ATLAS Internal Note
LARG-NO-003
16 November 1994

EDDY CURRENTS IN THE PLATES OF THE "ATLAS" HADRONIC CALORIMETER

semi-analytic approach

Guy Le Meur
Laboratoire de l'Accélérateur Linéaire, Orsay

The aim of this work is to estimate the forces, due to the eddy currents, acting on the plates of the hadronic calorimeter of the ATLAS detector in case of a quench in the supraconducting solenoidal coil.

Preliminarily to eventual computations using general eddy currents simulation tools, which would be anyway delicate and costly, we have tried to get realistic estimations of the phenomena by writing a simple program based on analytical formulas justified by specific approximations assuming an axisymmetric geometry.

We expose first, with some details, the numerical method, which lies on a matrix formulation, together with the encountered difficulties and the hypothesis we have assumed in order to avoid them.

After giving some results on test cases we provide the results for an "initial configuration" and a modified one. For the initial configuration the most important force acting on a plate has a value of 35000 N. For the other configuration this force is of the order of 10000 N.

SUMMARY

1. STATEMENT OF THE PROBLEM

A simplified geometry of the problem is shown on figure 1. A set of P coaxial copper plates (here P=50) is located in the stray field of the solenoid (at a distance of 3,45 m from the solenoid center). The thickness of each plate is e=30 mm. The spacing between plates is constant (h=8 mm). The total dc current in the solenoid coil is I_0. At a time t=0 the current breaks down following a law of the form $I(t)=I_0 f(t)$.

An other configuration is considered. It will be given beyond.

2. NUMERICAL APPROXIMATION

In order to use analytic formulas we replace each plate by a set of S concentric coplanar wires. The total number of plates is represented by P.S wires. A wire is assumed to consist of an infinitely thin conductor, without transverse dimension. A resistance R_α and a self inductance $L_{\alpha\alpha}$ is attributed to each wire (denoted by an index α) following some criteria which will be given later on. Mutual inductances $L_{\alpha\beta}$ will be evaluated as well. The problem consists of computing the induced currents in each wire.

The flux of the magnetic field through the wire α can be written :

$$\Phi_\alpha(t) = M_{0\alpha} I(t) + \sum_{\beta=1}^{P.S} L_{\alpha\beta} i_\beta$$

where $M_{0\alpha}$ is the mutual inductance between the solenoid and the wire α ; i_α is the current in the same wire α.

From the Lenz's and Ohm's laws we get the following relations :

$$-\frac{d\Phi_\alpha}{dt} = -M_{0\alpha}\frac{dI(t)}{dt} - \sum_{\beta=1}^{P.S} L_{\alpha\beta}\frac{di_\beta}{dt} = R_\alpha i_\alpha$$

Or, using matrix notation :

$$\mathcal{L}\frac{d\mathcal{J}}{dt} + \mathcal{R}\mathcal{J} = -\mathcal{M}\frac{dI(t)}{dt}$$

$\mathcal{L}$ is the symmetric square matrix of (self and mutual) inductances, $\mathcal{R}$ is the diagonal matrix of resistances, $\mathcal{M}$ is the vector of mutual solenoid-wire inductances , $\mathcal{J}$ the vector of the unknown currents.

This system of differential equations can be solved using standard methods. Starting from computed currents one can get the forces.

Remark :

The stored electromagnetic energy is W = 1/2 ${}^t\mathcal{J}\mathcal{L}\mathcal{J}$ (${}^t\mathcal{J}$ means $\mathcal{J}$ transposed). It follows that $\mathcal{J}$ must be positive definite.

Remark :

If the solenoid current breaks down instantaneously from I_0 to 0 (Heaviside distribution), the right hand side of the previous equation is a Dirac distribution. Integrating this differential equation between times -ε and ε (ε being an infinitely small time) one gets initial eddy currents values in the wires :

$$\mathcal{J} = -\mathcal{L}^{-1}\mathcal{M}I_0$$

In the following this case will be called "Dirac case"

3. IMPLEMENTATION

3.1 SPLITTING INTO WIRES AND RESISTANCE MATRIX COMPUTATION

Let r_{max} be the radius of each plate. One plate is splitted into a number S of wires. The mean radius of the wire numbered s is : $r_s=u/2+(s-1)\,u$, with $u=r_{max}/S$. The resistance of that wire is evaluated assuming that each wire represents a slice of the plate with radial width equal to u :

$$R_s=\rho\,(2\pi\, r_s)/(u\, e)$$

(we assumed a copper resistivity of : 1,7 10^{-8} Ω m)

3.2 INDUCTANCES

For two coaxial wires α and β, with respective radii r_α and r_β, located at positions z_α and z_β, the mutual inductance is (Durand p.175):

$$L_{\alpha\beta} = \frac{2\mu_0\sqrt{r_\alpha r_\beta}}{k}\left[\left(1-\frac{k^2}{2}\right)J_1(k) - J_2(k)\right]$$

with :

$$k = \sqrt{\frac{4r_\alpha r_\beta}{\left(r_\alpha + r_\beta\right)^2 + \left(z_\alpha - z_\beta\right)^2}}$$

J_1 and J_2 are the first and second kind elliptic functions :

$$J_1 = \int_0^{\pi/2} \frac{d\psi}{\sqrt{1-k^2\sin^2\psi}} \qquad J_2 = \int_0^{\pi/2} \sqrt{1-k^2\sin^2\psi}\,d\psi$$

Strictly speaking, the self inductance of a wire α is infinite. It can be verified on the expression of $L_{\alpha\beta}$ by putting $\alpha=\beta$: J_1 becomes infinite (k=1). Durand (p. 215) gives an asymptotic expression of $L_{\alpha\alpha}$ for a wire with a vanishing radial width λ :

$$L_{\alpha\alpha} = \mu_0 r_\alpha\left[\mathrm{Log}\frac{16r_\alpha}{\lambda} - \frac{7}{4}\right]$$

The problem is then to match λ. Intuitively λ must be less than u (u having here the same meaning as before). Let be η such that $\lambda=\eta.u$, with $0<\eta\leq 1$. If we choose η too close (or equal) to 1 the inductance matrix will not be diagonal dominant and consequently will not be necessarily positive definite. This fact will not be compatible with an interpretation of this matrix in terms of energy. At first glance taking η to small leads to very big self inductances and the previous equations will not be very representative of the behaviour of a plate. In order to fix η we made some tries (see below).

3.3 MUTUAL INDUCTANCES BETWEEN THE SOLENOID AND A WIRE

From the axisymmetric geometry, only the azimuthal component A_ϕ of the vector potential of the magnetic field produced by the solenoidal coil (radius r_0 and lenght 2 b) is different from zero. At a point (r,z) the value of A_ϕ is (Durand p. 99), choosing z=0 at the center of the solenoid :

$$A_\varphi(r,z) = \frac{1}{2\pi}\mu_0 K_\varphi[G(r,z;b) - G(r,z;-b)]$$

with :

$$G(r,z;\varsigma)=\frac{2r_0(z-\varsigma)}{r_1}\left[\frac{1}{k^2}(J_2-J_1)+\left(\frac{1}{m^2}-1\right)(J_3-J_1)\right]$$

and :

$$r_1=\sqrt{(r_0+r)^2+(z-\varsigma)^2}\quad k=\frac{2\sqrt{r_0 r}}{r_1}\quad m^2=\frac{4r_0 r}{(r_0+r)^2}$$

J_1 and J_2 are the first and second kind elliptical functions, J3 is the third kind elliptical function :

$$J_3(k,m)=\int_0^{\pi/2}\frac{d\psi}{\sqrt{1-k^2\sin^2\psi}\left(1-m^2\sin^2\psi\right)}$$

$K\Phi$ is the superficial current per unit of solenoid length.

On can get the flux of the solenoid magnetic field through any wire from this vector potential value using the relation :

$$\Phi_{0\alpha}=2\pi r_\alpha A_\varphi$$

dividing by the total current circulating in the coil we get the mutual inductance.

Remark :

If one wishes to work with a (computed or measured) field map, these mutual inductances will be evaluated through numerical integrations.

3.4 MATCHING THE PARAMETER η

We achieved some tries, in the Dirac case, for a unique plate with a radius of 2 m (thickness : 30 mm) in an homogeneous magnetic field of 1 T (constant B_z independent of the location, B_r=B_θ=0). Fig 2a shows the total eddy current in the plate versus the number of wires in the plate, for different values of η lying between 0.1 and 1. It can be checked that, for any value of η the total current converges to a unique value of 2.02 MA. The order of this value is confirmed by a rapid evaluation of the current which should circulate in a wire of radius 1 m for producing a magnetic field of 1 T : this last evaluation gives 1.67 MA.

In addition, the figure mentioned above shows that the best convergence is obtained for η=0.5.

Taking two plates (spacing between plates : 8 mm) we can test the importance of the mutual inductances between wires belonging to different plates. It can be noted (fig. 2b) that, for $\eta=1$, this matrix is positive definite only for those values of wire number per plate which are greather than 11, whereas the minimum wire number per plates is only 3 for $\eta=0.1$. This circonstance seems to indicate that the leading factor here is really the diagonal dominance : the mutual inductance (off diagonal term of the matrix) between wires having same radius but belonging to neighbouring plates is very high and decreasing η enforces the diagonal dominance.

It could be found better to minimize η but fig. 2b shows that it is not true because the convergence to the right current value as a function of the number of wires per plate becomes worse and at least 15 wires per plate will be necessary as well in order to get good values.

It can be seen in this two plates computation that, provided that the inductance matrix is positive definite, the solution converges to an unique value when the number of wires per plates increases : it is a good indication for the validity of the method.

Runs with 50 plates confirmed the value of 0.5 for η and gave an optimal number of wires per plate of 20.

3.5 TIME SCHEME

We have used a classical "θ-method" for the time discretization of the differential equation :

$$\mathcal{J}^{n+1} = \mathcal{J}^{n} + \Delta t \mathcal{L}^{-1}\left[-\theta\left(\mathcal{R}\mathcal{J}^{n+1} + \mathcal{M}\frac{dI}{dt}(t^{n+1})\right) - (1-\theta)\left(\mathcal{R}\mathcal{J}^{n} + \mathcal{M}\frac{dI}{dt}(t^{n})\right)\right]$$

where the currents are calculated at the discretization time n+1 starting from the current values at the previous time n ; Δt is the time step. After achieving some tests we adopted $\theta=0.5$. The method reduces then to the Crank-Nicholson scheme.

