possible concern for Lorentz covariance, it appears necessary [4] to perform more general measurement analyses, in which all of the coordinates of the spacetime point are considered, whereas here (and in most of the “quantum-gravity uncertainty principle” research) we focused on a single space coordinate and the time coordinate.

We have also set aside here, at the level of the analysis of the consequences of our analysis, the contributions of the type of the term  $Xv_p^{-1/2}$  in Eq. (52). These contributions appear to suggest that the localization can be rather poor (much worse than suggested by the  $\delta x \delta t \geq L_P^2$  limit) if the spacetime point under consideration is at a large distance from the position where the probe starts off for the measurement procedure. This type of behaviour is expected on the basis of some intuitions for the quantum-gravity problem, most notably the ones that favour a role for decoherence in quantum gravity [6, 7, 25, 26]. Also this possibility deserves further investigation.

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