3.6 COMPUTATION OF FORCES ACTING ON THE PLATES

This computation does not present any difficulty. The force acting on a plate consists of two parts : a) Laplace's force due to the fact that eddy currents circulating in the plate are immersed in the residual field of the solenoid, b) Laplace's force between eddy currents of the considered plate and the ones of the other plates ("mutual forces"). The first one can be obtained by derivating the vector potentiel written above. The second one is obtained as resultant of one-to-one forces whose analytical axpression is (with same notations as before ; Durand p. 177) :

$$F_{\alpha\beta} = -\frac{\mu_0 I_\alpha I_\beta (z_\alpha - z_\beta)}{\sqrt{(r_\alpha + r_\beta)^2 + (z_\alpha - z_\beta)^2}} \left[-J_1(k) + \frac{r_\alpha{}^2 + r_\beta{}^2 + (z_\alpha - z_\beta)^2}{(r_\alpha - r_\beta)^2 + (z_\alpha - z_\beta)^2} J_2(k) \right]$$

4. RESULTS FOR PLATES IN AN HOMOGENEOUS MAGNETIC FIELD

In order to check the validity of the method we made some computations for plates in an homogeneous magnetic field of 1 T in the Dirac case.

4.1 AN UNIQUE PLATE (FIG. 3)

One can see that the exterior wires have the highest eddy currents (fig.3a) ; that is very plausible. In addition, the shape of the time evolution of the total current (fig.3b) looks like a decreasing exponential law (a fit by an exponential fonction gave a relaxation time of the order of 0.85 s).

4.2 50 PLATES (FIG. 4)

At the initial time eddy currents in the plates located at the two extremities are much more important than in the plates located in the middle (fig.4a). It can be probably explained by the following arguments : eddy currents in a plate provide a magnetic flux through this plate opposite to the flux variation which causes these eddy currents. The middle plates cumulate their effects through the mutual inductances. That is not true for plates at the extremities where the compensation comes only from one side.

The middle plates have eddy currents circulating opposite to the eddy currents of the extreme plates (except for external wire). The currents decrease rapidly in time and go to an equilibrium by the influence of the mutual inductances and the inertia due to the resistance.

A fit of the time evolution of the total current in all the plates (fig.4b) by a decreasing exponential function gives a relaxation time much greather than the one we have got in the case of an unique plate : roughly 27 s.

The forces given on figure 4c (there actual values does not import here) provide the right symmetries : they are equal with opposit signs for corresponding symetric plates ; they become rapidly zero.

5. RESULTS FOR PLATES IN THE FIELD OF THE SOLENOID

5.1 "DIRAC CASE"

The plates are now assumed to be in the field of the solenoid. This field is computed as indicated before. The field breaks down instantaneously and the time evolution of the system is computed.

Figure 5a shows the repartition of the currents as a fonction of time. The first plate (closest to the solenoid) "sees" the greatest part of the solenoid field, that is why the currents have a much higher value in it. At the initial time, the currents in other plates circulate in opposite direction (except for external wire). The repartition becomes homogeneous during the evolution. If the decreasing of the current is fitted by an exponential function, the relaxation time is found to be a little bit lower than the previous one : 19 s. It can be probably explained by the fact that less plates are concerned with the phenomenon because of the decreasing shape of the solenoid field in the positive z direction.

Figure 5b represents the forces acting on the plates as a function of time. Obviously the maximum force arises at the initial time. It acts on the first plate, its value is approximately 10^6 N.

5.2 NOMINAL CASE

It is assumed that the current in the solenoid breaks down following a law of the form : $I=I_0 \exp(-t/\tau)$ with $\tau=40$ s. The time evolution is observed.

The total eddy current (fig. 6a) increases for approximately 30 s and decreases afterwards. Nevertheless if we have a look on the detail of the plates, in particular the first plate on which acts the most important force, it can be seen on this last a maximum of current at a time of 12 or 15 s. The force acting on this plate is maximum around 8 or 9 s and its value is 35 000 N (fig. 6d).

These figures show that the last plates (from 30 to 50) do not take part to the phenomenon. Some runs of the program have shown that it sufficient to take 20 plates in order to get approximately identical results. This consideration will be interesting in using sophisticated eddy current codes which will be more expensive in terms of computing ressources.

Remark :

We were asked to consider a case where the material of the plates has an almost zero resistivity. In this situation the matrix equation to be solved is exactly the same as the equation solved in the Dirac case, except that the integration has to be made between zero and infinity rather than between $-\varepsilon$ and $+\varepsilon$: the current, and consequently the force, follow an increasing exponential law with the same relaxation time as the decreasing of the solenoid current. The final values will be identical to the initial values of the Dirac case, namely 10^6 N for the force.

6. MODIFIED CONFIGURATION

The geometry of this configuration is shown on figure 7. The solenoid is shorter (5,4 m instead of 6,3 m). First elements where eddy currents are expected can be seen as two aluminium plates (thickness 30 mm, space between plates 30 mm, resistivity 0,28 10^{-7} Ω.m). The set of 50 copper plates is located 60 cm further. Figures 8 a,b,c give the behaviour of this system. Obviously the aluminium plates have the most important eddy current. The maximum force is now decreased to 10 000 N. This new value is essentially due to the lower stray field of the solenoid seen by the plates because of its shorter length.

7. TO GO FURTHER

The simplified semi-analytic approach we have implemented seems to provide realistic results, with the given specific hypothesis. We are now developping a more accurate formulation, starting from the Maxwell's equations for eddy currents. This formulation will be implemented in our Maxwell 2D and 3D codes, PRIAM and ANTIGONE. It will be possible to consider more sophisticated conductor shapes than axisymmetric plates and to evaluate eddy current in more complicated situations. These developments will be exposed in a following paper.

Reference : Durand, Magnétostatique (Masson-1968)

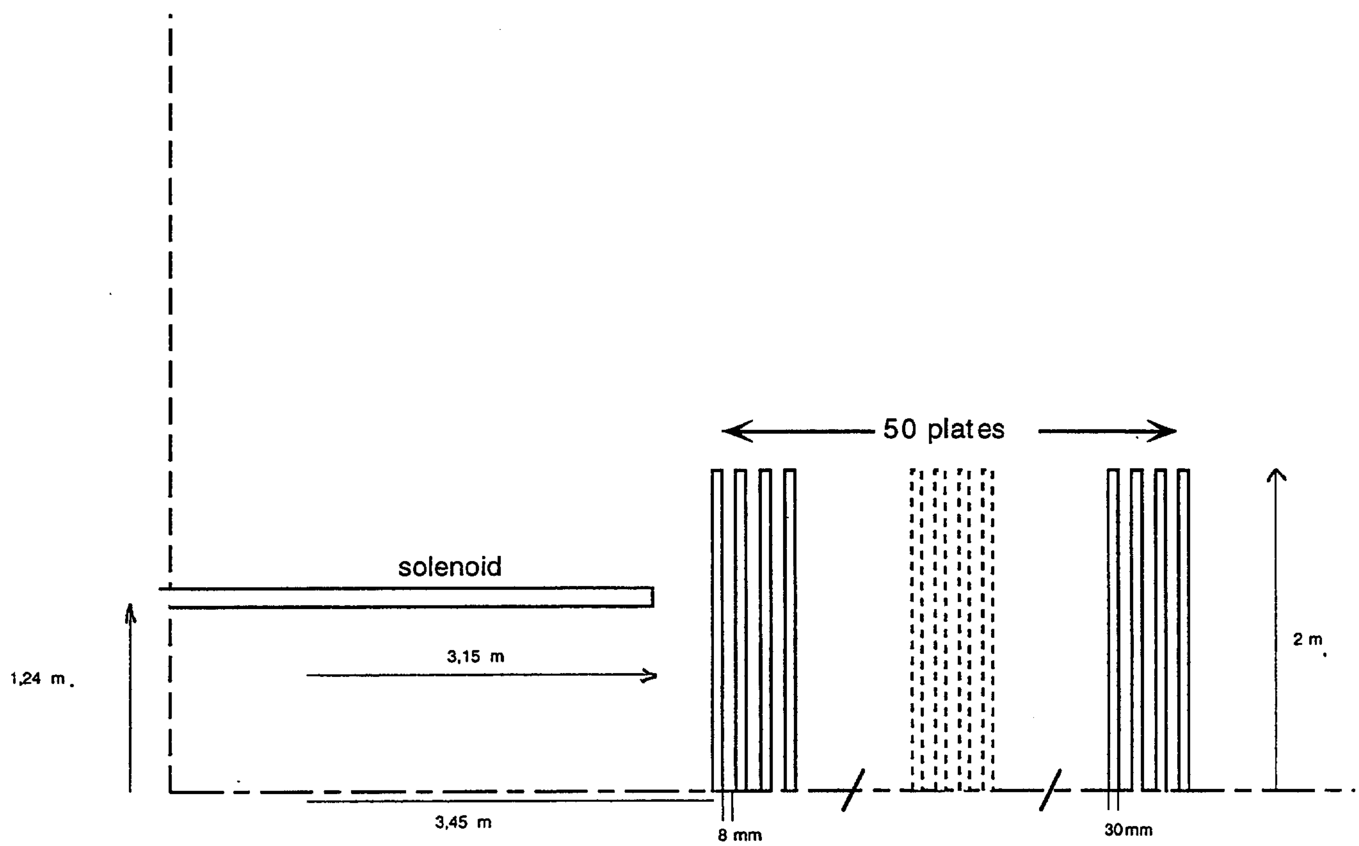

figure 1 : simplified geometry of the problem

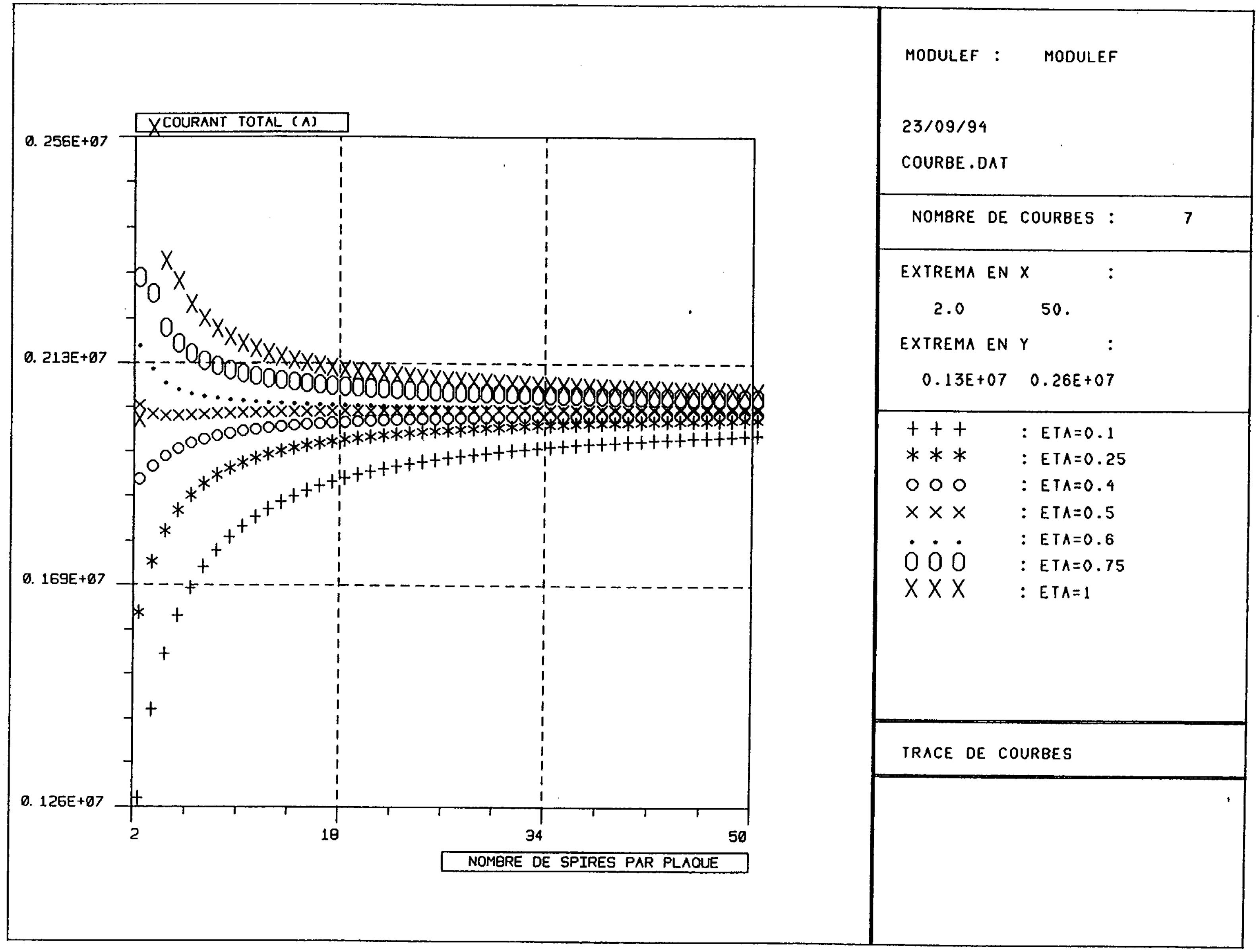

Figure 2a : matching of the parameter η (one plate)

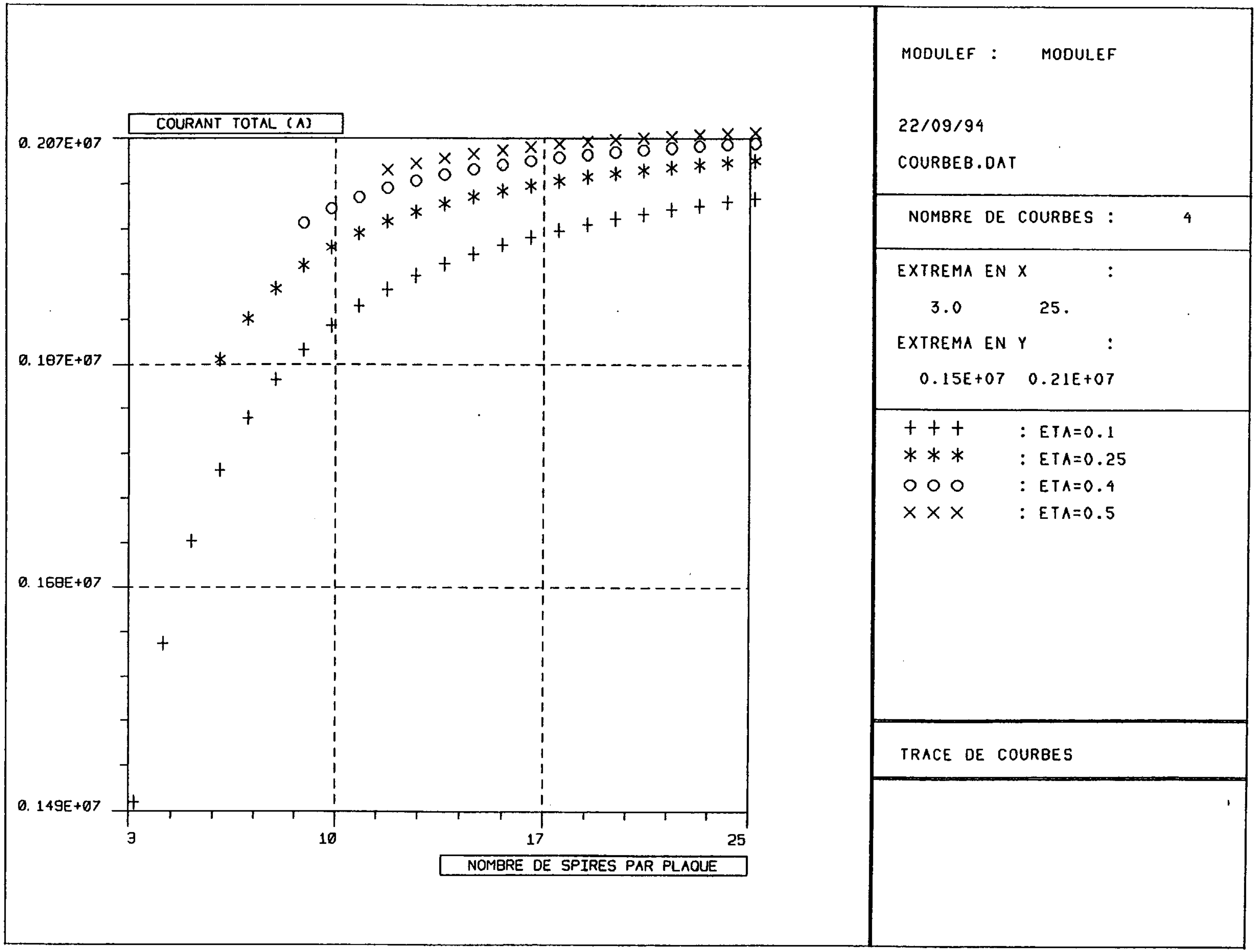

Figure 2b : matching of the parameter η (two plates)

(The absence of marker for the small values of the number of wires per plate means that the inductance matrix is not positive definite for these values)

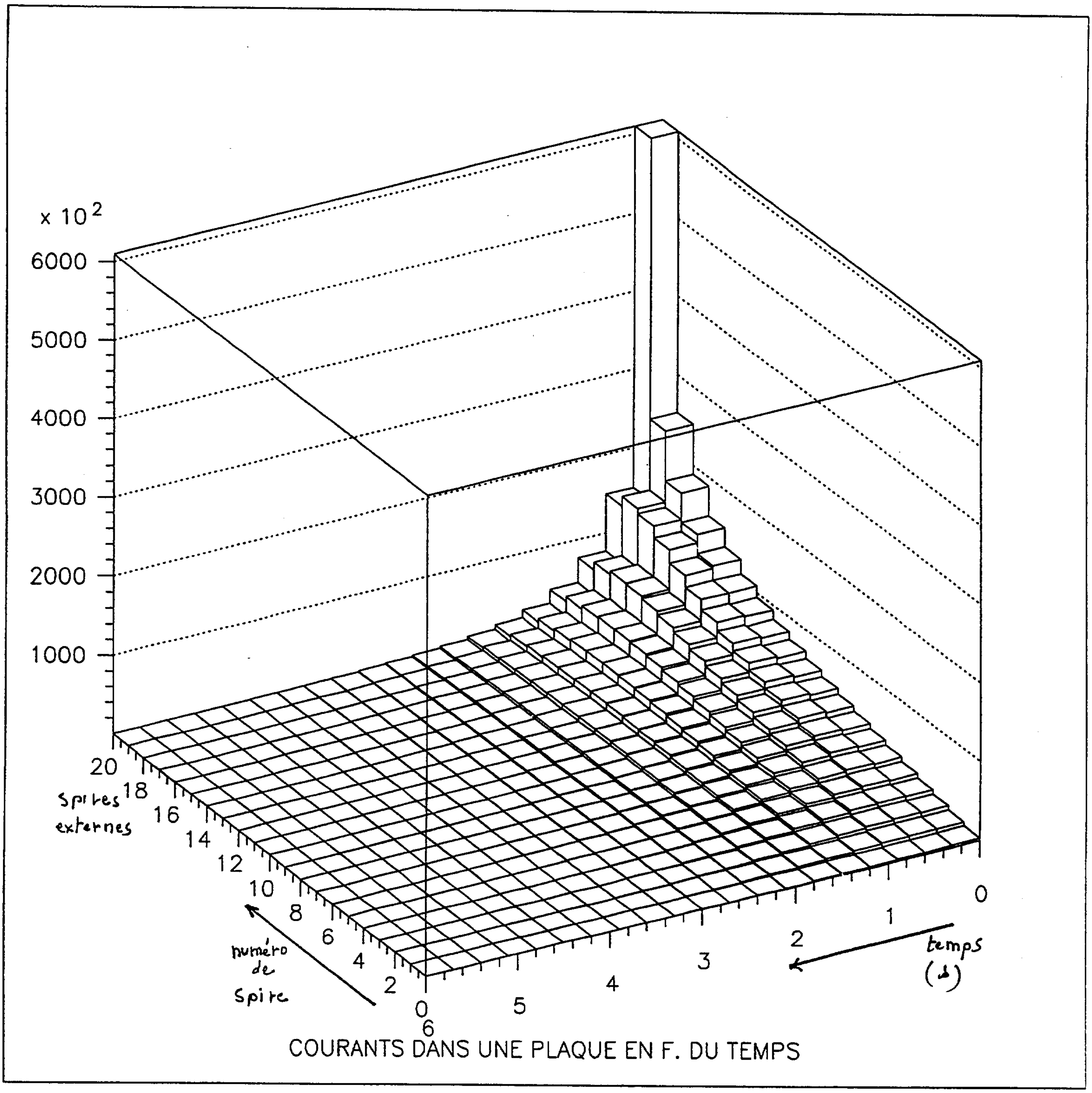

Figure 3a : "Dirac case" ; an unique plate in an homogeneous magnetic field : current in the wires.

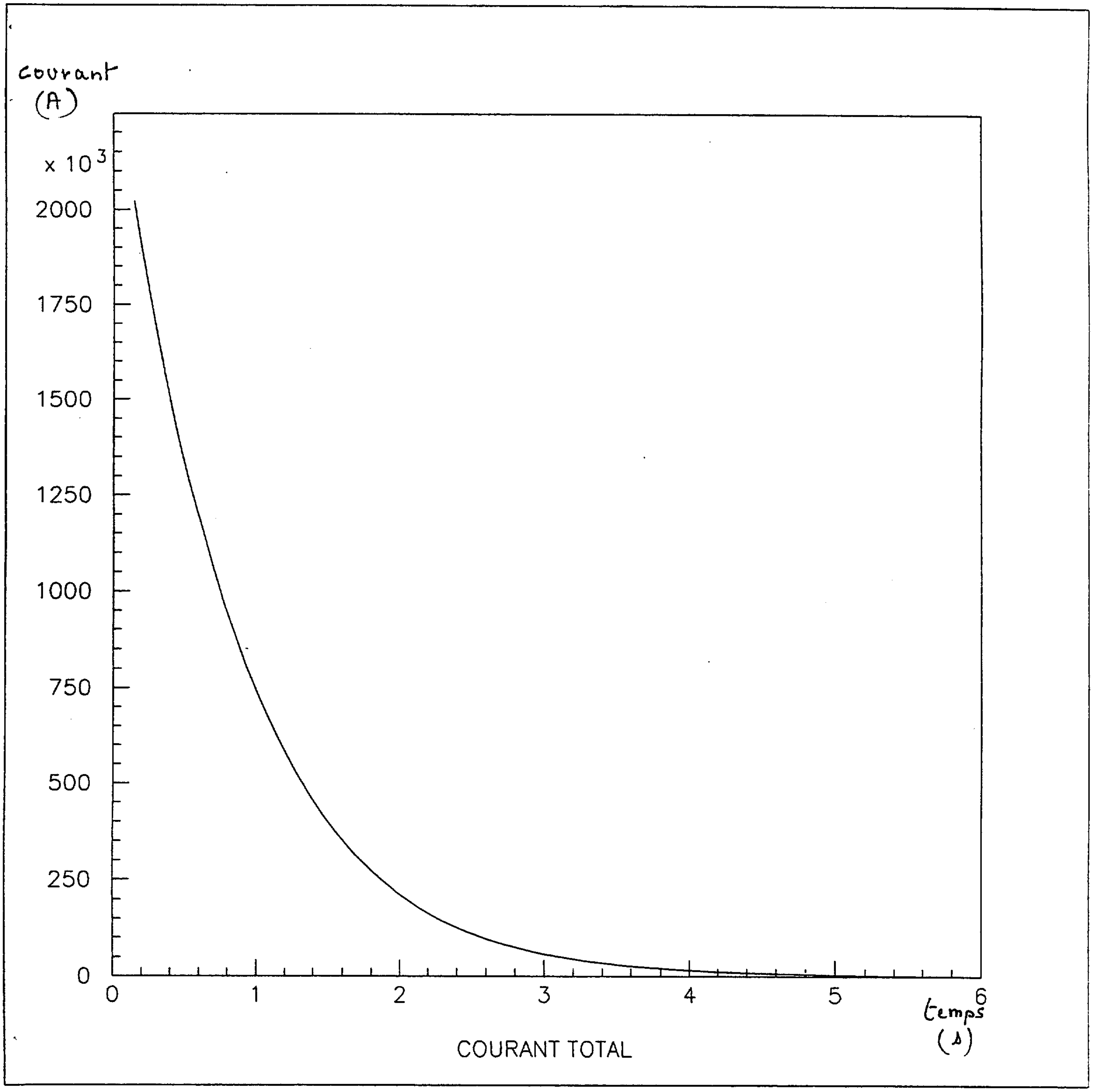

Figure 3b : “Dirac case” ; an unique plate in an homogeneous magnetic field : total current.

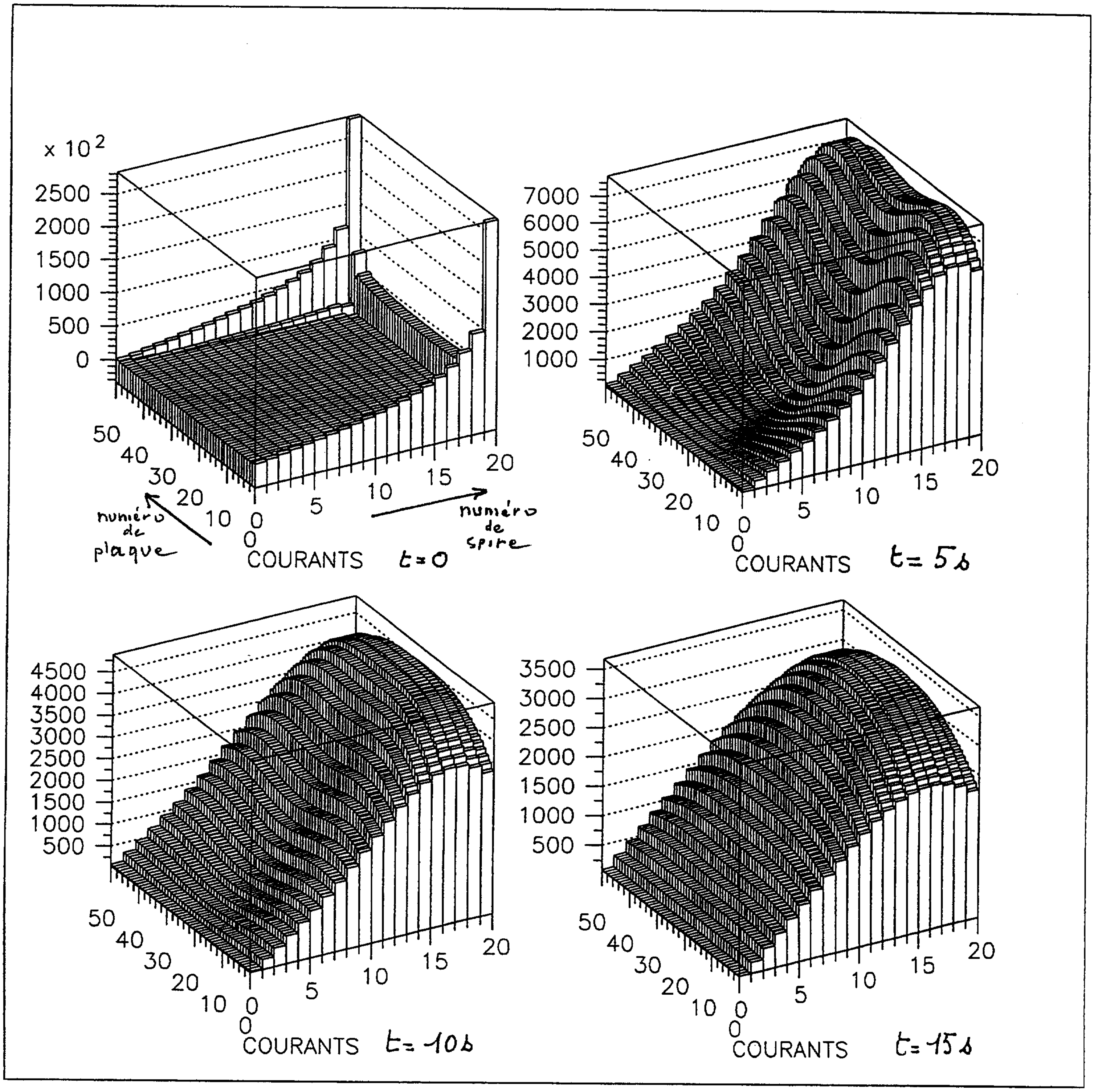

Figure 4a : "Dirac case" ; 50 plates in an homogeneous magnetic field : current in the wires as a function of time.

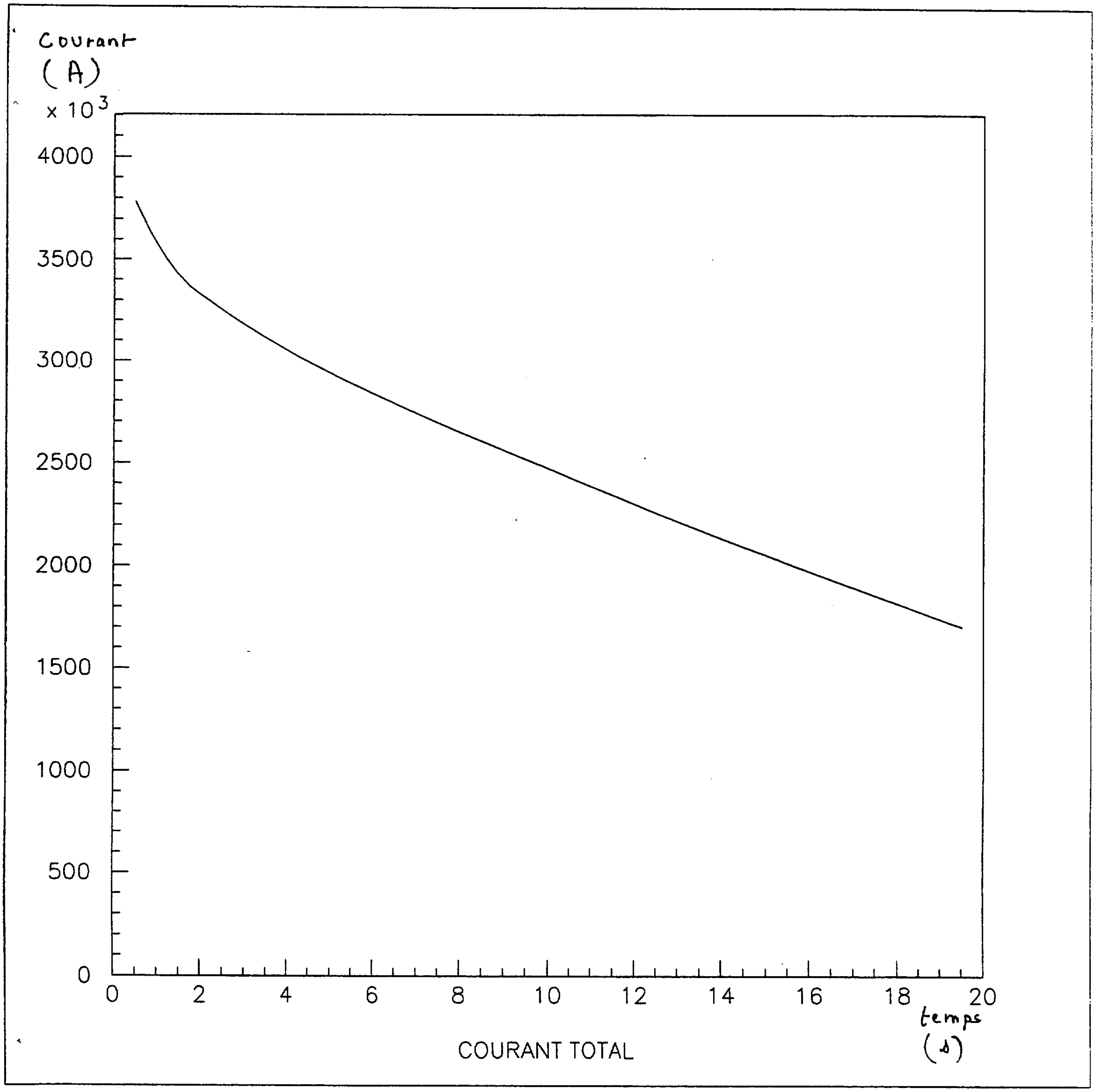

Figure 4b : cas "Dirac" ; 50 plates in an homogeneous magnetic field : total current.

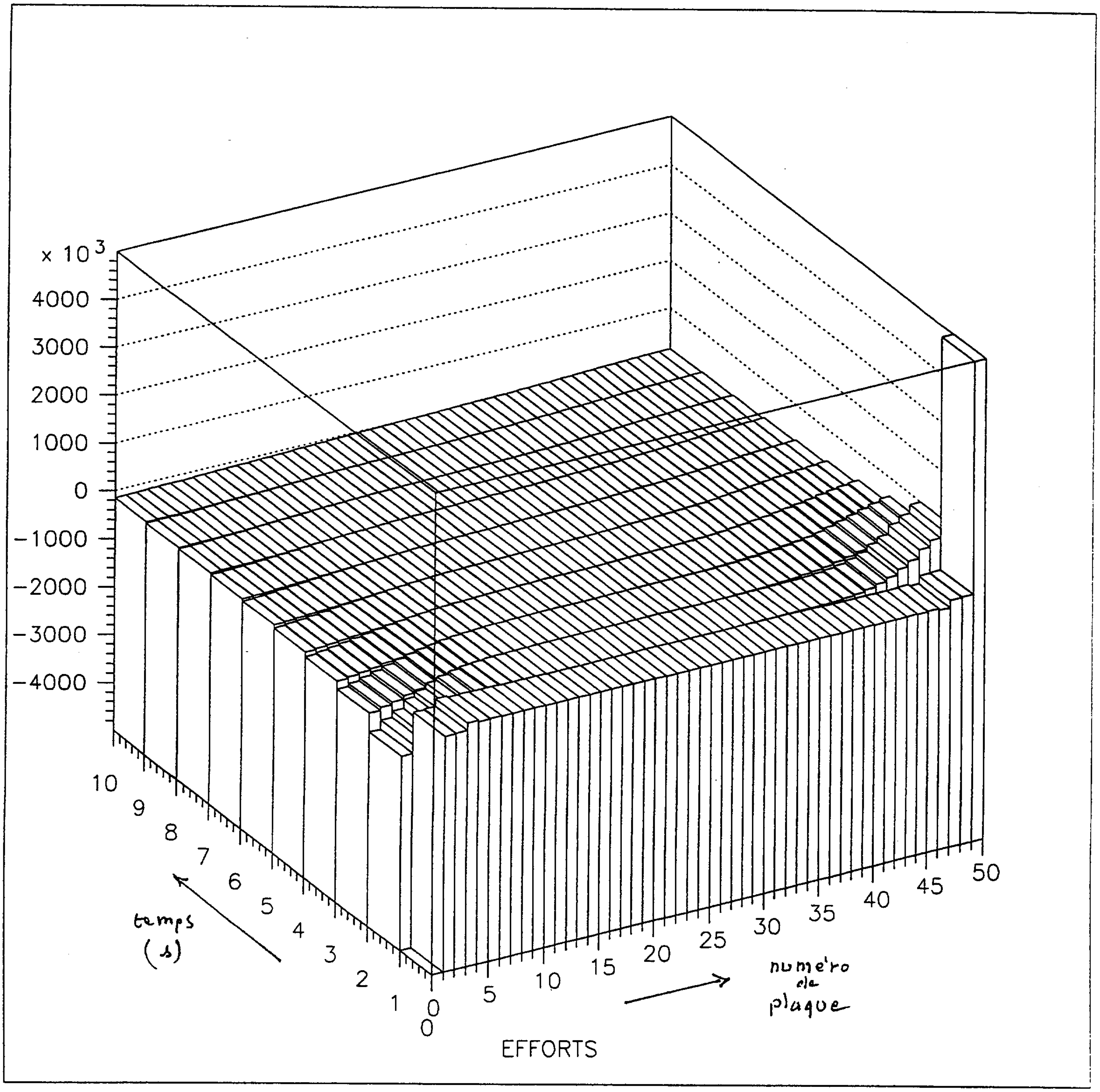

Figure 4c : "Dirac case" ; 50 plates in an homogeneous magnetic field : forces acting on plates as a function of time.

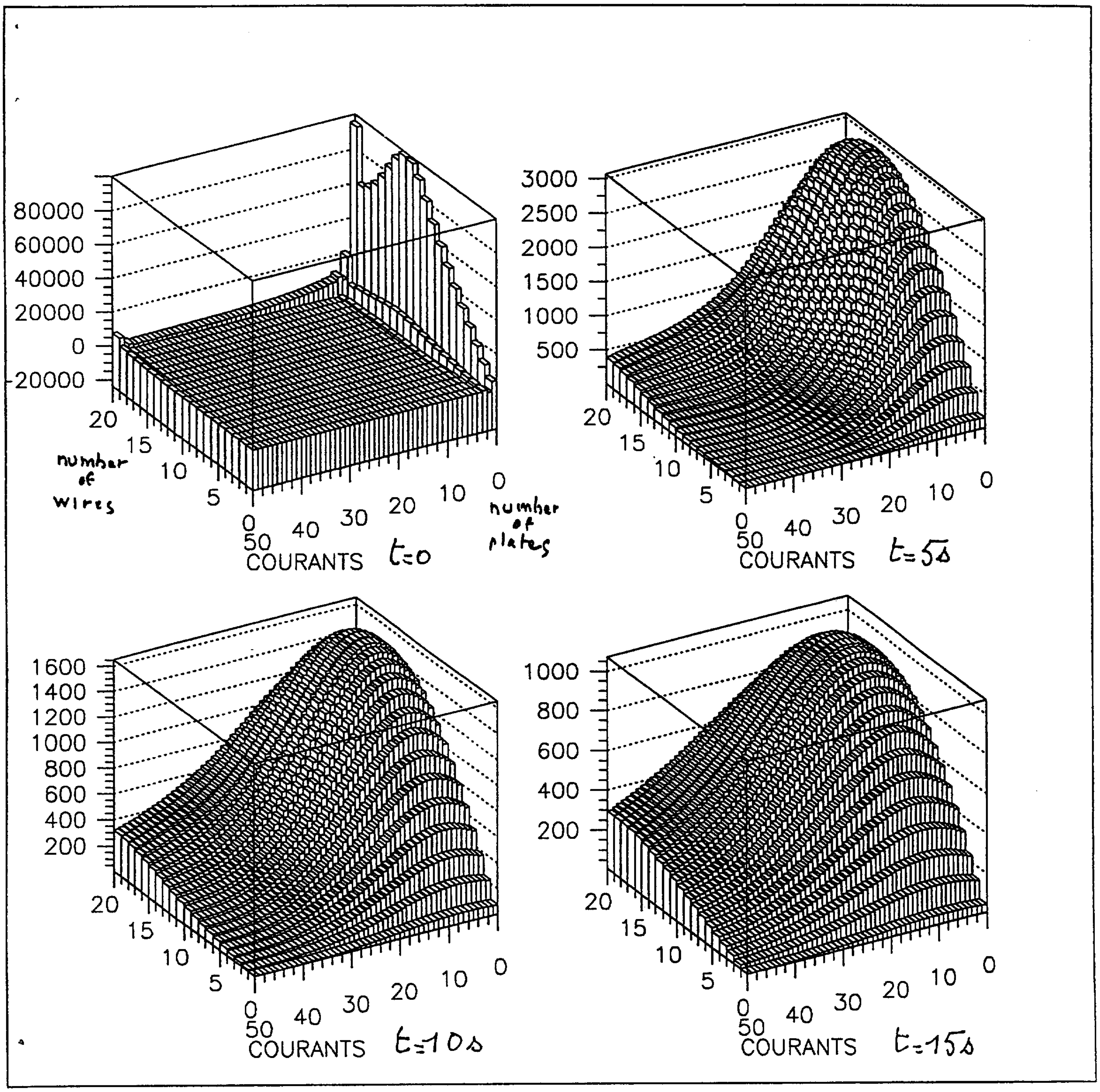

Figure 5a : "Dirac case" ; 50 plates in the solenoid field : current in the wires as a function of time.

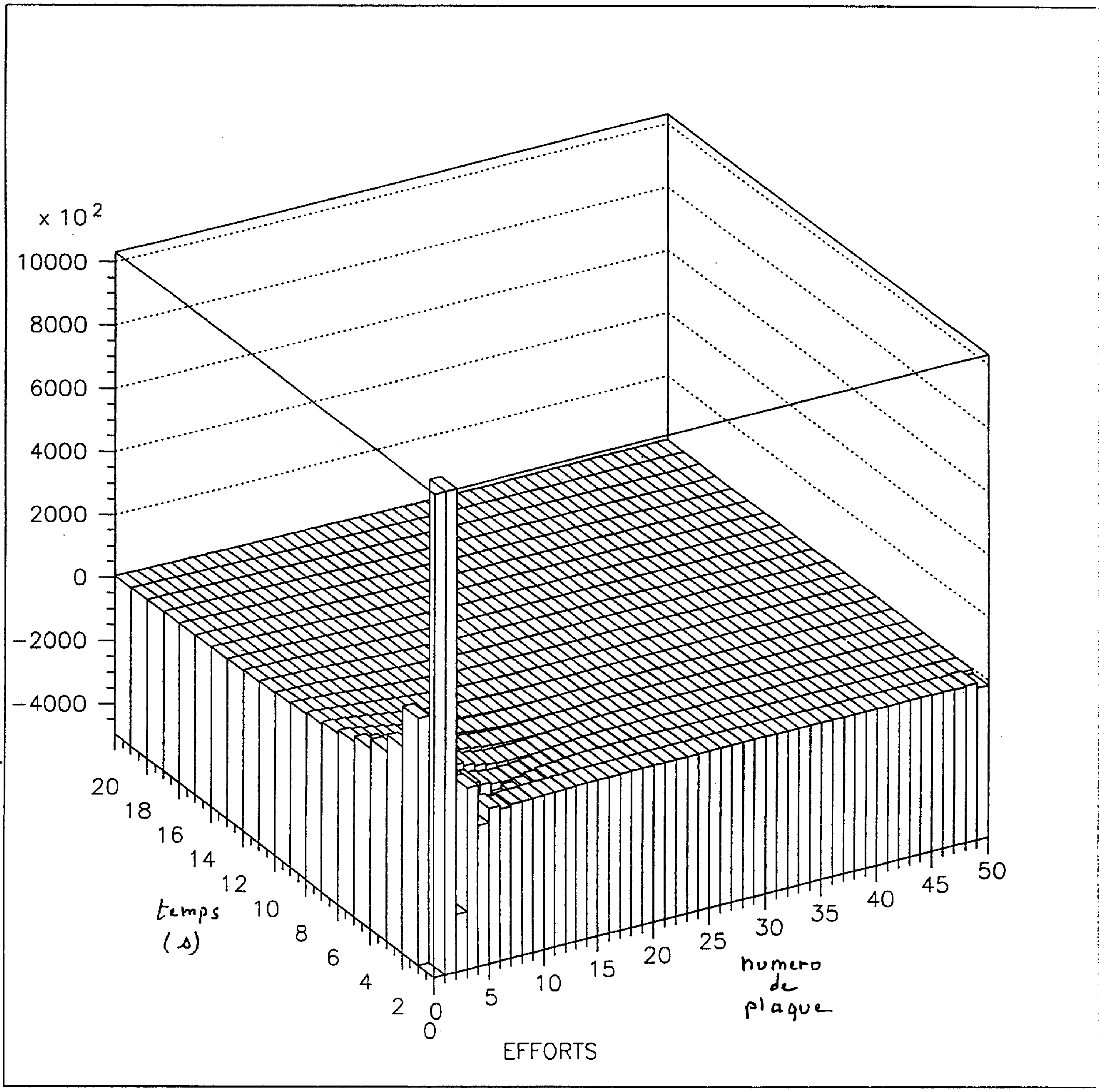

Figure 5b : "Dirac case" ; 50 plates in the solenoid field : forces acting on plates as a function of time.

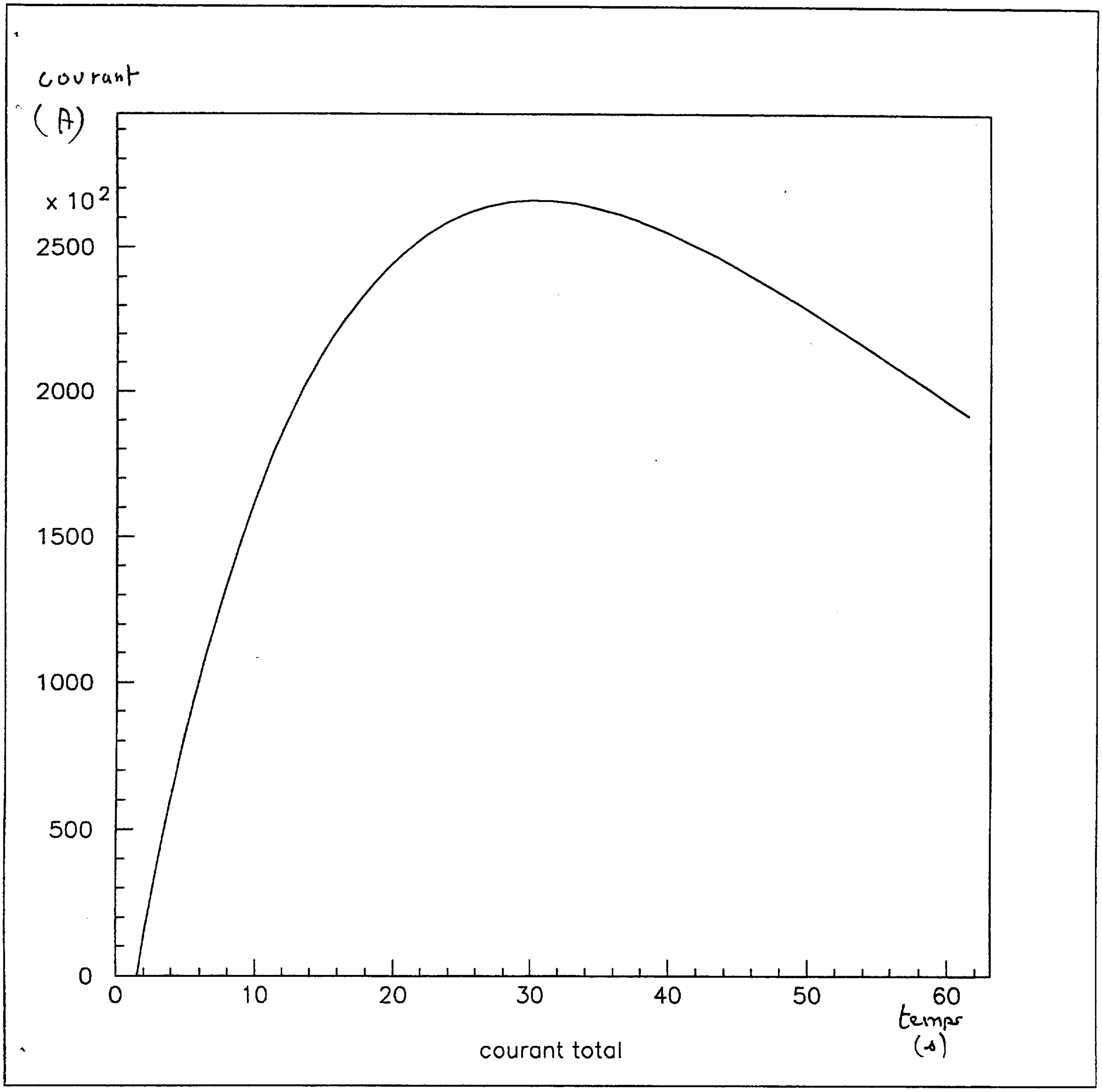

Figure 6a : nominal case ; total current as a function of time.

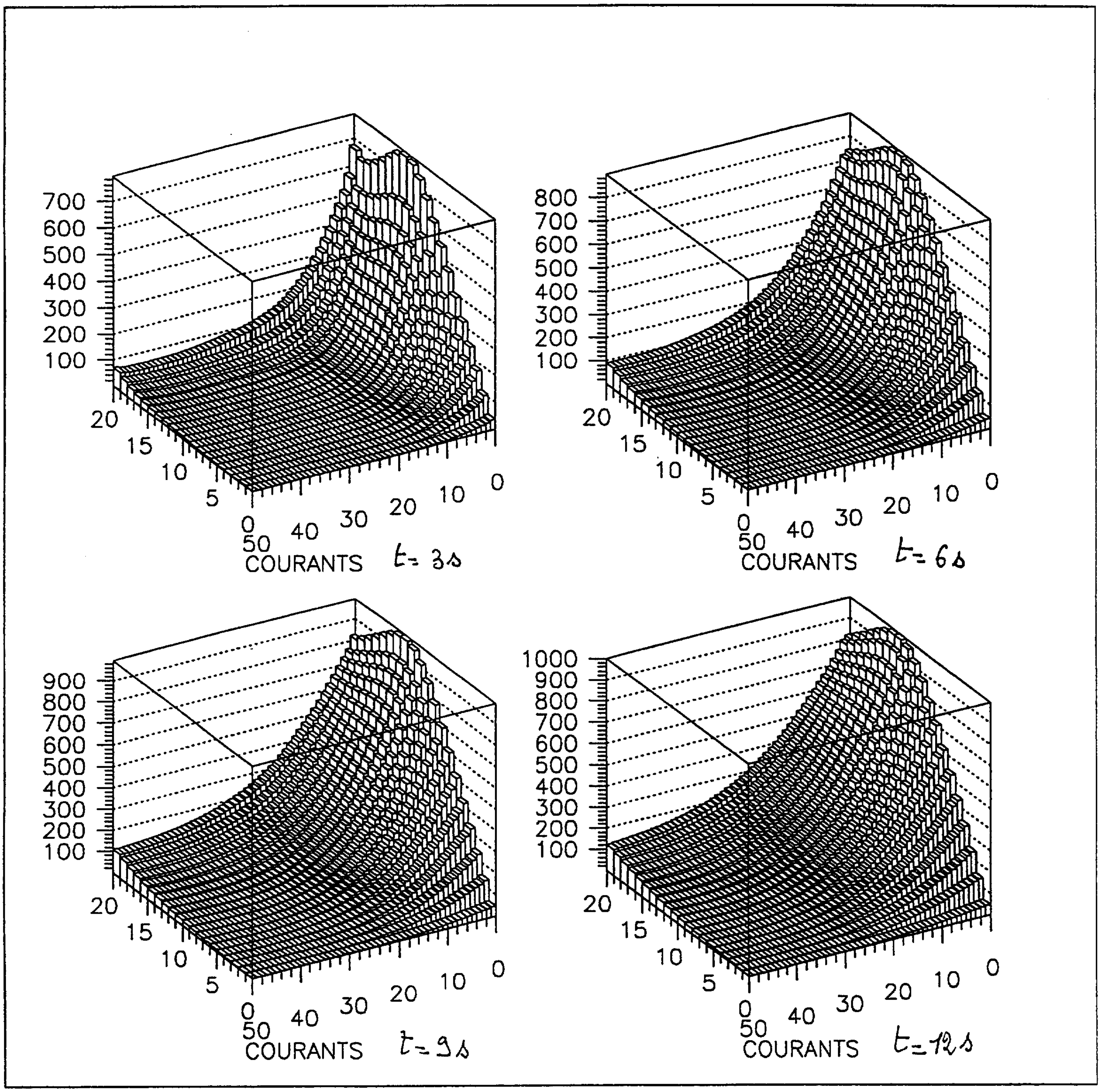

Figure 6b : nominal case ; current in the wires as a function of time.

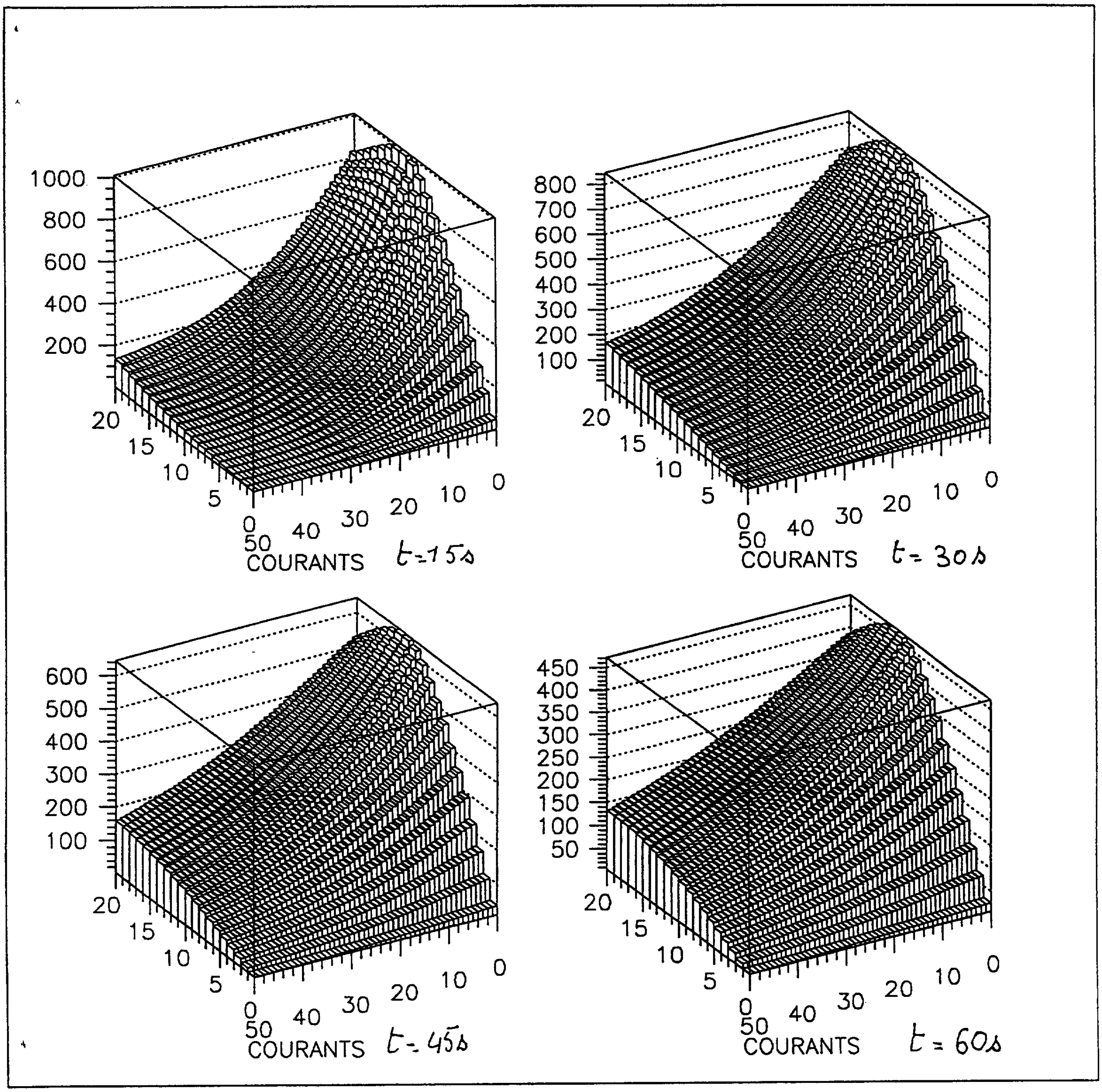

Figure 6c : nominal case; current in the wires as a function of time (6b continued).

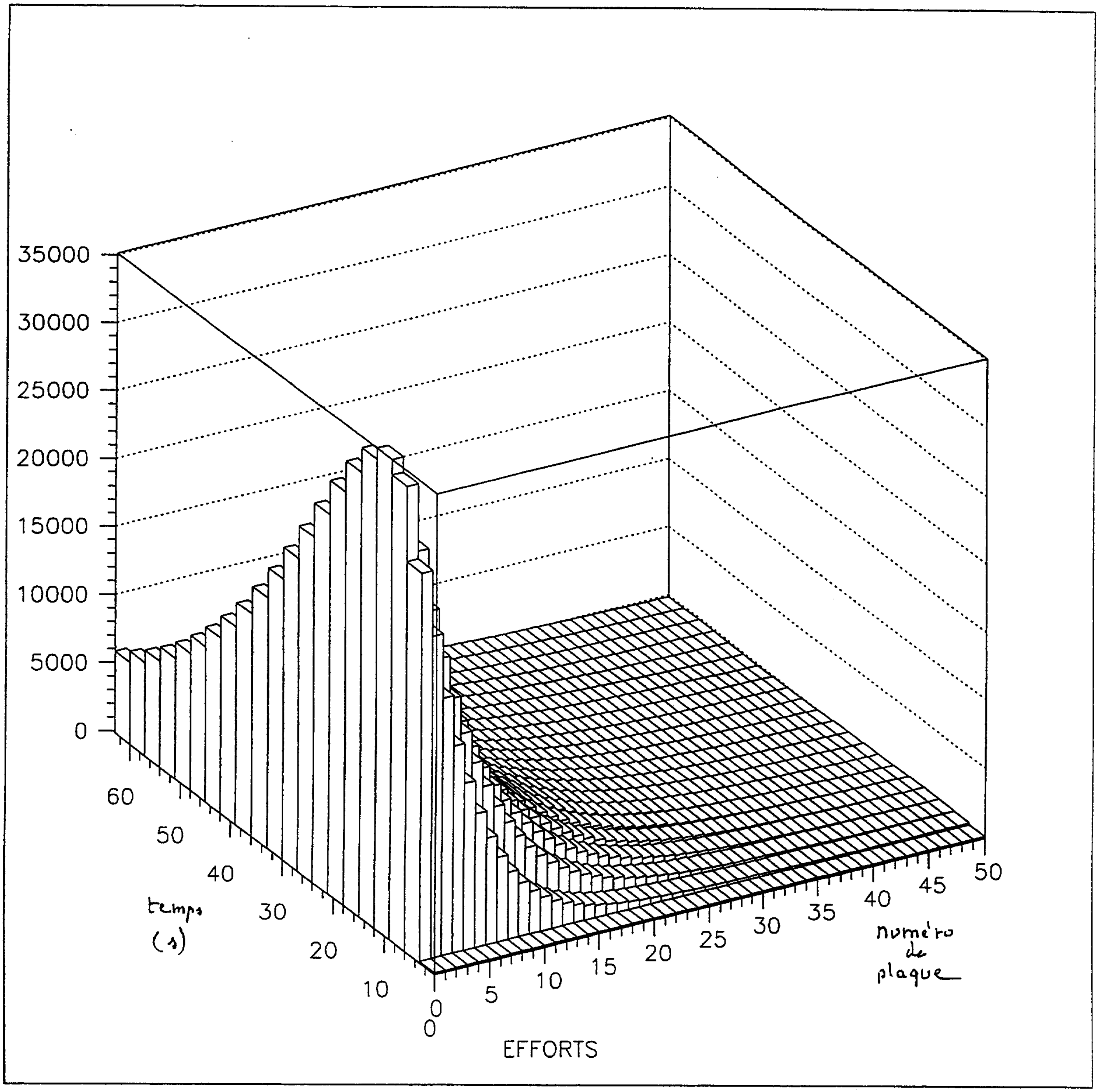

Figure 6d : nominal case ; forces acting on plates.

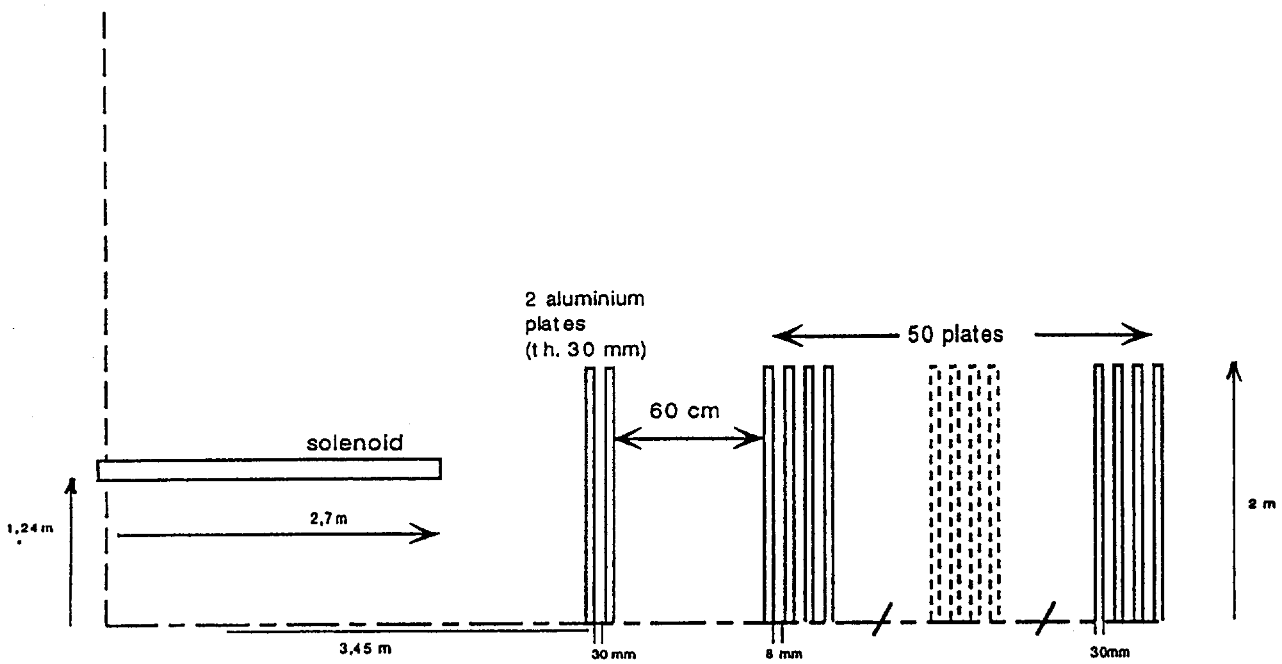

figure 7 : modified configuration

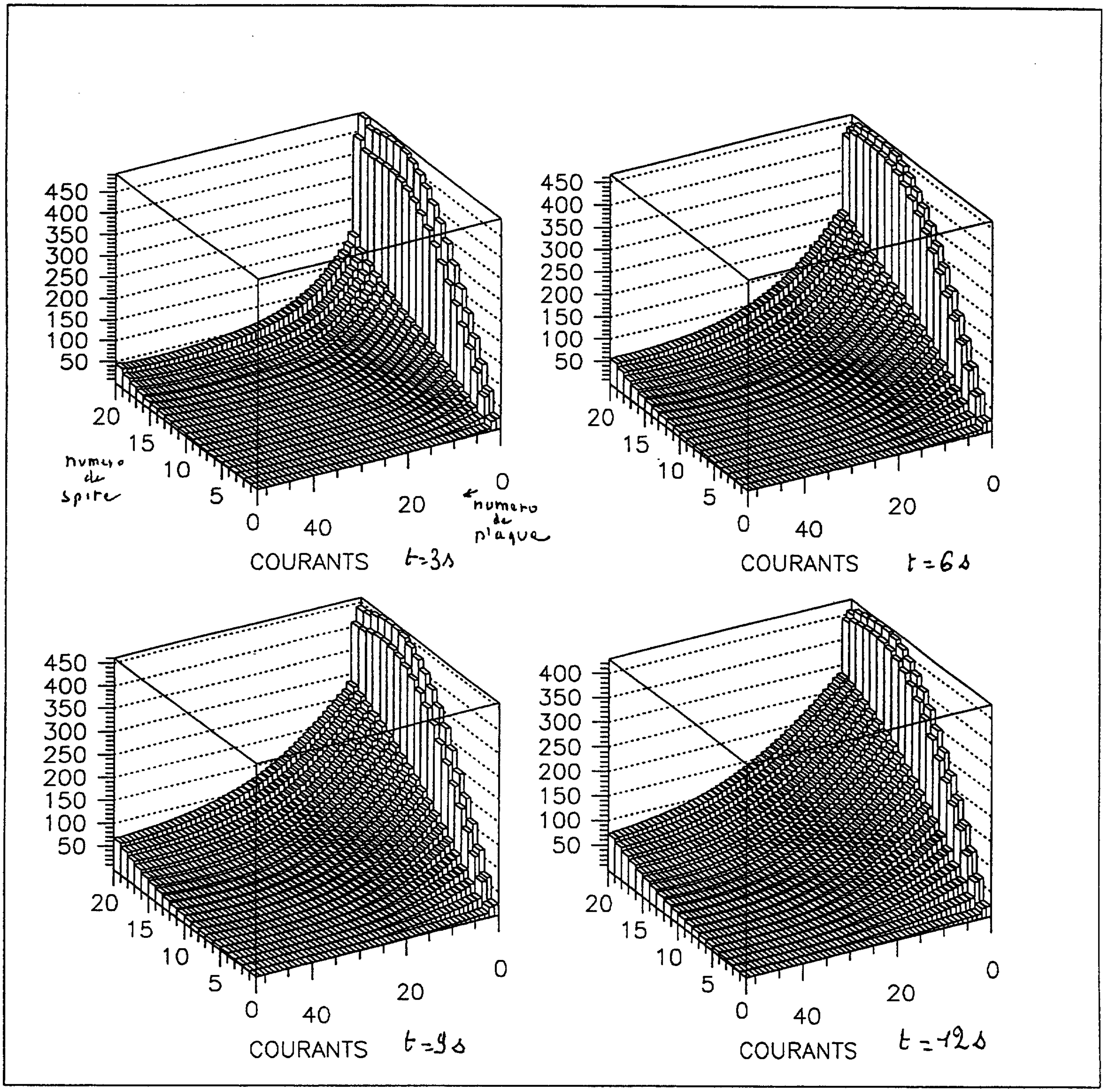

Figure 8a : modified configuration ; current in the wires as a function of time.

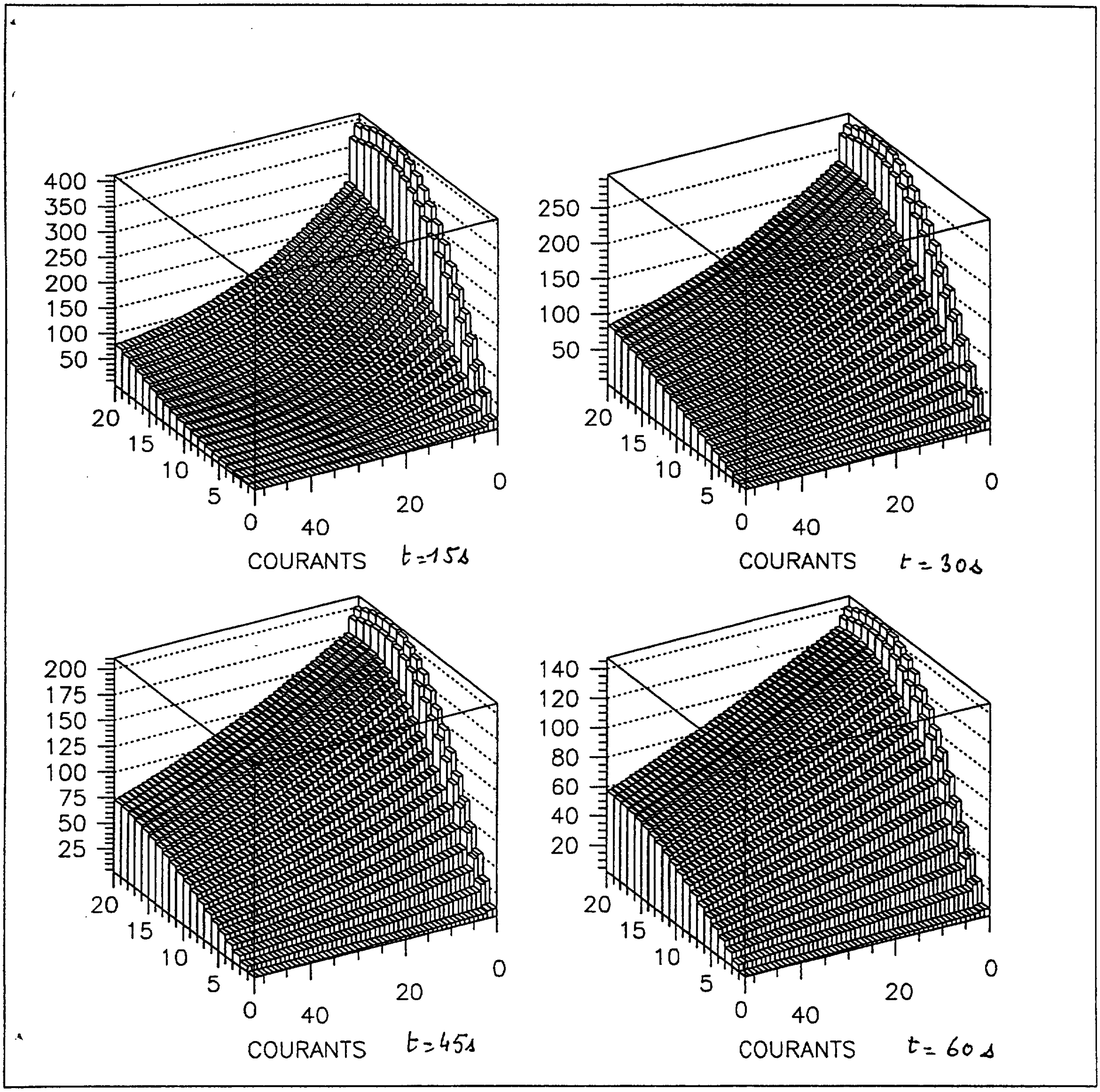

Figure 8b : modified configuration ; current in the wires as a function of time

(8a continued)

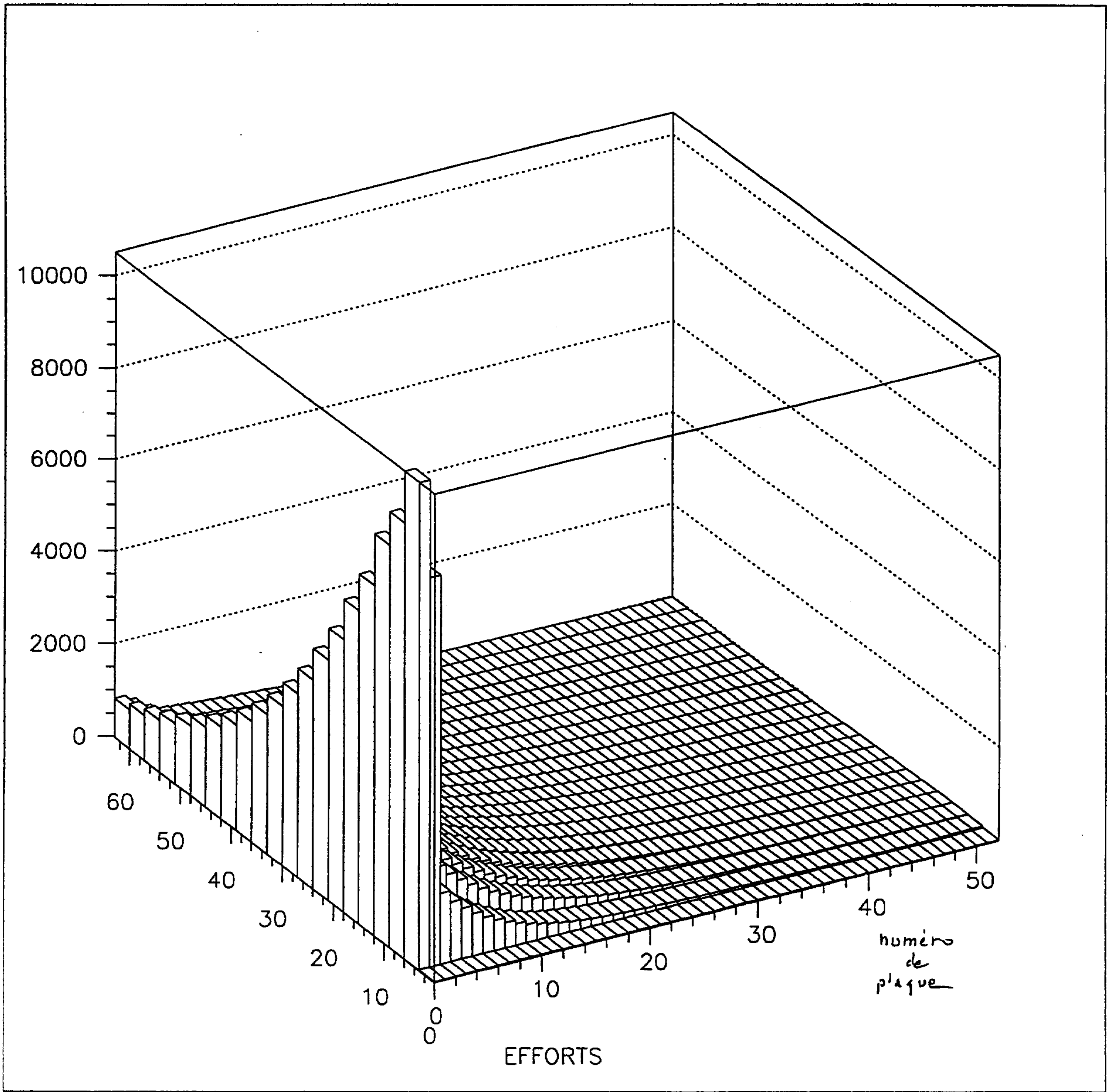

Figure 8c : modified configuration ; forces acting on plates as a function of time